Computer algebra independent integration tests

3-Logarithms/3.1.5-u-a+b-log-c-x^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [249]. This is test number [58].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (249)	% 0.00 (0)
Mathematica	% 97.59 (243)	% 2.41 (6)
Maple	% 31.33 (78)	% 68.67 (171)
Maxima	% 27.31 (68)	% 72.69 (181)
Fricas	% 36.14 (90)	% 63.86 (159)
Sympy	% 16.87 (42)	% 83.13 (207)
Giac	% 23.29 (58)	% 76.71 (191)
Mupad	% 18.47 (46)	% 81.53 (203)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

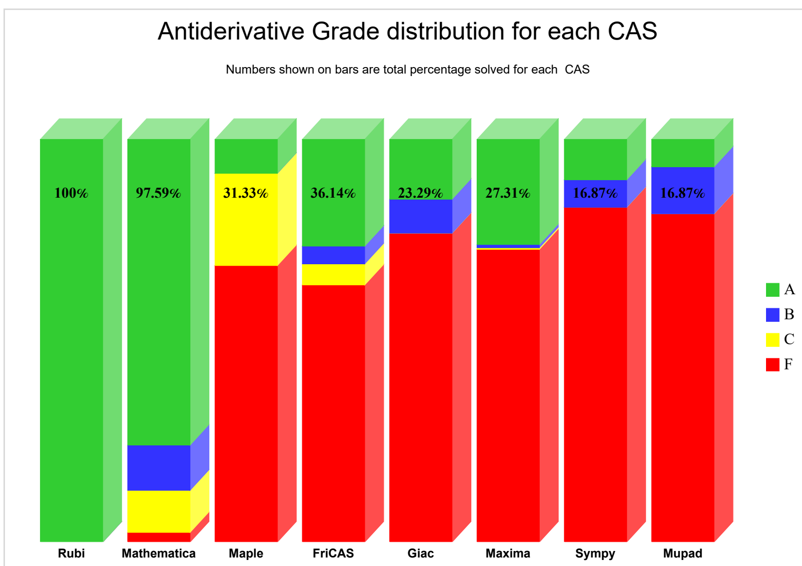
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

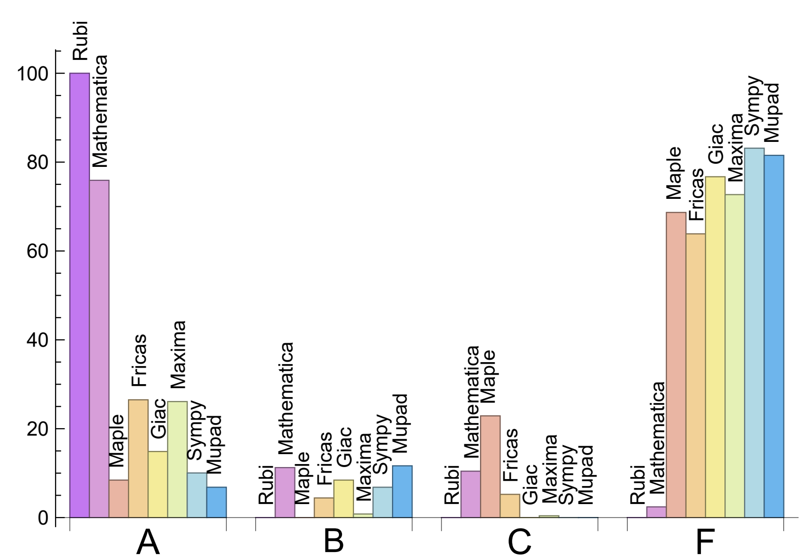
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	75.90	11.24	10.44	2.41
Maple	8.43	0.00	22.89	68.67
Maxima	26.10	0.80	0.40	72.69
Fricas	26.51	4.42	5.22	63.86
Sympy	10.04	6.83	0.00	83.13
Giac	14.86	8.43	0.00	76.71
Mupad	6.83	11.65	0.00	81.53

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	6	100.00 %	0.00 %	0.00 %
Maple	171	95.32 %	2.34 %	2.34 %
Maxima	181	91.71 %	0.55 %	7.73 %
Fricas	159	100.00 %	0.00 %	0.00 %
Sympy	207	22.22 %	77.78 %	0.00 %
Giac	191	94.76 %	1.05 %	4.19 %
Mupad	203	97.04 %	2.96 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

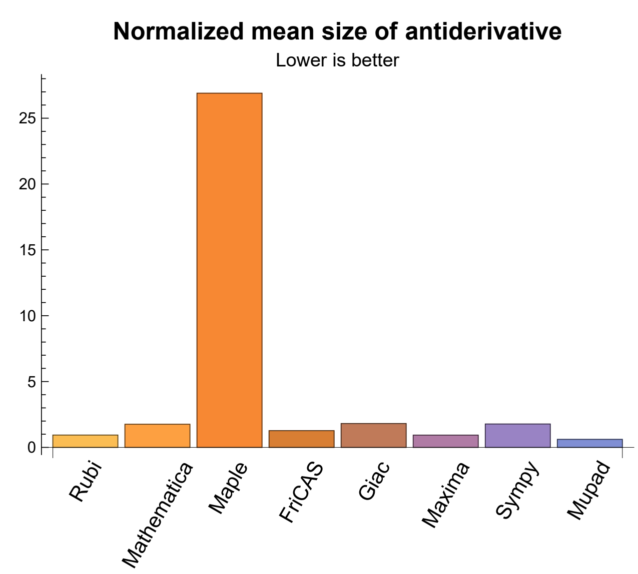
1.3 Performance

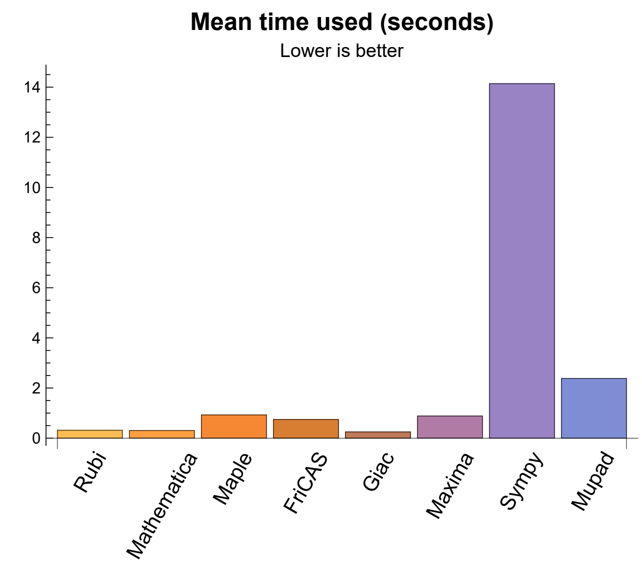
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	256.18	0.94	195.00	1.00
Mathematica	0.30	436.26	1.76	252.00	1.07
Maple	0.93	4177.64	26.90	1477.00	9.20
Maxima	0.88	108.97	0.93	67.50	1.10
Fricas	0.75	161.22	1.27	107.50	1.13
Sympy	14.14	165.52	1.78	116.50	1.77
Giac	0.25	178.71	1.81	87.50	1.43
Mupad	2.38	48.11	0.61	23.00	0.86

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{68, 69, 138, 142, 143, 144, 145, 146, 148, 149, 198, 202, 203, 204, 220, 221, 222}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {138, 144, 145, 146, 148, 149, 220}

Maple {220, 221, 222}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {138, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 194, 195, 196, 197, 220}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

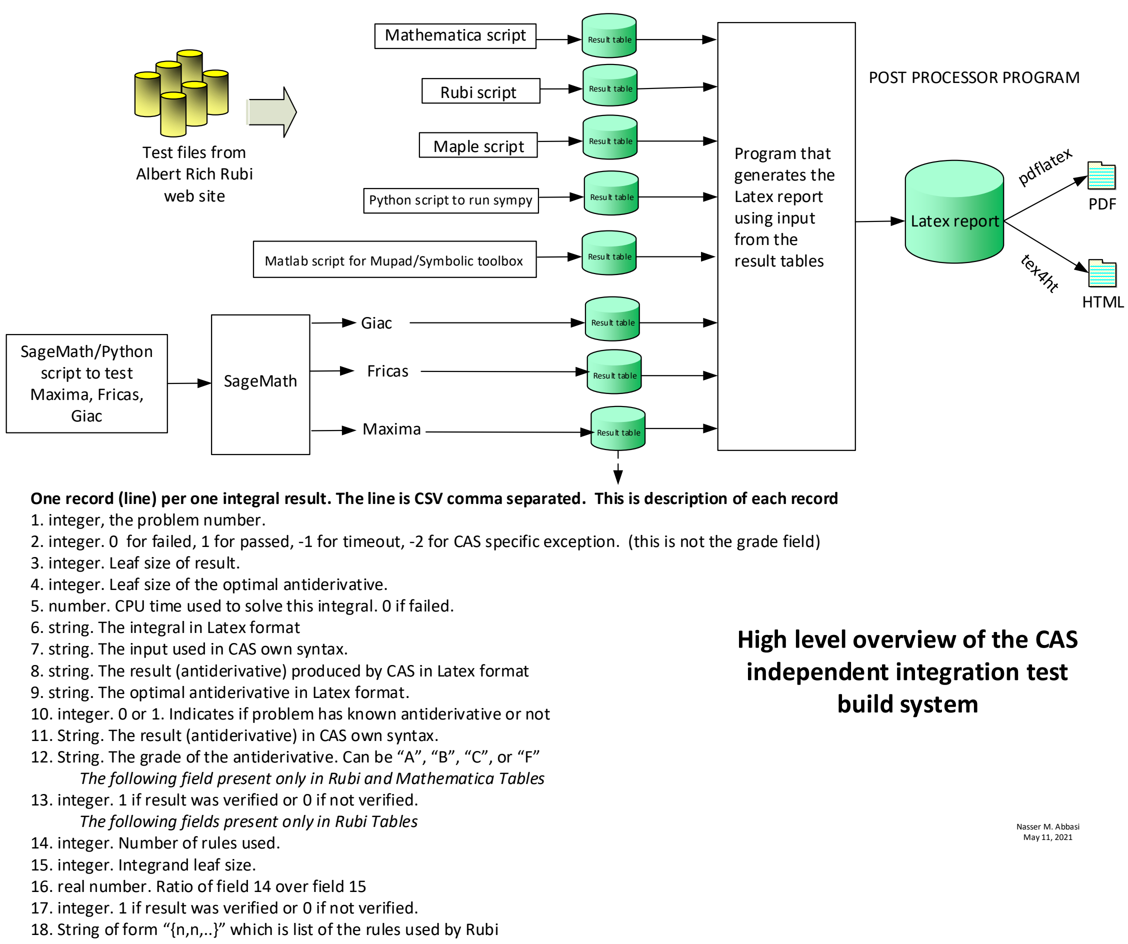
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 28, 29, 30, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 95, 96, 97, 104, 105, 106, 107, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 217, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249 }

B grade: { 22, 23, 62, 63, 64, 65, 66, 81, 82, 83, 84, 85, 86, 87, 88, 89, 111, 112, 113, 114, 128, 129, 130, 139, 140, 141, 147, 166 }

C grade: { 24, 25, 27, 31, 32, 33, 34, 35, 40, 41, 42, 43, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 108, 109, 110 }

F grade: { 207, 214, 215, 216, 218, 219 }

2.1.3 Maple

A grade: { 68, 69, 138, 142, 143, 144, 145, 146, 148, 149, 198, 202, 203, 204, 220, 221, 222, 225, 226, 233, 246 }

B grade: { }

C grade: { 1, 2, 3, 4, 5, 7, 8, 9, 24, 25, 27, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 81, 82, 87, 88, 90, 91, 93, 94, 95, 96, 97, 98, 99, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 172, 173, 176, 177, 182, 186, 187 }

F grade: { 6, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 78, 79, 80, 83, 84, 85, 86, 89, 92, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 147, 150, 151, 152, 153, 154, 155, 170, 171, 174, 175, 178, 179, 180, 181, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249 }

2.1.4 Maxima

A grade: { 2, 3, 4, 5, 7, 8, 9, 68, 69, 70, 71, 72, 73, 75, 76, 77, 138, 142, 143, 144, 145, 146, 148, 149, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 173, 177, 182, 193, 198, 202, 203, 204, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 246 }

B grade: { 166, 237 }

C grade: { 192 }

F grade: { 1, 6, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 147, 150, 151, 152, 153, 154, 155, 170, 171, 172, 174, 175, 176, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 240, 241, 242, 243, 244, 245, 247, 248, 249 }

2.1.5 FriCAS

A grade: { 68, 69, 138, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 181, 186, 187, 190, 191, 198, 202, 203, 204, 208, 209, 210, 212, 213, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 234, 235, 236, 239, 240, 241, 245, 246 }

B grade: { 163, 164, 165, 166, 182, 230, 231, 232, 233, 237, 238 }

C grade: { 64, 65, 66, 67, 139, 140, 141, 147, 214, 215, 216, 218, 219 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 178, 179, 180, 183, 184, 185, 188, 189, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 211, 217, 242, 243, 244, 247, 248, 249 }

2.1.6 Sympy

A grade: { 146, 158, 177, 188, 189, 198, 202, 203, 204, 205, 209, 210, 211, 217, 222, 223, 224, 225, 226, 227, 228, 229, 233, 239, 246 }

B grade: { 156, 157, 160, 161, 162, 163, 164, 165, 167, 168, 169, 230, 231, 232, 234, 235, 236 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 166, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 206, 207, 208, 212, 213, 214, 215, 216, 218, 219, 220, 221, 237, 238, 240, 241, 242, 243, 244, 245, 247, 248, 249 }

2.1.7 Giac

A grade: { 68, 69, 138, 142, 143, 144, 145, 146, 148, 149, 158, 159, 160, 161, 162, 170, 171, 172, 173, 177, 198, 202, 203, 204, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 234, 240, 246 }

B grade: { 156, 157, 163, 164, 165, 166, 167, 168, 169, 176, 182, 230, 231, 232, 233, 235, 236, 237, 238, 239, 241 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 147, 150, 151, 152, 153, 154, 155, 174, 175, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 242, 243, 244, 245, 247, 248, 249 }

2.1.8 Mupad

A grade: { 68, 69, 138, 142, 143, 144, 145, 146, 148, 149, 198, 202, 203, 204, 220, 221, 222 }

B grade: { 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 246 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 147, 150, 151, 152, 153, 154, 155, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	157	555	0	0	0	0	-1
normalized size	1	1.00	0.91	3.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.172	0.220	0.000	0.656	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	188	1014	259	0	0	0	-1
normalized size	1	1.00	0.90	4.83	1.23	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.119	0.093	0.296	1.472	0.611	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	161	870	220	0	0	0	-1
normalized size	1	1.00	0.90	4.89	1.24	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.075	0.294	1.353	0.703	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	131	725	178	0	0	0	-1
normalized size	1	1.00	0.90	4.97	1.22	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.067	0.293	1.401	0.618	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	90	557	126	0	0	0	-1
normalized size	1	1.00	1.22	7.53	1.70	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.034	0.252	1.247	0.771	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	34	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.009	0.447	0.000	0.776	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	69	481	128	0	0	0	-1
normalized size	1	1.00	0.64	4.50	1.20	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.065	0.245	1.395	0.572	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	215	647	194	0	0	0	-1
normalized size	1	1.00	1.32	3.97	1.19	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.073	0.250	1.369	0.501	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	206	796	232	0	0	0	-1
normalized size	1	1.00	1.06	4.08	1.19	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.085	0.254	1.184	0.782	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	456	594	0	0	0	0	0	-1
normalized size	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	0.209	0.571	0.000	0.625	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	506	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.169	0.513	0.000	0.577	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	416	0	0	0	0	0	-1
normalized size	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.140	0.665	0.000	0.842	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	294	0	0	0	0	0	-1
normalized size	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.335	0.101	0.570	0.000	0.599	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.078	0.468	0.000	0.688	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	220	183	0	0	0	0	0	-1
normalized size	1	1.08	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	0.217	0.473	0.000	0.665	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	310	513	0	0	0	0	0	-1
normalized size	1	1.08	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	0.202	0.530	0.000	0.780	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	710	710	1144	0	0	0	0	0	-1
normalized size	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.777	0.367	0.515	0.000	0.949	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	975	0	0	0	0	0	-1
normalized size	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.638	0.303	0.514	0.000	0.798	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	806	0	0	0	0	0	-1
normalized size	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	0.276	0.512	0.000	0.854	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	584	0	0	0	0	0	-1
normalized size	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.763	0.192	0.646	0.000	0.809	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.121	0.367	0.000	0.778	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	360	770	0	0	0	0	0	-1
normalized size	1	1.05	2.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.601	0.318	0.467	0.000	0.768	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	499	1047	0	0	0	0	0	-1
normalized size	1	1.06	2.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.820	0.404	0.509	0.000	0.688	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	348	827	0	0	0	0	-1
normalized size	1	1.00	1.93	4.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.106	0.310	0.000	0.675	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	267	820	0	0	0	0	-1
normalized size	1	1.00	2.34	7.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.050	0.268	0.000	0.530	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.032	0.011	0.408	0.000	0.643	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	241	619	0	0	0	0	-1
normalized size	1	1.00	1.71	4.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.096	0.203	0.000	0.716	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	364	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.093	0.543	0.000	0.637	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	254	0	0	0	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.096	0.430	0.000	0.852	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	221	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.095	0.485	0.000	0.822	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	285	0	0	0	0	0	-1
normalized size	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.140	0.188	0.392	0.000	0.833	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	654	0	0	0	0	0	-1
normalized size	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	0.358	0.496	0.000	0.680	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	519	0	0	0	0	0	-1
normalized size	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.505	0.266	0.576	0.000	0.694	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	484	0	0	0	0	0	-1
normalized size	1	1.00	6.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.210	0.500	0.000	0.765	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	488	0	0	0	0	0	-1
normalized size	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	0.364	0.370	0.000	0.583	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	612	612	703	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.034	0.615	0.452	0.000	0.672	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	544	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.801	0.333	0.529	0.000	0.780	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	414	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.556	0.310	0.408	0.000	0.769	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	585	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.866	0.546	0.421	0.000	0.592	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	1234	0	0	0	0	0	-1
normalized size	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.734	1.091	0.535	0.000	0.715	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	1004	0	0	0	0	0	-1
normalized size	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.042	0.602	0.459	0.000	0.916	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	754	0	0	0	0	0	-1
normalized size	1	1.00	7.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.325	0.404	0.000	1.073	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	940	0	0	0	0	0	-1
normalized size	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.583	0.391	0.427	0.000	0.549	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	938	938	1027	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.546	0.717	0.650	0.000	0.523	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	849	849	794	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.040	0.433	0.487	0.000	0.839	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	263	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.307	0.077	0.000	1.175	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	191	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.218	0.059	0.000	0.663	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	117	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.160	0.059	0.000	0.780	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.032	0.008	0.061	0.000	0.768	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	124	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.196	0.116	0.000	0.620	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	207	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.255	0.116	0.000	0.627	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	288	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	0.359	0.118	0.000	0.970	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	708	708	995	0	0	0	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.638	0.589	0.064	0.000	0.570	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	769	0	0	0	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	0.415	0.066	0.000	1.041	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	527	0	0	0	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	0.330	0.066	0.000	0.672	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.147	0.064	0.000	0.731	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	627	0	0	0	0	0	-1
normalized size	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.409	0.414	0.118	0.000	0.656	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	881	0	0	0	0	0	-1
normalized size	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.550	0.537	0.118	0.000	0.843	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	858	858	1432	0	0	0	0	0	-1
normalized size	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.935	0.638	0.096	0.000	0.946	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	604	986	0	0	0	0	0	-1
normalized size	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	0.510	0.118	0.000	0.939	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	98	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.212	0.069	0.000	0.833	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	1455	0	0	0	0	0	-1
normalized size	1	1.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.778	0.853	0.120	0.000	0.889	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	849	849	2009	0	0	0	0	0	-1
normalized size	1	1.00	2.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.144	1.073	0.173	0.000	0.738	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	1700	38574	0	523	0	0	-1
normalized size	1	1.00	12.41	281.56	0.00	3.82	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.702	1.891	0.000	0.891	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	1035	11734	0	285	0	0	-1
normalized size	1	1.00	9.86	111.75	0.00	2.71	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.407	1.270	0.000	0.742	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	526	2578	0	131	0	0	-1
normalized size	1	1.00	7.21	35.32	0.00	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.243	1.077	0.000	0.859	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	52	308	0	42	0	0	-1
normalized size	1	1.00	1.30	7.70	0.00	1.05	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.010	0.941	0.000	0.857	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.040	0.061	1.795	0.000	0.901	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.042	1.992	4.700	0.000	0.839	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	290	2403	381	0	0	0	-1
normalized size	1	1.00	1.02	8.49	1.35	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.223	0.872	2.304	0.805	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	252	2222	328	0	0	0	-1
normalized size	1	1.00	1.04	9.14	1.35	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.154	0.747	2.387	0.652	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	208	2041	269	0	0	0	-1
normalized size	1	1.00	1.02	10.05	1.33	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.126	0.119	0.726	2.277	0.670	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	152	1762	188	0	0	0	-1
normalized size	1	1.00	1.30	15.06	1.61	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.074	0.641	2.093	0.704	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	147	1795	0	0	0	0	-1
normalized size	1	1.00	1.47	17.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.068	0.473	0.000	0.928	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	117	1892	199	0	0	0	-1
normalized size	1	1.00	0.71	11.54	1.21	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.112	0.631	2.236	0.754	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	232	2100	285	0	0	0	-1
normalized size	1	1.00	0.99	8.97	1.22	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.148	0.691	2.373	0.536	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	280	2282	342	0	0	0	-1
normalized size	1	1.00	1.02	8.33	1.25	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.173	0.798	2.336	0.655	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	788	0	0	0	0	0	-1
normalized size	1	1.00	1.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.683	0.348	4.812	0.000	0.782	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	674	0	0	0	0	0	-1
normalized size	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	0.293	6.902	0.000	0.539	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	507	0	0	0	0	0	-1
normalized size	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.227	5.154	0.000	0.901	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	329	21792	0	0	0	0	-1
normalized size	1	1.00	2.51	166.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.178	1.306	0.000	0.885	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	283	600	10991	0	0	0	0	-1
normalized size	1	1.14	2.42	44.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	0.346	1.271	0.000	0.619	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	385	796	0	0	0	0	0	-1
normalized size	1	1.12	2.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.586	0.392	5.039	0.000	0.708	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	462	909	0	0	0	0	0	-1
normalized size	1	1.10	2.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.724	0.453	5.428	0.000	0.845	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	603	603	1431	0	0	0	0	0	-1
normalized size	1	1.00	2.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.974	0.597	87.770	0.000	0.688	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	1122	0	0	0	0	0	-1
normalized size	1	1.00	2.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.652	0.442	11.850	0.000	0.910	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	602	60520	0	0	0	0	-1
normalized size	1	1.00	3.74	375.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.262	2.956	0.000	0.639	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	459	1347	42181	0	0	0	0	-1
normalized size	1	1.12	3.28	102.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	0.691	2.660	0.000	0.670	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	614	1736	0	0	0	0	0	-1
normalized size	1	1.11	3.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.015	0.786	11.453	0.000	0.723	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	324	2259	0	0	0	0	-1
normalized size	1	1.00	1.47	10.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.154	0.908	0.000	1.194	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	266	2068	0	0	0	0	-1
normalized size	1	1.00	1.80	13.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.085	0.723	0.000	0.731	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	297	0	0	0	0	0	-1
normalized size	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.088	3.566	0.000	0.673	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	298	2101	0	0	0	0	-1
normalized size	1	1.00	1.53	10.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.126	0.677	0.000	0.906	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	363	2313	0	0	0	0	-1
normalized size	1	1.00	1.46	9.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.135	0.733	0.000	0.770	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	389	2321	0	0	0	0	-1
normalized size	1	1.00	1.55	9.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.141	0.566	0.000	0.865	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	332	2001	0	0	0	0	-1
normalized size	1	1.00	1.71	10.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.088	0.501	0.000	0.778	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	305	1972	0	0	0	0	-1
normalized size	1	1.00	1.70	11.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.089	0.552	0.000	0.765	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	362	2204	0	0	0	0	-1
normalized size	1	1.00	1.59	9.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.117	0.593	0.000	1.034	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	399	2385	0	0	0	0	-1
normalized size	1	1.00	1.49	8.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	0.175	0.594	0.000	0.705	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	814	0	0	0	0	0	-1
normalized size	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.536	0.272	5.869	0.000	0.992	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	736	0	0	0	0	0	-1
normalized size	1	1.00	5.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.237	5.164	0.000	1.048	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	946	0	0	0	0	0	-1
normalized size	1	1.00	3.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.330	0.439	5.346	0.000	0.810	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	408	1111	0	0	0	0	0	-1
normalized size	1	1.15	3.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.672	0.475	5.537	0.000	0.910	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	1128	0	0	0	0	0	-1
normalized size	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.065	0.446	111.553	0.000	1.053	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	993	0	0	0	0	0	-1
normalized size	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.806	0.337	130.676	0.000	0.651	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	917	0	0	0	0	0	-1
normalized size	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.515	0.329	69.215	0.000	0.854	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	1083	0	0	0	0	0	-1
normalized size	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.931	0.431	113.911	0.000	0.853	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	1911	0	0	0	0	0	-1
normalized size	1	1.00	3.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.924	0.577	10.374	0.000	0.902	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	1348	0	0	0	0	0	-1
normalized size	1	1.00	7.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.373	7.036	0.000	0.918	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	2248	0	0	0	0	0	-1
normalized size	1	1.00	4.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.569	0.915	11.546	0.000	0.930	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1092	2544	0	0	0	0	0	-1
normalized size	1	1.00	2.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.807	1.007	185.479	0.000	0.858	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	977	977	2302	0	0	0	0	0	-1
normalized size	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.499	0.751	123.980	0.000	0.763	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	879	879	2166	0	0	0	0	0	-1
normalized size	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.106	0.713	112.495	0.000	0.826	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1007	1007	2488	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.702	0.879	146.864	0.000	0.818	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	434	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	0.475	0.187	0.000	0.686	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	336	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.359	0.086	0.000	0.635	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	218	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.231	0.100	0.000	0.752	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	186	0	0	0	0	0	-1
normalized size	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.170	0.069	0.000	0.757	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	250	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.316	0.069	0.000	0.648	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	359	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.392	0.066	0.000	0.651	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	457	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.350	0.498	0.066	0.000	0.800	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	750	750	1319	0	0	0	0	0	-1
normalized size	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.846	0.891	0.096	0.000	0.601	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	960	0	0	0	0	0	-1
normalized size	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.656	0.507	0.068	0.000	0.623	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	718	0	0	0	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.441	0.402	0.071	0.000	0.644	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	263	0	0	0	0	0	-1
normalized size	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.251	0.092	0.000	0.645	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	821	0	0	0	0	0	-1
normalized size	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.633	0.518	0.153	0.000	0.559	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	1078	0	0	0	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.784	0.600	0.161	0.000	0.722	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	1968	0	0	0	0	0	-1
normalized size	1	1.00	2.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.309	0.883	0.168	0.000	0.741	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	639	639	1522	0	0	0	0	0	-1
normalized size	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.856	0.683	0.087	0.000	0.696	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	403	0	0	0	0	0	-1
normalized size	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.416	0.131	0.000	0.609	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	976	0	0	0	0	0	-1
normalized size	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.182	1.191	0.337	0.000	0.771	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	914	914	1549	0	0	0	0	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.520	2.304	0.326	0.000	0.745	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	394	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.416	0.086	0.000	0.624	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	296	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.228	0.307	0.070	0.000	0.706	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	145	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.413	0.072	0.000	0.572	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	326	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.399	0.069	0.000	0.978	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	422	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	0.451	0.071	0.000	0.711	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	304	0	0	0	0	0	-1
normalized size	1	0.00	9.81	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.348	0.392	0.000	0.752	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	1395	0	0	765	0	0	-1
normalized size	1	1.00	7.54	0.00	0.00	4.14	0.00	0.00	-0.01
time (sec)	N/A	0.299	0.648	0.300	0.000	0.654	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	741	0	0	406	0	0	-1
normalized size	1	1.00	4.94	0.00	0.00	2.71	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.378	0.129	0.000	0.769	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	277	0	0	173	0	0	-1
normalized size	1	1.00	2.43	0.00	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.167	0.130	0.000	0.701	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.034	0.161	0.118	0.000	0.518	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.034	1.905	0.120	0.000	0.743	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	292	0	0	0	0	0	-1
normalized size	1	0.00	10.07	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	0.189	0.146	0.000	0.652	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	292	0	0	0	0	0	-1
normalized size	1	0.00	10.81	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.176	0.130	0.000	0.906	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	165	0	0	0	0	0	-1
normalized size	1	0.00	6.35	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.005	0.181	0.130	0.000	0.634	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	277	0	0	173	0	0	-1
normalized size	1	1.00	2.43	0.00	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.168	0.128	0.000	0.690	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	282	0	0	0	0	0	-1
normalized size	1	0.00	9.72	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.171	0.129	0.000	0.712	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	292	0	0	0	0	0	-1
normalized size	1	0.00	10.07	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	0.162	0.134	0.000	0.729	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	410	0	0	368	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.85	0.00	0.00	-0.00
time (sec)	N/A	0.603	0.418	0.299	0.000	0.651	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	352	0	0	301	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.418	0.453	0.303	0.000	1.004	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	268	0	0	196	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.245	0.232	0.281	0.000	0.752	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	162	0	0	239	0	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.305	0.360	0.275	0.000	0.927	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	302	0	0	338	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.521	0.366	0.299	0.000	0.671	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	358	0	0	403	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.701	0.424	0.298	0.000	0.995	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	1640	104	134	202	161	82
normalized size	1	1.00	0.85	19.52	1.24	1.60	2.40	1.92	0.98
time (sec)	N/A	0.075	0.079	0.533	0.590	0.754	23.751	0.352	4.038

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	68	1640	102	128	199	161	82
normalized size	1	1.00	0.81	19.52	1.21	1.52	2.37	1.92	0.98
time (sec)	N/A	0.052	0.068	0.486	0.545	0.702	8.221	0.254	4.003

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	58	1503	82	110	151	122	66
normalized size	1	1.00	0.75	19.52	1.06	1.43	1.96	1.58	0.86
time (sec)	N/A	0.036	0.023	0.447	0.635	0.679	2.831	0.267	3.779

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	72	1597	73	62	0	85	73
normalized size	1	1.00	1.26	28.02	1.28	1.09	0.00	1.49	1.28
time (sec)	N/A	0.072	0.063	0.619	0.625	0.675	0.000	0.292	3.868

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	57	1443	94	91	153	108	75
normalized size	1	1.00	0.79	20.04	1.31	1.26	2.12	1.50	1.04
time (sec)	N/A	0.071	0.070	0.347	0.750	0.596	2.595	0.318	3.809

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	64	1442	93	95	201	116	83
normalized size	1	1.00	0.77	17.37	1.12	1.14	2.42	1.40	1.00
time (sec)	N/A	0.073	0.074	0.335	0.755	0.851	7.576	0.244	3.941

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	69	1451	99	105	204	121	83
normalized size	1	1.00	0.83	17.48	1.19	1.27	2.46	1.46	1.00
time (sec)	N/A	0.074	0.082	0.352	0.659	0.684	22.197	0.246	3.935

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	157	9271	250	388	654	506	189
normalized size	1	1.00	0.76	44.79	1.21	1.87	3.16	2.44	0.91
time (sec)	N/A	0.203	0.158	0.865	0.835	0.836	70.848	0.407	3.989

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	154	9262	247	386	600	497	187
normalized size	1	1.00	0.75	44.96	1.20	1.87	2.91	2.41	0.91
time (sec)	N/A	0.166	0.148	0.865	0.611	0.756	26.505	0.360	4.146

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	141	8701	213	345	534	425	165
normalized size	1	1.00	0.96	59.19	1.45	2.35	3.63	2.89	1.12
time (sec)	N/A	0.088	0.113	0.832	0.766	0.893	9.257	0.345	3.879

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	129	9164	163	170	0	223	124
normalized size	1	1.00	2.26	160.77	2.86	2.98	0.00	3.91	2.18
time (sec)	N/A	0.095	0.137	1.554	0.661	0.852	0.000	0.363	3.988

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	138	8407	221	311	536	392	181
normalized size	1	1.00	0.76	46.45	1.22	1.72	2.96	2.17	1.00
time (sec)	N/A	0.193	0.160	0.849	0.975	0.655	9.403	0.359	4.013

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	151	8407	224	326	602	403	186
normalized size	1	1.00	0.74	41.21	1.10	1.60	2.95	1.98	0.91
time (sec)	N/A	0.206	0.177	0.842	0.678	0.679	9.007	0.431	4.082

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	155	8407	230	329	656	403	190
normalized size	1	1.00	0.76	41.01	1.12	1.60	3.20	1.97	0.93
time (sec)	N/A	0.211	0.178	0.895	0.940	0.686	24.957	0.388	4.197

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	93	0	0	92	0	206	-1
normalized size	1	1.00	0.66	0.00	0.00	0.65	0.00	1.46	-0.01
time (sec)	N/A	0.180	0.168	1.419	0.000	0.786	0.000	0.427	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	93	0	0	92	0	206	-1
normalized size	1	1.00	0.66	0.00	0.00	0.65	0.00	1.46	-0.01
time (sec)	N/A	0.154	0.150	7.428	0.000	0.780	0.000	0.510	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	86	2329	0	84	0	179	-1
normalized size	1	1.00	0.66	17.92	0.00	0.65	0.00	1.38	-0.01
time (sec)	N/A	0.122	0.130	1.335	0.000	0.766	0.000	0.428	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	58	1744	118	51	0	85	-1
normalized size	1	1.00	0.82	24.56	1.66	0.72	0.00	1.20	-0.01
time (sec)	N/A	0.106	0.073	0.437	0.674	0.746	0.000	0.392	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	87	0	0	81	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.61	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.127	1.269	0.000	0.614	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	94	0	0	88	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.135	1.562	0.000	0.789	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	135	87	370	0	154	0	661	-1
normalized size	1	1.52	0.98	4.16	0.00	1.73	0.00	7.43	-0.01
time (sec)	N/A	0.138	0.146	0.647	0.000	0.700	0.000	0.516	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	131	32	16	32	17	-1
normalized size	1	1.00	0.97	4.52	1.10	0.55	1.10	0.59	-0.03
time (sec)	N/A	0.053	0.018	0.186	0.619	0.726	10.185	0.315	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	179	0	0	0	0	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	0.624	2.060	0.000	0.752	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	156	0	0	0	0	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	0.398	1.441	0.000	0.693	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	156	0	0	0	0	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	0.377	1.632	0.000	0.867	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	146	0	0	131	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.48	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.312	1.419	0.000	0.842	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	854	95	222	0	246	-1
normalized size	1	1.00	1.00	12.03	1.34	3.13	0.00	3.46	-0.01
time (sec)	N/A	0.155	0.135	1.188	0.658	0.623	0.000	0.397	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	141	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	0.322	1.500	0.000	0.680	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	154	0	0	0	0	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	0.382	0.939	0.000	0.790	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	154	0	0	0	0	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.381	0.923	0.000	0.775	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	248	6894	0	221	0	0	-1
normalized size	1	1.00	1.01	28.02	0.00	0.90	0.00	0.00	-0.00
time (sec)	N/A	0.233	0.182	8.327	0.000	1.267	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	248	5618	0	308	0	0	-1
normalized size	1	1.00	1.01	22.93	0.00	1.26	0.00	0.00	-0.00
time (sec)	N/A	0.227	0.188	11.062	0.000	1.344	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	165	0	0	0	221	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	1.21	0.00	-0.01
time (sec)	N/A	0.164	0.119	180.000	0.000	0.608	91.781	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-1)	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	178	0	0	0	231	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	1.27	0.00	-0.01
time (sec)	N/A	0.148	0.124	180.000	0.000	0.842	95.102	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	240	0	0	307	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	1.26	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.163	180.000	0.000	1.098	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	145	0	0	275	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.239	180.000	0.000	1.045	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	167	0	354	0	0	0	-1
normalized size	1	1.00	0.93	0.00	1.97	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.141	180.000	1.083	0.754	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	178	0	319	0	0	0	-1
normalized size	1	1.00	0.99	0.00	1.77	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.128	180.000	0.949	0.655	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	456	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.734	0.854	10.291	0.000	0.756	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	564	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	0.822	11.590	0.000	0.702	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	516	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	0.726	180.000	0.000	0.766	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	619	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.546	3.878	180.000	0.000	0.684	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	0.049	0.069	0.000	0.887	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	99	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.048	0.081	0.000	0.742	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.018	0.060	0.000	0.670	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	51	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.004	0.060	0.000	0.566	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.032	0.043	0.056	0.000	0.775	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.045	0.059	0.000	0.806	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.048	0.060	0.000	0.527	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	0	15	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	-0.05
time (sec)	N/A	0.024	0.002	0.036	0.000	0.695	2.326	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.044	0.003	0.047	0.000	0.769	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.110	0.156	0.107	0.000	0.769	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	196	0	0	247	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.548	0.278	0.000	0.686	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	168	0	0	207	264	0	-1
normalized size	1	1.00	0.91	0.00	0.00	1.12	1.43	0.00	-0.01
time (sec)	N/A	0.132	0.338	0.250	0.000	0.563	83.201	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	113	0	0	137	172	0	-1
normalized size	1	1.00	1.07	0.00	0.00	1.29	1.62	0.00	-0.01
time (sec)	N/A	0.114	0.076	0.221	0.000	1.044	23.638	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	0	0	0	26	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	1.00	0.00	-0.04
time (sec)	N/A	0.029	0.003	0.237	0.000	0.563	15.635	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	115	0	0	134	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.160	0.294	0.000	0.689	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	163	0	0	190	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.157	0.197	0.369	0.000	0.941	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	0	0	0	296	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	1.17	0.00	0.00	-0.00
time (sec)	N/A	0.254	0.141	0.431	0.000	0.790	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	257	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	1.16	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.122	0.424	0.000	0.879	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	172	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.082	0.347	0.000	0.663	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	0	0	0	26	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	1.00	0.00	-0.04
time (sec)	N/A	0.028	0.003	0.391	0.000	0.581	9.052	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	0	0	0	156	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.134	0.347	0.000	0.619	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	0	0	0	221	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.125	0.456	0.000	0.551	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	266	844	0	0	0	0	-1
normalized size	1	0.00	8.87	28.13	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.243	0.866	0.000	0.682	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	0	0	867	0	0	0	0	-1
normalized size	1	0.00	0.00	4.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.112	0.352	0.000	0.451	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	0	0	1065	0	0	0	0	-1
normalized size	1	0.00	0.00	4.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.067	2.282	0.000	0.808	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	23	32	37	32	23
normalized size	1	1.00	1.00	0.00	0.85	1.19	1.37	1.19	0.85
time (sec)	N/A	0.031	0.003	0.563	1.105	0.735	2.213	0.349	3.838

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	23	32	37	32	23
normalized size	1	1.00	1.00	0.00	0.85	1.19	1.37	1.19	0.85
time (sec)	N/A	0.017	0.001	0.061	1.163	0.645	0.948	0.315	3.799

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	21	24	21	17
normalized size	1	1.00	1.00	1.06	1.00	1.17	1.33	1.17	0.94
time (sec)	N/A	0.007	0.001	0.040	1.390	0.484	0.413	0.365	0.027

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	19	37	20	20
normalized size	1	1.00	1.00	0.95	0.91	0.86	1.68	0.91	0.91
time (sec)	N/A	0.030	0.001	0.037	1.201	0.674	2.024	0.302	3.801

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	23	20	26	25	19
normalized size	1	1.00	1.00	0.00	1.00	0.87	1.13	1.09	0.83
time (sec)	N/A	0.031	0.002	0.079	1.053	0.661	1.001	0.242	3.881

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	23	24	39	28	23
normalized size	1	1.00	1.00	0.00	0.85	0.89	1.44	1.04	0.85
time (sec)	N/A	0.029	0.002	0.065	1.113	0.585	2.316	0.289	3.842

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	23	24	39	28	23
normalized size	1	1.00	1.00	0.00	0.85	0.89	1.44	1.04	0.85
time (sec)	N/A	0.030	0.001	0.063	1.254	0.692	5.094	0.277	3.868

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	46	113	150	115	46
normalized size	1	1.00	1.00	0.00	0.88	2.17	2.88	2.21	0.88
time (sec)	N/A	0.071	0.003	0.060	1.082	0.711	5.000	0.275	3.798

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	0	46	113	133	112	46
normalized size	1	1.00	0.83	0.00	0.88	2.17	2.56	2.15	0.88
time (sec)	N/A	0.045	0.006	0.062	1.137	0.758	2.310	0.384	3.848

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	0	39	90	116	92	39
normalized size	1	1.00	0.95	0.00	1.00	2.31	2.97	2.36	1.00
time (sec)	N/A	0.022	0.003	0.065	1.171	0.574	1.031	0.287	3.844

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	54	41	59	20
normalized size	1	1.00	1.00	0.95	0.91	2.45	1.86	2.68	0.91
time (sec)	N/A	0.050	0.002	0.035	1.135	0.679	1.565	0.321	3.749

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	0	46	81	117	90	40
normalized size	1	1.00	0.87	0.00	1.00	1.76	2.54	1.96	0.87
time (sec)	N/A	0.070	0.005	0.080	1.142	0.635	1.039	0.289	3.851

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	0	46	87	134	94	46
normalized size	1	1.00	0.83	0.00	0.88	1.67	2.58	1.81	0.88
time (sec)	N/A	0.070	0.007	0.062	1.136	0.625	2.436	0.238	3.785

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	46	88	151	95	46
normalized size	1	1.00	1.00	0.00	0.88	1.69	2.90	1.83	0.88
time (sec)	N/A	0.070	0.002	0.064	1.171	0.511	5.229	0.245	3.881

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	91	0	282	1358	0	1811	-1
normalized size	1	1.00	0.67	0.00	2.09	10.06	0.00	13.41	-0.01
time (sec)	N/A	0.221	0.059	0.659	1.133	0.792	0.000	0.494	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	90	0	149	391	0	561	-1
normalized size	1	1.00	0.97	0.00	1.60	4.20	0.00	6.03	-0.01
time (sec)	N/A	0.126	0.043	0.128	1.045	0.588	0.000	0.353	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	37	0	62	72	112	111	-1
normalized size	1	1.00	0.73	0.00	1.22	1.41	2.20	2.18	-0.02
time (sec)	N/A	0.046	0.013	0.131	0.994	0.609	10.312	0.337	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	85	0	0	105	0	140	-1
normalized size	1	1.00	0.99	0.00	0.00	1.22	0.00	1.63	-0.01
time (sec)	N/A	0.184	0.206	0.092	0.000	0.651	0.000	0.427	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	112	0	0	202	0	1540	-1
normalized size	1	1.00	0.88	0.00	0.00	1.59	0.00	12.13	-0.01
time (sec)	N/A	0.242	0.308	0.092	0.000	0.662	0.000	0.840	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	133	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.203	0.284	0.000	0.906	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.155	0.089	0.000	0.818	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.144	0.089	0.000	1.008	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	0	0	73	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.129	0.091	0.000	0.924	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	33	49	80	36	33
normalized size	1	1.00	1.00	1.03	1.00	1.48	2.42	1.09	1.00
time (sec)	N/A	0.094	0.009	0.038	1.245	0.538	2.846	0.245	4.072

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.135	0.102	0.000	0.925	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.139	0.089	0.000	0.928	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.090	0.300	0.000	0.883	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [20] had the largest ratio of [.8421]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	3	1.00	23	0.130
2	A	6	4	1.00	20	0.200
3	A	6	4	1.00	20	0.200
4	A	6	4	1.00	18	0.222
5	A	7	7	1.00	17	0.412
6	A	2	2	1.00	20	0.100
7	A	8	7	1.00	20	0.350
8	A	7	5	1.00	20	0.250
9	A	7	5	1.00	20	0.250
10	A	15	9	1.00	22	0.409
11	A	14	9	1.00	22	0.409
12	A	13	9	1.00	20	0.450
13	A	14	12	1.00	19	0.632
14	A	3	3	1.00	22	0.136
15	A	15	14	1.08	22	0.636
16	A	19	13	1.08	22	0.591
17	A	29	12	1.00	22	0.546
18	A	26	12	1.00	22	0.546
19	A	23	12	1.00	20	0.600
20	A	24	16	1.00	19	0.842
21	A	4	3	1.00	22	0.136
22	A	22	15	1.05	22	0.682
23	A	30	14	1.06	22	0.636
24	A	7	5	1.00	26	0.192
25	A	8	9	1.00	24	0.375
26	A	2	2	1.00	26	0.077
27	A	9	8	1.00	26	0.308
28	A	9	7	1.00	26	0.269
29	A	8	7	1.00	23	0.304
30	A	7	6	1.00	26	0.231
31	A	8	7	1.00	26	0.269
32	A	13	9	1.00	28	0.321
33	A	15	16	1.00	26	0.615
34	A	3	3	1.00	28	0.107
35	A	11	11	1.00	28	0.393

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	30	17	1.00	28	0.607
37	A	26	16	1.00	25	0.640
38	A	16	12	1.00	28	0.429
39	A	24	15	1.00	28	0.536
40	A	22	11	1.00	28	0.393
41	A	24	21	1.00	26	0.808
42	A	4	3	1.00	28	0.107
43	A	15	12	1.00	28	0.429
44	A	42	17	1.00	25	0.680
45	A	26	13	1.00	28	0.464
46	A	7	5	1.00	28	0.179
47	A	7	5	1.00	26	0.192
48	A	7	5	1.00	25	0.200
49	A	2	2	1.00	28	0.071
50	A	8	6	1.00	28	0.214
51	A	8	6	1.00	28	0.214
52	A	8	6	1.00	28	0.214
53	A	18	10	1.00	30	0.333
54	A	16	10	1.00	28	0.357
55	A	14	9	1.00	27	0.333
56	A	3	3	1.00	30	0.100
57	A	17	14	1.00	30	0.467
58	A	19	14	1.00	30	0.467
59	A	30	13	1.00	28	0.464
60	A	24	12	1.00	27	0.444
61	A	4	3	1.00	30	0.100
62	A	28	16	1.00	30	0.533
63	A	34	16	1.00	30	0.533
64	A	5	3	1.00	28	0.107
65	A	4	3	1.00	28	0.107
66	A	3	3	1.00	28	0.107
67	A	2	2	1.00	26	0.077
68	A	0	0	0.00	0	0.000
69	A	0	0	0.00	0	0.000
70	A	7	5	1.00	24	0.208
71	A	7	5	1.00	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	7	5	1.00	22	0.227
73	A	8	8	1.00	21	0.381
74	A	4	4	1.00	24	0.167
75	A	9	8	1.00	24	0.333
76	A	8	6	1.00	24	0.250
77	A	8	6	1.00	24	0.250
78	A	24	12	1.00	26	0.462
79	A	21	12	1.00	24	0.500
80	A	18	11	1.00	23	0.478
81	A	5	5	1.00	26	0.192
82	A	15	14	1.14	26	0.538
83	A	19	13	1.12	26	0.500
84	A	22	13	1.10	26	0.500
85	A	34	13	1.00	24	0.542
86	A	28	12	1.00	23	0.522
87	A	6	5	1.00	26	0.192
88	A	22	15	1.12	26	0.577
89	A	30	14	1.11	26	0.538
90	A	9	6	1.00	26	0.231
91	A	9	10	1.00	24	0.417
92	A	4	4	1.00	26	0.154
93	A	11	9	1.00	26	0.346
94	A	10	7	1.00	26	0.269
95	A	9	6	1.00	26	0.231
96	A	8	6	1.00	23	0.261
97	A	7	5	1.00	26	0.192
98	A	8	6	1.00	26	0.231
99	A	9	6	1.00	26	0.231
100	A	17	11	1.00	26	0.423
101	A	5	5	1.00	28	0.179
102	A	11	11	1.00	28	0.393
103	A	20	14	1.15	28	0.500
104	A	30	17	1.00	28	0.607
105	A	26	16	1.00	25	0.640
106	A	16	12	1.00	28	0.429
107	A	24	15	1.00	28	0.536

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	26	12	1.00	26	0.462
109	A	6	5	1.00	28	0.179
110	A	15	12	1.00	28	0.429
111	A	49	18	1.00	28	0.643
112	A	42	17	1.00	25	0.680
113	A	26	13	1.00	28	0.464
114	A	39	16	1.00	28	0.571
115	A	9	6	1.00	28	0.214
116	A	9	6	1.00	26	0.231
117	A	9	7	1.00	25	0.280
118	A	4	4	1.00	28	0.143
119	A	10	7	1.00	28	0.250
120	A	10	7	1.00	28	0.250
121	A	10	7	1.00	28	0.250
122	A	22	13	1.00	28	0.464
123	A	20	13	1.00	26	0.500
124	A	18	13	1.00	25	0.520
125	A	5	5	1.00	28	0.179
126	A	21	17	1.00	28	0.607
127	A	23	17	1.00	28	0.607
128	A	36	16	1.00	26	0.615
129	A	30	16	1.00	25	0.640
130	A	6	5	1.00	28	0.179
131	A	34	19	1.00	28	0.679
132	A	40	19	1.00	28	0.679
133	A	9	6	1.00	30	0.200
134	A	9	6	1.00	30	0.200
135	A	11	9	1.00	30	0.300
136	A	10	7	1.00	30	0.233
137	A	10	7	1.00	30	0.233
138	A	0	0	0.00	0	0.000
139	A	6	5	1.00	28	0.179
140	A	5	5	1.00	28	0.179
141	A	4	4	1.00	26	0.154
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
144	A	0	0	0.00	0	0.000
145	A	0	0	0.00	0	0.000
146	A	0	0	0.00	0	0.000
147	A	4	4	1.00	26	0.154
148	A	0	0	0.00	0	0.000
149	A	0	0	0.00	0	0.000
150	A	18	12	1.00	32	0.375
151	A	16	12	1.00	32	0.375
152	A	14	11	1.00	30	0.367
153	A	15	12	1.00	32	0.375
154	A	16	12	1.00	32	0.375
155	A	18	12	1.00	32	0.375
156	A	3	3	1.00	24	0.125
157	A	3	3	1.00	22	0.136
158	A	3	2	1.00	21	0.095
159	A	4	5	1.00	24	0.208
160	A	2	2	1.00	24	0.083
161	A	3	3	1.00	24	0.125
162	A	3	3	1.00	24	0.125
163	A	7	5	1.00	26	0.192
164	A	7	5	1.00	24	0.208
165	A	6	3	1.00	23	0.130
166	A	4	4	1.00	26	0.154
167	A	6	4	1.00	26	0.154
168	A	7	5	1.00	26	0.192
169	A	7	5	1.00	26	0.192
170	A	6	6	1.00	26	0.231
171	A	6	6	1.00	24	0.250
172	A	6	6	1.00	23	0.261
173	A	5	6	1.00	26	0.231
174	A	6	6	1.00	26	0.231
175	A	6	6	1.00	26	0.231
176	A	7	4	1.52	23	0.174
177	A	2	4	1.00	18	0.222
178	A	8	7	1.00	28	0.250
179	A	7	7	1.00	26	0.269

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	7	7	1.00	24	0.292
181	A	7	7	1.00	23	0.304
182	A	4	4	1.00	26	0.154
183	A	7	7	1.00	26	0.269
184	A	7	7	1.00	26	0.269
185	A	7	7	1.00	26	0.269
186	A	17	11	1.00	18	0.611
187	A	17	11	1.00	18	0.611
188	A	9	10	1.00	18	0.556
189	A	9	10	1.00	18	0.556
190	A	17	11	1.00	18	0.611
191	A	12	11	1.00	18	0.611
192	A	9	10	1.00	18	0.556
193	A	9	10	1.00	18	0.556
194	A	21	14	1.00	20	0.700
195	A	21	14	1.00	20	0.700
196	A	21	14	1.00	20	0.700
197	A	21	14	1.00	20	0.700
198	A	0	0	0.00	0	0.000
199	A	4	2	1.00	23	0.087
200	A	3	2	1.00	23	0.087
201	A	2	2	1.00	21	0.095
202	A	0	0	0.00	0	0.000
203	A	0	0	0.00	0	0.000
204	A	0	0	0.00	0	0.000
205	A	2	2	1.00	11	0.182
206	A	3	2	1.00	13	0.154
207	A	2	1	1.00	57	0.018
208	A	10	5	1.00	19	0.263
209	A	10	5	1.00	17	0.294
210	A	10	8	1.00	16	0.500
211	A	2	2	1.00	19	0.105
212	A	13	8	1.00	19	0.421
213	A	11	6	1.00	19	0.316
214	A	15	6	1.00	19	0.316
215	A	15	6	1.00	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	14	9	1.00	16	0.562
217	A	2	2	1.00	19	0.105
218	A	19	9	1.00	19	0.474
219	A	16	7	1.00	19	0.368
220	A	0	0	0.00	0	0.000
221	A	0	0	0.00	0	0.000
222	A	0	0	0.00	0	0.000
223	A	2	2	1.00	14	0.143
224	A	2	2	1.00	12	0.167
225	A	2	2	1.00	10	0.200
226	A	2	2	1.00	14	0.143
227	A	2	2	1.00	14	0.143
228	A	2	2	1.00	14	0.143
229	A	2	2	1.00	14	0.143
230	A	3	3	1.00	16	0.188
231	A	3	3	1.00	14	0.214
232	A	3	3	1.00	12	0.250
233	A	3	3	1.00	16	0.188
234	A	3	3	1.00	16	0.188
235	A	3	3	1.00	16	0.188
236	A	3	3	1.00	16	0.188
237	A	4	3	1.00	22	0.136
238	A	3	3	1.00	22	0.136
239	A	2	2	1.00	20	0.100
240	A	3	3	1.00	22	0.136
241	A	4	4	1.00	22	0.182
242	A	3	3	1.00	22	0.136
243	A	3	3	1.00	20	0.150
244	A	3	3	1.00	18	0.167
245	A	3	3	1.00	16	0.188
246	A	3	3	1.00	20	0.150
247	A	3	3	1.00	20	0.150
248	A	3	3	1.00	20	0.150
249	A	6	6	1.00	24	0.250

Chapter 3

Listing of integrals

3.1 $\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx$

Optimal. Leaf size=173

$$\frac{\log\left(\frac{2fx}{e-\sqrt{e^2-4df}}+1\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e^2-4df}}-\frac{\log\left(\frac{2fx}{\sqrt{e^2-4df}+e}+1\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e^2-4df}}+\frac{bn\text{Li}_2\left(-\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}-\frac{bn\text{Li}_2\left(-\frac{}{e+}\right.}{\sqrt{e^2-}}$$

```
[Out] (a+b*ln(c*x^n))*ln(1+2*f*x/(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)-(a+b*
ln(c*x^n))*ln(1+2*f*x/(e+(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)+b*n*polylo
g(2,-2*f*x/(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)-b*n*polylog(2,-2*f*x/
(e+(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2357, 2317, 2391}

$$\frac{bn\text{PolyLog}\left(2,-\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}-\frac{bn\text{PolyLog}\left(2,-\frac{2fx}{\sqrt{e^2-4df}+e}\right)}{\sqrt{e^2-4df}}+\frac{\log\left(\frac{2fx}{e-\sqrt{e^2-4df}}+1\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e^2-4df}}-\frac{\log\left(\frac{2fx}{\sqrt{e^2-4d}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x + f*x^2),x]
```

```
[Out] ((a + b*Log[c*x^n])*Log[1 + (2*f*x)/(e - Sqrt[e^2 - 4*d*f])])/Sqrt[e^2 - 4*
d*f] - ((a + b*Log[c*x^n])*Log[1 + (2*f*x)/(e + Sqrt[e^2 - 4*d*f])])/Sqrt[e
^2 - 4*d*f] + (b*n*PolyLog[2, (-2*f*x)/(e - Sqrt[e^2 - 4*d*f])])/Sqrt[e^2 -
 4*d*f] - (b*n*PolyLog[2, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f])])/Sqrt[e^2 - 4*d
*f]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx = \int \left(\frac{2f\left(a+b\log(cx^n)\right)}{\sqrt{e^2-4df}\left(e-\sqrt{e^2-4df}+2fx\right)} - \frac{2f\left(a+b\log(cx^n)\right)}{\sqrt{e^2-4df}\left(e+\sqrt{e^2-4df}+2fx\right)}\right)dx$$
$$= \frac{(2f)\int \frac{a+b\log(cx^n)}{e-\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{a+b\log(cx^n)}{e+\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}}$$
$$= \frac{\left(a+b\log(cx^n)\right)\log\left(1+\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}} - \frac{\left(a+b\log(cx^n)\right)\log\left(1+\frac{2fx}{e+\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}} - \frac{(bn)\int \frac{\log}{}}{\sqrt{e}}$$
$$= \frac{\left(a+b\log(cx^n)\right)\log\left(1+\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}} - \frac{\left(a+b\log(cx^n)\right)\log\left(1+\frac{2fx}{e+\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}} + \frac{bn\mathrm{Li}_2\left(-\right.}{\sqrt{e^2-}}$$

Mathematica [A] time = 0.17, size = 157, normalized size = 0.91

$$\frac{\left(\log\left(\frac{-\sqrt{e^2-4df}+e+2fx}{e-\sqrt{e^2-4df}}\right)-\log\left(\frac{\sqrt{e^2-4df}+e+2fx}{\sqrt{e^2-4df}+e}\right)\right)\left(a+b\log(cx^n)\right)+bn\mathrm{Li}_2\left(\frac{2fx}{\sqrt{e^2-4df}-e}\right)-bn\mathrm{Li}_2\left(-\frac{2fx}{e+\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x + f*x^2),x]
[Out] ((a + b*Log[c*x^n])*(Log[(e - Sqrt[e^2 - 4*d*f] + 2*f*x)/(e - Sqrt[e^2 - 4*
d*f])] - Log[(e + Sqrt[e^2 - 4*d*f] + 2*f*x)/(e + Sqrt[e^2 - 4*d*f])]) + b*
n*PolyLog[2, (2*f*x)/(-e + Sqrt[e^2 - 4*d*f])] - b*n*PolyLog[2, (-2*f*x)/(e
 + Sqrt[e^2 - 4*d*f])])/Sqrt[e^2 - 4*d*f]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\log(cx^n)+a}{fx^2+ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(f*x^2 + e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log(cx^n)+a}{fx^2+ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(f*x^2 + e*x + d), x)
```

maple [C] time = 0.22, size = 555, normalized size = 3.21

$$-\frac{i\pi b\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)}{\sqrt{4df-e^2}}+\frac{i\pi b\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2}{\sqrt{4df-e^2}}+\frac{i\pi b\operatorname{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)/(f*x^2+e*x+d),x)
```

```
[Out] -2*b/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*n*ln(x)+2*b/(4*d
*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*ln(x^n)+b*n/(-4*d*f+e^2)^
(1/2)*ln(x)*ln((-2*f*x+(-4*d*f+e^2)^(1/2)-e)/(-e+(-4*d*f+e^2)^(1/2)))-b*n/(
-4*d*f+e^2)^(1/2)*ln(x)*ln((2*f*x+(-4*d*f+e^2)^(1/2)+e)/(e+(-4*d*f+e^2)^(1/
2)))+b*n/(-4*d*f+e^2)^(1/2)*dilog((-2*f*x+(-4*d*f+e^2)^(1/2)-e)/(-e+(-4*d*f
+e^2)^(1/2)))-b*n/(-4*d*f+e^2)^(1/2)*dilog((2*f*x+(-4*d*f+e^2)^(1/2)+e)/(e+
(-4*d*f+e^2)^(1/2)))+I/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2)
)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I/(4*d*f-e^2)^(1/2
)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*b*Pi*csgn(I*c*x^n)^3+I/(4*d*f-e^2)^(1
/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+2/(4
*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*b*ln(c)+2*a/(4*d*f-e^2)
^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,x^n\right)}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*log(c*x^n))/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(cx^n\right)}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(d + e*x + f*x**2), x)
```

3.2 $\int x^3 \left(a+b\log\left(cx^n\right)\right)\log(1+ex)\,dx$

Optimal. Leaf size=210

$$-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{4e^4}+\frac{x\left(a+b\log\left(cx^n\right)\right)}{4e^3}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{8e^2}+\frac{1}{4}x^4\log(ex+1)\left(a+b\log\left(cx^n\right)\right)+\frac{x^3\left(a+\right.}{}$$

```
[Out] -5/16*b*n*x/e^3+3/32*b*n*x^2/e^2-7/144*b*n*x^3/e+1/32*b*n*x^4+1/4*x*(a+b*ln
(c*x^n))/e^3-1/8*x^2*(a+b*ln(c*x^n))/e^2+1/12*x^3*(a+b*ln(c*x^n))/e-1/16*x^
4*(a+b*ln(c*x^n))+1/16*b*n*ln(e*x+1)/e^4-1/16*b*n*x^4*ln(e*x+1)-1/4*(a+b*ln
(c*x^n))*ln(e*x+1)/e^4+1/4*x^4*(a+b*ln(c*x^n))*ln(e*x+1)-1/4*b*n*polylog(2,
-e*x)/e^4
```

Rubi [A] time = 0.12, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2395, 43, 2376, 2391}

$$-\frac{bn\text{PolyLog}(2,-ex)}{4e^4}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{8e^2}+\frac{x\left(a+b\log\left(cx^n\right)\right)}{4e^3}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{4e^4}+\frac{1}{4}x^4\log(ex+1)\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (-5*b*n*x)/(16*e^3) + (3*b*n*x^2)/(32*e^2) - (7*b*n*x^3)/(144*e) + (b*n*x^4
)/32 + (x*(a + b*Log[c*x^n]))/(4*e^3) - (x^2*(a + b*Log[c*x^n]))/(8*e^2) +
(x^3*(a + b*Log[c*x^n]))/(12*e) - (x^4*(a + b*Log[c*x^n]))/16 + (b*n*Log[1
+ e*x])/(16*e^4) - (b*n*x^4*Log[1 + e*x])/16 - ((a + b*Log[c*x^n])*Log[1 +
e*x])/(4*e^4) + (x^4*(a + b*Log[c*x^n])*Log[1 + e*x])/4 - (b*n*PolyLog[2, -
(e*x)])/(4*e^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x^3 \left(a + b \log \left(cx^n\right)\right) \log(1 + ex)\, dx = \frac{x\left(a + b \log \left(cx^n\right)\right)}{4e^3} - \frac{x^2\left(a + b \log \left(cx^n\right)\right)}{8e^2} + \frac{x^3\left(a + b \log \left(cx^n\right)\right)}{12e} - \frac{1}{16}x$$

$$= -\frac{bnx}{4e^3} + \frac{bnx^2}{16e^2} - \frac{bnx^3}{36e} + \frac{1}{64}bnx^4 + \frac{x\left(a + b \log \left(cx^n\right)\right)}{4e^3} - \frac{x^2\left(a + b \log\right.}{8e^2}$$

$$= -\frac{bnx}{4e^3} + \frac{bnx^2}{16e^2} - \frac{bnx^3}{36e} + \frac{1}{64}bnx^4 + \frac{x\left(a + b \log \left(cx^n\right)\right)}{4e^3} - \frac{x^2\left(a + b \log\right.}{8e^2}$$

$$= -\frac{bnx}{4e^3} + \frac{bnx^2}{16e^2} - \frac{bnx^3}{36e} + \frac{1}{64}bnx^4 + \frac{x\left(a + b \log \left(cx^n\right)\right)}{4e^3} - \frac{x^2\left(a + b \log\right.}{8e^2}$$

$$= -\frac{5bnx}{16e^3} + \frac{3bnx^2}{32e^2} - \frac{7bnx^3}{144e} + \frac{1}{32}bnx^4 + \frac{x\left(a + b \log \left(cx^n\right)\right)}{4e^3} - \frac{x^2\left(a + b\,l\right.}{8e}$$

Mathematica [A] time = 0.09, size = 188, normalized size = 0.90

$$-18ae^4x^4 + 72ae^4x^4 \log(ex + 1) + 24ae^3x^3 - 36ae^2x^2 + 72aex - 72a \log(ex + 1) + 6b\left(12\left(e^4x^4 - 1\right)\log(ex + 1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (72*a*e*x - 90*b*e*n*x - 36*a*e^2*x^2 + 27*b*e^2*n*x^2 + 24*a*e^3*x^3 - 14*
b*e^3*n*x^3 - 18*a*e^4*x^4 + 9*b*e^4*n*x^4 - 72*a*Log[1 + e*x] + 18*b*n*Log
[1 + e*x] + 72*a*e^4*x^4*Log[1 + e*x] - 18*b*e^4*n*x^4*Log[1 + e*x] + 6*b*L
og[c*x^n]*(e*x*(12 - 6*e*x + 4*e^2*x^2 - 3*e^3*x^3) + 12*(-1 + e^4*x^4)*Log
[1 + e*x]) - 72*b*n*PolyLog[2, -(e*x)])/(288*e^4)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(bx^3 \log\left(cx^n\right)\log\left(ex + 1\right) + ax^3 \log\left(ex + 1\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b*x^3*log(c*x^n)*log(e*x + 1) + a*x^3*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^n\right) + a\right)x^3 \log\left(ex + 1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3*log(e*x + 1), x)
```

maple [C] time = 0.30, size = 1014, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*x^n)+a)*ln(e*x+1),x)
```

```
[Out] 1/12*a/e*x^3+25/48*a/e^4-1/16*a*x^4+(1/4*b*x^4*ln(e*x+1)-1/48*b*(3*e^4*x^4-
4*e^3*x^3+6*e^2*x^2-12*e*x+12*ln(e*x+1))/e^4)*ln(x^n)+25/96*I/e^4*Pi*b*csgn
```

```
(I*x^n)*csgn(I*c*x^n)^2+25/96*I/e^4*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/16*ln(
c)*b*x^4+1/32*b*n*x^4+25/48/e^4*b*ln(c)+1/12*b/e*x^3*ln(c)-1/8*b/e^2*x^2*ln
(c)-35/72*b*n/e^4+1/4*b/e^3*x*ln(c)-1/4/e^4*ln(e*x+1)*b*ln(c)+1/4*b*ln(c)*l
n(e*x+1)*x^4+1/4*a*ln(e*x+1)*x^4-1/8*a/e^2*x^2-1/4*a/e^4*ln(e*x+1)+1/4*a/e^
3*x+1/32*I*Pi*b*x^4*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/24*I/e*x^3*Pi*b*c
sgn(I*x^n)*csgn(I*c*x^n)^2+1/8*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(e*x+1)*x
^4-1/8*I/e^4*ln(e*x+1)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-7/144*b/e*n*x^3+3/32*
b/e^2*n*x^2-5/16*b/e^3*n*x+1/8*I/e^4*ln(e*x+1)*Pi*b*csgn(I*c*x^n)^3-1/8*I*P
i*b*csgn(I*c*x^n)^3*ln(e*x+1)*x^4-1/8*I/e^4*ln(e*x+1)*Pi*b*csgn(I*x^n)*csgn
(I*c*x^n)^2-25/96*I/e^4*Pi*b*csgn(I*c*x^n)^3+1/32*I*Pi*b*x^4*csgn(I*c*x^n)^
3-1/8*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(e*x+1)*x^4+1/8*I/e^4*ln
(e*x+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/24*I/e*x^3*Pi*b*csgn(I*x
^n)*csgn(I*c*x^n)*csgn(I*c)+1/16*I/e^2*x^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*c
sgn(I*c)-1/4*b*n/e^4*dilog(e*x+1)-1/8*I/e^3*x*Pi*b*csgn(I*x^n)*csgn(I*c*x^n
)*csgn(I*c)-1/24*I/e*x^3*Pi*b*csgn(I*c*x^n)^3+1/16*I/e^2*x^2*Pi*b*csgn(I*c*
x^n)^3-1/8*I/e^3*x*Pi*b*csgn(I*c*x^n)^3-1/32*I*Pi*b*x^4*csgn(I*x^n)*csgn(I*
c*x^n)^2-1/32*I*Pi*b*x^4*csgn(I*c*x^n)^2*csgn(I*c)+1/8*I*Pi*b*csgn(I*x^n)*c
sgn(I*c*x^n)^2*ln(e*x+1)*x^4-25/96*I/e^4*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csg
n(I*c)+1/24*I/e*x^3*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+1/8*I/e^3*x*Pi*b*csgn(I*
c*x^n)^2*csgn(I*c)-1/16*I/e^2*x^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/16*b*n
*ln(e*x+1)/e^4-1/16*b*n*x^4*ln(e*x+1)-1/16*I/e^2*x^2*Pi*b*csgn(I*c*x^n)^2*c
sgn(I*c)+1/8*I/e^3*x*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2
```

maxima [A] time = 1.47, size = 259, normalized size = 1.23

$$-\frac{\left(\log\left(ex+1\right)\log(x)+\mathrm{Li}_2\left(-ex\right)\right)bn}{4\,e^4}+\frac{\left(b\left(n-4\,\log(c)\right)-4\,a\right)\log\left(ex+1\right)}{16\,e^4}-\frac{9\left(2\,ae^4-\left(e^4n-2\,e^4\log(c)\right)b\right)x^4-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/4*(log(e*x + 1)*log(x) + dilog(-e*x))*b*n/e^4 + 1/16*(b*(n - 4*log(c)) -
 4*a)*log(e*x + 1)/e^4 - 1/288*(9*(2*a*e^4 - (e^4*n - 2*e^4*log(c))*b)*x^4
- 2*(12*a*e^3 - (7*e^3*n - 12*e^3*log(c))*b)*x^3 + 9*(4*a*e^2 - (3*e^2*n -
4*e^2*log(c))*b)*x^2 + 18*((5*e*n - 4*e*log(c))*b - 4*a*e)*x - 18*((4*a*e^4
 - (e^4*n - 4*e^4*log(c))*b)*x^4 + 4*b*n*log(x))*log(e*x + 1) + 6*(3*b*e^4*
x^4 - 4*b*e^3*x^3 + 6*b*e^2*x^2 - 12*b*e*x - 12*(b*e^4*x^4 - b)*log(e*x + 1
))*log(x^n))/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(e*x + 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^3*log(e*x + 1)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.3 $\int x^2 \left(a+b\log\left(cx^n\right)\right)\log(1+ex)\,dx$

Optimal. Leaf size=178

$$\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{x\left(a+b\log\left(cx^n\right)\right)}{3e^2}+\frac{1}{3}x^3\log(ex+1)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{6e}-\frac{1}{9}x^3\left(\right.$$

```
[Out] 4/9*b*n*x/e^2-5/36*b*n*x^2/e+2/27*b*n*x^3-1/3*x*(a+b*ln(c*x^n))/e^2+1/6*x^2
*(a+b*ln(c*x^n))/e-1/9*x^3*(a+b*ln(c*x^n))-1/9*b*n*ln(e*x+1)/e^3-1/9*b*n*x^
3*ln(e*x+1)+1/3*(a+b*ln(c*x^n))*ln(e*x+1)/e^3+1/3*x^3*(a+b*ln(c*x^n))*ln(e*
x+1)+1/3*b*n*polylog(2,-e*x)/e^3
```

Rubi [A] time = 0.10, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2395, 43, 2376, 2391}

$$\frac{bn\text{PolyLog}(2,-ex)}{3e^3}-\frac{x\left(a+b\log\left(cx^n\right)\right)}{3e^2}+\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+\frac{1}{3}x^3\log(ex+1)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (4*b*n*x)/(9*e^2) - (5*b*n*x^2)/(36*e) + (2*b*n*x^3)/27 - (x*(a + b*Log[c*x
^n]))/(3*e^2) + (x^2*(a + b*Log[c*x^n]))/(6*e) - (x^3*(a + b*Log[c*x^n]))/9
 - (b*n*Log[1 + e*x])/(9*e^3) - (b*n*x^3*Log[1 + e*x])/9 + ((a + b*Log[c*x^
n])*Log[1 + e*x])/(3*e^3) + (x^3*(a + b*Log[c*x^n])*Log[1 + e*x])/3 + (b*n*
PolyLog[2, -(e*x)])/(3*e^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(cx^n\right)\right)\log(1+ex)\,dx = -\frac{x\left(a+b\log\left(cx^n\right)\right)}{3e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{6e}-\frac{1}{9}x^3\left(a+b\log\left(cx^n\right)\right)+\frac{\left(a+\right.}{}$$

$$=\frac{bnx}{3e^2}-\frac{bnx^2}{12e}+\frac{1}{27}bnx^3-\frac{x\left(a+b\log\left(cx^n\right)\right)}{3e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{6e}-\frac{1}{9}x^3$$

$$=\frac{bnx}{3e^2}-\frac{bnx^2}{12e}+\frac{1}{27}bnx^3-\frac{x\left(a+b\log\left(cx^n\right)\right)}{3e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{6e}-\frac{1}{9}x^3$$

$$=\frac{bnx}{3e^2}-\frac{bnx^2}{12e}+\frac{1}{27}bnx^3-\frac{x\left(a+b\log\left(cx^n\right)\right)}{3e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{6e}-\frac{1}{9}x^3$$

$$=\frac{4bnx}{9e^2}-\frac{5bnx^2}{36e}+\frac{2}{27}bnx^3-\frac{x\left(a+b\log\left(cx^n\right)\right)}{3e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{6e}-\frac{1}{9}$$

Mathematica [A] time = 0.08, size = 161, normalized size = 0.90

$$\frac{-12ae^3x^3+36ae^3x^3\log(ex+1)+18ae^2x^2-36aex+36a\log(ex+1)+6b\left(6\left(e^3x^3+1\right)\log(ex+1)+ex\left(-2e^2x^2\right.\right.}{108}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (-36*a*e*x + 48*b*e*n*x + 18*a*e^2*x^2 - 15*b*e^2*n*x^2 - 12*a*e^3*x^3 + 8*
b*e^3*n*x^3 + 36*a*Log[1 + e*x] - 12*b*n*Log[1 + e*x] + 36*a*e^3*x^3*Log[1
+ e*x] - 12*b*e^3*n*x^3*Log[1 + e*x] + 6*b*Log[c*x^n]*(e*x*(-6 + 3*e*x - 2*
e^2*x^2) + 6*(1 + e^3*x^3)*Log[1 + e*x]) + 36*b*n*PolyLog[2, -(e*x)])/(108*
e^3)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(bx^2\log\left(cx^n\right)\log\left(ex+1\right)+ax^2\log\left(ex+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b*x^2*log(c*x^n)*log(e*x + 1) + a*x^2*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x^2\log\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log(e*x + 1), x)
```

maple [C] time = 0.29, size = 870, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)*ln(e*x+1),x)
```

```
[Out] -11/18*a/e^3-1/9*a*x^3+(1/3*b*x^3*ln(e*x+1)+1/18*b*(-2*e^3*x^3+3*e^2*x^2-6*
e*x+6*ln(e*x+1))/e^3)*ln(x^n)-1/9*ln(c)*b*x^3+1/6*a/e*x^2+2/27*b*n*x^3+1/3*
```

```
b*ln(c)*ln(e*x+1)*x^3+71/108/e^3*b*n+1/6*b/e*x^2*ln(c)+1/3/e^3*ln(e*x+1)*b*
ln(c)+1/3/e^3*b*n*dilog(e*x+1)+1/3*x^3*ln(e*x+1)*a-1/3*b/e^2*x*ln(c)-11/18/
e^3*b*ln(c)+1/3*a/e^3*ln(e*x+1)+1/18*I*Pi*b*x^3*csgn(I*c*x^n)^3+11/36*I/e^3
*Pi*b*csgn(I*c*x^n)^3+4/9*b/e^2*n*x-5/36*b/e*n*x^2-1/3*a/e^2*x-1/6*I/e^2*x*
Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/6*I/e^2*x*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)
+1/6*I/e^3*ln(e*x+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/6*I/e^3*ln(e*x+1)*P
i*b*csgn(I*c*x^n)^2*csgn(I*c)+1/6*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*x
+1)*x^3+1/6*I/e^2*x*Pi*b*csgn(I*c*x^n)^3-1/6*I/e^3*ln(e*x+1)*Pi*b*csgn(I*c*
x^n)^3-1/18*I*Pi*b*x^3*csgn(I*x^n)*csgn(I*c*x^n)^2-1/18*I*Pi*b*x^3*csgn(I*c
*x^n)^2*csgn(I*c)-1/12*I/e*x^2*Pi*b*csgn(I*c*x^n)^3-1/6*I*Pi*b*csgn(I*c*x^n
)^3*ln(e*x+1)*x^3+1/6*I/e^2*x*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/6*
I/e^3*ln(e*x+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/6*I*Pi*b*csgn(I*
x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(e*x+1)*x^3-1/12*I/e*x^2*Pi*b*csgn(I*x^n)*cs
gn(I*c*x^n)*csgn(I*c)-11/36*I/e^3*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-11/36*I/
e^3*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+1/6*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(
e*x+1)*x^3+1/18*I*Pi*b*x^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/12*I/e*x^2
*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/12*I/e*x^2*Pi*b*csgn(I*c*x^n)^2*csgn(I*
c)+11/36*I/e^3*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/9*b*n*ln(e*x+1)/e
^3-1/9*b*n*x^3*ln(e*x+1)
```

maxima [A] time = 1.35, size = 220, normalized size = 1.24

$$\frac{\left(\log\left(ex+1\right)\log(x)+\mathrm{Li}_2\left(-ex\right)\right)bn}{3\,e^3}-\frac{\left(b\left(n-3\,\log(c)\right)-3\,a\right)\log\left(ex+1\right)}{9\,e^3}-\frac{4\left(3\,ae^3-\left(2\,e^3n-3\,e^3\log(c)\right)b\right)x^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

```
[Out] 1/3*(log(e*x + 1)*log(x) + dilog(-e*x))*b*n/e^3 - 1/9*(b*(n - 3*log(c)) - 3
*a)*log(e*x + 1)/e^3 - 1/108*(4*(3*a*e^3 - (2*e^3*n - 3*e^3*log(c))*b)*x^3
- 3*(6*a*e^2 - (5*e^2*n - 6*e^2*log(c))*b)*x^2 - 12*((4*e*n - 3*e*log(c))*b
 - 3*a*e)*x - 12*((3*a*e^3 - (e^3*n - 3*e^3*log(c))*b)*x^3 - 3*b*n*log(x))*
log(e*x + 1) + 6*(2*b*e^3*x^3 - 3*b*e^2*x^2 + 6*b*e*x - 6*(b*e^3*x^3 + b)*l
og(e*x + 1))*log(x^n))/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(e*x + 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*log(e*x + 1)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.4 $\int x\left(a+b\log\left(cx^n\right)\right)\log(1+ex)\,dx$

Optimal. Leaf size=146

$$-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{x\left(a+b\log\left(cx^n\right)\right)}{2e}+\frac{1}{2}x^2\log(ex+1)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right)-\frac{bn\text{Li}_2}{2}$$

```
[Out] -3/4*b*n*x/e+1/4*b*n*x^2+1/2*x*(a+b*ln(c*x^n))/e-1/4*x^2*(a+b*ln(c*x^n))+1/
4*b*n*ln(e*x+1)/e^2-1/4*b*n*x^2*ln(e*x+1)-1/2*(a+b*ln(c*x^n))*ln(e*x+1)/e^2
+1/2*x^2*(a+b*ln(c*x^n))*ln(e*x+1)-1/2*b*n*polylog(2,-e*x)/e^2
```

Rubi [A] time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2395, 43, 2376, 2391}

$$-\frac{bn\text{PolyLog}(2,-ex)}{2e^2}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{1}{2}x^2\log(ex+1)\left(a+b\log\left(cx^n\right)\right)+\frac{x\left(a+b\log\left(cx^n\right)\right)}{2e}-\frac{1}{4}x^2\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (-3*b*n*x)/(4*e) + (b*n*x^2)/4 + (x*(a + b*Log[c*x^n]))/(2*e) - (x^2*(a + b
*Log[c*x^n]))/4 + (b*n*Log[1 + e*x])/(4*e^2) - (b*n*x^2*Log[1 + e*x])/4 - (
(a + b*Log[c*x^n])*Log[1 + e*x])/(2*e^2) + (x^2*(a + b*Log[c*x^n])*Log[1 +
e*x])/2 - (b*n*PolyLog[2, -(e*x)])/(2*e^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)\log(1+ex)\,dx = \frac{x\left(a+b\log\left(cx^n\right)\right)}{2e} - \frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right) - \frac{\left(a+b\log\left(cx^n\right)\right)\log(1+e}{2e^2}$$

$$= -\frac{bnx}{2e} + \frac{1}{8}bnx^2 + \frac{x\left(a+b\log\left(cx^n\right)\right)}{2e} - \frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right) - \frac{\left(a+b\log\right.}{}$$

$$= -\frac{bnx}{2e} + \frac{1}{8}bnx^2 + \frac{x\left(a+b\log\left(cx^n\right)\right)}{2e} - \frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right) - \frac{1}{4}bnx^2\log$$

$$= -\frac{bnx}{2e} + \frac{1}{8}bnx^2 + \frac{x\left(a+b\log\left(cx^n\right)\right)}{2e} - \frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right) - \frac{1}{4}bnx^2\log$$

$$= -\frac{3bnx}{4e} + \frac{1}{4}bnx^2 + \frac{x\left(a+b\log\left(cx^n\right)\right)}{2e} - \frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right) + \frac{bn\log(}{4}$$

Mathematica [A] time = 0.07, size = 131, normalized size = 0.90

$$\frac{-ae^2x^2 + 2ae^2x^2\log(ex+1) + 2aex - 2a\log(ex+1) + b\left(2\left(e^2x^2-1\right)\log(ex+1) + ex(2-ex)\right)\log\left(cx^n\right) + be^2}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (2*a*e*x - 3*b*e*n*x - a*e^2*x^2 + b*e^2*n*x^2 - 2*a*Log[1 + e*x] + b*n*Log
[1 + e*x] + 2*a*e^2*x^2*Log[1 + e*x] - b*e^2*n*x^2*Log[1 + e*x] + b*Log[c*x
^n]*(e*x*(2 - e*x) + 2*(-1 + e^2*x^2)*Log[1 + e*x]) - 2*b*n*PolyLog[2, -(e*
x)])/(4*e^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(bx\log\left(cx^n\right)\log\left(ex+1\right) + ax\log\left(ex+1\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b*x*log(c*x^n)*log(e*x + 1) + a*x*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right) + a\right)x\log\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*log(e*x + 1), x)
```

maple [C] time = 0.29, size = 725, normalized size = 4.97

$$\frac{3a}{4e^2} - \frac{a\,x^2}{4} + \left(\frac{b\,x^2\ln\left(ex+1\right)}{2} - \frac{\left(e^2x^2 - 2ex + 2\ln\left(ex+1\right)\right)b}{4e^2}\right)\ln\left(x^n\right) - \frac{bn\,\text{dilog}\left(ex+1\right)}{2e^2} - \frac{bn}{e^2} - \frac{b\,x^2\ln(c)}{4} + \frac{bn\,x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)*ln(e*x+1),x)
```

```
[Out] 3/4*a/e^2-1/4*a*x^2+(1/2*b*x^2*ln(e*x+1)-1/4*b*(e^2*x^2-2*e*x+2*ln(e*x+1))/
e^2)*ln(x^n)-1/e^2*b*n-1/4*ln(c)*b*x^2+1/4*b*n*x^2+1/2*b/e*x*ln(c)-1/2/e^2*
```

```
ln(e*x+1)*b*ln(c)-1/2/e^2*b*n*dilog(e*x+1)+3/4/e^2*b*ln(c)-1/2*a/e^2*ln(e*x
+1)+1/2*x^2*ln(e*x+1)*a+1/2*b*ln(c)*ln(e*x+1)*x^2+1/4*I/e*x*Pi*b*csgn(I*x^n
)*csgn(I*c*x^n)^2+1/4*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(e*x+1)*x^2+1/4*I/
e^2*ln(e*x+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*I/e*x*Pi*b*csgn(
I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3/8*I/e^2*Pi*b*csgn(I*c*x^n)^3+1/8*I*Pi*b*x^
2*csgn(I*c*x^n)^3+1/8*I*Pi*b*x^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/4*I*
Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*x+1)*x^2+1/4*I/e*x*Pi*b*csgn(I*c*x^n)
^2*csgn(I*c)-1/4*I/e^2*ln(e*x+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+3/8*I/e^2
*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I*Pi*b*x^2*csgn(I*x^n)*csgn(I*c*x^n)^
2-1/4*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(e*x+1)*x^2+1/4*I/e^2*ln
(e*x+1)*Pi*b*csgn(I*c*x^n)^3-1/4*I/e*x*Pi*b*csgn(I*c*x^n)^3-1/4*I*Pi*b*csgn
(I*c*x^n)^3*ln(e*x+1)*x^2+3/8*I/e^2*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/8*I*Pi
*b*x^2*csgn(I*c*x^n)^2*csgn(I*c)-1/4*I/e^2*ln(e*x+1)*Pi*b*csgn(I*c*x^n)^2*c
sgn(I*c)-3/8*I/e^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/2*a/e*x-3/4*b
/e*n*x+1/4*b*n*ln(e*x+1)/e^2-1/4*b*n*x^2*ln(e*x+1)
```

maxima [A] time = 1.40, size = 178, normalized size = 1.22

$$-\frac{\left(\log\left(ex+1\right)\log(x)+\mathrm{Li}_2\left(-ex\right)\right)bn}{2\,e^2}+\frac{\left(b\left(n-2\,\log(c)\right)-2\,a\right)\log\left(ex+1\right)}{4\,e^2}-\frac{\left(ae^2-\left(e^2n-e^2\log(c)\right)b\right)x^2+\left(\left(3\,en\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*(log(e*x + 1)*log(x) + dilog(-e*x))*b*n/e^2 + 1/4*(b*(n - 2*log(c)) -
2*a)*log(e*x + 1)/e^2 - 1/4*((a*e^2 - (e^2*n - e^2*log(c))*b)*x^2 + ((3*e*n
 - 2*e*log(c))*b - 2*a*e)*x - ((2*a*e^2 - (e^2*n - 2*e^2*log(c))*b)*x^2 + 2
*b*n*log(x))*log(e*x + 1) + (b*e^2*x^2 - 2*b*e*x - 2*(b*e^2*x^2 - b)*log(e*
x + 1))*log(x^n))/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(e*x + 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*log(e*x + 1)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.5 $\int \left(a+b\log\left(cx^n\right)\right)\log(1+ex)\,dx$

Optimal. Leaf size=74

$$\frac{(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{e}-x\left(a+b\log\left(cx^n\right)\right)+\frac{bn\text{Li}_2(-ex)}{e}-\frac{bn(ex+1)\log(ex+1)}{e}+2bnx$$

```
[Out] 2*b*n*x-x*(a+b*ln(c*x^n))-b*n*(e*x+1)*ln(e*x+1)/e+(e*x+1)*(a+b*ln(c*x^n))*l
n(e*x+1)/e+b*n*polylog(2,-e*x)/e
```

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2389, 2295, 2370, 2411, 43, 2351, 2315}

$$\frac{bn\text{PolyLog}(2,-ex)}{e}+\frac{(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{e}-x\left(a+b\log\left(cx^n\right)\right)-\frac{bn(ex+1)\log(ex+1)}{e}+2bnx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] 2*b*n*x - x*(a + b*Log[c*x^n]) - (b*n*(1 + e*x)*Log[1 + e*x])/e + ((1 + e*x
)*(a + b*Log[c*x^n])*Log[1 + e*x])/e + (b*n*PolyLog[2, -(e*x)])/e
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\int \left(a+b\log \left(cx^n\right)\right)\log(1+ex)\,dx = -x\left(a+b\log \left(cx^n\right)\right)+\frac{(1+ex)\left(a+b\log \left(cx^n\right)\right)\log(1+ex)}{e}-(bn)\int \left(-1+\right.$$

$$= bnx-x\left(a+b\log \left(cx^n\right)\right)+\frac{(1+ex)\left(a+b\log \left(cx^n\right)\right)\log(1+ex)}{e}-\frac{(bn)\int \frac{(1}{}}{}$$

$$= bnx-x\left(a+b\log \left(cx^n\right)\right)+\frac{(1+ex)\left(a+b\log \left(cx^n\right)\right)\log(1+ex)}{e}-\frac{(bn)\,\mathrm{Sub}}{}$$

$$= bnx-x\left(a+b\log \left(cx^n\right)\right)+\frac{(1+ex)\left(a+b\log \left(cx^n\right)\right)\log(1+ex)}{e}-\frac{(bn)\,\mathrm{Sub}}{}$$

$$= bnx-x\left(a+b\log \left(cx^n\right)\right)+\frac{(1+ex)\left(a+b\log \left(cx^n\right)\right)\log(1+ex)}{e}-\frac{(bn)\,\mathrm{Sub}}{}$$

$$= 2bnx-x\left(a+b\log \left(cx^n\right)\right)-\frac{bn(1+ex)\log(1+ex)}{e}+\frac{(1+ex)\left(a+b\log (cx\right.}{e}$$

Mathematica [A] time = 0.03, size = 90, normalized size = 1.22

$$\frac{-aex+aex\log(ex+1)+a\log(ex+1)+b((ex+1)\log(ex+1)-ex)\log \left(cx^n\right)+bn\mathrm{Li}_2(-ex)+2benx-benx\log(}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (-(a*e*x) + 2*b*e*n*x + a*Log[1 + e*x] - b*n*Log[1 + e*x] + a*e*x*Log[1 + e
*x] - b*e*n*x*Log[1 + e*x] + b*Log[c*x^n]*(-(e*x) + (1 + e*x)*Log[1 + e*x])
 + b*n*PolyLog[2, -(e*x)])/e
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b\log \left(cx^n\right)\log \left(ex+1\right)+a\log \left(ex+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b*log(c*x^n)*log(e*x + 1) + a*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log \left(cx^n\right)+a\right)\log \left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log(e*x + 1), x)
```

maple [C] time = 0.25, size = 557, normalized size = 7.53

$$-\frac{bn\ln(ex+1)}{e}-\frac{a}{e}-ax+\left(bx\ln(ex+1)+\frac{(-ex+\ln(ex+1))\,b}{e}\right)\ln(x^n)+\frac{2bn}{e}-bx\ln(c)+\frac{bn\,\mathrm{dilog}(ex+1)}{e}+2bn$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(e*x+1),x)
```

```
[Out] -1/e*b*n*ln(e*x+1)-a/e-a*x+(b*x*ln(e*x+1)+b*(-e*x+ln(e*x+1))/e)*ln(x^n)+2/e
*b*n-1/2*I/e*ln(e*x+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-b*x*ln(c)+1
/2*I/e*Pi*b*csgn(I*c*x^n)^3+2*b*n*x+a/e*ln(e*x+1)-1/e*b*ln(c)+1/e*b*n*dilog
(e*x+1)+1/e*ln(e*x+1)*b*ln(c)+ln(e*x+1)*ln(c)*x*b-n*b*x*ln(e*x+1)+x*ln(e*x+
1)*a-1/2*I*ln(e*x+1)*Pi*x*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/2*I*Pi*b*
csgn(I*c*x^n)^3*x+1/2*I/e*ln(e*x+1)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+1/2*I/e*
ln(e*x+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*
x^n)*csgn(I*c)*x+1/2*I/e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/2*I*ln(
e*x+1)*Pi*x*b*csgn(I*c*x^n)^2*csgn(I*c)-1/2*I*ln(e*x+1)*Pi*x*b*csgn(I*c*x^n
)^3+1/2*I*ln(e*x+1)*Pi*x*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*csgn(I*c*
x^n)^2*csgn(I*c)*x-1/2*I/e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I/e*Pi*b*cs
gn(I*c*x^n)^2*csgn(I*c)-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*x-1/2*I/e*ln
(e*x+1)*Pi*b*csgn(I*c*x^n)^3
```

maxima [A] time = 1.25, size = 126, normalized size = 1.70

$$\frac{\left(\log(ex+1)\log(x)+\mathrm{Li}_2(-ex)\right)bn}{e}-\frac{\left(b\left(n-\log(c)\right)-a\right)\log(ex+1)}{e}+\frac{\left(\left(2\,en-e\log(c)\right)b-ae\right)x-\left(bn\log(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

```
[Out] (log(e*x + 1)*log(x) + dilog(-e*x))*b*n/e - (b*(n - log(c)) - a)*log(e*x +
1)/e + (((2*e*n - e*log(c))*b - a*e)*x - (b*n*log(x) + ((e*n - e*log(c))*b
- a*e)*x)*log(e*x + 1) - (b*e*x - (b*e*x + b)*log(e*x + 1))*log(x^n))/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*x + 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(e*x + 1)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.6 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx$

Optimal. Leaf size=28

$$bn\text{Li}_3(-ex) - \text{Li}_2(-ex)\left(a + b\log\left(cx^n\right)\right)$$

```
[Out] -(a+b*ln(c*x^n))*polylog(2,-e*x)+b*n*polylog(3,-e*x)
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2374, 6589}

$$bn\text{PolyLog}(3,-ex) - \text{PolyLog}(2,-ex)\left(a + b\log\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])*PolyLog[2, -(e*x)]) + b*n*PolyLog[3, -(e*x)]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x}\,dx &= -\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(-ex)+(bn)\int \frac{\text{Li}_2(-ex)}{x}\,dx\\ &= -\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(-ex)+bn\text{Li}_3(-ex)\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.21

$$-a\text{Li}_2(-ex) - b\text{Li}_2(-ex)\log\left(cx^n\right) + bn\text{Li}_3(-ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x,x]
```

```
[Out] -(a*PolyLog[2, -(e*x)]) - b*Log[c*x^n]*PolyLog[2, -(e*x)] + b*n*PolyLog[3,
-(e*x)]
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(cx^n\right)\log\left(ex+1\right)+a\log\left(ex+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n)*log(e*x + 1) + a*log(e*x + 1))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right) \log\left(ex + 1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log(e*x + 1)/x, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\, x^{n}\right) + a\right) \ln\left(ex + 1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(e*x+1)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(e*x+1)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right) \log\left(ex + 1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)*log(e*x + 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(e\, x + 1\right)\, \left(a + b\, \ln\left(c\, x^{n}\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(e*x+1)/x,x)
```

```
[Out] Timed out
```

3.7 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx$

Optimal. Leaf size=107

$$e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\log(ex+1)\left(a+b\log\left(cx^n\right)\right)-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{x}-ben\mathrm{Li}_2(-ex)-\frac{1}{2}ben\log^2(x)$$

[Out] b*e*n*ln(x)-1/2*b*e*n*ln(x)^2+e*ln(x)*(a+b*ln(c*x^n))-b*e*n*ln(e*x+1)-b*n*ln(e*x+1)/x-e*(a+b*ln(c*x^n))*ln(e*x+1)-(a+b*ln(c*x^n))*ln(e*x+1)/x-b*e*n*polylog(2,-e*x)

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2395, 36, 29, 31, 2376, 2301, 2391}

$$-ben\text{PolyLog}(2,-ex)+e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\log(ex+1)\left(a+b\log\left(cx^n\right)\right)-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{x}-\frac{1}{2}b\ldots$$

Antiderivative was successfully verified.

[In] Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x^2,x]

[Out] b*e*n*Log[x] - (b*e*n*Log[x]^2)/2 + e*Log[x]*(a + b*Log[c*x^n]) - b*e*n*Log[1 + e*x] - (b*n*Log[1 + e*x])/x - e*(a + b*Log[c*x^n])*Log[1 + e*x] - ((a + b*Log[c*x^n])*Log[1 + e*x])/x - b*e*n*PolyLog[2, -(e*x)]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x^2}\,dx = e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\left(a+b\log\left(cx^n\right)\right)\log(1+ex)-\frac{\left(a+b\log(c\right.}{}$$

$$= e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\left(a+b\log\left(cx^n\right)\right)\log(1+ex)-\frac{\left(a+b\log(c\right.}{}$$

$$= -\frac{1}{2}ben\log^2(x)+e\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{bn\log(1+ex)}{x}-e\left(a+b\log\right.$$

$$= -\frac{1}{2}ben\log^2(x)+e\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{bn\log(1+ex)}{x}-e\left(a+b\log\right.$$

$$= ben\log(x)-\frac{1}{2}ben\log^2(x)+e\log(x)\left(a+b\log\left(cx^n\right)\right)-ben\log(1+ex)$$

Mathematica [A] time = 0.06, size = 69, normalized size = 0.64

$$e\log(x)\left(a+b\log\left(cx^n\right)+bn\right)-\frac{(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)+bn\right)}{x}-ben\text{Li}_2(-ex)-\frac{1}{2}ben\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x^2,x]
```

```
[Out] -1/2*(b*e*n*Log[x]^2) + e*Log[x]*(a + b*n + b*Log[c*x^n]) - ((1 + e*x)*(a +
 b*n + b*Log[c*x^n])*Log[1 + e*x])/x - b*e*n*PolyLog[2, -(e*x)]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(cx^n\right)\log\left(ex+1\right)+a\log\left(ex+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n)*log(e*x + 1) + a*log(e*x + 1))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(ex+1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log(e*x + 1)/x^2, x)
```

maple [C] time = 0.24, size = 481, normalized size = 4.50

$$-\frac{i\pi be\,\text{csgn}\left(ic\right)\text{csgn}\left(ix^n\right)\text{csgn}\left(ic\,x^n\right)\ln\left(ex\right)}{2}+\frac{i\pi be\,\text{csgn}\left(ic\right)\text{csgn}\left(ix^n\right)\text{csgn}\left(ic\,x^n\right)\ln\left(ex+1\right)}{2}+\frac{i\pi be\,\text{csgn}\left(ic\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(e*x+1)/x^2,x)
```

```
[Out] (-b/x*ln(e*x+1)+b*e*ln(x)-b*e*ln(e*x+1))*ln(x^n)-1/2*b*e*n*ln(x)^2-e*b*n*di
log(e*x+1)+n*b*e*ln(e*x)-b*e*n*ln(e*x+1)-b*n*ln(e*x+1)/x+1/2*I*e*Pi*b*csgn(
I*x^n)*csgn(I*c*x^n)^2*ln(e*x)-1/2*I*e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(
I*c)*ln(e*x)-1/2*I*e*Pi*b*csgn(I*c*x^n)^3*ln(e*x)-1/2*I*e*Pi*b*csgn(I*x^n)*
csgn(I*c*x^n)^2*ln(e*x+1)-1/2*I*e*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(e*x+1)+
1/2*I*e*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(e*x)+1/2*I*Pi*b*csgn(I*c*x^n)^3*l
n(e*x+1)/x-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*x+1)/x-1/2*I*Pi*b*cs
gn(I*c*x^n)^2*csgn(I*c)*ln(e*x+1)/x+1/2*I*e*Pi*b*csgn(I*c*x^n)^3*ln(e*x+1)+
1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(e*x+1)/x+1/2*I*e*Pi*b*csg
n(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(e*x+1)+e*b*ln(c)*ln(e*x)-e*b*ln(c)*ln(e
*x+1)-b*ln(c)*ln(e*x+1)/x+a*e*ln(e*x)-a*e*ln(e*x+1)-ln(e*x+1)/x*a
```

maxima [A] time = 1.40, size = 128, normalized size = 1.20

$$-\left(\log\left(ex+1\right)\log(x)+\mathrm{Li}_2\left(-ex\right)\right)ben-\left(\left(en+e\log(c)\right)b+ae\right)\log\left(ex+1\right)+\left(\left(en+e\log(c)\right)b+ae\right)\log(x)-\frac{benx\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -(log(e*x + 1)*log(x) + dilog(-e*x))*b*e*n - ((e*n + e*log(c))*b + a*e)*log
(e*x + 1) + ((e*n + e*log(c))*b + a*e)*log(x) - 1/2*(b*e*n*x*log(x)^2 - 2*(
b*e*n*x*log(x) - b*(n + log(c)) - a)*log(e*x + 1) - 2*(b*e*x*log(x) - (b*e*
x + b)*log(e*x + 1))*log(x^n))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(e*x+1)/x**2,x)
```

```
[Out] Timed out
```

3.8 $\int \frac{\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x^3}\,dx$

Optimal. Leaf size=163

$$-\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}e^2\log(ex+1)\left(a+b\log\left(cx^n\right)\right)-\frac{e\left(a+b\log\left(cx^n\right)\right)}{2x}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{2x^2}$$

```
[Out] -3/4*b*e*n/x-1/4*b*e^2*n*ln(x)+1/4*b*e^2*n*ln(x)^2-1/2*e*(a+b*ln(c*x^n))/x-
1/2*e^2*ln(x)*(a+b*ln(c*x^n))+1/4*b*e^2*n*ln(e*x+1)-1/4*b*n*ln(e*x+1)/x^2+1
/2*e^2*(a+b*ln(c*x^n))*ln(e*x+1)-1/2*(a+b*ln(c*x^n))*ln(e*x+1)/x^2+1/2*b*e^
2*n*polylog(2,-e*x)
```

Rubi [A] time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2395, 44, 2376, 2301, 2391}

$$\frac{1}{2}be^2n\text{PolyLog}(2,-ex)-\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}e^2\log(ex+1)\left(a+b\log\left(cx^n\right)\right)-\frac{e\left(a+b\log\left(cx^n\right)\right)}{2x}-\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x^3,x]
```

```
[Out] (-3*b*e*n)/(4*x) - (b*e^2*n*Log[x])/4 + (b*e^2*n*Log[x]^2)/4 - (e*(a + b*Lo
g[c*x^n]))/(2*x) - (e^2*Log[x]*(a + b*Log[c*x^n]))/2 + (b*e^2*n*Log[1 + e*x
])/4 - (b*n*Log[1 + e*x])/(4*x^2) + (e^2*(a + b*Log[c*x^n])*Log[1 + e*x])/2
 - ((a + b*Log[c*x^n])*Log[1 + e*x])/(2*x^2) + (b*e^2*n*PolyLog[2, -(e*x)])
/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x^3}\,dx = -\frac{e\left(a+b\log\left(cx^n\right)\right)}{2x}-\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}e^2\left(a+b\log\left(cx^n\right)\right)\mathrm{lo}$$

$$=-\frac{ben}{2x}-\frac{e\left(a+b\log\left(cx^n\right)\right)}{2x}-\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}e^2\left(a+b\log\left(c\right.\right.$$

$$=-\frac{ben}{2x}+\frac{1}{4}be^2n\log^2(x)-\frac{e\left(a+b\log\left(cx^n\right)\right)}{2x}-\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-$$

$$=-\frac{ben}{2x}+\frac{1}{4}be^2n\log^2(x)-\frac{e\left(a+b\log\left(cx^n\right)\right)}{2x}-\frac{1}{2}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-$$

$$=-\frac{3ben}{4x}-\frac{1}{4}be^2n\log(x)+\frac{1}{4}be^2n\log^2(x)-\frac{e\left(a+b\log\left(cx^n\right)\right)}{2x}-\frac{1}{2}e^2\log(x)\left(a\right.$$

Mathematica [A] time = 0.07, size = 215, normalized size = 1.32

$$-\frac{a\log(ex+1)}{2x^2}+\frac{1}{2}ae\left(-e\log(x)+e\log(ex+1)-\frac{1}{x}\right)-\frac{1}{4}be^2\log(x)\left(2\left(\log\left(cx^n\right)-n\log(x)\right)+n\right)+\frac{1}{4}be^2\log(ex+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x^3,x]
```

```
[Out] -1/4*(b*e^2*Log[x]*(n + 2*(-(n*Log[x]) + Log[c*x^n]))) + (b*(-(e*n) - 2*e*(
-(n*Log[x]) + Log[c*x^n])))/(4*x) - (a*Log[1 + e*x])/(2*x^2) + (b*e^2*(n +
2*(-(n*Log[x]) + Log[c*x^n]))*Log[1 + e*x])/4 - (b*(n + 2*n*Log[x] + 2*(-(n
*Log[x]) + Log[c*x^n]))*Log[1 + e*x])/(4*x^2) + (a*e*(-x^(-1) - e*Log[x] +
e*Log[1 + e*x]))/2 + (b*e*n*(-x^(-1) - Log[x]/x - (e*Log[x]^2)/2 + e^2*((Lo
g[x]*Log[1 + e*x])/e + PolyLog[2, -(e*x)]/e)))/2
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\log\left(cx^n\right)\log\left(ex+1\right)+a\log\left(ex+1\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n)*log(e*x + 1) + a*log(e*x + 1))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(ex+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log(e*x + 1)/x^3, x)
```

maple [C] time = 0.25, size = 647, normalized size = 3.97

$$\left(\frac{\left(-ex\ln(x)+ex\ln\left(ex+1\right)-1\right)be}{2x}-\frac{b\ln\left(ex+1\right)}{2x^2}\right)\ln\left(x^n\right)+\frac{b\,e^2n\,\mathrm{dilog}\left(ex+1\right)}{2}-\frac{b\ln(c)\ln\left(ex+1\right)}{2x^2}-\frac{a\ln\left(ex+1\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(e*x+1)/x^3,x)
```

```
[Out] (-1/2*b/x^2*ln(e*x+1)+1/2*b*e*(e*ln(e*x+1)*x-e*x*ln(x)-1)/x)*ln(x^n)-1/2*b*
ln(c)*ln(e*x+1)/x^2+1/2*b*e^2*n*dilog(e*x+1)-1/2*ln(e*x+1)/x^2*a-1/2*a*e^2*
ln(e*x)-1/2*a*e/x+1/2*a*e^2*ln(e*x+1)-1/4*n*b*e^2*ln(e*x)-1/4*I*e*Pi*b*csgn
(I*x^n)*csgn(I*c*x^n)^2/x-1/4*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(e*x+1)/x^
2+1/4*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*e^2*ln(e*x+1)-1/4*I*e*Pi*b*csgn(I*c*
x^n)^2*csgn(I*c)/x+1/4*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*e^2*ln(e*x+1)-1/4
*I*e^2*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(e*x)-1/4*I*Pi*b*csgn(I*x^n)*csgn(I
*c*x^n)^2*ln(e*x+1)/x^2-1/4*I*e^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*x)+
1/4*I*e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)/x-1/4*I*Pi*b*csgn(I*x^n)*c
sgn(I*c*x^n)*csgn(I*c)*e^2*ln(e*x+1)+1/4*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*c
sgn(I*c)*ln(e*x+1)/x^2+1/4*I*e^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*l
n(e*x)+1/4*I*e*Pi*b*csgn(I*c*x^n)^3/x-1/4*I*Pi*b*csgn(I*c*x^n)^3*e^2*ln(e*x
+1)+1/4*I*e^2*Pi*b*csgn(I*c*x^n)^3*ln(e*x)+1/4*I*Pi*b*csgn(I*c*x^n)^3*ln(e*
x+1)/x^2-1/2*e*b*ln(c)/x+1/4*b*e^2*n*ln(x)^2-1/2*e^2*b*ln(c)*ln(e*x)-3/4*b*
e*n/x+1/4*b*e^2*n*ln(e*x+1)-1/4*b*n*ln(e*x+1)/x^2+1/2*b*ln(c)*e^2*ln(e*x+1)
```

maxima [A] time = 1.37, size = 194, normalized size = 1.19

$$\frac{1}{2}\left(\log\left(ex+1\right)\log(x)+\mathrm{Li}_2\left(-ex\right)\right)be^2n+\frac{1}{4}\left(2\,ae^2+\left(e^2n+2\,e^2\log(c)\right)b\right)\log\left(ex+1\right)+\frac{be^2nx^2\log(x)^2-\left(2\,ae^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(log(e*x + 1)*log(x) + dilog(-e*x))*b*e^2*n + 1/4*(2*a*e^2 + (e^2*n + 2
*e^2*log(c))*b)*log(e*x + 1) + 1/4*(b*e^2*n*x^2*log(x)^2 - (2*a*e^2 + (e^2*
n + 2*e^2*log(c))*b)*x^2*log(x) - ((3*e*n + 2*e*log(c))*b + 2*a*e)*x - (2*b
*e^2*n*x^2*log(x) + b*(n + 2*log(c)) + 2*a)*log(e*x + 1) - 2*(b*e^2*x^2*log
(x) + b*e*x - (b*e^2*x^2 - b)*log(e*x + 1))*log(x^n))/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(e*x+1)/x**3,x)
```

```
[Out] Timed out
```

3.9 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx$

Optimal. Leaf size=195

$$\frac{1}{3}e^3\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{3}e^3\log(ex+1)\left(a+b\log\left(cx^n\right)\right)+\frac{e^2\left(a+b\log\left(cx^n\right)\right)}{3x}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{3x^3}-$$

```
[Out] -5/36*b*e*n/x^2+4/9*b*e^2*n/x+1/9*b*e^3*n*ln(x)-1/6*b*e^3*n*ln(x)^2-1/6*e*(
a+b*ln(c*x^n))/x^2+1/3*e^2*(a+b*ln(c*x^n))/x+1/3*e^3*ln(x)*(a+b*ln(c*x^n))-
1/9*b*e^3*n*ln(e*x+1)-1/9*b*n*ln(e*x+1)/x^3-1/3*e^3*(a+b*ln(c*x^n))*ln(e*x+
1)-1/3*(a+b*ln(c*x^n))*ln(e*x+1)/x^3-1/3*b*e^3*n*polylog(2,-e*x)
```

Rubi [A] time = 0.11, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2395, 44, 2376, 2301, 2391}

$$-\frac{1}{3}be^3n\text{PolyLog}(2,-ex)+\frac{1}{3}e^3\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{3}e^3\log(ex+1)\left(a+b\log\left(cx^n\right)\right)+\frac{e^2\left(a+b\log\left(cx^n\right)\right)}{3x}-\frac{e\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x^4,x]
```

```
[Out] (-5*b*e*n)/(36*x^2) + (4*b*e^2*n)/(9*x) + (b*e^3*n*Log[x])/9 - (b*e^3*n*Log
[x]^2)/6 - (e*(a + b*Log[c*x^n]))/(6*x^2) + (e^2*(a + b*Log[c*x^n]))/(3*x)
+ (e^3*Log[x]*(a + b*Log[c*x^n]))/3 - (b*e^3*n*Log[1 + e*x])/9 - (b*n*Log[1
 + e*x])/(9*x^3) - (e^3*(a + b*Log[c*x^n])*Log[1 + e*x])/3 - ((a + b*Log[c*
x^n])*Log[1 + e*x])/(3*x^3) - (b*e^3*n*PolyLog[2, -(e*x)])/3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x^4}\,dx = -\frac{e\left(a+b\log\left(cx^n\right)\right)}{6x^2}+\frac{e^2\left(a+b\log\left(cx^n\right)\right)}{3x}+\frac{1}{3}e^3\log(x)\left(a+b\log\left(cx^n\right)\right)$$
$$= -\frac{ben}{12x^2}+\frac{be^2n}{3x}-\frac{e\left(a+b\log\left(cx^n\right)\right)}{6x^2}+\frac{e^2\left(a+b\log\left(cx^n\right)\right)}{3x}+\frac{1}{3}e^3\log(x)\left(\right.$$
$$= -\frac{ben}{12x^2}+\frac{be^2n}{3x}-\frac{1}{6}be^3n\log^2(x)-\frac{e\left(a+b\log\left(cx^n\right)\right)}{6x^2}+\frac{e^2\left(a+b\log\left(cx^n\right.\right.}{3x}$$
$$= -\frac{ben}{12x^2}+\frac{be^2n}{3x}-\frac{1}{6}be^3n\log^2(x)-\frac{e\left(a+b\log\left(cx^n\right)\right)}{6x^2}+\frac{e^2\left(a+b\log\left(cx^n\right.\right.}{3x}$$
$$= -\frac{5ben}{36x^2}+\frac{4be^2n}{9x}+\frac{1}{9}be^3n\log(x)-\frac{1}{6}be^3n\log^2(x)-\frac{e\left(a+b\log\left(cx^n\right)\right)}{6x^2}+$$

Mathematica [A] time = 0.08, size = 206, normalized size = 1.06

$$-\frac{-4e^3x^3\log(x)\left(3a+3b\log\left(cx^n\right)+bn\right)+12ae^3x^3\log(ex+1)-12ae^2x^2+6aex+12a\log(ex+1)+12be^3x^3\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x^4,x]
```

```
[Out] -1/36*(6*a*e*x + 5*b*e*n*x - 12*a*e^2*x^2 - 16*b*e^2*n*x^2 + 6*b*e^3*n*x^3*
Log[x]^2 + 6*b*e*x*Log[c*x^n] - 12*b*e^2*x^2*Log[c*x^n] - 4*e^3*x^3*Log[x]*
(3*a + b*n + 3*b*Log[c*x^n]) + 12*a*Log[1 + e*x] + 4*b*n*Log[1 + e*x] + 12*
a*e^3*x^3*Log[1 + e*x] + 4*b*e^3*n*x^3*Log[1 + e*x] + 12*b*Log[c*x^n]*Log[1
 + e*x] + 12*b*e^3*x^3*Log[c*x^n]*Log[1 + e*x] + 12*b*e^3*n*x^3*PolyLog[2,
-(e*x)])/x^3
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(cx^n\right)\log\left(ex+1\right)+a\log\left(ex+1\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n)*log(e*x + 1) + a*log(e*x + 1))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(ex+1\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log(e*x + 1)/x^4, x)
```

maple [C] time = 0.25, size = 796, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(e*x+1)/x^4,x)
```

```
[Out] -1/6*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*e^2/x+(-1/3*b/x^3*ln(e*x+1)
-1/6*b*e*(2*e^2*ln(e*x+1)*x^2-2*e^2*x^2*ln(x)-2*e*x+1)/x^2)*ln(x^n)+1/3*e^3
*b*ln(c)*ln(e*x)-1/3*b*ln(c)*ln(e*x+1)/x^3+1/3*b*ln(c)*e^2/x-1/6*e*b*ln(c)/
x^2-1/3*b*ln(c)*e^3*ln(e*x+1)+1/9*n*e^3*b*ln(e*x)-1/3*b*e^3*n*dilog(e*x+1)+
1/12*I*e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)/x^2-1/3*ln(e*x+1)/x^3*a+1
/3*a*e^3*ln(e*x)+1/3*a*e^2/x-1/6*a*e/x^2-1/3*a*e^3*ln(e*x+1)+1/6*I*Pi*b*csg
n(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*e^3*ln(e*x+1)-5/36*b*e*n/x^2+1/6*I*Pi*b*cs
gn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(e*x+1)/x^3-1/6*I*Pi*b*csgn(I*x^n)*csgn
(I*c*x^n)^2*ln(e*x+1)/x^3+1/6*I*e^3*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*x
)-1/6*I*e^3*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(e*x)-1/6*I*Pi*b*csg
n(I*c*x^n)^2*csgn(I*c)*e^3*ln(e*x+1)-1/6*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*l
n(e*x+1)/x^3+1/6*I*e^3*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(e*x)-1/12*I*e*Pi*b
*csgn(I*c*x^n)^2*csgn(I*c)/x^2+1/6*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*e^2/x
-1/6*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*e^3*ln(e*x+1)-1/12*I*e*Pi*b*csgn(I*
x^n)*csgn(I*c*x^n)^2/x^2+1/6*I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*e^2/x+1/12*I*
e*Pi*b*csgn(I*c*x^n)^3/x^2-1/6*I*Pi*b*csgn(I*c*x^n)^3*e^2/x+1/6*I*Pi*b*csgn
(I*c*x^n)^3*e^3*ln(e*x+1)+1/6*I*Pi*b*csgn(I*c*x^n)^3*ln(e*x+1)/x^3-1/6*I*e^
3*Pi*b*csgn(I*c*x^n)^3*ln(e*x)-1/6*b*e^3*n*ln(x)^2+4/9*b*e^2*n/x-1/9*b*e^3*
n*ln(e*x+1)-1/9*b*n*ln(e*x+1)/x^3
```

maxima [A] time = 1.18, size = 232, normalized size = 1.19

$$-\frac{1}{3}\left(\log\left(ex+1\right)\log(x)+\mathrm{Li}_2\left(-ex\right)\right)be^3n-\frac{1}{9}\left(3\,ae^3+\left(e^3n+3\,e^3\log(c)\right)b\right)\log\left(ex+1\right)-\frac{6\,be^3nx^3\log(x)^2-4\left(3\,ae^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(log(e*x + 1)*log(x) + dilog(-e*x))*b*e^3*n - 1/9*(3*a*e^3 + (e^3*n +
3*e^3*log(c))*b)*log(e*x + 1) - 1/36*(6*b*e^3*n*x^3*log(x)^2 - 4*(3*a*e^3 +
 (e^3*n + 3*e^3*log(c))*b)*x^3*log(x) - 4*(3*a*e^2 + (4*e^2*n + 3*e^2*log(c
))*b)*x^2 + ((5*e*n + 6*e*log(c))*b + 6*a*e)*x - 4*(3*b*e^3*n*x^3*log(x) -
b*(n + 3*log(c)) - 3*a)*log(e*x + 1) - 6*(2*b*e^3*x^3*log(x) + 2*b*e^2*x^2
- b*e*x - 2*(b*e^3*x^3 + b)*log(e*x + 1))*log(x^n))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(e*x+1)/x**4,x)
```

```
[Out] Timed out
```

3.10 $\int x^3 \left(a+b\log\left(cx^n\right)\right)^2 \log(1+ex)\,dx$

Optimal. Leaf size=456

$$-\frac{bn\text{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^4}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^2}{4e^4}+\frac{bn\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{8e^4}+\frac{x\left(a+b\log\left(c\right.\right.}{4e^3}$$

```
[Out] -1/2*a*b*n*x/e^3+21/32*b^2*n^2*x/e^3-7/64*b^2*n^2*x^2/e^2+37/864*b^2*n^2*x^
3/e-3/128*b^2*n^2*x^4-1/2*b^2*n*x*ln(c*x^n)/e^3-1/8*b*n*x*(a+b*ln(c*x^n))/e
^3+3/16*b*n*x^2*(a+b*ln(c*x^n))/e^2-7/72*b*n*x^3*(a+b*ln(c*x^n))/e+1/16*b*n
*x^4*(a+b*ln(c*x^n))+1/4*x*(a+b*ln(c*x^n))^2/e^3-1/8*x^2*(a+b*ln(c*x^n))^2/
e^2+1/12*x^3*(a+b*ln(c*x^n))^2/e-1/16*x^4*(a+b*ln(c*x^n))^2-1/32*b^2*n^2*ln
(e*x+1)/e^4+1/32*b^2*n^2*x^4*ln(e*x+1)+1/8*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/e^
4-1/8*b*n*x^4*(a+b*ln(c*x^n))*ln(e*x+1)-1/4*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^4
+1/4*x^4*(a+b*ln(c*x^n))^2*ln(e*x+1)+1/8*b^2*n^2*polylog(2,-e*x)/e^4-1/2*b*
n*(a+b*ln(c*x^n))*polylog(2,-e*x)/e^4+1/2*b^2*n^2*polylog(3,-e*x)/e^4
```

Rubi [A] time = 0.33, antiderivative size = 456, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2395, 43, 2377, 2295, 2304, 2374, 6589, 2376, 2391}

$$-\frac{bn\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^4}+\frac{b^2n^2\text{PolyLog}(2,-ex)}{8e^4}+\frac{b^2n^2\text{PolyLog}(3,-ex)}{2e^4}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)^2}{8e^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] -(a*b*n*x)/(2*e^3) + (21*b^2*n^2*x)/(32*e^3) - (7*b^2*n^2*x^2)/(64*e^2) + (
37*b^2*n^2*x^3)/(864*e) - (3*b^2*n^2*x^4)/128 - (b^2*n*x*Log[c*x^n])/(2*e^3
) - (b*n*x*(a + b*Log[c*x^n]))/(8*e^3) + (3*b*n*x^2*(a + b*Log[c*x^n]))/(16
*e^2) - (7*b*n*x^3*(a + b*Log[c*x^n]))/(72*e) + (b*n*x^4*(a + b*Log[c*x^n])
)/16 + (x*(a + b*Log[c*x^n])^2)/(4*e^3) - (x^2*(a + b*Log[c*x^n])^2)/(8*e^2
) + (x^3*(a + b*Log[c*x^n])^2)/(12*e) - (x^4*(a + b*Log[c*x^n])^2)/16 - (b^
2*n^2*Log[1 + e*x])/(32*e^4) + (b^2*n^2*x^4*Log[1 + e*x])/32 + (b*n*(a + b*
Log[c*x^n])*Log[1 + e*x])/(8*e^4) - (b*n*x^4*(a + b*Log[c*x^n])*Log[1 + e*x
])/8 - ((a + b*Log[c*x^n])^2*Log[1 + e*x])/(4*e^4) + (x^4*(a + b*Log[c*x^n]
)^2*Log[1 + e*x])/4 + (b^2*n^2*PolyLog[2, -(e*x)])/(8*e^4) - (b*n*(a + b*Lo
g[c*x^n])*PolyLog[2, -(e*x)])/(2*e^4) + (b^2*n^2*PolyLog[3, -(e*x)])/(2*e^4
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\left(a+b\log\left(cx^n\right)\right)^2\log(1+ex)\,dx=\frac{x\left(a+b\log\left(cx^n\right)\right)^2}{4e^3}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)^2}{8e^2}+\frac{x^3\left(a+b\log\left(cx^n\right)\right)^2}{12e}-$$

$$=\frac{x\left(a+b\log\left(cx^n\right)\right)^2}{4e^3}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)^2}{8e^2}+\frac{x^3\left(a+b\log\left(cx^n\right)\right)^2}{12e}-$$

$$=-\frac{abnx}{2e^3}-\frac{b^2n^2x^2}{16e^2}+\frac{b^2n^2x^3}{54e}-\frac{1}{128}b^2n^2x^4-\frac{bnx\left(a+b\log\left(cx^n\right)\right)}{8e^3}+3b$$

$$=-\frac{abnx}{2e^3}+\frac{5b^2n^2x}{8e^3}-\frac{3b^2n^2x^2}{32e^2}+\frac{7b^2n^2x^3}{216e}-\frac{1}{64}b^2n^2x^4-\frac{b^2nx\log\left(cx^n\right)}{2e^3}$$

$$=-\frac{abnx}{2e^3}+\frac{5b^2n^2x}{8e^3}-\frac{3b^2n^2x^2}{32e^2}+\frac{7b^2n^2x^3}{216e}-\frac{1}{64}b^2n^2x^4-\frac{b^2nx\log\left(cx^n\right)}{2e^3}$$

$$=-\frac{abnx}{2e^3}+\frac{5b^2n^2x}{8e^3}-\frac{3b^2n^2x^2}{32e^2}+\frac{7b^2n^2x^3}{216e}-\frac{1}{64}b^2n^2x^4-\frac{b^2nx\log\left(cx^n\right)}{2e^3}$$

$$=-\frac{abnx}{2e^3}+\frac{21b^2n^2x}{32e^3}-\frac{7b^2n^2x^2}{64e^2}+\frac{37b^2n^2x^3}{864e}-\frac{3}{128}b^2n^2x^4-\frac{b^2nx\log(c}{2e^3}$$

Mathematica [A] time = 0.21, size = 594, normalized size = 1.30

$$-216a^2e^4x^4+864a^2e^4x^4\log(ex+1)+288a^2e^3x^3-432a^2e^2x^2+864a^2ex-864a^2\log(ex+1)-432abe^4x^4\log$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] (864*a^2*e*x - 2160*a*b*e*n*x + 2268*b^2*e*n^2*x - 432*a^2*e^2*x^2 + 648*a*
b*e^2*n*x^2 - 378*b^2*e^2*n^2*x^2 + 288*a^2*e^3*x^3 - 336*a*b*e^3*n*x^3 + 1
48*b^2*e^3*n^2*x^3 - 216*a^2*e^4*x^4 + 216*a*b*e^4*n*x^4 - 81*b^2*e^4*n^2*x
^4 + 1728*a*b*e*x*Log[c*x^n] - 2160*b^2*e*n*x*Log[c*x^n] - 864*a*b*e^2*x^2*
Log[c*x^n] + 648*b^2*e^2*n*x^2*Log[c*x^n] + 576*a*b*e^3*x^3*Log[c*x^n] - 33
6*b^2*e^3*n*x^3*Log[c*x^n] - 432*a*b*e^4*x^4*Log[c*x^n] + 216*b^2*e^4*n*x^4
*Log[c*x^n] + 864*b^2*e*x*Log[c*x^n]^2 - 432*b^2*e^2*x^2*Log[c*x^n]^2 + 288
*b^2*e^3*x^3*Log[c*x^n]^2 - 216*b^2*e^4*x^4*Log[c*x^n]^2 - 864*a^2*Log[1 +
e*x] + 432*a*b*n*Log[1 + e*x] - 108*b^2*n^2*Log[1 + e*x] + 864*a^2*e^4*x^4*
Log[1 + e*x] - 432*a*b*e^4*n*x^4*Log[1 + e*x] + 108*b^2*e^4*n^2*x^4*Log[1 +
 e*x] - 1728*a*b*Log[c*x^n]*Log[1 + e*x] + 432*b^2*n*Log[c*x^n]*Log[1 + e*x
] + 1728*a*b*e^4*x^4*Log[c*x^n]*Log[1 + e*x] - 432*b^2*e^4*n*x^4*Log[c*x^n]
*Log[1 + e*x] - 864*b^2*Log[c*x^n]^2*Log[1 + e*x] + 864*b^2*e^4*x^4*Log[c*x
^n]^2*Log[1 + e*x] + 432*b*n*(-4*a + b*n - 4*b*Log[c*x^n])*PolyLog[2, -(e*x
)] + 1728*b^2*n^2*PolyLog[3, -(e*x)])/(3456*e^4)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^3\log\left(cx^n\right)^2\log\left(ex+1\right)+2\,abx^3\log\left(cx^n\right)\log\left(ex+1\right)+a^2x^3\log\left(ex+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^2*x^3*log(c*x^n)^2*log(e*x + 1) + 2*a*b*x^3*log(c*x^n)*log(e*x +
 1) + a^2*x^3*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^2x^3\log\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^3*log(e*x + 1), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int (b \ln (c x^n) + a)^2 x^3 \ln (ex + 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*x^n)+a)^2*ln(e*x+1),x)
```

```
[Out] int(x^3*(b*ln(c*x^n)+a)^2*ln(e*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3 b^2 e^4 x^4 - 4 b^2 e^3 x^3 + 6 b^2 e^2 x^2 - 12 b^2 ex - 12 \left(b^2 e^4 x^4 - b^2\right) \log (ex + 1)\right) \log (x^n)^2}{48 e^4} + \frac{-\frac{3}{16} b^2 e^4 n^2 x^4 + \frac{3}{4} b^2 e^4 n x^4 \text{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/48*(3*b^2*e^4*x^4 - 4*b^2*e^3*x^3 + 6*b^2*e^2*x^2 - 12*b^2*e*x - 12*(b^2
*e^4*x^4 - b^2)*log(e*x + 1))*log(x^n)^2/e^4 + 1/24*integrate((24*(b^2*e^4*
log(c)^2 + 2*a*b*e^4*log(c) + a^2*e^4)*x^4*log(e*x + 1) + (3*b^2*e^4*n*x^4
- 4*b^2*e^3*n*x^3 + 6*b^2*e^2*n*x^2 - 12*b^2*e*n*x + 12*((4*a*b*e^4 - (e^4*
n - 4*e^4*log(c))*b^2)*x^4 + b^2*n)*log(e*x + 1))*log(x^n))/x, x)/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \ln (e\,x + 1)\,\left(a + b\,\ln (c\,x^n)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))**2*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.11 $\int x^2 \left(a+b\log\left(cx^n\right)\right)^2 \log(1+ex)\,dx$

Optimal. Leaf size=396

$$\frac{2bn\text{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^2}{3e^3}-\frac{2bn\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{9e^3}-\frac{x\left(a+b\log(\right.}{3e^2}$$

```
[Out] 2/3*a*b*n*x/e^2-26/27*b^2*n^2*x/e^2+19/108*b^2*n^2*x^2/e-2/27*b^2*n^2*x^3+2
/3*b^2*n*x*ln(c*x^n)/e^2+2/9*b*n*x*(a+b*ln(c*x^n))/e^2-5/18*b*n*x^2*(a+b*ln
(c*x^n))/e+4/27*b*n*x^3*(a+b*ln(c*x^n))-1/3*x*(a+b*ln(c*x^n))^2/e^2+1/6*x^2
*(a+b*ln(c*x^n))^2/e-1/9*x^3*(a+b*ln(c*x^n))^2+2/27*b^2*n^2*ln(e*x+1)/e^3+2
/27*b^2*n^2*x^3*ln(e*x+1)-2/9*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/e^3-2/9*b*n*x^3
*(a+b*ln(c*x^n))*ln(e*x+1)+1/3*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^3+1/3*x^3*(a+b
*ln(c*x^n))^2*ln(e*x+1)-2/9*b^2*n^2*polylog(2,-e*x)/e^3+2/3*b*n*(a+b*ln(c*x
^n))*polylog(2,-e*x)/e^3-2/3*b^2*n^2*polylog(3,-e*x)/e^3
```

Rubi [A] time = 0.29, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2395, 43, 2377, 2295, 2304, 2374, 6589, 2376, 2391}

$$\frac{2bn\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{2b^2n^2\text{PolyLog}(2,-ex)}{9e^3}-\frac{2b^2n^2\text{PolyLog}(3,-ex)}{3e^3}-\frac{x\left(a+b\log\left(cx^n\right)\right)^2}{3e^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] (2*a*b*n*x)/(3*e^2) - (26*b^2*n^2*x)/(27*e^2) + (19*b^2*n^2*x^2)/(108*e) -
(2*b^2*n^2*x^3)/27 + (2*b^2*n*x*Log[c*x^n])/(3*e^2) + (2*b*n*x*(a + b*Log[c
*x^n]))/(9*e^2) - (5*b*n*x^2*(a + b*Log[c*x^n]))/(18*e) + (4*b*n*x^3*(a + b
*Log[c*x^n]))/27 - (x*(a + b*Log[c*x^n])^2)/(3*e^2) + (x^2*(a + b*Log[c*x^n
])^2)/(6*e) - (x^3*(a + b*Log[c*x^n])^2)/9 + (2*b^2*n^2*Log[1 + e*x])/(27*e
^3) + (2*b^2*n^2*x^3*Log[1 + e*x])/27 - (2*b*n*(a + b*Log[c*x^n])*Log[1 + e
*x])/(9*e^3) - (2*b*n*x^3*(a + b*Log[c*x^n])*Log[1 + e*x])/9 + ((a + b*Log[
c*x^n])^2*Log[1 + e*x])/(3*e^3) + (x^3*(a + b*Log[c*x^n])^2*Log[1 + e*x])/3
 - (2*b^2*n^2*PolyLog[2, -(e*x)])/(9*e^3) + (2*b*n*(a + b*Log[c*x^n])*PolyL
og[2, -(e*x)])/(3*e^3) - (2*b^2*n^2*PolyLog[3, -(e*x)])/(3*e^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\left(a+b\log\left(cx^n\right)\right)^2\log(1+ex)\,dx = -\frac{x\left(a+b\log\left(cx^n\right)\right)^2}{3e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)^2}{6e}-\frac{1}{9}x^3\left(a+b\log\left(cx^n\right)\right)^2$$

$$= -\frac{x\left(a+b\log\left(cx^n\right)\right)^2}{3e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)^2}{6e}-\frac{1}{9}x^3\left(a+b\log\left(cx^n\right)\right)^2$$

$$= \frac{2abnx}{3e^2}+\frac{b^2n^2x^2}{12e}-\frac{2}{81}b^2n^2x^3+\frac{2bnx\left(a+b\log\left(cx^n\right)\right)}{9e^2}-\frac{5bnx^2\left(a+b\right.}{18}$$

$$= \frac{2abnx}{3e^2}-\frac{8b^2n^2x}{9e^2}+\frac{5b^2n^2x^2}{36e}-\frac{4}{81}b^2n^2x^3+\frac{2b^2nx\log\left(cx^n\right)}{3e^2}+\frac{2bnx\left(\right.}{}$$

$$= \frac{2abnx}{3e^2}-\frac{8b^2n^2x}{9e^2}+\frac{5b^2n^2x^2}{36e}-\frac{4}{81}b^2n^2x^3+\frac{2b^2nx\log\left(cx^n\right)}{3e^2}+\frac{2bnx\left(\right.}{}$$

$$= \frac{2abnx}{3e^2}-\frac{8b^2n^2x}{9e^2}+\frac{5b^2n^2x^2}{36e}-\frac{4}{81}b^2n^2x^3+\frac{2b^2nx\log\left(cx^n\right)}{3e^2}+\frac{2bnx\left(\right.}{}$$

$$= \frac{2abnx}{3e^2}-\frac{26b^2n^2x}{27e^2}+\frac{19b^2n^2x^2}{108e}-\frac{2}{27}b^2n^2x^3+\frac{2b^2nx\log\left(cx^n\right)}{3e^2}+\frac{2bnx}{}$$

Mathematica [A] time = 0.17, size = 506, normalized size = 1.28

$$\frac{-12a^2e^3x^3+36a^2e^3x^3\log(ex+1)+18a^2e^2x^2-36a^2ex+36a^2\log(ex+1)-24abe^3x^3\log\left(cx^n\right)+72abe^3x^3\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] (-36*a^2*e*x + 96*a*b*e*n*x - 104*b^2*e*n^2*x + 18*a^2*e^2*x^2 - 30*a*b*e^2
*n*x^2 + 19*b^2*e^2*n^2*x^2 - 12*a^2*e^3*x^3 + 16*a*b*e^3*n*x^3 - 8*b^2*e^3
*n^2*x^3 - 72*a*b*e*x*Log[c*x^n] + 96*b^2*e*n*x*Log[c*x^n] + 36*a*b*e^2*x^2
*Log[c*x^n] - 30*b^2*e^2*n*x^2*Log[c*x^n] - 24*a*b*e^3*x^3*Log[c*x^n] + 16*
b^2*e^3*n*x^3*Log[c*x^n] - 36*b^2*e*x*Log[c*x^n]^2 + 18*b^2*e^2*x^2*Log[c*x
^n]^2 - 12*b^2*e^3*x^3*Log[c*x^n]^2 + 36*a^2*Log[1 + e*x] - 24*a*b*n*Log[1
+ e*x] + 8*b^2*n^2*Log[1 + e*x] + 36*a^2*e^3*x^3*Log[1 + e*x] - 24*a*b*e^3*
n*x^3*Log[1 + e*x] + 8*b^2*e^3*n^2*x^3*Log[1 + e*x] + 72*a*b*Log[c*x^n]*Log
[1 + e*x] - 24*b^2*n*Log[c*x^n]*Log[1 + e*x] + 72*a*b*e^3*x^3*Log[c*x^n]*Lo
g[1 + e*x] - 24*b^2*e^3*n*x^3*Log[c*x^n]*Log[1 + e*x] + 36*b^2*Log[c*x^n]^2
*Log[1 + e*x] + 36*b^2*e^3*x^3*Log[c*x^n]^2*Log[1 + e*x] + 24*b*n*(3*a - b*
n + 3*b*Log[c*x^n])*PolyLog[2, -(e*x)] - 72*b^2*n^2*PolyLog[3, -(e*x)])/(10
8*e^3)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\log\left(cx^n\right)^2\log\left(ex+1\right)+2\,abx^2\log\left(cx^n\right)\log\left(ex+1\right)+a^2x^2\log\left(ex+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*x^n)^2*log(e*x + 1) + 2*a*b*x^2*log(c*x^n)*log(e*x +
 1) + a^2*x^2*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^2x^2\log\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log(e*x + 1), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c\,x^{n}\right)+a\right)^{2} x^{2} \ln \left(ex+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)^2*ln(e*x+1),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)^2*ln(e*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,b^2e^3x^3-3\,b^2e^2x^2+6\,b^2ex-6\left(b^2e^3x^3+b^2\right)\log\left(ex+1\right)\right)\log\left(x^n\right)^2}{18\,e^3}+\frac{-\frac{2}{9}\,b^2e^3n^2x^3+\frac{2}{3}\,b^2e^3nx^3\log\left(x^n\right)+\frac{3}{4}\,b^2e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/18*(2*b^2*e^3*x^3 - 3*b^2*e^2*x^2 + 6*b^2*e*x - 6*(b^2*e^3*x^3 + b^2)*lo
g(e*x + 1))*log(x^n)^2/e^3 + 1/9*integrate((9*(b^2*e^3*log(c)^2 + 2*a*b*e^3
*log(c) + a^2*e^3)*x^3*log(e*x + 1) + (2*b^2*e^3*n*x^3 - 3*b^2*e^2*n*x^2 +
6*b^2*e*n*x + 6*((3*a*b*e^3 - (e^3*n - 3*e^3*log(c))*b^2)*x^3 - b^2*n)*log(
e*x + 1))*log(x^n))/x, x)/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.12 $\int x \left(a+b \log \left(cx^n\right)\right)^2 \log(1+ex)\,dx$

Optimal. Leaf size=327

$$-\frac{bn\mathrm{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)}{e^2}+\frac{bn\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{2e^2}-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)^2}{2e^2}-\frac{bnx\left(a+b\log\right.}{2e}$$

```
[Out] -a*b*n*x/e+7/4*b^2*n^2*x/e-3/8*b^2*n^2*x^2-b^2*n*x*ln(c*x^n)/e-1/2*b*n*x*(a
+b*ln(c*x^n))/e+1/2*b*n*x^2*(a+b*ln(c*x^n))+1/2*x*(a+b*ln(c*x^n))^2/e-1/4*x
^2*(a+b*ln(c*x^n))^2-1/4*b^2*n^2*ln(e*x+1)/e^2+1/4*b^2*n^2*x^2*ln(e*x+1)+1/
2*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/e^2-1/2*b*n*x^2*(a+b*ln(c*x^n))*ln(e*x+1)-1
/2*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^2+1/2*x^2*(a+b*ln(c*x^n))^2*ln(e*x+1)+1/2*
b^2*n^2*polylog(2,-e*x)/e^2-b*n*(a+b*ln(c*x^n))*polylog(2,-e*x)/e^2+b^2*n^2
*polylog(3,-e*x)/e^2
```

Rubi [A] time = 0.22, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {2395, 43, 2377, 2295, 2304, 2374, 6589, 2376, 2391}

$$-\frac{bn\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)}{e^2}+\frac{b^2n^2\text{PolyLog}(2,-ex)}{2e^2}+\frac{b^2n^2\text{PolyLog}(3,-ex)}{e^2}+\frac{bn\log(ex+1)\left(a+b\,\mathrm{lo}\right.}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] -((a*b*n*x)/e) + (7*b^2*n^2*x)/(4*e) - (3*b^2*n^2*x^2)/8 - (b^2*n*x*Log[c*x
^n])/e - (b*n*x*(a + b*Log[c*x^n]))/(2*e) + (b*n*x^2*(a + b*Log[c*x^n]))/2
+ (x*(a + b*Log[c*x^n])^2)/(2*e) - (x^2*(a + b*Log[c*x^n])^2)/4 - (b^2*n^2*
Log[1 + e*x])/(4*e^2) + (b^2*n^2*x^2*Log[1 + e*x])/4 + (b*n*(a + b*Log[c*x^
n])*Log[1 + e*x])/(2*e^2) - (b*n*x^2*(a + b*Log[c*x^n])*Log[1 + e*x])/2 - (
(a + b*Log[c*x^n])^2*Log[1 + e*x])/(2*e^2) + (x^2*(a + b*Log[c*x^n])^2*Log[
1 + e*x])/2 + (b^2*n^2*PolyLog[2, -(e*x)])/(2*e^2) - (b*n*(a + b*Log[c*x^n]
)*PolyLog[2, -(e*x)])/e^2 + (b^2*n^2*PolyLog[3, -(e*x)])/e^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

```
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)^2\log(1+ex)\,dx = \frac{x\left(a+b\log\left(cx^n\right)\right)^2}{2e} - \frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right)^2 - \frac{\left(a+b\log\left(cx^n\right)\right)^2\log(1}{2e^2}$$

$$= \frac{x\left(a+b\log\left(cx^n\right)\right)^2}{2e} - \frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right)^2 - \frac{\left(a+b\log\left(cx^n\right)\right)^2\log(1}{2e^2}$$

$$= -\frac{abnx}{e} - \frac{1}{8}b^2n^2x^2 - \frac{bnx\left(a+b\log\left(cx^n\right)\right)}{2e} + \frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right) +$$

$$= -\frac{abnx}{e} + \frac{3b^2n^2x}{2e} - \frac{1}{4}b^2n^2x^2 - \frac{b^2nx\log\left(cx^n\right)}{e} - \frac{bnx\left(a+b\log\left(cx^n\right)\right)}{2e}$$

$$= -\frac{abnx}{e} + \frac{3b^2n^2x}{2e} - \frac{1}{4}b^2n^2x^2 - \frac{b^2nx\log\left(cx^n\right)}{e} - \frac{bnx\left(a+b\log\left(cx^n\right)\right)}{2e}$$

$$= -\frac{abnx}{e} + \frac{3b^2n^2x}{2e} - \frac{1}{4}b^2n^2x^2 - \frac{b^2nx\log\left(cx^n\right)}{e} - \frac{bnx\left(a+b\log\left(cx^n\right)\right)}{2e}$$

$$= -\frac{abnx}{e} + \frac{7b^2n^2x}{4e} - \frac{3}{8}b^2n^2x^2 - \frac{b^2nx\log\left(cx^n\right)}{e} - \frac{bnx\left(a+b\log\left(cx^n\right)\right)}{2e}$$

Mathematica [A] time = 0.14, size = 416, normalized size = 1.27

$$\frac{-2a^2e^2x^2 + 4a^2e^2x^2\log(ex+1) + 4a^2ex - 4a^2\log(ex+1) - 4abe^2x^2\log\left(cx^n\right) + 8abe^2x^2\log(ex+1)\log\left(cx^n\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] (4*a^2*e*x - 12*a*b*e*n*x + 14*b^2*e*n^2*x - 2*a^2*e^2*x^2 + 4*a*b*e^2*n*x^
2 - 3*b^2*e^2*n^2*x^2 + 8*a*b*e*x*Log[c*x^n] - 12*b^2*e*n*x*Log[c*x^n] - 4*
a*b*e^2*x^2*Log[c*x^n] + 4*b^2*e^2*n*x^2*Log[c*x^n] + 4*b^2*e*x*Log[c*x^n]^
2 - 2*b^2*e^2*x^2*Log[c*x^n]^2 - 4*a^2*Log[1 + e*x] + 4*a*b*n*Log[1 + e*x]
- 2*b^2*n^2*Log[1 + e*x] + 4*a^2*e^2*x^2*Log[1 + e*x] - 4*a*b*e^2*n*x^2*Log
[1 + e*x] + 2*b^2*e^2*n^2*x^2*Log[1 + e*x] - 8*a*b*Log[c*x^n]*Log[1 + e*x]
+ 4*b^2*n*Log[c*x^n]*Log[1 + e*x] + 8*a*b*e^2*x^2*Log[c*x^n]*Log[1 + e*x] -
 4*b^2*e^2*n*x^2*Log[c*x^n]*Log[1 + e*x] - 4*b^2*Log[c*x^n]^2*Log[1 + e*x]
+ 4*b^2*e^2*x^2*Log[c*x^n]^2*Log[1 + e*x] + 4*b*n*(-2*a + b*n - 2*b*Log[c*x
^n])*PolyLog[2, -(e*x)] + 8*b^2*n^2*PolyLog[3, -(e*x)])/(8*e^2)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x\log\left(cx^n\right)^2\log\left(ex+1\right) + 2\,abx\log\left(cx^n\right)\log\left(ex+1\right) + a^2x\log\left(ex+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*x^n)^2*log(e*x + 1) + 2*a*b*x*log(c*x^n)*log(e*x + 1)
+ a^2*x*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^2x\log\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log(e*x + 1), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int (b \ln (c x^n) + a)^2 x \ln (ex + 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^2*ln(e*x+1),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^2*ln(e*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^2e^2x^2 - 2\,b^2ex - 2\left(b^2e^2x^2 - b^2\right)\log(ex+1)\right)\log(x^n)^2}{4\,e^2} + \frac{-\frac{1}{4}\,b^2e^2n^2x^2 + \frac{1}{2}\,b^2e^2nx^2\log(x^n) + \frac{1}{2}\Big(2\,x^2\log(ex+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*e^2*x^2 - 2*b^2*e*x - 2*(b^2*e^2*x^2 - b^2)*log(e*x + 1))*log(x^n
)^2/e^2 + 1/2*integrate((2*(b^2*e^2*log(c)^2 + 2*a*b*e^2*log(c) + a^2*e^2)*
x^2*log(e*x + 1) + (b^2*e^2*n*x^2 - 2*b^2*e*n*x + 2*(b^2*n + (2*a*b*e^2 - (
e^2*n - 2*e^2*log(c))*b^2)*x^2)*log(e*x + 1))*log(x^n))/x, x)/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(e*x + 1)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x*log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.13 $\int \left(a+b\log\left(cx^n\right)\right)^2\log(1+ex)\,dx$

Optimal. Leaf size=193

$$\frac{2bn\mathrm{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{2bn(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{e}+\frac{(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{e}$$

```
[Out] 2*a*b*n*x-6*b^2*n^2*x+2*b^2*n*x*ln(c*x^n)+2*b*n*x*(a+b*ln(c*x^n))-x*(a+b*ln
(c*x^n))^2+2*b^2*n^2*(e*x+1)*ln(e*x+1)/e-2*b*n*(e*x+1)*(a+b*ln(c*x^n))*ln(e
*x+1)/e+(e*x+1)*(a+b*ln(c*x^n))^2*ln(e*x+1)/e-2*b^2*n^2*polylog(2,-e*x)/e+2
*b*n*(a+b*ln(c*x^n))*polylog(2,-e*x)/e-2*b^2*n^2*polylog(3,-e*x)/e
```

Rubi [A] time = 0.33, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {2389, 2295, 2370, 2346, 2301, 6742, 2411, 43, 2351, 2315, 2374, 6589}

$$\frac{2bn\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{2b^2n^2\text{PolyLog}(2,-ex)}{e}-\frac{2b^2n^2\text{PolyLog}(3,-ex)}{e}-\frac{2bn(ex+1)\log(ex+}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] 2*a*b*n*x - 6*b^2*n^2*x + 2*b^2*n*x*Log[c*x^n] + 2*b*n*x*(a + b*Log[c*x^n])
 - x*(a + b*Log[c*x^n])^2 + (2*b^2*n^2*(1 + e*x)*Log[1 + e*x])/e - (2*b*n*(
1 + e*x)*(a + b*Log[c*x^n])*Log[1 + e*x])/e + ((1 + e*x)*(a + b*Log[c*x^n])
^2*Log[1 + e*x])/e - (2*b^2*n^2*PolyLog[2, -(e*x)])/e + (2*b*n*(a + b*Log[c
*x^n])*PolyLog[2, -(e*x)])/e - (2*b^2*n^2*PolyLog[3, -(e*x)])/e
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)^2 \log(1+ex)\,dx = -x\left(a+b\log\left(cx^n\right)\right)^2 + \frac{(1+ex)\left(a+b\log\left(cx^n\right)\right)^2\log(1+ex)}{e} - (2bn)\int$$

$$= 2abnx - x\left(a+b\log\left(cx^n\right)\right)^2 + \frac{(1+ex)\left(a+b\log\left(cx^n\right)\right)^2\log(1+ex)}{e} + ($$

$$= 2abnx - 2b^2n^2x + 2b^2nx\log\left(cx^n\right) - x\left(a+b\log\left(cx^n\right)\right)^2 + \frac{(1+ex)\left(a+\right.}{}$$

$$= 2abnx - 2b^2n^2x + 2b^2nx\log\left(cx^n\right) - x\left(a+b\log\left(cx^n\right)\right)^2 + \frac{(1+ex)\left(a+\right.}{}$$

$$= 2abnx - 2b^2n^2x + 2b^2nx\log\left(cx^n\right) + 2bnx\left(a+b\log\left(cx^n\right)\right) - x\left(a+b\,\mathrm{lo}\right.$$

$$= 2abnx - 4b^2n^2x + 2b^2nx\log\left(cx^n\right) + 2bnx\left(a+b\log\left(cx^n\right)\right) - x\left(a+b\,\mathrm{lo}\right.$$

$$= 2abnx - 4b^2n^2x + 2b^2nx\log\left(cx^n\right) + 2bnx\left(a+b\log\left(cx^n\right)\right) - x\left(a+b\,\mathrm{lo}\right.$$

$$= 2abnx - 4b^2n^2x + 2b^2nx\log\left(cx^n\right) + 2bnx\left(a+b\log\left(cx^n\right)\right) - x\left(a+b\,\mathrm{lo}\right.$$

$$= 2abnx - 4b^2n^2x + 2b^2nx\log\left(cx^n\right) + 2bnx\left(a+b\log\left(cx^n\right)\right) - x\left(a+b\,\mathrm{lo}\right.$$

$$= 2abnx - 6b^2n^2x + 2b^2nx\log\left(cx^n\right) + 2bnx\left(a+b\log\left(cx^n\right)\right) - x\left(a+b\,\mathrm{lo}\right.$$

Mathematica [A] time = 0.10, size = 294, normalized size = 1.52

$$\frac{a^2(-e)x + a^2ex\log(ex+1) + a^2\log(ex+1) + 2bn\mathrm{Li}_2(-ex)\left(a+b\log\left(cx^n\right)-bn\right) - 2abex\log\left(cx^n\right) + 2ab\log(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] (-(a^2*e*x) + 4*a*b*e*n*x - 6*b^2*e*n^2*x - 2*a*b*e*x*Log[c*x^n] + 4*b^2*e*
n*x*Log[c*x^n] - b^2*e*x*Log[c*x^n]^2 + a^2*Log[1 + e*x] - 2*a*b*n*Log[1 +
e*x] + 2*b^2*n^2*Log[1 + e*x] + a^2*e*x*Log[1 + e*x] - 2*a*b*e*n*x*Log[1 +
e*x] + 2*b^2*e*n^2*x*Log[1 + e*x] + 2*a*b*Log[c*x^n]*Log[1 + e*x] - 2*b^2*n
*Log[c*x^n]*Log[1 + e*x] + 2*a*b*e*x*Log[c*x^n]*Log[1 + e*x] - 2*b^2*e*n*x*
Log[c*x^n]*Log[1 + e*x] + b^2*Log[c*x^n]^2*Log[1 + e*x] + b^2*e*x*Log[c*x^n
]^2*Log[1 + e*x] + 2*b*n*(a - b*n + b*Log[c*x^n])*PolyLog[2, -(e*x)] - 2*b^
2*n^2*PolyLog[3, -(e*x)])/e
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^2\log\left(cx^n\right)^2\log\left(ex+1\right) + 2\,ab\log\left(cx^n\right)\log\left(ex+1\right) + a^2\log\left(ex+1\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*x^n)^2*log(e*x + 1) + 2*a*b*log(c*x^n)*log(e*x + 1) + a^
2*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)^2\log\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log(e*x + 1), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)^2\ln\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(e*x+1),x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(e*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^2ex-\left(b^2ex+b^2\right)\log\left(ex+1\right)\right)\log\left(x^n\right)^2}{e}+\frac{-2\,b^2en^2x+2\,b^2enx\log\left(x^n\right)-\left(ex-\left(ex+1\right)\log\left(ex+1\right)+1\right)b^2\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -(b^2*e*x - (b^2*e*x + b^2)*log(e*x + 1))*log(x^n)^2/e + integrate(((b^2*e*
log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x*log(e*x + 1) + 2*(b^2*e*n*x - (b^2*n +
 ((e*n - e*log(c))*b^2 - a*b*e)*x)*log(e*x + 1))*log(x^n))/x, x)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*x + 1)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.14 $\int \frac{\left(a+b\log(cx^n)\right)^2\log(1+ex)}{x}\,dx$

Optimal. Leaf size=55

$$2bn\text{Li}_3(-ex)\left(a+b\log\left(cx^n\right)\right)-\text{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)^2-2b^2n^2\text{Li}_4(-ex)$$

```
[Out] -(a+b*ln(c*x^n))^2*polylog(2,-e*x)+2*b*n*(a+b*ln(c*x^n))*polylog(3,-e*x)-2*
b^2*n^2*polylog(4,-e*x)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2374, 2383, 6589}

$$2bn\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)-\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)^2-2b^2n^2\text{PolyLog}(4,-ex)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])^2*PolyLog[2, -(e*x)]) + 2*b*n*(a + b*Log[c*x^n])*PolyL
og[3, -(e*x)] - 2*b^2*n^2*PolyLog[4, -(e*x)]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log(1+ex)}{x}\,dx=-\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2(-ex)+(2bn)\int \frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(-ex)}{x}\,dx$$
$$=-\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2(-ex)+2bn\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(-ex)-\left(2b^2n^2\right)$$
$$=-\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2(-ex)+2bn\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(-ex)-2b^2n^2\text{Li}$$

Mathematica [A] time = 0.08, size = 53, normalized size = 0.96

$$2bn\left(\text{Li}_3(-ex)\left(a+b\log\left(cx^n\right)\right)-bn\text{Li}_4(-ex)\right)-\text{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])^2*PolyLog[2, -(e*x)]) + 2*b*n*((a + b*Log[c*x^n])*Poly
Log[3, -(e*x)] - b*n*PolyLog[4, -(e*x)])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(cx^n\right)^2\log\left(ex+1\right)+2\,ab\log\left(cx^n\right)\log\left(ex+1\right)+a^2\log\left(ex+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2*log(e*x + 1) + 2*a*b*log(c*x^n)*log(e*x + 1) + a
^2*log(e*x + 1))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(ex+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log(e*x + 1)/x, x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right)+a\right)^2\ln\left(ex+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(e*x+1)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(e*x+1)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(ex+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log(e*x + 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(e*x+1)/x,x)
```

```
[Out] Timed out
```

3.15 $\int \frac{(a+b\log(cx^n))^2 \log(1+ex)}{x^2} dx$

Optimal. Leaf size=203

$$2ben\text{Li}_2\left(-\frac{1}{ex}\right)\left(a+b\log\left(cx^n\right)\right)-2ben\log\left(\frac{1}{ex}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{2bn\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{x}-e\log\left(\frac{1}{ex}\right.$$

```
[Out] 2*b^2*e*n^2*ln(x)-2*b*e*n*ln(1+1/e/x)*(a+b*ln(c*x^n))-e*ln(1+1/e/x)*(a+b*ln
(c*x^n))^2-2*b^2*e*n^2*ln(e*x+1)-2*b^2*n^2*ln(e*x+1)/x-2*b*n*(a+b*ln(c*x^n)
)*ln(e*x+1)/x-(a+b*ln(c*x^n))^2*ln(e*x+1)/x+2*b^2*e*n^2*polylog(2,-1/e/x)+2
*b*e*n*(a+b*ln(c*x^n))*polylog(2,-1/e/x)+2*b^2*e*n^2*polylog(3,-1/e/x)
```

Rubi [A] time = 0.34, antiderivative size = 220, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2305, 2304, 2378, 36, 29, 31, 2344, 2301, 2317, 2391, 2302, 30, 2374, 6589}

$$-2ben\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)-2b^2en^2\text{PolyLog}(2,-ex)+2b^2en^2\text{PolyLog}(3,-ex)+\frac{e\left(a+b\log\left(cx^n\right)\right)^3}{3bn}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^2,x]
```

```
[Out] 2*b^2*e*n^2*Log[x] + e*(a + b*Log[c*x^n])^2 + (e*(a + b*Log[c*x^n])^3)/(3*b
*n) - 2*b^2*e*n^2*Log[1 + e*x] - (2*b^2*n^2*Log[1 + e*x])/x - 2*b*e*n*(a +
b*Log[c*x^n])*Log[1 + e*x] - (2*b*n*(a + b*Log[c*x^n])*Log[1 + e*x])/x - e*
(a + b*Log[c*x^n])^2*Log[1 + e*x] - ((a + b*Log[c*x^n])^2*Log[1 + e*x])/x -
 2*b^2*e*n^2*PolyLog[2, -(e*x)] - 2*b*e*n*(a + b*Log[c*x^n])*PolyLog[2, -(e
*x)] + 2*b^2*e*n^2*PolyLog[3, -(e*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log(1+ex)}{x^2}\,dx = -\frac{2b^2n^2\log(1+ex)}{x} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x} - \frac{\left(a+b\log\left(cx^n\right)\right)}{x}$$

$$= -\frac{2b^2n^2\log(1+ex)}{x} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x} - \frac{\left(a+b\log\left(cx^n\right)\right)}{x}$$

$$= -\frac{2b^2n^2\log(1+ex)}{x} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x} - \frac{\left(a+b\log\left(cx^n\right)\right)}{x}$$

$$= 2b^2en^2\log(x) + e\left(a+b\log\left(cx^n\right)\right)^2 - 2b^2en^2\log(1+ex) - \frac{2b^2n^2\log(1+e}{x}$$

$$= 2b^2en^2\log(x) + e\left(a+b\log\left(cx^n\right)\right)^2 + \frac{e\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - 2b^2en^2\log(1+$$

$$= 2b^2en^2\log(x) + e\left(a+b\log\left(cx^n\right)\right)^2 + \frac{e\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - 2b^2en^2\log(1+$$

Mathematica [A] time = 0.22, size = 183, normalized size = 0.90

$$e\log(x)\left(a^2+2b(a+bn)\log\left(cx^n\right)+2abn+b^2\log^2\left(cx^n\right)+2b^2n^2\right) - \frac{(ex+1)\log(ex+1)\left(a^2+2b(a+bn)\log\left(cx^n\right.\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^2,x]
```

```
[Out] (b^2*e*n^2*Log[x]^3)/3 - b*e*n*Log[x]^2*(a + b*n + b*Log[c*x^n]) + e*Log[x]
*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)
- ((1 + e*x)*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*Log[c*x^n] + b^2*Lo
g[c*x^n]^2)*Log[1 + e*x])/x - 2*b*e*n*(a + b*n + b*Log[c*x^n])*PolyLog[2, -
(e*x)] + 2*b^2*e*n^2*PolyLog[3, -(e*x)]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(cx^n\right)^2\log\left(ex+1\right)+2\,ab\log\left(cx^n\right)\log\left(ex+1\right)+a^2\log\left(ex+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2*log(e*x + 1) + 2*a*b*log(c*x^n)*log(e*x + 1) + a
^2*log(e*x + 1))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(ex+1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log(e*x + 1)/x^2, x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right)+a\right)^2\ln\left(ex+1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(e*x+1)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(e*x+1)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^2ex\log(x) - \left(b^2ex + b^2\right)\log\left(ex+1\right)\right)\log\left(x^n\right)^2}{x} + \int \frac{\left(b^2\log(c)^2 + 2\,ab\log(c) + a^2\right)\log\left(ex+1\right) - 2\left(b^2enx\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*e*x*log(x) - (b^2*e*x + b^2)*log(e*x + 1))*log(x^n)^2/x + integrate(((
b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(e*x + 1) - 2*(b^2*e*n*x*log(x) - (b^
2*e*n*x + b^2*(n + log(c)) + a*b)*log(e*x + 1))*log(x^n))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^2,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(e*x+1)/x**2,x)
```

```
[Out] Timed out
```

3.16 $\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx$

Optimal. Leaf size=287

$$-be^2n\mathrm{Li}_2\left(-\frac{1}{ex}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}e^2\log\left(\frac{1}{ex}+1\right)\left(a+b\log\left(cx^n\right)\right)^2+\frac{1}{2}be^2n\log\left(\frac{1}{ex}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{e(a}{}$$

```
[Out] -7/4*b^2*e*n^2/x-1/4*b^2*e^2*n^2*ln(x)-3/2*b*e*n*(a+b*ln(c*x^n))/x+1/2*b*e^
2*n*ln(1+1/e/x)*(a+b*ln(c*x^n))-1/2*e*(a+b*ln(c*x^n))^2/x+1/2*e^2*ln(1+1/e/
x)*(a+b*ln(c*x^n))^2+1/4*b^2*e^2*n^2*ln(e*x+1)-1/4*b^2*n^2*ln(e*x+1)/x^2-1/
2*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/x^2-1/2*(a+b*ln(c*x^n))^2*ln(e*x+1)/x^2-1/2
*b^2*e^2*n^2*polylog(2,-1/e/x)-b*e^2*n*(a+b*ln(c*x^n))*polylog(2,-1/e/x)-b^
2*e^2*n^2*polylog(3,-1/e/x)
```

Rubi [A] time = 0.48, antiderivative size = 310, normalized size of antiderivative = 1.08, number of steps used = 19, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2305, 2304, 2378, 44, 2351, 2301, 2317, 2391, 2353, 2302, 30, 2374, 6589}

$$be^2n\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}b^2e^2n^2\text{PolyLog}(2,-ex)-b^2e^2n^2\text{PolyLog}(3,-ex)-\frac{e^2\left(a+b\log\left(cx^n\right)\right)^3}{6bn}-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^3,x]
```

```
[Out] (-7*b^2*e*n^2)/(4*x) - (b^2*e^2*n^2*Log[x])/4 - (3*b*e*n*(a + b*Log[c*x^n])
)/(2*x) - (e^2*(a + b*Log[c*x^n])^2)/4 - (e*(a + b*Log[c*x^n])^2)/(2*x) - (
e^2*(a + b*Log[c*x^n])^3)/(6*b*n) + (b^2*e^2*n^2*Log[1 + e*x])/4 - (b^2*n^2
*Log[1 + e*x])/(4*x^2) + (b*e^2*n*(a + b*Log[c*x^n])*Log[1 + e*x])/2 - (b*n
*(a + b*Log[c*x^n])*Log[1 + e*x])/(2*x^2) + (e^2*(a + b*Log[c*x^n])^2*Log[1
 + e*x])/2 - ((a + b*Log[c*x^n])^2*Log[1 + e*x])/(2*x^2) + (b^2*e^2*n^2*Pol
yLog[2, -(e*x)])/2 + b*e^2*n*(a + b*Log[c*x^n])*PolyLog[2, -(e*x)] - b^2*e^
2*n^2*PolyLog[3, -(e*x)]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log(1+ex)}{x^3}\,dx = -\frac{b^2n^2\log(1+ex)}{4x^2}-\frac{bn\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{2x^2}-\frac{\left(a+b\log\left(cx^n\right)\right)^2\log}{2x^2}$$

$$= -\frac{b^2n^2\log(1+ex)}{4x^2}-\frac{bn\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{2x^2}-\frac{\left(a+b\log\left(cx^n\right)\right)^2\log}{2x^2}$$

$$= -\frac{b^2n^2\log(1+ex)}{4x^2}-\frac{bn\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{2x^2}-\frac{\left(a+b\log\left(cx^n\right)\right)^2\log}{2x^2}$$

$$= -\frac{b^2en^2}{4x}-\frac{1}{4}b^2e^2n^2\log(x)+\frac{1}{4}b^2e^2n^2\log(1+ex)-\frac{b^2n^2\log(1+ex)}{4x^2}-\frac{bn\left(\right.}{}$$

$$= -\frac{3b^2en^2}{4x}-\frac{1}{4}b^2e^2n^2\log(x)-\frac{ben\left(a+b\log\left(cx^n\right)\right)}{2x}-\frac{1}{4}e^2\left(a+b\log\left(cx^n\right)\right)^2$$

$$= -\frac{7b^2en^2}{4x}-\frac{1}{4}b^2e^2n^2\log(x)-\frac{3ben\left(a+b\log\left(cx^n\right)\right)}{2x}-\frac{1}{4}e^2\left(a+b\log\left(cx^n\right)\right)$$

$$= -\frac{7b^2en^2}{4x}-\frac{1}{4}b^2e^2n^2\log(x)-\frac{3ben\left(a+b\log\left(cx^n\right)\right)}{2x}-\frac{1}{4}e^2\left(a+b\log\left(cx^n\right)\right)$$

Mathematica [A] time = 0.20, size = 513, normalized size = 1.79

$$-\frac{6a^2e^2x^2\log(x)-6a^2e^2x^2\log(ex+1)+6a^2ex+6a^2\log(ex+1)-6be^2nx^2\text{Li}_2(-ex)\left(2a+2b\log\left(cx^n\right)+bn\right)+12}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^3,x]
```

```
[Out] -1/12*(6*a^2*e*x + 18*a*b*e*n*x + 21*b^2*e*n^2*x + 6*a^2*e^2*x^2*Log[x] + 6
*a*b*e^2*n*x^2*Log[x] + 3*b^2*e^2*n^2*x^2*Log[x] - 6*a*b*e^2*n*x^2*Log[x]^2
 - 3*b^2*e^2*n^2*x^2*Log[x]^2 + 2*b^2*e^2*n^2*x^2*Log[x]^3 + 12*a*b*e*x*Log
[c*x^n] + 18*b^2*e*n*x*Log[c*x^n] + 12*a*b*e^2*x^2*Log[x]*Log[c*x^n] + 6*b^
2*e^2*n*x^2*Log[x]*Log[c*x^n] - 6*b^2*e^2*n*x^2*Log[x]^2*Log[c*x^n] + 6*b^2
*e*x*Log[c*x^n]^2 + 6*b^2*e^2*x^2*Log[x]*Log[c*x^n]^2 + 6*a^2*Log[1 + e*x]
+ 6*a*b*n*Log[1 + e*x] + 3*b^2*n^2*Log[1 + e*x] - 6*a^2*e^2*x^2*Log[1 + e*x
] - 6*a*b*e^2*n*x^2*Log[1 + e*x] - 3*b^2*e^2*n^2*x^2*Log[1 + e*x] + 12*a*b*
Log[c*x^n]*Log[1 + e*x] + 6*b^2*n*Log[c*x^n]*Log[1 + e*x] - 12*a*b*e^2*x^2*
Log[c*x^n]*Log[1 + e*x] - 6*b^2*e^2*n*x^2*Log[c*x^n]*Log[1 + e*x] + 6*b^2*L
og[c*x^n]^2*Log[1 + e*x] - 6*b^2*e^2*x^2*Log[c*x^n]^2*Log[1 + e*x] - 6*b*e^
2*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[2, -(e*x)] + 12*b^2*e^2*n^2*x^
2*PolyLog[3, -(e*x)])/x^2
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(cx^n\right)^2\log\left(ex+1\right)+2\,ab\log\left(cx^n\right)\log\left(ex+1\right)+a^2\log\left(ex+1\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2*log(e*x + 1) + 2*a*b*log(c*x^n)*log(e*x + 1) + a
^2*log(e*x + 1))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{2}\log\left(ex+1\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log(e*x + 1)/x^3, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^{n}\right)+a\right)^{2}\ln\left(ex+1\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(e*x+1)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(e*x+1)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^{2}e^{2}x^{2}\log(x)+b^{2}ex-\left(b^{2}e^{2}x^{2}-b^{2}\right)\log\left(ex+1\right)\right)\log\left(x^{n}\right)^{2}}{2\,x^{2}}-\int -\frac{\left(b^{2}\log(c)^{2}+2\,ab\log(c)+a^{2}\right)\log\left(ex+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*e^2*x^2*log(x) + b^2*e*x - (b^2*e^2*x^2 - b^2)*log(e*x + 1))*log(
x^n)^2/x^2 - integrate(-((b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(e*x + 1) +
 (b^2*e^2*n*x^2*log(x) + b^2*e*n*x - (b^2*e^2*n*x^2 - b^2*(n + 2*log(c)) -
2*a*b)*log(e*x + 1))*log(x^n))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(e*x+1)/x**3,x)
```

```
[Out] Timed out
```

3.17 $\int x^3 \left(a+b\log\left(cx^n\right)\right)^3 \log(1+ex)\,dx$

Optimal. Leaf size=710

$$\frac{3b^2n^2\text{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)}{8e^4}+\frac{3b^2n^2\text{Li}_3(-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^4}-\frac{3b^2n^2\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{32e^4}+\frac{3b^2n^2x\left(a\right.}{}$$

```
[Out] -9/128*b^2*n^2*x^4*(a+b*ln(c*x^n))+3/32*b*n*x^4*(a+b*ln(c*x^n))^2-255/128*b
^3*n^3*x/e^3+45/256*b^3*n^3*x^2/e^2-175/3456*b^3*n^3*x^3/e-3/32*b^3*n^3*pol
ylog(2,-e*x)/e^4-3/8*b^3*n^3*polylog(3,-e*x)/e^4-3/2*b^3*n^3*polylog(4,-e*x
)/e^4-1/16*x^4*(a+b*ln(c*x^n))^3+3/128*b^3*n^3*x^4+1/4*x*(a+b*ln(c*x^n))^3/
e^3-1/8*x^2*(a+b*ln(c*x^n))^3/e^2+1/12*x^3*(a+b*ln(c*x^n))^3/e-1/4*(a+b*ln(
c*x^n))^3*ln(e*x+1)/e^4+1/4*x^4*(a+b*ln(c*x^n))^3*ln(e*x+1)+3/128*b^3*n^3*l
n(e*x+1)/e^4-3/128*b^3*n^3*x^4*ln(e*x+1)+3/8*b^2*n^2*(a+b*ln(c*x^n))*polylo
g(2,-e*x)/e^4-3/4*b*n*(a+b*ln(c*x^n))^2*polylog(2,-e*x)/e^4+3/2*b^2*n^2*(a+
b*ln(c*x^n))*polylog(3,-e*x)/e^4+15/8*a*b^2*n^2*x/e^3+15/8*b^3*n^2*x*ln(c*x
^n)/e^3+3/32*b^2*n^2*x*(a+b*ln(c*x^n))/e^3-21/64*b^2*n^2*x^2*(a+b*ln(c*x^n)
)/e^2+37/288*b^2*n^2*x^3*(a+b*ln(c*x^n))/e-15/16*b*n*x*(a+b*ln(c*x^n))^2/e^
3+9/32*b*n*x^2*(a+b*ln(c*x^n))^2/e^2-7/48*b*n*x^3*(a+b*ln(c*x^n))^2/e-3/32*
b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/e^4+3/32*b^2*n^2*x^4*(a+b*ln(c*x^n))*ln(e
*x+1)+3/16*b*n*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^4-3/16*b*n*x^4*(a+b*ln(c*x^n))
^2*ln(e*x+1)
```

Rubi [A] time = 0.78, antiderivative size = 710, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2395, 43, 2377, 2296, 2295, 2305, 2304, 2374, 2383, 6589, 2376, 2391}

$$\frac{3b^2n^2\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)}{8e^4}+\frac{3b^2n^2\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^4}-\frac{3bn\text{PolyLog}(2,-ex)\left(a+b\log\right.}{4e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (15*a*b^2*n^2*x)/(8*e^3) - (255*b^3*n^3*x)/(128*e^3) + (45*b^3*n^3*x^2)/(25
6*e^2) - (175*b^3*n^3*x^3)/(3456*e) + (3*b^3*n^3*x^4)/128 + (15*b^3*n^2*x*L
og[c*x^n])/(8*e^3) + (3*b^2*n^2*x*(a + b*Log[c*x^n]))/(32*e^3) - (21*b^2*n^
2*x^2*(a + b*Log[c*x^n]))/(64*e^2) + (37*b^2*n^2*x^3*(a + b*Log[c*x^n]))/(2
88*e) - (9*b^2*n^2*x^4*(a + b*Log[c*x^n]))/128 - (15*b*n*x*(a + b*Log[c*x^n
])^2)/(16*e^3) + (9*b*n*x^2*(a + b*Log[c*x^n])^2)/(32*e^2) - (7*b*n*x^3*(a
+ b*Log[c*x^n])^2)/(48*e) + (3*b*n*x^4*(a + b*Log[c*x^n])^2)/32 + (x*(a + b
*Log[c*x^n])^3)/(4*e^3) - (x^2*(a + b*Log[c*x^n])^3)/(8*e^2) + (x^3*(a + b*
Log[c*x^n])^3)/(12*e) - (x^4*(a + b*Log[c*x^n])^3)/16 + (3*b^3*n^3*Log[1 +
e*x])/(128*e^4) - (3*b^3*n^3*x^4*Log[1 + e*x])/128 - (3*b^2*n^2*(a + b*Log[
c*x^n])*Log[1 + e*x])/(32*e^4) + (3*b^2*n^2*x^4*(a + b*Log[c*x^n])*Log[1 +
e*x])/32 + (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + e*x])/(16*e^4) - (3*b*n*x^4*
(a + b*Log[c*x^n])^2*Log[1 + e*x])/16 - ((a + b*Log[c*x^n])^3*Log[1 + e*x])
/(4*e^4) + (x^4*(a + b*Log[c*x^n])^3*Log[1 + e*x])/4 - (3*b^3*n^3*PolyLog[2
, -(e*x)])/(32*e^4) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/(8*
e^4) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(e*x)])/(4*e^4) - (3*b^3*n^3
*PolyLog[3, -(e*x)])/(8*e^4) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e
*x)])/(2*e^4) - (3*b^3*n^3*PolyLog[4, -(e*x)])/(2*e^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\left(a+b\log\left(cx^n\right)\right)^3\log(1+ex)\,dx=\frac{x\left(a+b\log\left(cx^n\right)\right)^3}{4e^3}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)^3}{8e^2}+\frac{x^3\left(a+b\log\left(cx^n\right)\right)^3}{12e}-\frac{1}{16}$$

$$=\frac{x\left(a+b\log\left(cx^n\right)\right)^3}{4e^3}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)^3}{8e^2}+\frac{x^3\left(a+b\log\left(cx^n\right)\right)^3}{12e}-\frac{1}{16}$$

$$=-\frac{15bnx\left(a+b\log\left(cx^n\right)\right)^2}{16e^3}+\frac{9bnx^2\left(a+b\log\left(cx^n\right)\right)^2}{32e^2}-\frac{7bnx^3\left(a+b\log\right.}{48e}$$

$$=\frac{3ab^2n^2x}{2e^3}+\frac{3b^3n^3x^2}{32e^2}-\frac{b^3n^3x^3}{54e}+\frac{3}{512}b^3n^3x^4-\frac{3b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}{16e^2}$$

$$=\frac{15ab^2n^2x}{8e^3}-\frac{3b^3n^3x}{2e^3}+\frac{9b^3n^3x^2}{64e^2}-\frac{7b^3n^3x^3}{216e}+\frac{3}{256}b^3n^3x^4+\frac{3b^3n^2x\log(}{2e^3}$$

$$=\frac{15ab^2n^2x}{8e^3}-\frac{63b^3n^3x}{32e^3}+\frac{21b^3n^3x^2}{128e^2}-\frac{37b^3n^3x^3}{864e}+\frac{9}{512}b^3n^3x^4+\frac{15b^3n^2x\,l}{8e}$$

$$=\frac{15ab^2n^2x}{8e^3}-\frac{63b^3n^3x}{32e^3}+\frac{21b^3n^3x^2}{128e^2}-\frac{37b^3n^3x^3}{864e}+\frac{9}{512}b^3n^3x^4+\frac{15b^3n^2x\,l}{8e}$$

$$=\frac{15ab^2n^2x}{8e^3}-\frac{63b^3n^3x}{32e^3}+\frac{21b^3n^3x^2}{128e^2}-\frac{37b^3n^3x^3}{864e}+\frac{9}{512}b^3n^3x^4+\frac{15b^3n^2x\,l}{8e}$$

$$=\frac{15ab^2n^2x}{8e^3}-\frac{255b^3n^3x}{128e^3}+\frac{45b^3n^3x^2}{256e^2}-\frac{175b^3n^3x^3}{3456e}+\frac{3}{128}b^3n^3x^4+\frac{15b^3n^2}{}$$

Mathematica [A] time = 0.37, size = 1144, normalized size = 1.61

$$\frac{-432a^3e^4x^4+162b^3e^4n^3x^4-432b^3e^4\log^3\left(cx^n\right)x^4-486ab^2e^4n^2x^4-1296ab^2e^4\log^2\left(cx^n\right)x^4+648b^3e^4n\log^2\left(cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (1728*a^3*e*x - 6480*a^2*b*e*n*x + 13608*a*b^2*e*n^2*x - 13770*b^3*e*n^3*x
- 864*a^3*e^2*x^2 + 1944*a^2*b*e^2*n*x^2 - 2268*a*b^2*e^2*n^2*x^2 + 1215*b^
3*e^2*n^3*x^2 + 576*a^3*e^3*x^3 - 1008*a^2*b*e^3*n*x^3 + 888*a*b^2*e^3*n^2*
x^3 - 350*b^3*e^3*n^3*x^3 - 432*a^3*e^4*x^4 + 648*a^2*b*e^4*n*x^4 - 486*a*b
^2*e^4*n^2*x^4 + 162*b^3*e^4*n^3*x^4 + 5184*a^2*b*e*x*Log[c*x^n] - 12960*a*
b^2*e*n*x*Log[c*x^n] + 13608*b^3*e*n^2*x*Log[c*x^n] - 2592*a^2*b*e^2*x^2*Lo
g[c*x^n] + 3888*a*b^2*e^2*n*x^2*Log[c*x^n] - 2268*b^3*e^2*n^2*x^2*Log[c*x^n
] + 1728*a^2*b*e^3*x^3*Log[c*x^n] - 2016*a*b^2*e^3*n*x^3*Log[c*x^n] + 888*b
^3*e^3*n^2*x^3*Log[c*x^n] - 1296*a^2*b*e^4*x^4*Log[c*x^n] + 1296*a*b^2*e^4*
n*x^4*Log[c*x^n] - 486*b^3*e^4*n^2*x^4*Log[c*x^n] + 5184*a*b^2*e*x*Log[c*x^
n]^2 - 6480*b^3*e*n*x*Log[c*x^n]^2 - 2592*a*b^2*e^2*x^2*Log[c*x^n]^2 + 1944
*b^3*e^2*n*x^2*Log[c*x^n]^2 + 1728*a*b^2*e^3*x^3*Log[c*x^n]^2 - 1008*b^3*e^
3*n*x^3*Log[c*x^n]^2 - 1296*a*b^2*e^4*x^4*Log[c*x^n]^2 + 648*b^3*e^4*n*x^4*
Log[c*x^n]^2 + 1728*b^3*e*x*Log[c*x^n]^3 - 864*b^3*e^2*x^2*Log[c*x^n]^3 + 5
76*b^3*e^3*x^3*Log[c*x^n]^3 - 432*b^3*e^4*x^4*Log[c*x^n]^3 - 1728*a^3*Log[1
 + e*x] + 1296*a^2*b*n*Log[1 + e*x] - 648*a*b^2*n^2*Log[1 + e*x] + 162*b^3*
n^3*Log[1 + e*x] + 1728*a^3*e^4*x^4*Log[1 + e*x] - 1296*a^2*b*e^4*n*x^4*Log
[1 + e*x] + 648*a*b^2*e^4*n^2*x^4*Log[1 + e*x] - 162*b^3*e^4*n^3*x^4*Log[1
+ e*x] - 5184*a^2*b*Log[c*x^n]*Log[1 + e*x] + 2592*a*b^2*n*Log[c*x^n]*Log[1
 + e*x] - 648*b^3*n^2*Log[c*x^n]*Log[1 + e*x] + 5184*a^2*b*e^4*x^4*Log[c*x^
n]*Log[1 + e*x] - 2592*a*b^2*e^4*n*x^4*Log[c*x^n]*Log[1 + e*x] + 648*b^3*e^
4*n^2*x^4*Log[c*x^n]*Log[1 + e*x] - 5184*a*b^2*Log[c*x^n]^2*Log[1 + e*x] +
1296*b^3*n*Log[c*x^n]^2*Log[1 + e*x] + 5184*a*b^2*e^4*x^4*Log[c*x^n]^2*Log[
1 + e*x] - 1296*b^3*e^4*n*x^4*Log[c*x^n]^2*Log[1 + e*x] - 1728*b^3*Log[c*x^
n]^3*Log[1 + e*x] + 1728*b^3*e^4*x^4*Log[c*x^n]^3*Log[1 + e*x] - 648*b*n*(8
*a^2 - 4*a*b*n + b^2*n^2 - 4*b*(-4*a + b*n)*Log[c*x^n] + 8*b^2*Log[c*x^n]^2
)*PolyLog[2, -(e*x)] + 2592*b^2*n^2*(4*a - b*n + 4*b*Log[c*x^n])*PolyLog[3,
 -(e*x)] - 10368*b^3*n^3*PolyLog[4, -(e*x)])/(6912*e^4)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\log\left(cx^n\right)^3\log\left(ex+1\right)+3\,ab^2x^3\log\left(cx^n\right)^2\log\left(ex+1\right)+3\,a^2bx^3\log\left(cx^n\right)\log\left(ex+1\right)+a^3x^3\,\text{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*x^3*log(c*x^n)^2*log(e
*x + 1) + 3*a^2*b*x^3*log(c*x^n)*log(e*x + 1) + a^3*x^3*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^3x^3\log\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x^3*log(e*x + 1), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^3x^3\ln\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*x^n)+a)^3*ln(e*x+1),x)
```

```
[Out] int(x^3*(b*ln(c*x^n)+a)^3*ln(e*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3\,b^3e^4x^4 - 4\,b^3e^3x^3 + 6\,b^3e^2x^2 - 12\,b^3ex - 12\left(b^3e^4x^4 - b^3\right)\log\left(ex+1\right)\right)\log\left(x^n\right)^3}{48\,e^4} + \frac{\frac{1}{3}\left(12\,x^4\log\left(ex+1\right) - e\left(\frac{3\,e^3}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/48*(3*b^3*e^4*x^4 - 4*b^3*e^3*x^3 + 6*b^3*e^2*x^2 - 12*b^3*e*x - 12*(b^3
*e^4*x^4 - b^3)*log(e*x + 1))*log(x^n)^3/e^4 + 1/16*integrate((48*(b^3*e^4*
log(c)^2 + 2*a*b^2*e^4*log(c) + a^2*b*e^4)*x^4*log(e*x + 1)*log(x^n) + 16*(
b^3*e^4*log(c)^3 + 3*a*b^2*e^4*log(c)^2 + 3*a^2*b*e^4*log(c) + a^3*e^4)*x^4
*log(e*x + 1) + (3*b^3*e^4*n*x^4 - 4*b^3*e^3*n*x^3 + 6*b^3*e^2*n*x^2 - 12*b
^3*e*n*x + 12*((4*a*b^2*e^4 - (e^4*n - 4*e^4*log(c))*b^3)*x^4 + b^3*n)*log(
e*x + 1))*log(x^n)^2)/x, x)/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.18 $\int x^2 \left(a+b\log\left(cx^n\right)\right)^3 \log(1+ex)\,dx$

Optimal. Leaf size=615

$$-\frac{2b^2n^2\text{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{2b^2n^2\text{Li}_3(-ex)\left(a+b\log\left(cx^n\right)\right)}{e^3}+\frac{2b^2n^2\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{9e^3}-\frac{2b^2n}{}$$

```
[Out] -2/9*b^2*n^2*x^3*(a+b*ln(c*x^n))+2/9*b*n*x^3*(a+b*ln(c*x^n))^2+80/27*b^3*n^
3*x/e^2-65/216*b^3*n^3*x^2/e+2/9*b^3*n^3*polylog(2,-e*x)/e^3+2/3*b^3*n^3*po
lylog(3,-e*x)/e^3+2*b^3*n^3*polylog(4,-e*x)/e^3-1/9*x^3*(a+b*ln(c*x^n))^3+8
/81*b^3*n^3*x^3+1/3*x^3*(a+b*ln(c*x^n))^3*ln(e*x+1)-1/3*x*(a+b*ln(c*x^n))^3
/e^2+1/6*x^2*(a+b*ln(c*x^n))^3/e+1/3*(a+b*ln(c*x^n))^3*ln(e*x+1)/e^3-2/27*b
^3*n^3*ln(e*x+1)/e^3-2/27*b^3*n^3*x^3*ln(e*x+1)+b*n*(a+b*ln(c*x^n))^2*polyl
og(2,-e*x)/e^3+4/3*b*n*x*(a+b*ln(c*x^n))^2/e^2-5/12*b*n*x^2*(a+b*ln(c*x^n))
^2/e+2/9*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/e^3+2/9*b^2*n^2*x^3*(a+b*ln(c*x^
n))*ln(e*x+1)-1/3*b*n*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^3-1/3*b*n*x^3*(a+b*ln(c
*x^n))^2*ln(e*x+1)-2/3*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-e*x)/e^3-2*b^2*n^
2*(a+b*ln(c*x^n))*polylog(3,-e*x)/e^3-8/3*b^3*n^2*x*ln(c*x^n)/e^2-2/9*b^2*n
^2*x*(a+b*ln(c*x^n))/e^2+19/36*b^2*n^2*x^2*(a+b*ln(c*x^n))/e-8/3*a*b^2*n^2*
x/e^2
```

Rubi [A] time = 0.64, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2395, 43, 2377, 2296, 2295, 2305, 2304, 2374, 2383, 6589, 2376, 2391}

$$-\frac{2b^2n^2\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{2b^2n^2\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)}{e^3}+\frac{bn\text{PolyLog}(2,-ex)\left(a+b\right.}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (-8*a*b^2*n^2*x)/(3*e^2) + (80*b^3*n^3*x)/(27*e^2) - (65*b^3*n^3*x^2)/(216*
e) + (8*b^3*n^3*x^3)/81 - (8*b^3*n^2*x*Log[c*x^n])/(3*e^2) - (2*b^2*n^2*x*(
a + b*Log[c*x^n]))/(9*e^2) + (19*b^2*n^2*x^2*(a + b*Log[c*x^n]))/(36*e) - (
2*b^2*n^2*x^3*(a + b*Log[c*x^n]))/9 + (4*b*n*x*(a + b*Log[c*x^n])^2)/(3*e^2
) - (5*b*n*x^2*(a + b*Log[c*x^n])^2)/(12*e) + (2*b*n*x^3*(a + b*Log[c*x^n])
^2)/9 - (x*(a + b*Log[c*x^n])^3)/(3*e^2) + (x^2*(a + b*Log[c*x^n])^3)/(6*e)
 - (x^3*(a + b*Log[c*x^n])^3)/9 - (2*b^3*n^3*Log[1 + e*x])/(27*e^3) - (2*b^
3*n^3*x^3*Log[1 + e*x])/27 + (2*b^2*n^2*(a + b*Log[c*x^n])*Log[1 + e*x])/(9
*e^3) + (2*b^2*n^2*x^3*(a + b*Log[c*x^n])*Log[1 + e*x])/9 - (b*n*(a + b*Log
[c*x^n])^2*Log[1 + e*x])/(3*e^3) - (b*n*x^3*(a + b*Log[c*x^n])^2*Log[1 + e*
x])/3 + ((a + b*Log[c*x^n])^3*Log[1 + e*x])/(3*e^3) + (x^3*(a + b*Log[c*x^n
])^3*Log[1 + e*x])/3 + (2*b^3*n^3*PolyLog[2, -(e*x)])/(9*e^3) - (2*b^2*n^2*
(a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/(3*e^3) + (b*n*(a + b*Log[c*x^n])^2*
PolyLog[2, -(e*x)])/e^3 + (2*b^3*n^3*PolyLog[3, -(e*x)])/(3*e^3) - (2*b^2*n
^2*(a + b*Log[c*x^n])*PolyLog[3, -(e*x)])/e^3 + (2*b^3*n^3*PolyLog[4, -(e*x
)])/e^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(cx^n\right)\right)^3 \log(1+ex)\,dx = -\frac{x\left(a+b\log\left(cx^n\right)\right)^3}{3e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)^3}{6e}-\frac{1}{9}x^3\left(a+b\log\left(cx^n\right)\right)^3$$

$$= -\frac{x\left(a+b\log\left(cx^n\right)\right)^3}{3e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)^3}{6e}-\frac{1}{9}x^3\left(a+b\log\left(cx^n\right)\right)^3$$

$$= \frac{4bnx\left(a+b\log\left(cx^n\right)\right)^2}{3e^2}-\frac{5bnx^2\left(a+b\log\left(cx^n\right)\right)^2}{12e}+\frac{2}{9}bnx^3\left(a+b\log\right.$$

$$= -\frac{2ab^2n^2x}{e^2}-\frac{b^3n^3x^2}{8e}+\frac{2}{81}b^3n^3x^3+\frac{b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}{4e}-\frac{2}{27}b^2n^2$$

$$= -\frac{8ab^2n^2x}{3e^2}+\frac{2b^3n^3x}{e^2}-\frac{5b^3n^3x^2}{24e}+\frac{4}{81}b^3n^3x^3-\frac{2b^3n^2x\log\left(cx^n\right)}{e^2}-2b$$

$$= -\frac{8ab^2n^2x}{3e^2}+\frac{26b^3n^3x}{9e^2}-\frac{19b^3n^3x^2}{72e}+\frac{2}{27}b^3n^3x^3-\frac{8b^3n^2x\log\left(cx^n\right)}{3e^2}-$$

$$= -\frac{8ab^2n^2x}{3e^2}+\frac{26b^3n^3x}{9e^2}-\frac{19b^3n^3x^2}{72e}+\frac{2}{27}b^3n^3x^3-\frac{8b^3n^2x\log\left(cx^n\right)}{3e^2}-$$

$$= -\frac{8ab^2n^2x}{3e^2}+\frac{26b^3n^3x}{9e^2}-\frac{19b^3n^3x^2}{72e}+\frac{2}{27}b^3n^3x^3-\frac{8b^3n^2x\log\left(cx^n\right)}{3e^2}-$$

$$= -\frac{8ab^2n^2x}{3e^2}+\frac{80b^3n^3x}{27e^2}-\frac{65b^3n^3x^2}{216e}+\frac{8}{81}b^3n^3x^3-\frac{8b^3n^2x\log\left(cx^n\right)}{3e^2}-$$

Mathematica [A] time = 0.30, size = 975, normalized size = 1.59

$$\frac{-72e^3x^3a^3+108e^2x^2a^3-216exa^3+216e^3x^3\log(ex+1)a^3+216\log(ex+1)a^3+144be^3nx^3a^2-270be^2nx^2a^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (-216*a^3*e*x + 864*a^2*b*e*n*x - 1872*a*b^2*e*n^2*x + 1920*b^3*e*n^3*x + 1
08*a^3*e^2*x^2 - 270*a^2*b*e^2*n*x^2 + 342*a*b^2*e^2*n^2*x^2 - 195*b^3*e^2*
n^3*x^2 - 72*a^3*e^3*x^3 + 144*a^2*b*e^3*n*x^3 - 144*a*b^2*e^3*n^2*x^3 + 64
*b^3*e^3*n^3*x^3 - 648*a^2*b*e*x*Log[c*x^n] + 1728*a*b^2*e*n*x*Log[c*x^n] -
```

```
1872*b^3*e*n^2*x*Log[c*x^n] + 324*a^2*b*e^2*x^2*Log[c*x^n] - 540*a*b^2*e^2
*n*x^2*Log[c*x^n] + 342*b^3*e^2*n^2*x^2*Log[c*x^n] - 216*a^2*b*e^3*x^3*Log[
c*x^n] + 288*a*b^2*e^3*n*x^3*Log[c*x^n] - 144*b^3*e^3*n^2*x^3*Log[c*x^n] -
648*a*b^2*e*x*Log[c*x^n]^2 + 864*b^3*e*n*x*Log[c*x^n]^2 + 324*a*b^2*e^2*x^2
*Log[c*x^n]^2 - 270*b^3*e^2*n*x^2*Log[c*x^n]^2 - 216*a*b^2*e^3*x^3*Log[c*x^
n]^2 + 144*b^3*e^3*n*x^3*Log[c*x^n]^2 - 216*b^3*e*x*Log[c*x^n]^3 + 108*b^3*
e^2*x^2*Log[c*x^n]^3 - 72*b^3*e^3*x^3*Log[c*x^n]^3 + 216*a^3*Log[1 + e*x] -
 216*a^2*b*n*Log[1 + e*x] + 144*a*b^2*n^2*Log[1 + e*x] - 48*b^3*n^3*Log[1 +
 e*x] + 216*a^3*e^3*x^3*Log[1 + e*x] - 216*a^2*b*e^3*n*x^3*Log[1 + e*x] + 1
44*a*b^2*e^3*n^2*x^3*Log[1 + e*x] - 48*b^3*e^3*n^3*x^3*Log[1 + e*x] + 648*a
^2*b*Log[c*x^n]*Log[1 + e*x] - 432*a*b^2*n*Log[c*x^n]*Log[1 + e*x] + 144*b^
3*n^2*Log[c*x^n]*Log[1 + e*x] + 648*a^2*b*e^3*x^3*Log[c*x^n]*Log[1 + e*x] -
 432*a*b^2*e^3*n*x^3*Log[c*x^n]*Log[1 + e*x] + 144*b^3*e^3*n^2*x^3*Log[c*x^
n]*Log[1 + e*x] + 648*a*b^2*Log[c*x^n]^2*Log[1 + e*x] - 216*b^3*n*Log[c*x^n
]^2*Log[1 + e*x] + 648*a*b^2*e^3*x^3*Log[c*x^n]^2*Log[1 + e*x] - 216*b^3*e^
3*n*x^3*Log[c*x^n]^2*Log[1 + e*x] + 216*b^3*Log[c*x^n]^3*Log[1 + e*x] + 216
*b^3*e^3*x^3*Log[c*x^n]^3*Log[1 + e*x] + 72*b*n*(9*a^2 - 6*a*b*n + 2*b^2*n^
2 - 6*b*(-3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2)*PolyLog[2, -(e*x)] +
432*b^2*n^2*(-3*a + b*n - 3*b*Log[c*x^n])*PolyLog[3, -(e*x)] + 1296*b^3*n^3
*PolyLog[4, -(e*x)])/(648*e^3)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^2\log(cx^n)^3\log(ex+1)+3\,ab^2x^2\log(cx^n)^2\log(ex+1)+3\,a^2bx^2\log(cx^n)\log(ex+1)+a^3x^2\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*x^2*log(c*x^n)^2*log(e
*x + 1) + 3*a^2*b*x^2*log(c*x^n)*log(e*x + 1) + a^3*x^2*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log(cx^n)+a\right)^3x^2\log(ex+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x^2*log(e*x + 1), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int\left(b\ln(c\,x^n)+a\right)^3x^2\ln(ex+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)^3*ln(e*x+1),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)^3*ln(e*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,b^3e^3x^3-3\,b^3e^2x^2+6\,b^3ex-6\left(b^3e^3x^3+b^3\right)\log(ex+1)\right)\log(x^n)^3}{18\,e^3}+\frac{\frac{1}{3}\left(6\,x^3\log(ex+1)-e\left(\frac{2\,e^2x^3-3\,ex^2+6\,x}{e^3}-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/18*(2*b^3*e^3*x^3 - 3*b^3*e^2*x^2 + 6*b^3*e*x - 6*(b^3*e^3*x^3 + b^3)*lo
g(e*x + 1))*log(x^n)^3/e^3 + 1/6*integrate((18*(b^3*e^3*log(c)^2 + 2*a*b^2*
e^3*log(c) + a^2*b*e^3)*x^3*log(e*x + 1)*log(x^n) + 6*(b^3*e^3*log(c)^3 + 3
*a*b^2*e^3*log(c)^2 + 3*a^2*b*e^3*log(c) + a^3*e^3)*x^3*log(e*x + 1) + (2*b
^3*e^3*n*x^3 - 3*b^3*e^2*n*x^2 + 6*b^3*e*n*x - 6*(b^3*n - (3*a*b^2*e^3 - (e
^3*n - 3*e^3*log(c))*b^3)*x^3)*log(e*x + 1))*log(x^n)^2)/x, x)/e^3
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.19 $\int x\left(a+b\log\left(cx^n\right)\right)^3\log(1+ex)\,dx$

Optimal. Leaf size=530

$$\frac{3b^2n^2\text{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{3b^2n^2\text{Li}_3(-ex)\left(a+b\log\left(cx^n\right)\right)}{e^2}-\frac{3b^2n^2\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{4e^2}+\frac{3b^2n^2x\,(a}{}$$

```
[Out] 9/2*a*b^2*n^2*x/e-45/8*b^3*n^3*x/e+3/4*b^3*n^3*x^2+9/2*b^3*n^2*x*ln(c*x^n)/
e+3/4*b^2*n^2*x*(a+b*ln(c*x^n))/e-9/8*b^2*n^2*x^2*(a+b*ln(c*x^n))-9/4*b*n*x
*(a+b*ln(c*x^n))^2/e+3/4*b*n*x^2*(a+b*ln(c*x^n))^2+1/2*x*(a+b*ln(c*x^n))^3/
e-1/4*x^2*(a+b*ln(c*x^n))^3+3/8*b^3*n^3*ln(e*x+1)/e^2-3/8*b^3*n^3*x^2*ln(e*
x+1)-3/4*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/e^2+3/4*b^2*n^2*x^2*(a+b*ln(c*x^
n))*ln(e*x+1)+3/4*b*n*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^2-3/4*b*n*x^2*(a+b*ln(c
*x^n))^2*ln(e*x+1)-1/2*(a+b*ln(c*x^n))^3*ln(e*x+1)/e^2+1/2*x^2*(a+b*ln(c*x^
n))^3*ln(e*x+1)-3/4*b^3*n^3*polylog(2,-e*x)/e^2+3/2*b^2*n^2*(a+b*ln(c*x^n))
*polylog(2,-e*x)/e^2-3/2*b*n*(a+b*ln(c*x^n))^2*polylog(2,-e*x)/e^2-3/2*b^3*
n^3*polylog(3,-e*x)/e^2+3*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-e*x)/e^2-3*b^3
*n^3*polylog(4,-e*x)/e^2
```

Rubi [A] time = 0.49, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2395, 43, 2377, 2296, 2295, 2305, 2304, 2374, 2383, 6589, 2376, 2391}

$$\frac{3b^2n^2\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{3b^2n^2\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)}{e^2}-\frac{3bn\text{PolyLog}(2,-ex)\left(a+b\log\right.}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (9*a*b^2*n^2*x)/(2*e) - (45*b^3*n^3*x)/(8*e) + (3*b^3*n^3*x^2)/4 + (9*b^3*n
^2*x*Log[c*x^n])/(2*e) + (3*b^2*n^2*x*(a + b*Log[c*x^n]))/(4*e) - (9*b^2*n^
2*x^2*(a + b*Log[c*x^n]))/8 - (9*b*n*x*(a + b*Log[c*x^n])^2)/(4*e) + (3*b*n
*x^2*(a + b*Log[c*x^n])^2)/4 + (x*(a + b*Log[c*x^n])^3)/(2*e) - (x^2*(a + b
*Log[c*x^n])^3)/4 + (3*b^3*n^3*Log[1 + e*x])/(8*e^2) - (3*b^3*n^3*x^2*Log[1
 + e*x])/8 - (3*b^2*n^2*(a + b*Log[c*x^n])*Log[1 + e*x])/(4*e^2) + (3*b^2*n
^2*x^2*(a + b*Log[c*x^n])*Log[1 + e*x])/4 + (3*b*n*(a + b*Log[c*x^n])^2*Log
[1 + e*x])/(4*e^2) - (3*b*n*x^2*(a + b*Log[c*x^n])^2*Log[1 + e*x])/4 - ((a
+ b*Log[c*x^n])^3*Log[1 + e*x])/(2*e^2) + (x^2*(a + b*Log[c*x^n])^3*Log[1 +
 e*x])/2 - (3*b^3*n^3*PolyLog[2, -(e*x)])/(4*e^2) + (3*b^2*n^2*(a + b*Log[c
*x^n])*PolyLog[2, -(e*x)])/(2*e^2) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2,
 -(e*x)])/(2*e^2) - (3*b^3*n^3*PolyLog[3, -(e*x)])/(2*e^2) + (3*b^2*n^2*(a
+ b*Log[c*x^n])*PolyLog[3, -(e*x)])/e^2 - (3*b^3*n^3*PolyLog[4, -(e*x)])/e^
2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

```
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)^3\log(1+ex)\,dx = \frac{x\left(a+b\log\left(cx^n\right)\right)^3}{2e} - \frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right)^3 - \frac{\left(a+b\log\left(cx^n\right)\right)^3\log(1+}{2e^2}$$

$$= \frac{x\left(a+b\log\left(cx^n\right)\right)^3}{2e} - \frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right)^3 - \frac{\left(a+b\log\left(cx^n\right)\right)^3\log(1+}{2e^2}$$

$$= -\frac{9bnx\left(a+b\log\left(cx^n\right)\right)^2}{4e} + \frac{3}{4}bnx^2\left(a+b\log\left(cx^n\right)\right)^2 + \frac{x\left(a+b\log\left(cx^n\right)\right)}{2e}$$

$$= \frac{3ab^2n^2x}{e} + \frac{3}{16}b^3n^3x^2 - \frac{3}{8}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right) - \frac{9bnx\left(a+b\log\left(cx^n\right)\right)}{4e}$$

$$= \frac{9ab^2n^2x}{2e} - \frac{3b^3n^3x}{e} + \frac{3}{8}b^3n^3x^2 + \frac{3b^3n^2x\log\left(cx^n\right)}{e} + \frac{3b^2n^2x\left(a+b\log\left(c\right.\right.}{4e}$$

$$= \frac{9ab^2n^2x}{2e} - \frac{21b^3n^3x}{4e} + \frac{9}{16}b^3n^3x^2 + \frac{9b^3n^2x\log\left(cx^n\right)}{2e} + \frac{3b^2n^2x\left(a+b\log\right.}{4e}$$

$$= \frac{9ab^2n^2x}{2e} - \frac{21b^3n^3x}{4e} + \frac{9}{16}b^3n^3x^2 + \frac{9b^3n^2x\log\left(cx^n\right)}{2e} + \frac{3b^2n^2x\left(a+b\log\right.}{4e}$$

$$= \frac{9ab^2n^2x}{2e} - \frac{21b^3n^3x}{4e} + \frac{9}{16}b^3n^3x^2 + \frac{9b^3n^2x\log\left(cx^n\right)}{2e} + \frac{3b^2n^2x\left(a+b\log\right.}{4e}$$

$$= \frac{9ab^2n^2x}{2e} - \frac{45b^3n^3x}{8e} + \frac{3}{4}b^3n^3x^2 + \frac{9b^3n^2x\log\left(cx^n\right)}{2e} + \frac{3b^2n^2x\left(a+b\log\left(\right.\right.}{4e}$$

Mathematica [A] time = 0.28, size = 806, normalized size = 1.52

$$-2e^2x^2a^3 + 4exa^3 + 4e^2x^2\log(ex+1)a^3 - 4\log(ex+1)a^3 + 6be^2nx^2a^2 - 18benxa^2 - 6be^2x^2\log\left(cx^n\right)a^2 + 12bex$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (4*a^3*e*x - 18*a^2*b*e*n*x + 42*a*b^2*e*n^2*x - 45*b^3*e*n^3*x - 2*a^3*e^2
*x^2 + 6*a^2*b*e^2*n*x^2 - 9*a*b^2*e^2*n^2*x^2 + 6*b^3*e^2*n^3*x^2 + 12*a^2
*b*e*x*Log[c*x^n] - 36*a*b^2*e*n*x*Log[c*x^n] + 42*b^3*e*n^2*x*Log[c*x^n] -
 6*a^2*b*e^2*x^2*Log[c*x^n] + 12*a*b^2*e^2*n*x^2*Log[c*x^n] - 9*b^3*e^2*n^2
*x^2*Log[c*x^n] + 12*a*b^2*e*x*Log[c*x^n]^2 - 18*b^3*e*n*x*Log[c*x^n]^2 - 6
*a*b^2*e^2*x^2*Log[c*x^n]^2 + 6*b^3*e^2*n*x^2*Log[c*x^n]^2 + 4*b^3*e*x*Log[
c*x^n]^3 - 2*b^3*e^2*x^2*Log[c*x^n]^3 - 4*a^3*Log[1 + e*x] + 6*a^2*b*n*Log[
1 + e*x] - 6*a*b^2*n^2*Log[1 + e*x] + 3*b^3*n^3*Log[1 + e*x] + 4*a^3*e^2*x^
2*Log[1 + e*x] - 6*a^2*b*e^2*n*x^2*Log[1 + e*x] + 6*a*b^2*e^2*n^2*x^2*Log[1
```

```
 + e*x] - 3*b^3*e^2*n^3*x^2*Log[1 + e*x] - 12*a^2*b*Log[c*x^n]*Log[1 + e*x]
 + 12*a*b^2*n*Log[c*x^n]*Log[1 + e*x] - 6*b^3*n^2*Log[c*x^n]*Log[1 + e*x] +
 12*a^2*b*e^2*x^2*Log[c*x^n]*Log[1 + e*x] - 12*a*b^2*e^2*n*x^2*Log[c*x^n]*L
og[1 + e*x] + 6*b^3*e^2*n^2*x^2*Log[c*x^n]*Log[1 + e*x] - 12*a*b^2*Log[c*x^
n]^2*Log[1 + e*x] + 6*b^3*n*Log[c*x^n]^2*Log[1 + e*x] + 12*a*b^2*e^2*x^2*Lo
g[c*x^n]^2*Log[1 + e*x] - 6*b^3*e^2*n*x^2*Log[c*x^n]^2*Log[1 + e*x] - 4*b^3
*Log[c*x^n]^3*Log[1 + e*x] + 4*b^3*e^2*x^2*Log[c*x^n]^3*Log[1 + e*x] - 6*b*
n*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^
n]^2)*PolyLog[2, -(e*x)] + 12*b^2*n^2*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[
3, -(e*x)] - 24*b^3*n^3*PolyLog[4, -(e*x)])/(8*e^2)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x\log\left(cx^n\right)^3\log\left(ex+1\right)+3\,ab^2x\log\left(cx^n\right)^2\log\left(ex+1\right)+3\,a^2bx\log\left(cx^n\right)\log\left(ex+1\right)+a^3x\log\left(e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*x*log(c*x^n)^2*log(e*x +
 1) + 3*a^2*b*x*log(c*x^n)*log(e*x + 1) + a^3*x*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^3x\log\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log(e*x + 1), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^3x\ln\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^3*ln(e*x+1),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^3*ln(e*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^3e^2x^2-2\,b^3ex-2\left(b^3e^2x^2-b^3\right)\log\left(ex+1\right)\right)\log\left(x^n\right)^3}{4\,e^2}+\frac{\left(2\,x^2\log\left(ex+1\right)-e\left(\frac{ex^2-2\,x}{e^2}+\frac{2\log(ex+1)}{e^3}\right)\right)b^3e^2\log\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*e^2*x^2 - 2*b^3*e*x - 2*(b^3*e^2*x^2 - b^3)*log(e*x + 1))*log(x^n
)^3/e^2 + 1/4*integrate((12*(b^3*e^2*log(c)^2 + 2*a*b^2*e^2*log(c) + a^2*b*
e^2)*x^2*log(e*x + 1)*log(x^n) + 4*(b^3*e^2*log(c)^3 + 3*a*b^2*e^2*log(c)^2
 + 3*a^2*b*e^2*log(c) + a^3*e^2)*x^2*log(e*x + 1) + 3*(b^3*e^2*n*x^2 - 2*b^
3*e*n*x + 2*(b^3*n + (2*a*b^2*e^2 - (e^2*n - 2*e^2*log(c))*b^3)*x^2)*log(e*
x + 1))*log(x^n)^2)/x, x)/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x*log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.20 $\int \left(a+b\log\left(cx^n\right)\right)^3\log(1+ex)\,dx$

Optimal. Leaf size=327

$$-\frac{6b^2n^2\mathrm{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{6b^2n^2\mathrm{Li}_3(-ex)\left(a+b\log\left(cx^n\right)\right)}{e}+\frac{6b^2n^2(ex+1)\log(ex+1)\left(a+b\log\left(cx^n\right)\right.}{e}$$

```
[Out] -12*a*b^2*n^2*x+24*b^3*n^3*x-12*b^3*n^2*x*ln(c*x^n)-6*b^2*n^2*x*(a+b*ln(c*x
^n))+6*b*n*x*(a+b*ln(c*x^n))^2-x*(a+b*ln(c*x^n))^3-6*b^3*n^3*(e*x+1)*ln(e*x
+1)/e+6*b^2*n^2*(e*x+1)*(a+b*ln(c*x^n))*ln(e*x+1)/e-3*b*n*(e*x+1)*(a+b*ln(c
*x^n))^2*ln(e*x+1)/e+(e*x+1)*(a+b*ln(c*x^n))^3*ln(e*x+1)/e+6*b^3*n^3*polylo
g(2,-e*x)/e-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-e*x)/e+3*b*n*(a+b*ln(c*x^n
))^2*polylog(2,-e*x)/e+6*b^3*n^3*polylog(3,-e*x)/e-6*b^2*n^2*(a+b*ln(c*x^n)
)*polylog(3,-e*x)/e+6*b^3*n^3*polylog(4,-e*x)/e
```

Rubi [A] time = 0.76, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.842, Rules used = {2389, 2295, 2370, 2296, 2346, 2302, 30, 6742, 2301, 2411, 43, 2351, 2315, 2374, 6589, 2383}

$$-\frac{6b^2n^2\mathrm{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{6b^2n^2\mathrm{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)}{e}+\frac{3bn\mathrm{PolyLog}(2,-ex)\left(a+\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] -12*a*b^2*n^2*x + 24*b^3*n^3*x - 12*b^3*n^2*x*Log[c*x^n] - 6*b^2*n^2*x*(a +
 b*Log[c*x^n]) + 6*b*n*x*(a + b*Log[c*x^n])^2 - x*(a + b*Log[c*x^n])^3 - (6
*b^3*n^3*(1 + e*x)*Log[1 + e*x])/e + (6*b^2*n^2*(1 + e*x)*(a + b*Log[c*x^n]
)*Log[1 + e*x])/e - (3*b*n*(1 + e*x)*(a + b*Log[c*x^n])^2*Log[1 + e*x])/e +
 ((1 + e*x)*(a + b*Log[c*x^n])^3*Log[1 + e*x])/e + (6*b^3*n^3*PolyLog[2, -(
e*x)])/e - (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/e + (3*b*n*(a
+ b*Log[c*x^n])^2*PolyLog[2, -(e*x)])/e + (6*b^3*n^3*PolyLog[3, -(e*x)])/e
- (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e*x)])/e + (6*b^3*n^3*PolyLog[
4, -(e*x)])/e
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)^3\log(1+ex)\,dx = -x\left(a+b\log\left(cx^n\right)\right)^3+\frac{(1+ex)\left(a+b\log\left(cx^n\right)\right)^3\log(1+ex)}{e}-(3bn)\int \Bigg(-$$

$$= -x\left(a+b\log\left(cx^n\right)\right)^3+\frac{(1+ex)\left(a+b\log\left(cx^n\right)\right)^3\log(1+ex)}{e}+(3bn)\int \big(a$$

$$= 3bnx\left(a+b\log\left(cx^n\right)\right)^2-x\left(a+b\log\left(cx^n\right)\right)^3+\frac{(1+ex)\left(a+b\log\left(cx^n\right)\right)^3 \mathrm{l}}{e}$$

$$= -6ab^2n^2x+3bnx\left(a+b\log\left(cx^n\right)\right)^2-x\left(a+b\log\left(cx^n\right)\right)^3+\frac{(1+ex)\big(a+b}{}$$

$$= -6ab^2n^2x+6b^3n^3x-6b^3n^2x\log\left(cx^n\right)+6bnx\left(a+b\log\left(cx^n\right)\right)^2-x\big(a+b$$

$$= -12ab^2n^2x+6b^3n^3x-6b^3n^2x\log\left(cx^n\right)+6bnx\left(a+b\log\left(cx^n\right)\right)^2-x\big(a+$$

$$= -12ab^2n^2x+12b^3n^3x-12b^3n^2x\log\left(cx^n\right)+6bnx\left(a+b\log\left(cx^n\right)\right)^2-x\big(a$$

$$= -12ab^2n^2x+12b^3n^3x-12b^3n^2x\log\left(cx^n\right)+6bnx\left(a+b\log\left(cx^n\right)\right)^2-x\big(a$$

$$= -12ab^2n^2x+12b^3n^3x-12b^3n^2x\log\left(cx^n\right)-6b^2n^2x\left(a+b\log\left(cx^n\right)\right)+6bn$$

$$= -12ab^2n^2x+18b^3n^3x-12b^3n^2x\log\left(cx^n\right)-6b^2n^2x\left(a+b\log\left(cx^n\right)\right)+6bn$$

$$= -12ab^2n^2x+18b^3n^3x-12b^3n^2x\log\left(cx^n\right)-6b^2n^2x\left(a+b\log\left(cx^n\right)\right)+6bn$$

$$= -12ab^2n^2x+18b^3n^3x-12b^3n^2x\log\left(cx^n\right)-6b^2n^2x\left(a+b\log\left(cx^n\right)\right)+6bn$$

$$= -12ab^2n^2x+18b^3n^3x-12b^3n^2x\log\left(cx^n\right)-6b^2n^2x\left(a+b\log\left(cx^n\right)\right)+6bn$$

$$= -12ab^2n^2x+24b^3n^3x-12b^3n^2x\log\left(cx^n\right)-6b^2n^2x\left(a+b\log\left(cx^n\right)\right)+6bn$$

Mathematica [A] time = 0.19, size = 584, normalized size = 1.79

$$\frac{a^3(-e)x+a^3ex\log(ex+1)+a^3\log(ex+1)+3bn\mathrm{Li}_2(-ex)\left(a^2+2b(a-bn)\log\left(cx^n\right)-2abn+b^2\log^2\left(cx^n\right)+2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (-(a^3*e*x) + 6*a^2*b*e*n*x - 18*a*b^2*e*n^2*x + 24*b^3*e*n^3*x - 3*a^2*b*e
*x*Log[c*x^n] + 12*a*b^2*e*n*x*Log[c*x^n] - 18*b^3*e*n^2*x*Log[c*x^n] - 3*a
*b^2*e*x*Log[c*x^n]^2 + 6*b^3*e*n*x*Log[c*x^n]^2 - b^3*e*x*Log[c*x^n]^3 + a
^3*Log[1 + e*x] - 3*a^2*b*n*Log[1 + e*x] + 6*a*b^2*n^2*Log[1 + e*x] - 6*b^3
```

```
*n^3*Log[1 + e*x] + a^3*e*x*Log[1 + e*x] - 3*a^2*b*e*n*x*Log[1 + e*x] + 6*a
*b^2*e*n^2*x*Log[1 + e*x] - 6*b^3*e*n^3*x*Log[1 + e*x] + 3*a^2*b*Log[c*x^n]
*Log[1 + e*x] - 6*a*b^2*n*Log[c*x^n]*Log[1 + e*x] + 6*b^3*n^2*Log[c*x^n]*Lo
g[1 + e*x] + 3*a^2*b*e*x*Log[c*x^n]*Log[1 + e*x] - 6*a*b^2*e*n*x*Log[c*x^n]
*Log[1 + e*x] + 6*b^3*e*n^2*x*Log[c*x^n]*Log[1 + e*x] + 3*a*b^2*Log[c*x^n]^
2*Log[1 + e*x] - 3*b^3*n*Log[c*x^n]^2*Log[1 + e*x] + 3*a*b^2*e*x*Log[c*x^n]
^2*Log[1 + e*x] - 3*b^3*e*n*x*Log[c*x^n]^2*Log[1 + e*x] + b^3*Log[c*x^n]^3*
Log[1 + e*x] + b^3*e*x*Log[c*x^n]^3*Log[1 + e*x] + 3*b*n*(a^2 - 2*a*b*n + 2
*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -(e*x)]
- 6*b^2*n^2*(a - b*n + b*Log[c*x^n])*PolyLog[3, -(e*x)] + 6*b^3*n^3*PolyLog
[4, -(e*x)])/e
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3 \log\left(cx^n\right)^3 \log\left(ex+1\right) + 3\,ab^2 \log\left(cx^n\right)^2 \log\left(ex+1\right) + 3\,a^2 b \log\left(cx^n\right) \log\left(ex+1\right) + a^3 \log\left(ex+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*log(c*x^n)^2*log(e*x + 1)
+ 3*a^2*b*log(c*x^n)*log(e*x + 1) + a^3*log(e*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^n\right) + a\right)^3 \log\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log(e*x + 1), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\,x^n\right) + a\right)^3 \ln\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(e*x+1),x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(e*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^3ex - \left(b^3ex + b^3\right)\log\left(ex+1\right)\right)\log\left(x^n\right)^3}{e} + \frac{-\left(ex - (ex+1)\log\left(ex+1\right) + 1\right)b^3\log(c)^3 - 3\left(ex - (ex+1)\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -(b^3*e*x - (b^3*e*x + b^3)*log(e*x + 1))*log(x^n)^3/e + integrate((3*(b^3*
e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x*log(e*x + 1)*log(x^n) + (b^3*e*l
og(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x*log(e*x + 1) + 3
*(b^3*e*n*x - (b^3*n + ((e*n - e*log(c))*b^3 - a*b^2*e)*x)*log(e*x + 1))*lo
g(x^n)^2)/x, x)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\,x+1\right)\,\left(a + b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

```
[Out] Timed out
```

3.21 $\int \frac{(a+b\log(cx^n))^3 \log(1+ex)}{x}\,dx$

Optimal. Leaf size=81

$$-6b^2n^2\text{Li}_4(-ex)\left(a+b\log\left(cx^n\right)\right)+3bn\text{Li}_3(-ex)\left(a+b\log\left(cx^n\right)\right)^2-\text{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)^3+6b^3n^3\text{Li}_5(-ex)$$

```
[Out] -(a+b*ln(c*x^n))^3*polylog(2,-e*x)+3*b*n*(a+b*ln(c*x^n))^2*polylog(3,-e*x)-
6*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-e*x)+6*b^3*n^3*polylog(5,-e*x)
```

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2374, 2383, 6589}

$$-6b^2n^2\text{PolyLog}(4,-ex)\left(a+b\log\left(cx^n\right)\right)+3bn\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)^2-\text{PolyLog}(2,-ex)\left(a+b\log\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])^3*PolyLog[2, -(e*x)]) + 3*b*n*(a + b*Log[c*x^n])^2*Pol
yLog[3, -(e*x)] - 6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[4, -(e*x)] + 6*b^3*n
^3*PolyLog[5, -(e*x)]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log(1+ex)}{x}\,dx &= -\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2(-ex)+(3bn)\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2(-ex)}{x}\,dx\\ &= -\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2(-ex)+3bn\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_3(-ex)-\left(6b^2n^2\right.\\ &= -\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2(-ex)+3bn\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_3(-ex)-6b^2n^2\left(\right.\\ &= -\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2(-ex)+3bn\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_3(-ex)-6b^2n^2\left(\right.\end{aligned}$$

Mathematica [A] time = 0.12, size = 77, normalized size = 0.95

$$3bn\left(\text{Li}_3(-ex)\left(a+b\log\left(cx^n\right)\right)^2+2bn\left(bn\text{Li}_5(-ex)-\text{Li}_4(-ex)\left(a+b\log\left(cx^n\right)\right)\right)\right)-\text{Li}_2(-ex)\left(a+b\log\left(cx^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])^3*PolyLog[2, -(e*x)]) + 3*b*n*((a + b*Log[c*x^n])^2*Po
lyLog[3, -(e*x)] + 2*b*n*(-((a + b*Log[c*x^n])*PolyLog[4, -(e*x)]) + b*n*Po
lyLog[5, -(e*x)]))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(cx^n\right)^3\log\left(ex+1\right)+3\,ab^2\log\left(cx^n\right)^2\log\left(ex+1\right)+3\,a^2b\log\left(cx^n\right)\log\left(ex+1\right)+a^3\log\left(ex+1\right)}{x},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*log(c*x^n)^2*log(e*x + 1)
 + 3*a^2*b*log(c*x^n)*log(e*x + 1) + a^3*log(e*x + 1))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(ex+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log(e*x + 1)/x, x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^n\right)+a\right)^3\ln\left(ex+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(e*x+1)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(e*x+1)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(ex+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log(e*x + 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(e*x+1)/x,x)
```

```
[Out] Timed out
```

3.22 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx$

Optimal. Leaf size=342

$$6b^2en^2\text{Li}_2\left(-\frac{1}{ex}\right)\left(a+b\log\left(cx^n\right)\right)+6b^2en^2\text{Li}_3\left(-\frac{1}{ex}\right)\left(a+b\log\left(cx^n\right)\right)-6b^2en^2\log\left(\frac{1}{ex}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{6b^2\cdots}{\cdots}$$

```
[Out] 6*b^3*e*n^3*ln(x)-6*b^2*e*n^2*ln(1+1/e/x)*(a+b*ln(c*x^n))-3*b*e*n*ln(1+1/e/
x)*(a+b*ln(c*x^n))^2-e*ln(1+1/e/x)*(a+b*ln(c*x^n))^3-6*b^3*e*n^3*ln(e*x+1)-
6*b^3*n^3*ln(e*x+1)/x-6*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/x-3*b*n*(a+b*ln(c
*x^n))^2*ln(e*x+1)/x-(a+b*ln(c*x^n))^3*ln(e*x+1)/x+6*b^3*e*n^3*polylog(2,-1
/e/x)+6*b^2*e*n^2*(a+b*ln(c*x^n))*polylog(2,-1/e/x)+3*b*e*n*(a+b*ln(c*x^n))
^2*polylog(2,-1/e/x)+6*b^3*e*n^3*polylog(3,-1/e/x)+6*b^2*e*n^2*(a+b*ln(c*x^
n))*polylog(3,-1/e/x)+6*b^3*e*n^3*polylog(4,-1/e/x)
```

Rubi [A] time = 0.60, antiderivative size = 360, normalized size of antiderivative = 1.05, number of steps used = 22, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {2305, 2304, 2378, 36, 29, 31, 2344, 2301, 2317, 2391, 2302, 30, 2374, 6589, 2383}

$$-6b^2en^2\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)+6b^2en^2\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)-3ben\text{PolyLog}(2,-ex)\left(a+b\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^2,x]
```

```
[Out] 6*b^3*e*n^3*Log[x] + 3*b*e*n*(a + b*Log[c*x^n])^2 + e*(a + b*Log[c*x^n])^3
+ (e*(a + b*Log[c*x^n])^4)/(4*b*n) - 6*b^3*e*n^3*Log[1 + e*x] - (6*b^3*n^3*
Log[1 + e*x])/x - 6*b^2*e*n^2*(a + b*Log[c*x^n])*Log[1 + e*x] - (6*b^2*n^2*
(a + b*Log[c*x^n])*Log[1 + e*x])/x - 3*b*e*n*(a + b*Log[c*x^n])^2*Log[1 + e
*x] - (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + e*x])/x - e*(a + b*Log[c*x^n])^3*
Log[1 + e*x] - ((a + b*Log[c*x^n])^3*Log[1 + e*x])/x - 6*b^3*e*n^3*PolyLog[
2, -(e*x)] - 6*b^2*e*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(e*x)] - 3*b*e*n*(a
 + b*Log[c*x^n])^2*PolyLog[2, -(e*x)] + 6*b^3*e*n^3*PolyLog[3, -(e*x)] + 6*
b^2*e*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e*x)] - 6*b^3*e*n^3*PolyLog[4, -(
e*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
```

```
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log(1+ex)}{x^2}\,dx = -\frac{6b^3n^3\log(1+ex)}{x} - \frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x} - \frac{3bn\left(a+b\log\right.}{}$$

$$= -\frac{6b^3n^3\log(1+ex)}{x} - \frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x} - \frac{3bn\left(a+b\log\right.}{}$$

$$= -\frac{6b^3n^3\log(1+ex)}{x} - \frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{x} - \frac{3bn\left(a+b\log\right.}{}$$

$$= 6b^3en^3\log(x) + 3ben\left(a+b\log\left(cx^n\right)\right)^2 - 6b^3en^3\log(1+ex) - \frac{6b^3n^3\log(1}{x}$$

$$= 6b^3en^3\log(x) + 3ben\left(a+b\log\left(cx^n\right)\right)^2 + e\left(a+b\log\left(cx^n\right)\right)^3 + \frac{e\left(a+b\log\right.}{4bn}$$

$$= 6b^3en^3\log(x) + 3ben\left(a+b\log\left(cx^n\right)\right)^2 + e\left(a+b\log\left(cx^n\right)\right)^3 + \frac{e\left(a+b\log\right.}{4bn}$$

$$= 6b^3en^3\log(x) + 3ben\left(a+b\log\left(cx^n\right)\right)^2 + e\left(a+b\log\left(cx^n\right)\right)^3 + \frac{e\left(a+b\log\right.}{4bn}$$

Mathematica [B] time = 0.32, size = 770, normalized size = 2.25

$$a^3e\log(x)-a^3e\log(ex+1)-\frac{a^3\log(ex+1)}{x}-3ben\text{Li}_2(-ex)\left(a^2+2b(a+bn)\log\left(cx^n\right)+2abn+b^2\log^2\left(cx^n\right)+2b^2n\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^2,x]
```

```
[Out] a^3*e*Log[x] + 3*a^2*b*e*n*Log[x] + 6*a*b^2*e*n^2*Log[x] + 6*b^3*e*n^3*Log[
x] - (3*a^2*b*e*n*Log[x]^2)/2 - 3*a*b^2*e*n^2*Log[x]^2 - 3*b^3*e*n^3*Log[x]
^2 + a*b^2*e*n^2*Log[x]^3 + b^3*e*n^3*Log[x]^3 - (b^3*e*n^3*Log[x]^4)/4 + 3
*a^2*b*e*Log[x]*Log[c*x^n] + 6*a*b^2*e*n*Log[x]*Log[c*x^n] + 6*b^3*e*n^2*Lo
g[x]*Log[c*x^n] - 3*a*b^2*e*n*Log[x]^2*Log[c*x^n] - 3*b^3*e*n^2*Log[x]^2*Lo
g[c*x^n] + b^3*e*n^2*Log[x]^3*Log[c*x^n] + 3*a*b^2*e*Log[x]*Log[c*x^n]^2 +
3*b^3*e*n*Log[x]*Log[c*x^n]^2 - (3*b^3*e*n*Log[x]^2*Log[c*x^n]^2)/2 + b^3*e
*Log[x]*Log[c*x^n]^3 - a^3*e*Log[1 + e*x] - 3*a^2*b*e*n*Log[1 + e*x] - 6*a*
b^2*e*n^2*Log[1 + e*x] - 6*b^3*e*n^3*Log[1 + e*x] - (a^3*Log[1 + e*x])/x -
(3*a^2*b*n*Log[1 + e*x])/x - (6*a*b^2*n^2*Log[1 + e*x])/x - (6*b^3*n^3*Log[
1 + e*x])/x - 3*a^2*b*e*Log[c*x^n]*Log[1 + e*x] - 6*a*b^2*e*n*Log[c*x^n]*Lo
g[1 + e*x] - 6*b^3*e*n^2*Log[c*x^n]*Log[1 + e*x] - (3*a^2*b*Log[c*x^n]*Log[
1 + e*x])/x - (6*a*b^2*n*Log[c*x^n]*Log[1 + e*x])/x - (6*b^3*n^2*Log[c*x^n]
```

```
*Log[1 + e*x])/x - 3*a*b^2*e*Log[c*x^n]^2*Log[1 + e*x] - 3*b^3*e*n*Log[c*x^
n]^2*Log[1 + e*x] - (3*a*b^2*Log[c*x^n]^2*Log[1 + e*x])/x - (3*b^3*n*Log[c*
x^n]^2*Log[1 + e*x])/x - b^3*e*Log[c*x^n]^3*Log[1 + e*x] - (b^3*Log[c*x^n]^
3*Log[1 + e*x])/x - 3*b*e*n*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*Log[
c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -(e*x)] + 6*b^2*e*n^2*(a + b*n + b*Lo
g[c*x^n])*PolyLog[3, -(e*x)] - 6*b^3*e*n^3*PolyLog[4, -(e*x)]
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(cx^n\right)^3\log\left(ex+1\right)+3\,ab^2\log\left(cx^n\right)^2\log\left(ex+1\right)+3\,a^2b\log\left(cx^n\right)\log\left(ex+1\right)+a^3\log\left(ex+1\right)}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*log(c*x^n)^2*log(e*x + 1)
 + 3*a^2*b*log(c*x^n)*log(e*x + 1) + a^3*log(e*x + 1))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(ex+1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log(e*x + 1)/x^2, x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right)+a\right)^3\ln\left(ex+1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(e*x+1)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(e*x+1)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3ex\log(x)-\left(b^3ex+b^3\right)\log\left(ex+1\right)\right)\log\left(x^n\right)^3}{x}+\int \frac{3\left(b^3\log(c)^2+2\,ab^2\log(c)+a^2b\right)\log\left(ex+1\right)\log\left(x^n\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] (b^3*e*x*log(x) - (b^3*e*x + b^3)*log(e*x + 1))*log(x^n)^3/x + integrate((3
*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log(e*x + 1)*log(x^n) - 3*(b^3*e*n
*x*log(x) - (b^3*e*n*x + b^3*(n + log(c)) + a*b^2)*log(e*x + 1))*log(x^n)^2
 + (b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*log(e*x + 1))/x
^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^2,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(e*x+1)/x**2,x)
```

```
[Out] Timed out
```

3.23 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx$

Optimal. Leaf size=470

$$-\frac{3}{2}b^2e^2n^2\text{Li}_2\left(-\frac{1}{ex}\right)\left(a+b\log\left(cx^n\right)\right)-3b^2e^2n^2\text{Li}_3\left(-\frac{1}{ex}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{3}{4}b^2e^2n^2\log\left(\frac{1}{ex}+1\right)\left(a+b\log\left(cx\right.\right.$$

```
[Out] -45/8*b^3*e*n^3/x-3/8*b^3*e^2*n^3*ln(x)-21/4*b^2*e*n^2*(a+b*ln(c*x^n))/x+3/
4*b^2*e^2*n^2*ln(1+1/e/x)*(a+b*ln(c*x^n))-9/4*b*e*n*(a+b*ln(c*x^n))^2/x+3/4
*b*e^2*n*ln(1+1/e/x)*(a+b*ln(c*x^n))^2-1/2*e*(a+b*ln(c*x^n))^3/x+1/2*e^2*ln
(1+1/e/x)*(a+b*ln(c*x^n))^3+3/8*b^3*e^2*n^3*ln(e*x+1)-3/8*b^3*n^3*ln(e*x+1)
/x^2-3/4*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln
(e*x+1)/x^2-1/2*(a+b*ln(c*x^n))^3*ln(e*x+1)/x^2-3/4*b^3*e^2*n^3*polylog(2,-
1/e/x)-3/2*b^2*e^2*n^2*(a+b*ln(c*x^n))*polylog(2,-1/e/x)-3/2*b*e^2*n*(a+b*l
n(c*x^n))^2*polylog(2,-1/e/x)-3/2*b^3*e^2*n^3*polylog(3,-1/e/x)-3*b^2*e^2*n
^2*(a+b*ln(c*x^n))*polylog(3,-1/e/x)-3*b^3*e^2*n^3*polylog(4,-1/e/x)
```

Rubi [A] time = 0.82, antiderivative size = 499, normalized size of antiderivative = 1.06, number of steps used = 30, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2305, 2304, 2378, 44, 2351, 2301, 2317, 2391, 2353, 2302, 30, 2374, 6589, 2383}

$$\frac{3}{2}b^2e^2n^2\text{PolyLog}(2,-ex)\left(a+b\log\left(cx^n\right)\right)-3b^2e^2n^2\text{PolyLog}(3,-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}be^2n\text{PolyLog}(2,-ex)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^3,x]
```

```
[Out] (-45*b^3*e*n^3)/(8*x) - (3*b^3*e^2*n^3*Log[x])/8 - (21*b^2*e*n^2*(a + b*Log
[c*x^n]))/(4*x) - (3*b*e^2*n*(a + b*Log[c*x^n])^2)/8 - (9*b*e*n*(a + b*Log[
c*x^n])^2)/(4*x) - (e^2*(a + b*Log[c*x^n])^3)/4 - (e*(a + b*Log[c*x^n])^3)/
(2*x) - (e^2*(a + b*Log[c*x^n])^4)/(8*b*n) + (3*b^3*e^2*n^3*Log[1 + e*x])/8
 - (3*b^3*n^3*Log[1 + e*x])/(8*x^2) + (3*b^2*e^2*n^2*(a + b*Log[c*x^n])*Log
[1 + e*x])/4 - (3*b^2*n^2*(a + b*Log[c*x^n])*Log[1 + e*x])/(4*x^2) + (3*b*e
^2*n*(a + b*Log[c*x^n])^2*Log[1 + e*x])/4 - (3*b*n*(a + b*Log[c*x^n])^2*Log
[1 + e*x])/(4*x^2) + (e^2*(a + b*Log[c*x^n])^3*Log[1 + e*x])/2 - ((a + b*Lo
g[c*x^n])^3*Log[1 + e*x])/(2*x^2) + (3*b^3*e^2*n^3*PolyLog[2, -(e*x)])/4 +
(3*b^2*e^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/2 + (3*b*e^2*n*(a + b
*Log[c*x^n])^2*PolyLog[2, -(e*x)])/2 - (3*b^3*e^2*n^3*PolyLog[3, -(e*x)])/2
 - 3*b^2*e^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e*x)] + 3*b^3*e^2*n^3*Poly
Log[4, -(e*x)]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log(1+ex)}{x^3}\,dx = -\frac{3b^3n^3\log(1+ex)}{8x^2} - \frac{3b^2n^2\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{4x^2} - \frac{3bn\left(a+b\,l\right.}{}$$

$$= -\frac{3b^3n^3\log(1+ex)}{8x^2} - \frac{3b^2n^2\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{4x^2} - \frac{3bn\left(a+b\,l\right.}{}$$

$$= -\frac{3b^3n^3\log(1+ex)}{8x^2} - \frac{3b^2n^2\left(a+b\log\left(cx^n\right)\right)\log(1+ex)}{4x^2} - \frac{3bn\left(a+b\,l\right.}{}$$

$$= -\frac{3b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) + \frac{3}{8}b^3e^2n^3\log(1+ex) - \frac{3b^3n^3\log(1+ex)}{8x^2} -$$

$$= -\frac{9b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) - \frac{3b^2en^2\left(a+b\log\left(cx^n\right)\right)}{4x} - \frac{3}{8}be^2n\left(a+b\,\mathrm{lo}\right.$$

$$= -\frac{21b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) - \frac{9b^2en^2\left(a+b\log\left(cx^n\right)\right)}{4x} - \frac{3}{8}be^2n\left(a+b\,\mathrm{l}\right.$$

$$= -\frac{45b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) - \frac{21b^2en^2\left(a+b\log\left(cx^n\right)\right)}{4x} - \frac{3}{8}be^2n\left(a+b\right.$$

$$= -\frac{45b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) - \frac{21b^2en^2\left(a+b\log\left(cx^n\right)\right)}{4x} - \frac{3}{8}be^2n\left(a+b\right.$$

Mathematica [B] time = 0.40, size = 1047, normalized size = 2.23

$$-\frac{-b^3e^2n^3x^2\log^4(x) + 2b^3e^2n^3x^2\log^3(x) + 4ab^2e^2n^2x^2\log^3(x) + 4b^3e^2n^2x^2\log\left(cx^n\right)\log^3(x) - 3b^3e^2n^3x^2\log^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^3,x]
```

```
[Out] -1/8*(4*a^3*e*x + 18*a^2*b*e*n*x + 42*a*b^2*e*n^2*x + 45*b^3*e*n^3*x + 4*a^
3*e^2*x^2*Log[x] + 6*a^2*b*e^2*n*x^2*Log[x] + 6*a*b^2*e^2*n^2*x^2*Log[x] +
3*b^3*e^2*n^3*x^2*Log[x] - 6*a^2*b*e^2*n*x^2*Log[x]^2 - 6*a*b^2*e^2*n^2*x^2
*Log[x]^2 - 3*b^3*e^2*n^3*x^2*Log[x]^2 + 4*a*b^2*e^2*n^2*x^2*Log[x]^3 + 2*b
^3*e^2*n^3*x^2*Log[x]^3 - b^3*e^2*n^3*x^2*Log[x]^4 + 12*a^2*b*e*x*Log[c*x^n
] + 36*a*b^2*e*n*x*Log[c*x^n] + 42*b^3*e*n^2*x*Log[c*x^n] + 12*a^2*b*e^2*x^
```

```
2*Log[x]*Log[c*x^n] + 12*a*b^2*e^2*n*x^2*Log[x]*Log[c*x^n] + 6*b^3*e^2*n^2*
x^2*Log[x]*Log[c*x^n] - 12*a*b^2*e^2*n*x^2*Log[x]^2*Log[c*x^n] - 6*b^3*e^2*
n^2*x^2*Log[x]^2*Log[c*x^n] + 4*b^3*e^2*n^2*x^2*Log[x]^3*Log[c*x^n] + 12*a*
b^2*e*x*Log[c*x^n]^2 + 18*b^3*e*n*x*Log[c*x^n]^2 + 12*a*b^2*e^2*x^2*Log[x]*
Log[c*x^n]^2 + 6*b^3*e^2*n*x^2*Log[x]*Log[c*x^n]^2 - 6*b^3*e^2*n*x^2*Log[x]
^2*Log[c*x^n]^2 + 4*b^3*e*x*Log[c*x^n]^3 + 4*b^3*e^2*x^2*Log[x]*Log[c*x^n]^
3 + 4*a^3*Log[1 + e*x] + 6*a^2*b*n*Log[1 + e*x] + 6*a*b^2*n^2*Log[1 + e*x]
+ 3*b^3*n^3*Log[1 + e*x] - 4*a^3*e^2*x^2*Log[1 + e*x] - 6*a^2*b*e^2*n*x^2*L
og[1 + e*x] - 6*a*b^2*e^2*n^2*x^2*Log[1 + e*x] - 3*b^3*e^2*n^3*x^2*Log[1 +
e*x] + 12*a^2*b*Log[c*x^n]*Log[1 + e*x] + 12*a*b^2*n*Log[c*x^n]*Log[1 + e*x
] + 6*b^3*n^2*Log[c*x^n]*Log[1 + e*x] - 12*a^2*b*e^2*x^2*Log[c*x^n]*Log[1 +
 e*x] - 12*a*b^2*e^2*n*x^2*Log[c*x^n]*Log[1 + e*x] - 6*b^3*e^2*n^2*x^2*Log[
c*x^n]*Log[1 + e*x] + 12*a*b^2*Log[c*x^n]^2*Log[1 + e*x] + 6*b^3*n*Log[c*x^
n]^2*Log[1 + e*x] - 12*a*b^2*e^2*x^2*Log[c*x^n]^2*Log[1 + e*x] - 6*b^3*e^2*
n*x^2*Log[c*x^n]^2*Log[1 + e*x] + 4*b^3*Log[c*x^n]^3*Log[1 + e*x] - 4*b^3*e
^2*x^2*Log[c*x^n]^3*Log[1 + e*x] - 6*b*e^2*n*x^2*(2*a^2 + 2*a*b*n + b^2*n^2
 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, -(e*x)] + 12
*b^2*e^2*n^2*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[3, -(e*x)] - 24*b^3*e
^2*n^3*x^2*PolyLog[4, -(e*x)])/x^2
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \log(cx^n)^3 \log(ex+1) + 3\,ab^2 \log(cx^n)^2 \log(ex+1) + 3\,a^2 b \log(cx^n) \log(ex+1) + a^3 \log(ex+1)}{x^3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3*log(e*x + 1) + 3*a*b^2*log(c*x^n)^2*log(e*x + 1)
 + 3*a^2*b*log(c*x^n)*log(e*x + 1) + a^3*log(e*x + 1))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log(cx^n) + a\right)^3 \log(ex+1)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log(e*x + 1)/x^3, x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{(b \ln(c\,x^n) + a)^3 \ln(ex+1)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(e*x+1)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(e*x+1)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^3 e^2 x^2 \log(x) + b^3 ex - \left(b^3 e^2 x^2 - b^3\right) \log(ex+1)\right) \log(x^n)^3}{2\,x^2} - \frac{1}{2} \int -\frac{6\left(b^3 \log(c)^2 + 2\,ab^2 \log(c) + a^2 b\right) \log(ex\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*e^2*x^2*log(x) + b^3*e*x - (b^3*e^2*x^2 - b^3)*log(e*x + 1))*log(
x^n)^3/x^2 - 1/2*integrate(-(6*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log(
e*x + 1)*log(x^n) + 3*(b^3*e^2*n*x^2*log(x) + b^3*e*n*x - (b^3*e^2*n*x^2 -
b^3*(n + 2*log(c)) - 2*a*b^2)*log(e*x + 1))*log(x^n)^2 + 2*(b^3*log(c)^3 +
3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*log(e*x + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln (e\,x+1)\,\left(a+b\,\ln (c\,x^n)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^3,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(e*x+1)/x**3,x)
```

```
[Out] Timed out
```

3.24 $\int x^3 \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=180

$$-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{4d^2f^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{4df}+\frac{1}{4}x^4\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right.$$

```
[Out] -3/16*b*n*x^2/d/f+1/16*b*n*x^4+1/4*x^2*(a+b*ln(c*x^n))/d/f-1/8*x^4*(a+b*ln(
c*x^n))+1/16*b*n*ln(d*f*x^2+1)/d^2/f^2-1/16*b*n*x^4*ln(d*f*x^2+1)-1/4*(a+b*
ln(c*x^n))*ln(d*f*x^2+1)/d^2/f^2+1/4*x^4*(a+b*ln(c*x^n))*ln(d*f*x^2+1)-1/8*
b*n*polylog(2,-d*f*x^2)/d^2/f^2
```

Rubi [A] time = 0.17, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2454, 2395, 43, 2376, 2391}

$$-\frac{bn\text{PolyLog}\left(2,-dfx^2\right)}{8d^2f^2}-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{4d^2f^2}+\frac{1}{4}x^4\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)+\frac{x^2\left(a+b\log\left(c\right.\right.}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-3*b*n*x^2)/(16*d*f) + (b*n*x^4)/16 + (x^2*(a + b*Log[c*x^n]))/(4*d*f) - (
x^4*(a + b*Log[c*x^n]))/8 + (b*n*Log[1 + d*f*x^2])/(16*d^2*f^2) - (b*n*x^4*
Log[1 + d*f*x^2])/16 - ((a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(4*d^2*f^2) +
(x^4*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/4 - (b*n*PolyLog[2, -(d*f*x^2)])/
(8*d^2*f^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx=\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{4df}-\frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right)-\frac{\left(a+b\log\left(cx^n\right)\right)}{4d^2}$$

$$=-\frac{bnx^2}{8df}+\frac{1}{32}bnx^4+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{4df}-\frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right)$$

$$=-\frac{bnx^2}{8df}+\frac{1}{32}bnx^4+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{4df}-\frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right)$$

$$=-\frac{bnx^2}{8df}+\frac{1}{32}bnx^4+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{4df}-\frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right)$$

$$=-\frac{bnx^2}{8df}+\frac{1}{32}bnx^4+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{4df}-\frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right)$$

$$=-\frac{3bnx^2}{16df}+\frac{1}{16}bnx^4+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{4df}-\frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right.$$

Mathematica [C] time = 0.11, size = 348, normalized size = 1.93

$$-\frac{a\log\left(dfx^2+1\right)}{4d^2f^2}+\frac{ax^2}{4df}+\frac{1}{4}ax^4\log\left(dfx^2+1\right)-\frac{ax^4}{8}+\frac{b\left(n-4\left(\log\left(cx^n\right)-n\log(x)\right)\right)\log\left(dfx^2+1\right)}{16d^2f^2}+\frac{bx^2\left(4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (a*x^2)/(4*d*f) - (a*x^4)/8 + (b*x^4*(n - 4*(-(n*Log[x]) + Log[c*x^n])))/32
 + (b*x^2*(-n + 4*(-(n*Log[x]) + Log[c*x^n])))/(16*d*f) - (a*Log[1 + d*f*x^
2])/(4*d^2*f^2) + (a*x^4*Log[1 + d*f*x^2])/4 + (b*(n - 4*(-(n*Log[x]) + Log
[c*x^n]))*Log[1 + d*f*x^2])/(16*d^2*f^2) + (b*x^4*(-n + 4*n*Log[x] + 4*(-(n
*Log[x]) + Log[c*x^n]))*Log[1 + d*f*x^2])/16 - (b*d*f*n*(-((-1/4*x^2 + (x^2
*Log[x])/2)/(d^2*f^2)) + (-1/16*x^4 + (x^4*Log[x])/4)/(d*f) + (Log[x]*Log[1
 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(2*d^3*f^3) +
 (Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(2
*d^3*f^3)))/2
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(bx^3\log\left(dfx^2+1\right)\log\left(cx^n\right)+ax^3\log\left(dfx^2+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b*x^3*log(d*f*x^2 + 1)*log(c*x^n) + a*x^3*log(d*f*x^2 + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.31, size = 827, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*x^n)+a)*ln(d*(1/d+f*x^2)),x)
```

```
[Out] (1/4*b*x^4*ln(d*(1/d+f*x^2))-1/8*b*(d^2*f^2*x^4-2*d*f*x^2+2*ln(d*(1/d+f*x^2
)))/d^2/f^2)*ln(x^n)-1/8*a*x^4+1/4*b*ln(c)*x^4*ln(d*f*x^2+1)+1/4*a/d/f*x^2-
1/4*a/d^2/f^2*ln(d*f*x^2+1)-1/8*b*x^4*ln(c)+1/4*a*x^4*ln(d*f*x^2+1)+1/16*b*
n*x^4+1/16*b*n*ln(d*f*x^2+1)/d^2/f^2-1/8*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*c
sgn(I*c)*x^4*ln(d*f*x^2+1)+1/4*n*b/d^2/f^2*ln(x)*ln(d*f*x^2+1)-1/4/d^2/f^2*
b*n*dilog(1-x*(-d*f)^(1/2))-1/4/d^2/f^2*b*n*dilog(1+x*(-d*f)^(1/2))-1/4/d^2
/f^2*ln(d*f*x^2+1)*b*ln(c)+1/4/d/f*x^2*b*ln(c)-1/8*I/d^2/f^2*ln(d*f*x^2+1)*
Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+1/8*I/d/f*x^2*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)
-1/8*I/d^2/f^2*ln(d*f*x^2+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*I/d/f*x^2
*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/16*I*Pi*b*x^4*csgn(I*c*x^n)^3-1/4/d^2/f
^2*b*n*ln(x)*ln(1+x*(-d*f)^(1/2))-1/4/d^2/f^2*b*n*ln(x)*ln(1-x*(-d*f)^(1/2)
)-1/8*I*Pi*b*csgn(I*c*x^n)^3*x^4*ln(d*f*x^2+1)-1/16*I*Pi*b*x^4*csgn(I*x^n)*
csgn(I*c*x^n)^2-1/16*I*Pi*b*x^4*csgn(I*c*x^n)^2*csgn(I*c)-1/8*I/d/f*x^2*Pi*
b*csgn(I*c*x^n)^3+1/16*I*Pi*b*x^4*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*I
*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*x^4*ln(d*f*x^2+1)+1/8*I*Pi*b*csgn(I*x^n)*cs
gn(I*c*x^n)^2*x^4*ln(d*f*x^2+1)-1/8*I/d/f*x^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n
)*csgn(I*c)+1/8*I/d^2/f^2*ln(d*f*x^2+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn
(I*c)+1/8*I/d^2/f^2*ln(d*f*x^2+1)*Pi*b*csgn(I*c*x^n)^3-1/16*b*n*x^4*ln(d*f*
x^2+1)-3/16*b*n*x^2/d/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\left(4\,bx^4\log\left(x^n\right)-\left(b\left(n-4\,\log(c)\right)-4\,a\right)x^4\right)\log\left(dfx^2+1\right)-\int\frac{4\,bdfx^5\log\left(x^n\right)+\left(4\,adf-\left(dfn-4\,df\log(c)\right.\right.}{8\left(dfx^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/16*(4*b*x^4*log(x^n) - (b*(n - 4*log(c)) - 4*a)*x^4)*log(d*f*x^2 + 1) - i
ntegrate(1/8*(4*b*d*f*x^5*log(x^n) + (4*a*d*f - (d*f*n - 4*d*f*log(c))*b)*x
^5)/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

3.25 $\int x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=114

$$\frac{\left(dfx^2+1\right)\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{2df}-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{bn\text{Li}_2\left(-dfx^2\right)}{4df}-\frac{bn\left(dfx^2+1\right)\log\left(dfx^2+1\right)}{4df}$$

```
[Out] 1/2*b*n*x^2-1/2*x^2*(a+b*ln(c*x^n))-1/4*b*n*(d*f*x^2+1)*ln(d*f*x^2+1)/d/f+1
/2*(d*f*x^2+1)*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d/f+1/4*b*n*polylog(2,-d*f*x^2
)/d/f
```

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2454, 2389, 2295, 2376, 2475, 2411, 43, 2351, 2315}

$$\frac{bn\text{PolyLog}\left(2,-dfx^2\right)}{4df}+\frac{\left(dfx^2+1\right)\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{2df}-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)-\frac{bn\left(dfx^2+1\right)\log\left(dfx^2+1\right)}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (b*n*x^2)/2 - (x^2*(a + b*Log[c*x^n]))/2 - (b*n*(1 + d*f*x^2)*Log[1 + d*f*x
^2])/(4*d*f) + ((1 + d*f*x^2)*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(2*d*f)
+ (b*n*PolyLog[2, -(d*f*x^2)])/(4*d*f)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx=-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(1+dfx^2\right)\left(a+b\log\left(cx^n\right)\right)\log\left(1+d\right.}{2df}$$

$$=\frac{1}{4}bnx^2-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(1+dfx^2\right)\left(a+b\log\left(cx^n\right)\right)\log}{2df}$$

$$=\frac{1}{4}bnx^2-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(1+dfx^2\right)\left(a+b\log\left(cx^n\right)\right)\log}{2df}$$

$$=\frac{1}{4}bnx^2-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(1+dfx^2\right)\left(a+b\log\left(cx^n\right)\right)\log}{2df}$$

$$=\frac{1}{4}bnx^2-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(1+dfx^2\right)\left(a+b\log\left(cx^n\right)\right)\log}{2df}$$

$$=\frac{1}{4}bnx^2-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(1+dfx^2\right)\left(a+b\log\left(cx^n\right)\right)\log}{2df}$$

$$=\frac{1}{2}bnx^2-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)-\frac{bn\left(1+dfx^2\right)\log\left(1+dfx^2\right)}{4df}+$$

Mathematica [C] time = 0.05, size = 267, normalized size = 2.34

$$\frac{1}{2}a\left(\frac{\left(dfx^2+1\right)\log\left(dfx^2+1\right)}{df}-x^2\right)+\frac{1}{4}bx^2\left(2\left(\log\left(cx^n\right)-n\log(x)\right)+2n\log(x)-n\right)\log\left(dfx^2+1\right)+\frac{b\left(2\left(\log\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (b*x^2*(n - 2*(-(n*Log[x]) + Log[c*x^n])))/4 + (b*(-n + 2*(-(n*Log[x]) + Lo
g[c*x^n]))*Log[1 + d*f*x^2])/(4*d*f) + (b*x^2*(-n + 2*n*Log[x] + 2*(-(n*Log
[x]) + Log[c*x^n]))*Log[1 + d*f*x^2])/4 + (a*(-x^2 + ((1 + d*f*x^2)*Log[1 +
 d*f*x^2])/(d*f)))/2 - b*d*f*n*((-1/4*x^2 + (x^2*Log[x])/2)/(d*f) - (Log[x]
*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(2*d^2*
f^2) - (Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*
x])/(2*d^2*f^2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(bx\log\left(dfx^2+1\right)\log\left(cx^n\right)+ax\log\left(dfx^2+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b*x*log(d*f*x^2 + 1)*log(c*x^n) + a*x*log(d*f*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*log((f*x^2 + 1/d)*d), x)
```

maple [C] time = 0.27, size = 820, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d),x)
```

```
[Out] -1/4*b*n/d/f*ln(d*f*x^2+1)-1/2*a/d/f-1/2*a*x^2-1/4*n*b*x^2*ln(d*f*x^2+1)-1/
2/d/f*b*ln(c)+1/2*ln((f*x^2+1/d)*d)*ln(c)*x^2*b+(1/2*b*x^2*ln((f*x^2+1/d)*d
)+1/2*b*(-d*f*x^2+ln((f*x^2+1/d)*d))/d/f)*ln(x^n)+1/2/d/f*ln((f*x^2+1/d)*d)
*a-1/2*b*x^2*ln(c)+1/2*b*n*x^2+1/2*ln((f*x^2+1/d)*d)*x^2*a+1/4*I*Pi*b*x^2*c
sgn(I*c*x^n)^3+1/4*I/d/f*ln((f*x^2+1/d)*d)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+1
/2/f*b*n/d*ln(x)*ln((-d*f)^(1/2)*x+1)+1/2/f*b*n/d*ln(x)*ln(-(-d*f)^(1/2)*x+
1)+1/2/d/f*ln((f*x^2+1/d)*d)*ln(c)*b+1/2/f*b*n/d*dilog((-d*f)^(1/2)*x+1)+1/
2/f*b*n/d*dilog(-(-d*f)^(1/2)*x+1)+1/4*I*ln((f*x^2+1/d)*d)*Pi*x^2*b*csgn(I*
c*x^n)^2*csgn(I*c)+1/4*I/d/f*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/4*I
/d/f*ln((f*x^2+1/d)*d)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I*ln((f*x^2+1/d
)*d)*Pi*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*I/d/f*Pi*b*csgn(I*x^n
)*csgn(I*c*x^n)^2-1/4*I*Pi*b*x^2*csgn(I*c*x^n)^2*csgn(I*c)+1/4*I/d/f*Pi*b*c
sgn(I*c*x^n)^3-1/4*I*Pi*b*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*n*b/d/f*ln(x)
*ln(d*f*x^2+1)-1/4*I*ln((f*x^2+1/d)*d)*Pi*x^2*b*csgn(I*c*x^n)^3-1/4*I/d/f*l
n((f*x^2+1/d)*d)*Pi*b*csgn(I*c*x^n)^3+1/4*I*Pi*b*x^2*csgn(I*x^n)*csgn(I*c*x
```

```
^n)*csgn(I*c)-1/4*I/d/f*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+1/4*I*ln((f*x^2+1/d)
*d)*Pi*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I/d/f*ln((f*x^2+1/d)*d)*Pi*b*c
sgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(2\,bx^2\log\left(x^n\right)-\left(b\left(n-2\,\log(c)\right)-2\,a\right)x^2\right)\log\left(dfx^2+1\right)-\int\frac{2\,bdfx^3\log\left(x^n\right)+\left(2\,adf-\left(dfn-2\,df\log(\right.\right.}{2\left(dfx^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2*log(x^n) - (b*(n - 2*log(c)) - 2*a)*x^2)*log(d*f*x^2 + 1) - in
tegrate(1/2*(2*b*d*f*x^3*log(x^n) + (2*a*d*f - (d*f*n - 2*d*f*log(c))*b)*x^
3)/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

3.26 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx$

Optimal. Leaf size=39

$$\frac{1}{4}bn\text{Li}_3\left(-dfx^2\right)-\frac{1}{2}\text{Li}_2\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] -1/2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^2)+1/4*b*n*polylog(3,-d*f*x^2)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2374, 6589}

$$\frac{1}{4}bn\text{PolyLog}\left(3,-dfx^2\right)-\frac{1}{2}\text{PolyLog}\left(2,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^2)])/2 + (b*n*PolyLog[3, -(d*f*x^2)
])/4
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx &= -\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(-dfx^2\right)+\frac{1}{2}(bn)\int \frac{\text{Li}_2\left(-dfx^2\right)}{x}\,dx\\ &= -\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(-dfx^2\right)+\frac{1}{4}bn\text{Li}_3\left(-dfx^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.28

$$-\frac{1}{2}a\text{Li}_2\left(-dfx^2\right)-\frac{1}{2}b\log\left(cx^n\right)\text{Li}_2\left(-dfx^2\right)+\frac{1}{4}bn\text{Li}_3\left(-dfx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] -1/2*(a*PolyLog[2, -(d*f*x^2)]) - (b*Log[c*x^n]*PolyLog[2, -(d*f*x^2)])/2 +
 (b*n*PolyLog[3, -(d*f*x^2)])/4
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(dfx^2+1\right)\log\left(cx^n\right)+a\log\left(dfx^2+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x,x, algorithm="fricas")
[Out] integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^n\right)+a\right)\ln\left(\left(f\,x^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d)/x,x)
[Out] int((b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(bn\log(x)^2-2\,b\log(x)\log\left(x^n\right)-2\left(b\log(c)+a\right)\log(x)\right)\log\left(dfx^2+1\right)-\int-\frac{bdfnx\log(x)^2-2\,bdfx\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x,x, algorithm="maxima")
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log(d*f
*x^2 + 1) - integrate(-(b*d*f*n*x*log(x)^2 - 2*b*d*f*x*log(x)*log(x^n) - 2*
(b*d*f*log(c) + a*d*f)*x*log(x))/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x,x)
[Out] Timed out
```

3.27 $\int \frac{\left(a+b\log(cx^n)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=141

$$df\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}df\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{1}{4}bdfn\text{Li}_2\left(-$$

[Out] 1/2*b*d*f*n*ln(x)-1/2*b*d*f*n*ln(x)^2+d*f*ln(x)*(a+b*ln(c*x^n))-1/4*b*d*f*n*ln(d*f*x^2+1)-1/4*b*n*ln(d*f*x^2+1)/x^2-1/2*d*f*(a+b*ln(c*x^n))*ln(d*f*x^2+1)-1/2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^2-1/4*b*d*f*n*polylog(2,-d*f*x^2)

Rubi [A] time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2454, 2395, 36, 29, 31, 2376, 2301, 2391}

$$-\frac{1}{4}bdfn\text{PolyLog}\left(2,-dfx^2\right)+df\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}df\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{\log\left(dfx^2+1\right)\left(\right.}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] (b*d*f*n*Log[x])/2 - (b*d*f*n*Log[x]^2)/2 + d*f*Log[x]*(a + b*Log[c*x^n]) -
 (b*d*f*n*Log[1 + d*f*x^2])/4 - (b*n*Log[1 + d*f*x^2])/(4*x^2) - (d*f*(a +
b*Log[c*x^n])*Log[1 + d*f*x^2])/2 - ((a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(
2*x^2) - (b*d*f*n*PolyLog[2, -(d*f*x^2)])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = df\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}df\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)$$

$$= df\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}df\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)$$

$$= -\frac{1}{2}bdfn\log^2(x)+df\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}df\left(a+b\log\left(cx\right.\right.$$

$$= -\frac{1}{2}bdfn\log^2(x)+df\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{bn\log\left(1+dfx^2\right)}{4x^2}$$

$$= -\frac{1}{2}bdfn\log^2(x)+df\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{bn\log\left(1+dfx^2\right)}{4x^2}$$

$$= \frac{1}{2}bdfn\log(x)-\frac{1}{2}bdfn\log^2(x)+df\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}b$$

Mathematica [C] time = 0.10, size = 241, normalized size = 1.71

$$-\frac{1}{2}adf\log\left(dfx^2+1\right)-\frac{a\log\left(dfx^2+1\right)}{2x^2}+adf\log(x)+\frac{1}{2}bdf\log(x)\left(2\left(\log\left(cx^n\right)-n\log(x)\right)+n\right)-\frac{1}{4}bdf\left(2\left(\mathrm{lo}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] a*d*f*Log[x] + (b*d*f*Log[x]*(n + 2*(-(n*Log[x]) + Log[c*x^n])))/2 - (a*d*f
*Log[1 + d*f*x^2])/2 - (a*Log[1 + d*f*x^2])/(2*x^2) - (b*d*f*(n + 2*(-(n*Lo
g[x]) + Log[c*x^n]))*Log[1 + d*f*x^2])/4 - (b*(n + 2*n*Log[x] + 2*(-(n*Log[
x]) + Log[c*x^n]))*Log[1 + d*f*x^2])/(4*x^2) + b*d*f*n*(Log[x]^2/2 + (-(Log
[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/2 +
 (-(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])
/2)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\log\left(dfx^2+1\right)\log\left(cx^n\right)+a\log\left(dfx^2+1\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(fx^{2}+\frac{1}{d}\right)d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x^3, x)
```

maple [C] time = 0.20, size = 619, normalized size = 4.39

$$-\frac{b\ln(c)\ln\left(dfx^{2}+1\right)}{2x^{2}}+adf\ln(x)-\frac{adf\ln\left(dfx^{2}+1\right)}{2}+\left(bdf\ln(x)-\frac{bdf\ln\left(\left(fx^{2}+\frac{1}{d}\right)d\right)}{2}-\frac{b\ln\left(\left(fx^{2}+\frac{1}{d}\right)d\right)}{2x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d)/x^3,x)
```

```
[Out] 1/4*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)/x^2*ln(d*f*x^2+1)-1/4*I*Pi*b
*csgn(I*c*x^n)^2*csgn(I*c)/x^2*ln(d*f*x^2+1)-1/2*b*ln(c)/x^2*ln(d*f*x^2+1)+
a*d*f*ln(x)-1/2*a*d*f*ln(d*f*x^2+1)+(-1/2*b/x^2*ln((f*x^2+1/d)*d)+b*f*d*ln(
x)-1/2*b*f*d*ln((f*x^2+1/d)*d))*ln(x^n)+1/2*I*f*d*ln(x)*Pi*b*csgn(I*c*x^n)^
2*csgn(I*c)+1/2*I*f*d*ln(x)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*f*d*b*n*di
log(-(-d*f)^(1/2)*x+1)-1/2*a/x^2*ln(d*f*x^2+1)+1/2*b*d*f*n*ln(x)-1/2*b*d*f*
n*ln(x)^2-1/4*b*d*f*n*ln(d*f*x^2+1)+1/4*I*f*d*ln(d*f*x^2+1)*Pi*b*csgn(I*x^n
)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*f*d*ln(x)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*cs
gn(I*c)-1/4*I*f*d*ln(d*f*x^2+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*f*d*
ln(x)*Pi*b*csgn(I*c*x^n)^3-1/4*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2/x^2*ln(d*
f*x^2+1)+1/4*I*f*d*ln(d*f*x^2+1)*Pi*b*csgn(I*c*x^n)^3-1/4*I*f*d*ln(d*f*x^2+
1)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/2*f*d*b*n*dilog((-d*f)^(1/2)*x+1)+f*d*l
n(x)*ln(c)*b-1/2*f*d*ln(d*f*x^2+1)*ln(c)*b-1/2*f*d*b*n*ln(x)*ln(-(-d*f)^(1/
2)*x+1)+1/4*I*Pi*b*csgn(I*c*x^n)^3/x^2*ln(d*f*x^2+1)-1/2*f*d*b*n*ln(x)*ln((
-d*f)^(1/2)*x+1)+1/2*n*b*ln(d*f*x^2+1)*ln(x)*d*f-1/4*b*n*ln(d*f*x^2+1)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b\left(n+2\log(c)\right)+2b\log\left(x^{n}\right)+2a\right)\log\left(dfx^{2}+1\right)}{4x^{2}}+\int\frac{2bdf\log\left(x^{n}\right)+2adf+\left(dfn+2df\log(c)\right)b}{2\left(dfx^{3}+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*(n + 2*log(c)) + 2*b*log(x^n) + 2*a)*log(d*f*x^2 + 1)/x^2 + integra
te(1/2*(2*b*d*f*log(x^n) + 2*a*d*f + (d*f*n + 2*d*f*log(c))*b)/(d*f*x^3 + x
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(fx^{2}+\frac{1}{d}\right)\right)\left(a+b\ln\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^3,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x**3,x)
[Out] Timed out
```

3.28 $\int x^2 \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=241

$$-\frac{2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)}{3d^{3/2}f^{3/2}}+\frac{2x\left(a+b\log\left(cx^n\right)\right)}{3df}+\frac{1}{3}x^3\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{2}{9}x^3\left(a+b\log\right.$$

```
[Out] -8/9*b*n*x/d/f+4/27*b*n*x^3+2/9*b*n*arctan(x*d^(1/2)*f^(1/2))/d^(3/2)/f^(3/
2)+2/3*x*(a+b*ln(c*x^n))/d/f-2/9*x^3*(a+b*ln(c*x^n))-2/3*arctan(x*d^(1/2)*f
^(1/2))*(a+b*ln(c*x^n))/d^(3/2)/f^(3/2)-1/9*b*n*x^3*ln(d*f*x^2+1)+1/3*x^3*(
a+b*ln(c*x^n))*ln(d*f*x^2+1)+1/3*I*b*n*polylog(2,-I*x*d^(1/2)*f^(1/2))/d^(3
/2)/f^(3/2)-1/3*I*b*n*polylog(2,I*x*d^(1/2)*f^(1/2))/d^(3/2)/f^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2455, 302, 205, 2376, 4848, 2391, 203}

$$\frac{ibn\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)}{3d^{3/2}f^{3/2}}-\frac{ibn\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)}{3d^{3/2}f^{3/2}}-\frac{2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)}{3d^{3/2}f^{3/2}}+\frac{1}{3}x^3\log\left(dfx^2+1\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-8*b*n*x)/(9*d*f) + (4*b*n*x^3)/27 + (2*b*n*ArcTan[Sqrt[d]*Sqrt[f]*x])/(9*
d^(3/2)*f^(3/2)) + (2*x*(a + b*Log[c*x^n]))/(3*d*f) - (2*x^3*(a + b*Log[c*x
^n]))/9 - (2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]))/(3*d^(3/2)*f^(3/
2)) - (b*n*x^3*Log[1 + d*f*x^2])/9 + (x^3*(a + b*Log[c*x^n])*Log[1 + d*f*x^
2])/3 + ((I/3)*b*n*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(3/2)) -
((I/3)*b*n*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx = \frac{2x\left(a+b\log\left(cx^n\right)\right)}{3df}-\frac{2}{9}x^3\left(a+b\log\left(cx^n\right)\right)-\frac{2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)}{3d^3}$$

$$= -\frac{2bnx}{3df}+\frac{2}{27}bnx^3+\frac{2x\left(a+b\log\left(cx^n\right)\right)}{3df}-\frac{2}{9}x^3\left(a+b\log\left(cx^n\right)\right)$$

$$= -\frac{2bnx}{3df}+\frac{2}{27}bnx^3+\frac{2x\left(a+b\log\left(cx^n\right)\right)}{3df}-\frac{2}{9}x^3\left(a+b\log\left(cx^n\right)\right)$$

$$= -\frac{2bnx}{3df}+\frac{2}{27}bnx^3+\frac{2x\left(a+b\log\left(cx^n\right)\right)}{3df}-\frac{2}{9}x^3\left(a+b\log\left(cx^n\right)\right)$$

$$= -\frac{8bnx}{9df}+\frac{4}{27}bnx^3+\frac{2x\left(a+b\log\left(cx^n\right)\right)}{3df}-\frac{2}{9}x^3\left(a+b\log\left(cx^n\right)\right)$$

$$= -\frac{8bnx}{9df}+\frac{4}{27}bnx^3+\frac{2bn\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)}{9d^{3/2}f^{3/2}}+\frac{2x\left(a+b\log\left(cx^n\right)\right)}{3df}$$

Mathematica [A] time = 0.09, size = 364, normalized size = 1.51

$$-\frac{2a\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)}{3d^{3/2}f^{3/2}}+\frac{1}{3}ax^3\log\left(dfx^2+1\right)+\frac{2ax}{3df}-\frac{2ax^3}{9}-\frac{2b\left(3\left(\log\left(cx^n\right)-n\log(x)\right)-n\right)\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)}{9d^{3/2}f^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (2*a*x)/(3*d*f) - (2*a*x^3)/9 - (2*a*ArcTan[Sqrt[d]*Sqrt[f]*x])/(3*d^(3/2)*
f^(3/2)) + (2*b*x*(-n + 3*(-(n*Log[x]) + Log[c*x^n])))/(9*d*f) - (2*b*x^3*(
-n + 3*(-(n*Log[x]) + Log[c*x^n])))/27 - (2*b*ArcTan[Sqrt[d]*Sqrt[f]*x]*(-n
 + 3*(-(n*Log[x]) + Log[c*x^n])))/(9*d^(3/2)*f^(3/2)) + (a*x^3*Log[1 + d*f*
x^2])/3 + (b*x^3*(-n + 3*n*Log[x] + 3*(-(n*Log[x]) + Log[c*x^n]))*Log[1 + d
*f*x^2])/9 - (2*b*d*f*n*(-((x*(-1 + Log[x]))/(d^2*f^2)) + (-1/9*x^3 + (x^3*
Log[x])/3)/(d*f) - ((I/2)*(Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2,
 (-I)*Sqrt[d]*Sqrt[f]*x]))/(d^(5/2)*f^(5/2)) + ((I/2)*(Log[x]*Log[1 - I*Sqr
t[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]))/(d^(5/2)*f^(5/2))))/3
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(bx^2\log\left(dfx^2+1\right)\log\left(cx^n\right)+ax^2\log\left(dfx^2+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b*x^2*log(d*f*x^2 + 1)*log(c*x^n) + a*x^2*log(d*f*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^{n}\right) + a\right)x^{2} \log\left(\left(fx^{2} + \frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*x^2 + 1/d)*d), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\,x^{n}\right) + a\right) x^{2} \ln\left(\left(f\,x^{2} + \frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{9}\left(3\,bx^{3}\log\left(x^{n}\right) - \left(b\left(n - 3\,\log(c)\right) - 3\,a\right)x^{3}\right)\log\left(dfx^{2} + 1\right) - \int \frac{2\left(3\,bdfx^{4}\log\left(x^{n}\right) + \left(3\,adf - \left(dfn - 3\,df\log(c\right)\right)\right)\right)}{9\left(dfx^{2} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/9*(3*b*x^3*log(x^n) - (b*(n - 3*log(c)) - 3*a)*x^3)*log(d*f*x^2 + 1) - in
tegrate(2/9*(3*b*d*f*x^4*log(x^n) + (3*a*d*f - (d*f*n - 3*d*f*log(c))*b)*x^
4)/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{2}\,\ln\left(d\,\left(f\,x^{2} + \frac{1}{d}\right)\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

3.29 $\int \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=182

$$\frac{2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{d}\,\sqrt{f}}+x\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)-2x\left(a+b\log\left(cx^n\right)\right)-\frac{ibn\text{Li}_2\left(-i\sqrt{d}\,\sqrt{f}\right.}{\sqrt{d}\,\sqrt{f}}$$

```
[Out] 4*b*n*x-2*x*(a+b*ln(c*x^n))-b*n*x*ln(d*f*x^2+1)+x*(a+b*ln(c*x^n))*ln(d*f*x^
2+1)-2*b*n*arctan(x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)+2*arctan(x*d^(1/2)*f^(
1/2))*(a+b*ln(c*x^n))/d^(1/2)/f^(1/2)-I*b*n*polylog(2,-I*x*d^(1/2)*f^(1/2))
/d^(1/2)/f^(1/2)+I*b*n*polylog(2,I*x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2448, 321, 205, 2370, 4848, 2391, 203}

$$-\frac{ibn\text{PolyLog}\left(2,-i\sqrt{d}\,\sqrt{f}\,x\right)}{\sqrt{d}\,\sqrt{f}}+\frac{ibn\text{PolyLog}\left(2,i\sqrt{d}\,\sqrt{f}\,x\right)}{\sqrt{d}\,\sqrt{f}}+x\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)+\frac{2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\right.}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] 4*b*n*x - (2*b*n*ArcTan[Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - 2*x*(a + b*
Log[c*x^n]) + (2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]))/(Sqrt[d]*Sqr
t[f]) - b*n*x*Log[1 + d*f*x^2] + x*(a + b*Log[c*x^n])*Log[1 + d*f*x^2] - (I
*b*n*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) + (I*b*n*PolyLog
[2, I*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx = -2x\left(a+b\log\left(cx^n\right)\right)+\frac{2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{d}\sqrt{f}}+x\left(a+\right.$$

$$=2bnx-2x\left(a+b\log\left(cx^n\right)\right)+\frac{2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{d}\sqrt{f}}+$$

$$=2bnx-2x\left(a+b\log\left(cx^n\right)\right)+\frac{2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{d}\sqrt{f}}-$$

$$=4bnx-2x\left(a+b\log\left(cx^n\right)\right)+\frac{2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{d}\sqrt{f}}-$$

$$=4bnx-\frac{2bn\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}-2x\left(a+b\log\left(cx^n\right)\right)+\frac{2\tan^{-1}\left(\sqrt{d}\right.}{}$$

Mathematica [A] time = 0.10, size = 254, normalized size = 1.40

$$ax\log\left(dfx^2+1\right)+\frac{2a\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}-2ax+\frac{2b\left(\log\left(cx^n\right)+n(-\log(x))-n\right)\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}+bx\left(\log\left(cx^n\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] -2*a*x + (2*a*ArcTan[Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - 2*b*x*(-n - n*
Log[x] + Log[c*x^n]) + (2*b*ArcTan[Sqrt[d]*Sqrt[f]*x]*(-n - n*Log[x] + Log[
c*x^n]))/(Sqrt[d]*Sqrt[f]) + a*x*Log[1 + d*f*x^2] + b*x*(-n + Log[c*x^n])*L
og[1 + d*f*x^2] - 2*b*d*f*n*((x*(-1 + Log[x]))/(d*f) + ((I/2)*(Log[x]*Log[1
 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]))/(d^(3/2)*f^(
3/2)) - ((I/2)*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*
Sqrt[f]*x]))/(d^(3/2)*f^(3/2)))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(b\log\left(dfx^2+1\right)\log\left(cx^n\right)+a\log\left(dfx^2+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^{n}\right) + a\right) \log\left(\left(fx^{2} + \frac{1}{d}\right)d\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\,x^{n}\right) + a\right) \ln\left(\left(f\,x^{2} + \frac{1}{d}\right) d\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d),x)
[Out] int((b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(bx \log\left(x^{n}\right) - \left(b\left(n - \log(c)\right) - a\right)x\right) \log\left(dfx^{2} + 1\right) - \int \frac{2\left(bdfx^{2} \log\left(x^{n}\right) + \left(adf - \left(dfn - df\log(c)\right)b\right)x^{2}\right)}{dfx^{2} + 1} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
[Out] (b*x*log(x^n) - (b*(n - log(c)) - a)*x)*log(d*f*x^2 + 1) - integrate(2*(b*d
*f*x^2*log(x^n) + (a*d*f - (d*f*n - d*f*log(c))*b)*x^2)/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d\,\left(f\,x^{2} + \frac{1}{d}\right)\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
[Out] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
[Out] Timed out
```

3.30 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$

Optimal. Leaf size=169

$$2\sqrt{d}\sqrt{f}\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{x}-ib\sqrt{d}\sqrt{f}n\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)+ib\sqrt{d}\sqrt{f}$$

```
[Out] -b*n*ln(d*f*x^2+1)/x-(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x+2*b*n*arctan(x*d^(1/2)
*f^(1/2))*d^(1/2)*f^(1/2)+2*arctan(x*d^(1/2)*f^(1/2))*(a+b*ln(c*x^n))*d^(1/
2)*f^(1/2)-I*b*n*polylog(2,-I*x*d^(1/2)*f^(1/2))*d^(1/2)*f^(1/2)+I*b*n*poly
log(2,I*x*d^(1/2)*f^(1/2))*d^(1/2)*f^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2455, 205, 2376, 4848, 2391, 203}

$$-ib\sqrt{d}\sqrt{f}n\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)+ib\sqrt{d}\sqrt{f}n\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{x}+2\sqrt{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 2*b*Sqrt[d]*Sqrt[f]*n*ArcTan[Sqrt[d]*Sqrt[f]*x] + 2*Sqrt[d]*Sqrt[f]*ArcTan[
Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]) - (b*n*Log[1 + d*f*x^2])/x - ((a + b*
Log[c*x^n])*Log[1 + d*f*x^2])/x - I*b*Sqrt[d]*Sqrt[f]*n*PolyLog[2, (-I)*Sqr
t[d]*Sqrt[f]*x] + I*b*Sqrt[d]*Sqrt[f]*n*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = 2\sqrt{d}\sqrt{f}\ \tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right) - \frac{\left(a+b\log\left(cx^n\right)\right)\log}{x}$$

$$= 2\sqrt{d}\sqrt{f}\ \tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right) - \frac{\left(a+b\log\left(cx^n\right)\right)\log}{x}$$

$$= 2\sqrt{d}\sqrt{f}\ \tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right) - \frac{bn\log\left(1+dfx^2\right)}{x} -$$

$$= 2b\sqrt{d}\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 2\sqrt{d}\sqrt{f}\ \tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\log\right.$$

Mathematica [A] time = 0.09, size = 221, normalized size = 1.31

$$-\frac{a\log\left(dfx^2+1\right)}{x}+2a\sqrt{d}\sqrt{f}\ \tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)+2b\sqrt{d}\sqrt{f}\ \left(\log\left(cx^n\right)+n(-\log(x))+n\right)\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)-\frac{b(1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 2*a*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sqrt[f]*x] + 2*b*Sqrt[d]*Sqrt[f]*ArcTan[
Sqrt[d]*Sqrt[f]*x]*(n - n*Log[x] + Log[c*x^n]) - (a*Log[1 + d*f*x^2])/x - (
b*(n + Log[c*x^n])*Log[1 + d*f*x^2])/x + 2*b*d*f*n*(((-1/2*I)*(Log[x]*Log[1
 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]))/(Sqrt[d]*Sqr
t[f]) + ((I/2)*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*
Sqrt[f]*x]))/(Sqrt[d]*Sqrt[f]))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\log\left(dfx^2+1\right)\log\left(cx^n\right)+a\log\left(dfx^2+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x^2, x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c\,x^{n}\right)+a\right) \ln \left(\left(f\,x^{2}+\frac{1}{d}\right) d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b\left(n+\log(c)\right)+b\log\left(x^{n}\right)+a\right)\log\left(dfx^{2}+1\right)}{x}+\int \frac{2\left(bdf\log\left(x^{n}\right)+adf+\left(dfn+df\log(c)\right)b\right)}{dfx^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] -(b*(n + log(c)) + b*log(x^n) + a)*log(d*f*x^2 + 1)/x + integrate(2*(b*d*f*
log(x^n) + a*d*f + (d*f*n + d*f*log(c))*b)/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x**2,x)
```

```
[Out] Timed out
```

3.31 $\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx$

Optimal. Leaf size=211

$$-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)-\frac{2df\left(a+b\log\left(cx^n\right)\right)}{3x}-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{3x^3}+\frac{1}{3}ibd^{3/2}$$

```
[Out] -8/9*b*d*f*n/x-2/9*b*d^(3/2)*f^(3/2)*n*arctan(x*d^(1/2)*f^(1/2))-2/3*d*f*(a
+b*ln(c*x^n))/x-2/3*d^(3/2)*f^(3/2)*arctan(x*d^(1/2)*f^(1/2))*(a+b*ln(c*x^n
))-1/9*b*n*ln(d*f*x^2+1)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^3+1/3*I*b*
d^(3/2)*f^(3/2)*n*polylog(2,-I*x*d^(1/2)*f^(1/2))-1/3*I*b*d^(3/2)*f^(3/2)*n
*polylog(2,I*x*d^(1/2)*f^(1/2))
```

Rubi [A] time = 0.14, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2455, 325, 205, 2376, 4848, 2391, 203}

$$\frac{1}{3}ibd^{3/2}f^{3/2}n\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}\,x\right)-\frac{1}{3}ibd^{3/2}f^{3/2}n\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}\,x\right)-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\,l\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

```
[Out] (-8*b*d*f*n)/(9*x) - (2*b*d^(3/2)*f^(3/2)*n*ArcTan[Sqrt[d]*Sqrt[f]*x])/9 -
(2*d*f*(a + b*Log[c*x^n]))/(3*x) - (2*d^(3/2)*f^(3/2)*ArcTan[Sqrt[d]*Sqrt[f
]*x]*(a + b*Log[c*x^n]))/3 - (b*n*Log[1 + d*f*x^2])/(9*x^3) - ((a + b*Log[c
*x^n])*Log[1 + d*f*x^2])/(3*x^3) + (I/3)*b*d^(3/2)*f^(3/2)*n*PolyLog[2, (-I
)*Sqrt[d]*Sqrt[f]*x] - (I/3)*b*d^(3/2)*f^(3/2)*n*PolyLog[2, I*Sqrt[d]*Sqrt[
f]*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = -\frac{2df\left(a+b\log\left(cx^n\right)\right)}{3x}-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)$$

$$= -\frac{2bdfn}{3x}-\frac{2df\left(a+b\log\left(cx^n\right)\right)}{3x}-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\,\mathrm{l}\right.$$

$$= -\frac{2bdfn}{3x}-\frac{2df\left(a+b\log\left(cx^n\right)\right)}{3x}-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\,\mathrm{l}\right.$$

$$= -\frac{8bdfn}{9x}-\frac{2df\left(a+b\log\left(cx^n\right)\right)}{3x}-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\,\mathrm{l}\right.$$

$$= -\frac{8bdfn}{9x}-\frac{2}{9}bd^{3/2}f^{3/2}n\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)-\frac{2df\left(a+b\log\left(cx^n\right)\right)}{3x}-\frac{2}{3}$$

Mathematica [C] time = 0.19, size = 285, normalized size = 1.35

$$-\frac{2adf\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-dfx^2\right)}{3x}-\frac{a\log\left(dfx^2+1\right)}{3x^3}-\frac{2}{9}bd^{3/2}f^{3/2}\left(3\left(\log\left(cx^n\right)-n\log(x)\right)+n\right)\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)-\frac{2b\left(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

```
[Out] (-2*a*d*f*Hypergeometric2F1[-1/2, 1, 1/2, -(d*f*x^2)])/(3*x) - (2*b*d^(3/2)
*f^(3/2)*ArcTan[Sqrt[d]*Sqrt[f]*x]*(n + 3*(-(n*Log[x]) + Log[c*x^n])))/9 -
(2*b*(d*f*n + 3*d*f*(-(n*Log[x]) + Log[c*x^n])))/(9*x) - (a*Log[1 + d*f*x^2
])/(3*x^3) - (b*(n + 3*n*Log[x] + 3*(-(n*Log[x]) + Log[c*x^n]))*Log[1 + d*f
*x^2])/(9*x^3) + (2*b*d*f*n*(-x^(-1) - Log[x]/x + (I/2)*Sqrt[d]*Sqrt[f]*(Lo
g[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]) - (
I/2)*Sqrt[d]*Sqrt[f]*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sq
rt[d]*Sqrt[f]*x])))/3
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\log\left(dfx^2+1\right)\log\left(cx^n\right)+a\log\left(dfx^2+1\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^4,x, algorithm="fricas")
[Out] integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(fx^{2}+\frac{1}{d}\right)d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x^4, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^{n}\right)+a\right)\ln\left(\left(f\,x^{2}+\frac{1}{d}\right)d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d)/x^4,x)
[Out] int((b*ln(c*x^n)+a)*ln((f*x^2+1/d)*d)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b\left(n+3\,\log(c)\right)+3\,b\log\left(x^{n}\right)+3\,a\right)\log\left(dfx^{2}+1\right)}{9\,x^{3}}+\int \frac{2\left(3\,bdf\log\left(x^{n}\right)+3\,adf+\left(dfn+3\,df\log(c)\right)b\right)}{9\left(dfx^{4}+x^{2}\right)}\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^4,x, algorithm="maxima")
[Out] -1/9*(b*(n + 3*log(c)) + 3*b*log(x^n) + 3*a)*log(d*f*x^2 + 1)/x^3 + integra
te(2/9*(3*b*d*f*log(x^n) + 3*a*d*f + (d*f*n + 3*d*f*log(c))*b)/(d*f*x^4 + x
^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(f\,x^{2}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^4,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x**4,x)
[Out] Timed out
```

3.32 $\int x^3 \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right) dx$

Optimal. Leaf size=367

$$-\frac{bn\text{Li}_2\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{4d^2f^2}-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{4d^2f^2}+\frac{bn\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{8d^2f^2}+\frac{x^2\left(a\right.}{}$$

```
[Out] 7/32*b^2*n^2*x^2/d/f-3/64*b^2*n^2*x^4-3/8*b*n*x^2*(a+b*ln(c*x^n))/d/f+1/8*b
*n*x^4*(a+b*ln(c*x^n))+1/4*x^2*(a+b*ln(c*x^n))^2/d/f-1/8*x^4*(a+b*ln(c*x^n)
)^2-1/32*b^2*n^2*ln(d*f*x^2+1)/d^2/f^2+1/32*b^2*n^2*x^4*ln(d*f*x^2+1)+1/8*b
*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d^2/f^2-1/8*b*n*x^4*(a+b*ln(c*x^n))*ln(d*f
*x^2+1)-1/4*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d^2/f^2+1/4*x^4*(a+b*ln(c*x^n))
^2*ln(d*f*x^2+1)+1/16*b^2*n^2*polylog(2,-d*f*x^2)/d^2/f^2-1/4*b*n*(a+b*ln(c
*x^n))*polylog(2,-d*f*x^2)/d^2/f^2+1/8*b^2*n^2*polylog(3,-d*f*x^2)/d^2/f^2
```

Rubi [A] time = 0.36, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2454, 2395, 43, 2377, 2304, 2374, 6589, 2376, 2391}

$$-\frac{bn\text{PolyLog}\left(2,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{4d^2f^2}+\frac{b^2n^2\text{PolyLog}\left(2,-dfx^2\right)}{16d^2f^2}+\frac{b^2n^2\text{PolyLog}\left(3,-dfx^2\right)}{8d^2f^2}-\frac{\log\left(dfx^2+1\right)\left(\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (7*b^2*n^2*x^2)/(32*d*f) - (3*b^2*n^2*x^4)/64 - (3*b*n*x^2*(a + b*Log[c*x^n
]))/(8*d*f) + (b*n*x^4*(a + b*Log[c*x^n]))/8 + (x^2*(a + b*Log[c*x^n])^2)/(
4*d*f) - (x^4*(a + b*Log[c*x^n])^2)/8 - (b^2*n^2*Log[1 + d*f*x^2])/(32*d^2*
f^2) + (b^2*n^2*x^4*Log[1 + d*f*x^2])/32 + (b*n*(a + b*Log[c*x^n])*Log[1 +
d*f*x^2])/(8*d^2*f^2) - (b*n*x^4*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/8 - (
(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(4*d^2*f^2) + (x^4*(a + b*Log[c*x^n]
)^2*Log[1 + d*f*x^2])/4 + (b^2*n^2*PolyLog[2, -(d*f*x^2)])/(16*d^2*f^2) - (
b*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^2)])/(4*d^2*f^2) + (b^2*n^2*PolyL
og[3, -(d*f*x^2)])/(8*d^2*f^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx = \frac{x^2\left(a+b\log\left(cx^n\right)\right)^2}{4df} - \frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right)^2 - \frac{\left(a+b\log\left(cx^n\right)\right)^2}{4d^2\ldots}$$

$$= \frac{x^2\left(a+b\log\left(cx^n\right)\right)^2}{4df} - \frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right)^2 - \frac{\left(a+b\log\left(cx^n\right)\right)^2}{4d^2\ldots}$$

$$= \frac{b^2n^2x^2}{8df} - \frac{1}{64}b^2n^2x^4 - \frac{3bnx^2\left(a+b\log\left(cx^n\right)\right)}{8df} + \frac{1}{8}bnx^4\left(a+b\log\ldots\right.$$

$$= \frac{3b^2n^2x^2}{16df} - \frac{1}{32}b^2n^2x^4 - \frac{3bnx^2\left(a+b\log\left(cx^n\right)\right)}{8df} + \frac{1}{8}bnx^4\left(a+b\log\ldots\right.$$

$$= \frac{3b^2n^2x^2}{16df} - \frac{1}{32}b^2n^2x^4 - \frac{3bnx^2\left(a+b\log\left(cx^n\right)\right)}{8df} + \frac{1}{8}bnx^4\left(a+b\log\ldots\right.$$

$$= \frac{3b^2n^2x^2}{16df} - \frac{1}{32}b^2n^2x^4 - \frac{3bnx^2\left(a+b\log\left(cx^n\right)\right)}{8df} + \frac{1}{8}bnx^4\left(a+b\log\ldots\right.$$

$$= \frac{3b^2n^2x^2}{16df} - \frac{1}{32}b^2n^2x^4 - \frac{3bnx^2\left(a+b\log\left(cx^n\right)\right)}{8df} + \frac{1}{8}bnx^4\left(a+b\log\ldots\right.$$

$$= \frac{7b^2n^2x^2}{32df} - \frac{3}{64}b^2n^2x^4 - \frac{3bnx^2\left(a+b\log\left(cx^n\right)\right)}{8df} + \frac{1}{8}bnx^4\left(a+b\log\ldots\right.$$

Mathematica [C] time = 0.36, size = 654, normalized size = 1.78

$$\frac{-d^2f^2x^4\left(8a^2+16ab\left(\log\left(cx^n\right)-n\log(x)\right)-4abn+8b^2\left(\log\left(cx^n\right)-n\log(x)\right)^2+4b^2n\left(n\log(x)-\log\left(cx^n\right)\right)+\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (2*d*f*x^2*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*b^2*n*(n*Log[x] - Log[c*x^n]) + 1
6*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(-(n*Log[x]) + Log[c*x^n])^2) - d^
2*f^2*x^4*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*b^2*n*(n*Log[x] - Log[c*x^n]) + 16
*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(-(n*Log[x]) + Log[c*x^n])^2) + 2*d
^2*f^2*x^4*(8*a^2 - 4*a*b*n + b^2*n^2 - 4*b*(-4*a + b*n)*Log[c*x^n] + 8*b^2
*Log[c*x^n]^2)*Log[1 + d*f*x^2] - 2*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*b^2*n*(n
*Log[x] - Log[c*x^n]) + 16*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(-(n*Log[
x]) + Log[c*x^n])^2)*Log[1 + d*f*x^2] + b*n*(-4*a + b*n + 4*b*n*Log[x] - 4*
b*Log[c*x^n])*(4*d*f*x^2 - d^2*f^2*x^4 - 8*d*f*x^2*Log[x] + 4*d^2*f^2*x^4*L
og[x] + 8*Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + 8*Log[x]*Log[1 + I*Sqrt[d]*
Sqrt[f]*x] + 8*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 8*PolyLog[2, I*Sqrt[d]*
Sqrt[f]*x]) + 32*b^2*n^2*((d*f*x^2*(1 - 2*Log[x] + 2*Log[x]^2))/4 - (d^2*f^
2*x^4*(1 - 4*Log[x] + 8*Log[x]^2))/32 - (Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]
*x])/2 - (Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*PolyLog[2, (-I)
*Sqrt[d]*Sqrt[f]*x] - Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, (
-I)*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]))/(64*d^2*f^2)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^3\log\left(dfx^2+1\right)\log\left(cx^n\right)^2+2\,abx^3\log\left(dfx^2+1\right)\log\left(cx^n\right)+a^2x^3\log\left(dfx^2+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^2*x^3*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*x^3*log(d*f*x^2 + 1)
*log(c*x^n) + a^2*x^3*log(d*f*x^2 + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int (b\ln(c\,x^n)+a)^2\,x^3\ln\left(\left(f\,x^2+\frac{1}{d}\right)d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d),x)
```

```
[Out] int(x^3*(b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{32}\left(8\,b^2x^4\log(x^n)^2-4\left(b^2\left(n-4\,\log(c)\right)-4\,ab\right)x^4\log(x^n)+\left(\left(n^2-4\,n\log(c)+8\,\log(c)^2\right)b^2-4\,ab\left(n-4\,\mathrm{lo}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/32*(8*b^2*x^4*log(x^n)^2 - 4*(b^2*(n - 4*log(c)) - 4*a*b)*x^4*log(x^n) +
((n^2 - 4*n*log(c) + 8*log(c)^2)*b^2 - 4*a*b*(n - 4*log(c)) + 8*a^2)*x^4)*l
og(d*f*x^2 + 1) - integrate(1/16*(8*b^2*d*f*x^5*log(x^n)^2 + 4*(4*a*b*d*f -
 (d*f*n - 4*d*f*log(c))*b^2)*x^5*log(x^n) + (8*a^2*d*f - 4*(d*f*n - 4*d*f*l
og(c))*a*b + (d*f*n^2 - 4*d*f*n*log(c) + 8*d*f*log(c)^2)*b^2)*x^5)/(d*f*x^2
 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

3.33 $\int x\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx$

Optimal. Leaf size=241

$$\frac{bn\text{Li}_2\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{2df}-\frac{bn\left(dfx^2+1\right)\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{2df}+\frac{\left(dfx^2+1\right)\log\left(dfx^2+1\right)\left(a+\right.}{2df}$$

```
[Out] -3/4*b^2*n^2*x^2+b*n*x^2*(a+b*ln(c*x^n))-1/2*x^2*(a+b*ln(c*x^n))^2+1/4*b^2*
n^2*(d*f*x^2+1)*ln(d*f*x^2+1)/d/f-1/2*b*n*(d*f*x^2+1)*(a+b*ln(c*x^n))*ln(d*
f*x^2+1)/d/f+1/2*(d*f*x^2+1)*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d/f-1/4*b^2*n^
2*polylog(2,-d*f*x^2)/d/f+1/2*b*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^2)/d/f-1
/4*b^2*n^2*polylog(3,-d*f*x^2)/d/f
```

Rubi [A] time = 0.51, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 16, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2454, 2389, 2295, 2377, 2304, 14, 2351, 2301, 6742, 2374, 6589, 2376, 2475, 2411, 43, 2315}

$$\frac{bn\text{PolyLog}\left(2,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{2df}-\frac{b^2n^2\text{PolyLog}\left(2,-dfx^2\right)}{4df}-\frac{b^2n^2\text{PolyLog}\left(3,-dfx^2\right)}{4df}-\frac{bn\left(dfx^2+1\right)\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-3*b^2*n^2*x^2)/4 + b*n*x^2*(a + b*Log[c*x^n]) - (x^2*(a + b*Log[c*x^n])^2
)/2 + (b^2*n^2*(1 + d*f*x^2)*Log[1 + d*f*x^2])/(4*d*f) - (b*n*(1 + d*f*x^2)
*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(2*d*f) + ((1 + d*f*x^2)*(a + b*Log[c
*x^n])^2*Log[1 + d*f*x^2])/(2*d*f) - (b^2*n^2*PolyLog[2, -(d*f*x^2)])/(4*d*
f) + (b*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^2)])/(2*d*f) - (b^2*n^2*Pol
yLog[3, -(d*f*x^2)])/(4*d*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx = -\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2+\frac{\left(1+dfx^2\right)\left(a+b\log\left(cx^n\right)\right)^2\log\left(1-\right.}{2df}$$

$$= -\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2+\frac{\left(1+dfx^2\right)\left(a+b\log\left(cx^n\right)\right)^2\log\left(1-\right.}{2df}$$

$$= -\frac{1}{4}b^2n^2x^2+\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2+$$

$$= -\frac{1}{4}b^2n^2x^2+\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2+$$

$$= -\frac{1}{4}b^2n^2x^2+bnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2-\frac{b}{}$$

$$= -\frac{1}{2}b^2n^2x^2+bnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2-\frac{b}{}$$

$$= -\frac{1}{2}b^2n^2x^2+bnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2-\frac{b}{}$$

$$= -\frac{1}{2}b^2n^2x^2+bnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2-\frac{b}{}$$

$$= -\frac{1}{2}b^2n^2x^2+bnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2-\frac{b}{}$$

$$= -\frac{1}{2}b^2n^2x^2+bnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2-\frac{b}{}$$

$$= -\frac{3}{4}b^2n^2x^2+bnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2+\frac{b}{}$$

Mathematica [C] time = 0.27, size = 519, normalized size = 2.15

$$\frac{dfx^2\log\left(dfx^2+1\right)\left(2a^2-2b(bn-2a)\log\left(cx^n\right)-2abn+2b^2\log^2\left(cx^n\right)+b^2n^2\right)-dfx^2\left(2a^2+4ab\left(\log\left(cx^n\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-(d*f*x^2*(2*a^2 - 2*a*b*n + b^2*n^2 + 2*b^2*n*(n*Log[x] - Log[c*x^n]) + 4
*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Log[c*x^n])^2)) + d*
f*x^2*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*Log[
c*x^n]^2)*Log[1 + d*f*x^2] + (2*a^2 - 2*a*b*n + b^2*n^2 + 2*b^2*n*(n*Log[x]
 - Log[c*x^n]) + 4*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Lo
g[c*x^n])^2)*Log[1 + d*f*x^2] + 2*b*n*(2*a - b*n - 2*b*n*Log[x] + 2*b*Log[c
*x^n])*((d*f*x^2)/2 - d*f*x^2*Log[x] + Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x]
+ Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]
+ PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) - b^2*n^2*(d*f*x^2 - 2*d*f*x^2*Log[x] +
2*d*f*x^2*Log[x]^2 - 2*Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - 2*Log[x]^2*L
og[1 + I*Sqrt[d]*Sqrt[f]*x] - 4*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] -
```

```
4*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + 4*PolyLog[3, (-I)*Sqrt[d]*Sqrt[
f]*x] + 4*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]))/(4*d*f)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x\log\left(dfx^2+1\right)\log\left(cx^n\right)^2+2\,abx\log\left(dfx^2+1\right)\log\left(cx^n\right)+a^2x\log\left(dfx^2+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*x*log(d*f*x^2 + 1)*log
(c*x^n) + a^2*x*log(d*f*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^2x\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*x^2 + 1/d)*d), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^2x\ln\left(\left(f\,x^2+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(2\,b^2x^2\log\left(x^n\right)^2-2\left(b^2\left(n-2\,\log(c)\right)-2\,ab\right)x^2\log\left(x^n\right)+\left(\left(n^2-2\,n\log(c)+2\,\log(c)^2\right)b^2-2\,ab\left(n-2\,\log(c)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*x^2*log(x^n)^2 - 2*(b^2*(n - 2*log(c)) - 2*a*b)*x^2*log(x^n) + (
(n^2 - 2*n*log(c) + 2*log(c)^2)*b^2 - 2*a*b*(n - 2*log(c)) + 2*a^2)*x^2)*lo
g(d*f*x^2 + 1) - integrate(1/2*(2*b^2*d*f*x^3*log(x^n)^2 + 2*(2*a*b*d*f - (
d*f*n - 2*d*f*log(c))*b^2)*x^3*log(x^n) + (2*a^2*d*f - 2*(d*f*n - 2*d*f*log
(c))*a*b + (d*f*n^2 - 2*d*f*n*log(c) + 2*d*f*log(c)^2)*b^2)*x^3)/(d*f*x^2 +
 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
[Out] Timed out
```

3.34 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx$

Optimal. Leaf size=70

$$\frac{1}{2}bn\mathrm{Li}_3\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}\mathrm{Li}_2\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{4}b^2n^2\mathrm{Li}_4\left(-dfx^2\right)$$

```
[Out] -1/2*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^2)+1/2*b*n*(a+b*ln(c*x^n))*polylog(
3,-d*f*x^2)-1/4*b^2*n^2*polylog(4,-d*f*x^2)
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2374, 2383, 6589}

$$\frac{1}{2}bn\text{PolyLog}\left(3,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}\text{PolyLog}\left(2,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{4}b^2n^2\text{PolyLog}\left(4,-dfx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*x^2)])/2 + (b*n*(a + b*Log[c*x^n])*
PolyLog[3, -(d*f*x^2)])/2 - (b^2*n^2*PolyLog[4, -(d*f*x^2)])/4
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx=-\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)^2\mathrm{Li}_2\left(-dfx^2\right)+(bn)\int \frac{\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2\left(-c\right.}{x}$$

$$=-\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)^2\mathrm{Li}_2\left(-dfx^2\right)+\frac{1}{2}bn\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3\left(-dfx\right.$$

$$=-\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)^2\mathrm{Li}_2\left(-dfx^2\right)+\frac{1}{2}bn\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3\left(-dfx\right.$$

Mathematica [C] time = 0.21, size = 484, normalized size = 6.91

$$\frac{1}{3}\left(\log(x)\log\left(dfx^2+1\right)\left(-3bn\log(x)\left(a+b\log\left(cx^n\right)\right)+3\left(a+b\log\left(cx^n\right)\right)^2+b^2n^2\log^2(x)\right)+3bn\left(-2\mathrm{Li}_3\left(-i\sqrt{\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] (Log[x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a + b*Log[c*x^n]) + 3*(a + b*Log[
c*x^n])^2)*Log[1 + d*f*x^2] - 3*(a - b*n*Log[x] + b*Log[c*x^n])^2*(Log[x]*(
Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[1 + I*Sqrt[d]*Sqrt[f]*x]) + PolyLog[2, (
-I)*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) + 3*b*n*(-a + b*n
*Log[x] - b*Log[c*x^n])*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]^2*L
og[1 + I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] +
 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[
f]*x] - 2*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) - b^2*n^2*(Log[x]^3*Log[1 - I*Sq
rt[d]*Sqrt[f]*x] + Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyL
og[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]
- 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[3, I*Sqrt[
d]*Sqrt[f]*x] + 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, I*Sqrt[
d]*Sqrt[f]*x]))/3
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \log\left(dfx^2+1\right)\log\left(cx^n\right)^2 + 2\,ab\log\left(dfx^2+1\right)\log\left(cx^n\right) + a^2\log\left(dfx^2+1\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*
x^n) + a^2*log(d*f*x^2 + 1))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^2 \log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x, x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right)+a\right)^2 \ln\left(\left(f\,x^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(b^2n^2\log(x)^3 + 3\,b^2\log(x)\log\left(x^n\right)^2 - 3\left(b^2n\log(c)+abn\right)\log(x)^2 - 3\left(b^2n\log(x)^2 - 2\left(b^2\log(c)+ab\right)\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)*
log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*(b
^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log(d*f*x^2 + 1) - integrate(2/3*
(b^2*d*f*n^2*x*log(x)^3 + 3*b^2*d*f*x*log(x)*log(x^n)^2 - 3*(b^2*d*f*n*log(
c) + a*b*d*f*n)*x*log(x)^2 + 3*(b^2*d*f*log(c)^2 + 2*a*b*d*f*log(c) + a^2*d
*f)*x*log(x) - 3*(b^2*d*f*n*x*log(x)^2 - 2*(b^2*d*f*log(c) + a*b*d*f)*x*log
(x))*log(x^n))/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x,x)
```

```
[Out] Timed out
```

3.35 $\int \frac{\left(a+b\log(cx^n)\right)^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=257

$$\frac{1}{2}bdfn\text{Li}_2\left(-\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}bdfn\log\left(\frac{1}{dfx^2}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{bn\log\left(dfx^2+1\right)\left(a+b\log(c\right.}{2x^2}$$

```
[Out] 1/2*b^2*d*f*n^2*ln(x)-1/2*b*d*f*n*ln(1+1/d/f/x^2)*(a+b*ln(c*x^n))-1/2*d*f*l
n(1+1/d/f/x^2)*(a+b*ln(c*x^n))^2-1/4*b^2*d*f*n^2*ln(d*f*x^2+1)-1/4*b^2*n^2*
ln(d*f*x^2+1)/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^2-1/2*(a+b*ln(c*x
^n))^2*ln(d*f*x^2+1)/x^2+1/4*b^2*d*f*n^2*polylog(2,-1/d/f/x^2)+1/2*b*d*f*n*
(a+b*ln(c*x^n))*polylog(2,-1/d/f/x^2)+1/4*b^2*d*f*n^2*polylog(3,-1/d/f/x^2)
```

Rubi [A] time = 0.34, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2305, 2304, 2378, 266, 36, 29, 31, 2345, 2391, 2374, 6589}

$$\frac{1}{2}bdfn\text{PolyLog}\left(2,-\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}b^2dfn^2\text{PolyLog}\left(2,-\frac{1}{dfx^2}\right)+\frac{1}{4}b^2dfn^2\text{PolyLog}\left(3,-\frac{1}{dfx^2}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] (b^2*d*f*n^2*Log[x])/2 - (b*d*f*n*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n]))/
2 - (d*f*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n])^2)/2 - (b^2*d*f*n^2*Log[1
+ d*f*x^2])/4 - (b^2*n^2*Log[1 + d*f*x^2])/(4*x^2) - (b*n*(a + b*Log[c*x^n]
)*Log[1 + d*f*x^2])/(2*x^2) - ((a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(2*x^
2) + (b^2*d*f*n^2*PolyLog[2, -(1/(d*f*x^2))])/4 + (b*d*f*n*(a + b*Log[c*x^n
])*PolyLog[2, -(1/(d*f*x^2))])/2 + (b^2*d*f*n^2*PolyLog[3, -(1/(d*f*x^2))])
/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2345

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> -Simp[(Log[1 + d/(e*x^r)]*(a + b*Log[c*x^n])^p)/(d*r),
x] + Dist[(b*n*p)/(d*r), Int[(Log[1 + d/(e*x^r)]*(a + b*Log[c*x^n])^(p - 1)
)/x, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = -\frac{b^2n^2\log\left(1+dfx^2\right)}{4x^2} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)}{2x^2} - \frac{(a}{}$$

$$= -\frac{b^2n^2\log\left(1+dfx^2\right)}{4x^2} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)}{2x^2} - \frac{(a}{}$$

$$= -\frac{1}{2}bdfn\log\left(1+\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right) - \frac{1}{2}df\log\left(1+\frac{1}{dfx^2}\right)($$

$$= -\frac{1}{2}bdfn\log\left(1+\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right) - \frac{1}{2}df\log\left(1+\frac{1}{dfx^2}\right)($$

$$= \frac{1}{2}b^2dfn^2\log(x) - \frac{1}{2}bdfn\log\left(1+\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right) - \frac{1}{2}df$$

Mathematica [C] time = 0.36, size = 488, normalized size = 1.90

$$\frac{1}{4}\left(2df\log(x)\left(2a^2+4ab\left(\log\left(cx^n\right)-n\log(x)\right)+2abn+2b^2\left(\log\left(cx^n\right)-n\log(x)\right)^2+2b^2n\left(\log\left(cx^n\right)-n\log\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] (2*d*f*Log[x]*(2*a^2 + 2*a*b*n + b^2*n^2 + 4*a*b*(-(n*Log[x]) + Log[c*x^n])
 + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Log[c*x^n])^2)
 - ((2*a^2 + 2*a*b*n + b^2*n^2 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x
^n]^2)*Log[1 + d*f*x^2])/x^2 - d*f*(2*a^2 + 2*a*b*n + b^2*n^2 + 4*a*b*(-(n*
Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log
[x]) + Log[c*x^n])^2)*Log[1 + d*f*x^2] - 2*b*d*f*n*(-2*a - b*n + 2*b*n*Log[
x] - 2*b*Log[c*x^n])*(Log[x]*(Log[x] - Log[1 - I*Sqrt[d]*Sqrt[f]*x] - Log[1
 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - PolyLog[2,
I*Sqrt[d]*Sqrt[f]*x]) + (2*b^2*d*f*n^2*(2*Log[x]^3 - 3*Log[x]^2*Log[1 - I*S
qrt[d]*Sqrt[f]*x] - 3*Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*Poly
Log[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] +
 6*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]))
/3)/4
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(dfx^2+1\right)\log\left(cx^n\right)^2+2\,ab\log\left(dfx^2+1\right)\log\left(cx^n\right)+a^2\log\left(dfx^2+1\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*
x^n) + a^2*log(d*f*x^2 + 1))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x^3, x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(b \ln (c x^n) + a)^2 \ln \left(\left(f x^2 + \frac{1}{d}\right) d\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2 b^2 \log (x^n)^2 + \left(n^2 + 2 n \log(c) + 2 \log(c)^2\right) b^2 + 2 ab\left(n + 2 \log(c)\right) + 2 a^2 + 2 \left(b^2\left(n + 2 \log(c)\right) + 2 ab\right) \log (x}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b^2*log(x^n)^2 + (n^2 + 2*n*log(c) + 2*log(c)^2)*b^2 + 2*a*b*(n + 2
*log(c)) + 2*a^2 + 2*(b^2*(n + 2*log(c)) + 2*a*b)*log(x^n))*log(d*f*x^2 + 1
)/x^2 + integrate(1/2*(2*b^2*d*f*log(x^n)^2 + 2*a^2*d*f + 2*(d*f*n + 2*d*f*
log(c))*a*b + (d*f*n^2 + 2*d*f*n*log(c) + 2*d*f*log(c)^2)*b^2 + 2*(2*a*b*d*
f + (d*f*n + 2*d*f*log(c))*b^2)*log(x^n))/(d*f*x^3 + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(d \left(f x^2 + \frac{1}{d}\right)\right) (a + b \ln (c x^n))^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x**3,x)
```

```
[Out] Timed out
```

3.36 $\int x^2 \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=612

$$\frac{4bn\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)}{9d^{3/2}f^{3/2}}+\frac{2bn\text{Li}_2\left(-\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)}{3(-d)^{3/2}f^{3/2}}-\frac{2bn\text{Li}_2\left(\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\right.}{3(-d)^{3/2}f^{3/2}}$$

```
[Out] -16/9*a*b*n*x/d/f+52/27*b^2*n^2*x/d/f-4/27*b^2*n^2*x^3-4/27*b^2*n^2*arctan(
x*d^(1/2)*f^(1/2))/d^(3/2)/f^(3/2)-16/9*b^2*n*x*ln(c*x^n)/d/f+8/27*b*n*x^3*
(a+b*ln(c*x^n))+4/9*b*n*arctan(x*d^(1/2)*f^(1/2))*(a+b*ln(c*x^n))/d^(3/2)/f
^(3/2)+2/3*x*(a+b*ln(c*x^n))^2/d/f-2/9*x^3*(a+b*ln(c*x^n))^2+2/27*b^2*n^2*x
^3*ln(d*f*x^2+1)-2/9*b*n*x^3*(a+b*ln(c*x^n))*ln(d*f*x^2+1)+1/3*x^3*(a+b*ln(
c*x^n))^2*ln(d*f*x^2+1)-1/3*(a+b*ln(c*x^n))^2*ln(1-x*(-d)^(1/2)*f^(1/2))/(-
d)^(3/2)/f^(3/2)+1/3*(a+b*ln(c*x^n))^2*ln(1+x*(-d)^(1/2)*f^(1/2))/(-d)^(3/2
)/f^(3/2)+2/3*b*n*(a+b*ln(c*x^n))*polylog(2,-x*(-d)^(1/2)*f^(1/2))/(-d)^(3/
2)/f^(3/2)-2/3*b*n*(a+b*ln(c*x^n))*polylog(2,x*(-d)^(1/2)*f^(1/2))/(-d)^(3/
2)/f^(3/2)+2/9*I*b^2*n^2*polylog(2,I*x*d^(1/2)*f^(1/2))/d^(3/2)/f^(3/2)-2/9
*I*b^2*n^2*polylog(2,-I*x*d^(1/2)*f^(1/2))/d^(3/2)/f^(3/2)-2/3*b^2*n^2*poly
log(3,-x*(-d)^(1/2)*f^(1/2))/(-d)^(3/2)/f^(3/2)+2/3*b^2*n^2*polylog(3,x*(-d
)^(1/2)*f^(1/2))/(-d)^(3/2)/f^(3/2)
```

Rubi [A] time = 1.03, antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.607, Rules used = {2305, 2304, 2378, 302, 203, 2351, 2295, 2324, 12, 4848, 2391, 2353, 2296, 2330, 2317, 2374, 6589}

$$\frac{2bn\text{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)}{3(-d)^{3/2}f^{3/2}}-\frac{2bn\text{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)}{3(-d)^{3/2}f^{3/2}}-\frac{2ib^2n^2\text{PolyLog}}{9d^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-16*a*b*n*x)/(9*d*f) + (52*b^2*n^2*x)/(27*d*f) - (4*b^2*n^2*x^3)/27 - (4*b
^2*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x])/(27*d^(3/2)*f^(3/2)) - (16*b^2*n*x*Log[c*
x^n])/(9*d*f) + (8*b*n*x^3*(a + b*Log[c*x^n]))/27 + (4*b*n*ArcTan[Sqrt[d]*S
qrt[f]*x]*(a + b*Log[c*x^n]))/(9*d^(3/2)*f^(3/2)) + (2*x*(a + b*Log[c*x^n])
^2)/(3*d*f) - (2*x^3*(a + b*Log[c*x^n])^2)/9 - ((a + b*Log[c*x^n])^2*Log[1
- Sqrt[-d]*Sqrt[f]*x])/(3*(-d)^(3/2)*f^(3/2)) + ((a + b*Log[c*x^n])^2*Log[1
 + Sqrt[-d]*Sqrt[f]*x])/(3*(-d)^(3/2)*f^(3/2)) + (2*b^2*n^2*x^3*Log[1 + d*f
*x^2])/27 - (2*b*n*x^3*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/9 + (x^3*(a + b
*Log[c*x^n])^2*Log[1 + d*f*x^2])/3 + (2*b*n*(a + b*Log[c*x^n])*PolyLog[2, -
(Sqrt[-d]*Sqrt[f]*x)])/(3*(-d)^(3/2)*f^(3/2)) - (2*b*n*(a + b*Log[c*x^n])*P
olyLog[2, Sqrt[-d]*Sqrt[f]*x])/(3*(-d)^(3/2)*f^(3/2)) - (((2*I)/9)*b^2*n^2*
PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(3/2)) + (((2*I)/9)*b^2*n^2*
PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(3/2)) - (2*b^2*n^2*PolyLog[3,
-(Sqrt[-d]*Sqrt[f]*x)])/(3*(-d)^(3/2)*f^(3/2)) + (2*b^2*n^2*PolyLog[3, Sqrt
[-d]*Sqrt[f]*x])/(3*(-d)^(3/2)*f^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right) dx = \frac{2}{27}b^2n^2x^3\log\left(1+dfx^2\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right.$$

$$= \frac{2}{27}b^2n^2x^3\log\left(1+dfx^2\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right.$$

$$= \frac{2}{27}b^2n^2x^3\log\left(1+dfx^2\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right.$$

$$= \frac{4b^2n^2x}{27df}-\frac{4}{81}b^2n^2x^3+\frac{2}{27}b^2n^2x^3\log\left(1+dfx^2\right)-\frac{2}{9}bnx^3\left(a+b\,l\right.$$

$$= -\frac{4abnx}{9df}+\frac{4b^2n^2x}{27df}-\frac{8}{81}b^2n^2x^3-\frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{27d^{3/2}f^{3/2}}+\frac{4}{27}bn$$

$$= -\frac{16abnx}{9df}+\frac{16b^2n^2x}{27df}-\frac{4}{27}b^2n^2x^3-\frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{27d^{3/2}f^{3/2}}-\frac{4b^2}{}$$

$$= -\frac{16abnx}{9df}+\frac{52b^2n^2x}{27df}-\frac{4}{27}b^2n^2x^3-\frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{27d^{3/2}f^{3/2}}-\frac{16}{}$$

$$= -\frac{16abnx}{9df}+\frac{52b^2n^2x}{27df}-\frac{4}{27}b^2n^2x^3-\frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{27d^{3/2}f^{3/2}}-\frac{16}{}$$

$$= -\frac{16abnx}{9df}+\frac{52b^2n^2x}{27df}-\frac{4}{27}b^2n^2x^3-\frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{27d^{3/2}f^{3/2}}-\frac{16}{}$$

Mathematica [A] time = 0.62, size = 703, normalized size = 1.15

$$\frac{-2d^{3/2}f^{3/2}x^3\left(9a^2+18ab\left(\log\left(cx^n\right)-n\log(x)\right)-6abn+9b^2\left(\log\left(cx^n\right)-n\log(x)\right)^2+6b^2n\left(n\log(x)-\log\left(cx^n\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (6*Sqrt[d]*Sqrt[f]*x*(9*a^2 - 6*a*b*n + 2*b^2*n^2 + 6*b^2*n*(n*Log[x] - Log
[c*x^n]) + 18*a*b*(-(n*Log[x]) + Log[c*x^n]) + 9*b^2*(-(n*Log[x]) + Log[c*x
^n])^2) - 2*d^(3/2)*f^(3/2)*x^3*(9*a^2 - 6*a*b*n + 2*b^2*n^2 + 6*b^2*n*(n*L
og[x] - Log[c*x^n]) + 18*a*b*(-(n*Log[x]) + Log[c*x^n]) + 9*b^2*(-(n*Log[x]
) + Log[c*x^n])^2) - 6*ArcTan[Sqrt[d]*Sqrt[f]*x]*(9*a^2 - 6*a*b*n + 2*b^2*n
^2 + 6*b^2*n*(n*Log[x] - Log[c*x^n]) + 18*a*b*(-(n*Log[x]) + Log[c*x^n]) +
9*b^2*(-(n*Log[x]) + Log[c*x^n])^2) + 3*d^(3/2)*f^(3/2)*x^3*(9*a^2 - 6*a*b*
n + 2*b^2*n^2 - 6*b*(-3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2)*Log[1 + d
*f*x^2] - 18*b*n*(3*a - b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n])*(-2*Sqrt[d]*Sq
rt[f]*x*(-1 + Log[x]) + (2*d^(3/2)*f^(3/2)*x^3*(-1 + 3*Log[x]))/9 - I*(Log[
x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]) + I*(
Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])) + 5
4*b^2*n^2*(Sqrt[d]*Sqrt[f]*x*(2 - 2*Log[x] + Log[x]^2) - (d^(3/2)*f^(3/2)*x
^3*(2 - 6*Log[x] + 9*Log[x]^2))/27 + (I/2)*(Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt
[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sq
rt[d]*Sqrt[f]*x]) - (I/2)*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]
*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x])))/(81
*d^(3/2)*f^(3/2))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\log\left(dfx^2+1\right)\log\left(cx^n\right)^2+2\,abx^2\log\left(dfx^2+1\right)\log\left(cx^n\right)+a^2x^2\log\left(dfx^2+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*x^2*log(d*f*x^2 + 1)
*log(c*x^n) + a^2*x^2*log(d*f*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^2x^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*x^2 + 1/d)*d), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^2x^2\ln\left(\left(f\,x^2+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{27}\left(9\,b^2x^3\log\left(x^n\right)^2-6\left(b^2\left(n-3\,\log(c)\right)-3\,ab\right)x^3\log\left(x^n\right)+\left(\left(2\,n^2-6\,n\log(c)+9\,\log(c)^2\right)b^2-6\,ab\left(n-3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/27*(9*b^2*x^3*log(x^n)^2 - 6*(b^2*(n - 3*log(c)) - 3*a*b)*x^3*log(x^n) +
((2*n^2 - 6*n*log(c) + 9*log(c)^2)*b^2 - 6*a*b*(n - 3*log(c)) + 9*a^2)*x^3)
*log(d*f*x^2 + 1) - integrate(2/27*(9*b^2*d*f*x^4*log(x^n)^2 + 6*(3*a*b*d*f
 - (d*f*n - 3*d*f*log(c))*b^2)*x^4*log(x^n) + (9*a^2*d*f - 6*(d*f*n - 3*d*f
*log(c))*a*b + (2*d*f*n^2 - 6*d*f*n*log(c) + 9*d*f*log(c)^2)*b^2)*x^4)/(d*f
*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

3.37 $\int \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=519

$$\frac{2bn\text{Li}_2\left(-\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{-d}\sqrt{f}}-\frac{2bn\text{Li}_2\left(\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{-d}\sqrt{f}}-\frac{\log\left(1-\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(\right.\right.}{\sqrt{-d}\sqrt{f}}$$

```
[Out] 4*a*b*n*x-8*b^2*n^2*x+4*b*n*(-b*n+a)*x+8*b^2*n*x*ln(c*x^n)-2*x*(a+b*ln(c*x^
n))^2-2*a*b*n*x*ln(d*f*x^2+1)+2*b^2*n^2*x*ln(d*f*x^2+1)-2*b^2*n*x*ln(c*x^n)
*ln(d*f*x^2+1)+x*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)-(a+b*ln(c*x^n))^2*ln(1-x*(
-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+(a+b*ln(c*x^n))^2*ln(1+x*(-d)^(1/2)*f
^(1/2))/(-d)^(1/2)/f^(1/2)+2*b*n*(a+b*ln(c*x^n))*polylog(2,-x*(-d)^(1/2)*f^
(1/2))/(-d)^(1/2)/f^(1/2)-2*b*n*(a+b*ln(c*x^n))*polylog(2,x*(-d)^(1/2)*f^(1
/2))/(-d)^(1/2)/f^(1/2)-2*b^2*n^2*polylog(3,-x*(-d)^(1/2)*f^(1/2))/(-d)^(1/
2)/f^(1/2)+2*b^2*n^2*polylog(3,x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-4*b
*n*(-b*n+a)*arctan(x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)-4*b^2*n*arctan(x*d^(1
/2)*f^(1/2))*ln(c*x^n)/d^(1/2)/f^(1/2)+2*I*b^2*n^2*polylog(2,-I*x*d^(1/2)*f
^(1/2))/d^(1/2)/f^(1/2)-2*I*b^2*n^2*polylog(2,I*x*d^(1/2)*f^(1/2))/d^(1/2)/
f^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {2296, 2295, 2371, 6, 321, 203, 2351, 2324, 12, 4848, 2391, 2353, 2330, 2317, 2374, 6589}

$$\frac{2bn\text{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{-d}\sqrt{f}}-\frac{2bn\text{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{-d}\sqrt{f}}+\frac{2ib^2n^2\text{PolyLog}}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] 4*a*b*n*x - 8*b^2*n^2*x + 4*b*n*(a - b*n)*x - (4*b*n*(a - b*n)*ArcTan[Sqrt[
d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) + 8*b^2*n*x*Log[c*x^n] - (4*b^2*n*ArcTan[S
qrt[d]*Sqrt[f]*x]*Log[c*x^n])/(Sqrt[d]*Sqrt[f]) - 2*x*(a + b*Log[c*x^n])^2
- ((a + b*Log[c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + (
(a + b*Log[c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) - 2*a*
b*n*x*Log[1 + d*f*x^2] + 2*b^2*n^2*x*Log[1 + d*f*x^2] - 2*b^2*n*x*Log[c*x^n
]*Log[1 + d*f*x^2] + x*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2] + (2*b*n*(a +
b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) - (2*b*
n*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + (
(2*I)*b^2*n^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - ((2*I
)*b^2*n^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - (2*b^2*n^2*P
olyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) + (2*b^2*n^2*PolyLog[3
, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2371

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx = -2abnx\log\left(1+dfx^2\right)+2b^2n^2x\log\left(1+dfx^2\right)-2b^2nx\log\left(cx^n\right)\log$$

$$= -2abnx\log\left(1+dfx^2\right)+2b^2n^2x\log\left(1+dfx^2\right)-2b^2nx\log\left(cx^n\right)\log$$

$$= -2abnx\log\left(1+dfx^2\right)+2b^2n^2x\log\left(1+dfx^2\right)-2b^2nx\log\left(cx^n\right)\log$$

$$= 4bn(a-bn)x-2abnx\log\left(1+dfx^2\right)+2b^2n^2x\log\left(1+dfx^2\right)-2b^2$$

$$= 4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}-2abnx\log\left(1+dfx^2\right.$$

$$= -4b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}+4b^2nx\log$$

$$= 4abnx-4b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}+4$$

$$= 4abnx-8b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}+8$$

$$= 4abnx-8b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}+8$$

$$= 4abnx-8b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}+8$$

Mathematica [A] time = 0.33, size = 544, normalized size = 1.05

$$\frac{-2\sqrt{d}\sqrt{f}\,x\left(a^2+2ab\left(\log\left(cx^n\right)-n\log(x)\right)-2abn+b^2\left(\log\left(cx^n\right)-n\log(x)\right)^2+2b^2n\left(n\log(x)-\log\left(cx^n\right)\right)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[f]*x*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b^2*n*(n*Log[x] - Log[
c*x^n]) + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x]) + Log[c*x^n])
^2) + 2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b^2*n*(n*L
og[x] - Log[c*x^n]) + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x]) +
 Log[c*x^n])^2) + Sqrt[d]*Sqrt[f]*x*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b*(a - b
*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*Log[1 + d*f*x^2] + 2*b*n*(a - b*n - b*n*
Log[x] + b*Log[c*x^n])*(-2*Sqrt[d]*Sqrt[f]*x*(-1 + Log[x]) - I*(Log[x]*Log[
1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]) + I*(Log[x]*
Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])) - 2*b^2*n^
2*(Sqrt[d]*Sqrt[f]*x*(2 - 2*Log[x] + Log[x]^2) + (I/2)*(Log[x]^2*Log[1 + I*
Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLo
g[3, (-I)*Sqrt[d]*Sqrt[f]*x]) - (I/2)*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x
] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*Sqrt[
f]*x])))/(Sqrt[d]*Sqrt[f])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\log\left(dfx^2+1\right)\log\left(cx^n\right)^2+2\,ab\log\left(dfx^2+1\right)\log\left(cx^n\right)+a^2\log\left(dfx^2+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*x
^n) + a^2*log(d*f*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)^2\ln\left(\left(f\,x^2+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d),x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(b^2x\log\left(x^n\right)^2-2\left(b^2\left(n-\log(c)\right)-ab\right)x\log\left(x^n\right)+\left(\left(2\,n^2-2\,n\log(c)+\log(c)^2\right)b^2-2\,ab\left(n-\log(c)\right)+a^2\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] (b^2*x*log(x^n)^2 - 2*(b^2*(n - log(c)) - a*b)*x*log(x^n) + ((2*n^2 - 2*n*l
og(c) + log(c)^2)*b^2 - 2*a*b*(n - log(c)) + a^2)*x)*log(d*f*x^2 + 1) - int
egrate(2*(b^2*d*f*x^2*log(x^n)^2 + 2*(a*b*d*f - (d*f*n - d*f*log(c))*b^2)*x
^2*log(x^n) + (a^2*d*f - 2*(d*f*n - d*f*log(c))*a*b + (2*d*f*n^2 - 2*d*f*n*
log(c) + d*f*log(c)^2)*b^2)*x^2)/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

3.38 $\int \frac{\left(a+b\log(cx^n)\right)^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$

Optimal. Leaf size=459

$$-2b\sqrt{-d}\,\sqrt{f}\,n\text{Li}_2\left(-\sqrt{-d}\,\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)+2b\sqrt{-d}\,\sqrt{f}\,n\text{Li}_2\left(\sqrt{-d}\,\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)+\sqrt{-d}\,\sqrt{f}\,\log\left(1\right.$$

```
[Out] -2*b^2*n^2*ln(d*f*x^2+1)/x-2*b*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x-(a+b*ln(c*
x^n))^2*ln(d*f*x^2+1)/x+(a+b*ln(c*x^n))^2*ln(1-x*(-d)^(1/2)*f^(1/2))*(-d)^(
1/2)*f^(1/2)-(a+b*ln(c*x^n))^2*ln(1+x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2
)-2*b*n*(a+b*ln(c*x^n))*polylog(2,-x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)
+2*b*n*(a+b*ln(c*x^n))*polylog(2,x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+2
*b^2*n^2*polylog(3,-x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-2*b^2*n^2*poly
log(3,x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+4*b^2*n^2*arctan(x*d^(1/2)*f
^(1/2))*d^(1/2)*f^(1/2)+4*b*n*arctan(x*d^(1/2)*f^(1/2))*(a+b*ln(c*x^n))*d^(
1/2)*f^(1/2)-2*I*b^2*n^2*polylog(2,-I*x*d^(1/2)*f^(1/2))*d^(1/2)*f^(1/2)+2*
I*b^2*n^2*polylog(2,I*x*d^(1/2)*f^(1/2))*d^(1/2)*f^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2305, 2304, 2378, 203, 2324, 12, 4848, 2391, 2330, 2317, 2374, 6589}

$$-2b\sqrt{-d}\,\sqrt{f}\,n\text{PolyLog}\left(2,-\sqrt{-d}\,\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)+2b\sqrt{-d}\,\sqrt{f}\,n\text{PolyLog}\left(2,\sqrt{-d}\,\sqrt{f}\,x\right)\left(a+b\log\left(cx^n\right)\right)-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 4*b^2*Sqrt[d]*Sqrt[f]*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x] + 4*b*Sqrt[d]*Sqrt[f]*n
*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]) + Sqrt[-d]*Sqrt[f]*(a + b*Log
[c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x] - Sqrt[-d]*Sqrt[f]*(a + b*Log[c*x^n]
)^2*Log[1 + Sqrt[-d]*Sqrt[f]*x] - (2*b^2*n^2*Log[1 + d*f*x^2])/x - (2*b*n*(
a + b*Log[c*x^n])*Log[1 + d*f*x^2])/x - ((a + b*Log[c*x^n])^2*Log[1 + d*f*x
^2])/x - 2*b*Sqrt[-d]*Sqrt[f]*n*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sq
rt[f]*x)] + 2*b*Sqrt[-d]*Sqrt[f]*n*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d]*S
qrt[f]*x] - (2*I)*b^2*Sqrt[d]*Sqrt[f]*n^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x
] + (2*I)*b^2*Sqrt[d]*Sqrt[f]*n^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + 2*b^2*S
qrt[-d]*Sqrt[f]*n^2*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)] - 2*b^2*Sqrt[-d]*Sqrt
[f]*n^2*PolyLog[3, Sqrt[-d]*Sqrt[f]*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = -\frac{2b^2n^2\log\left(1+dfx^2\right)}{x} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)}{x} - \frac{(a}{}$$

$$= -\frac{2b^2n^2\log\left(1+dfx^2\right)}{x} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)}{x} - \frac{(a}{}$$

$$= 4b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 4b\sqrt{d}\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\right.$$

$$= 4b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 4b\sqrt{d}\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\right.$$

$$= 4b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 4b\sqrt{d}\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\right.$$

$$= 4b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 4b\sqrt{d}\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\right.$$

$$= 4b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 4b\sqrt{d}\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a+b\right.$$

Mathematica [A] time = 0.31, size = 414, normalized size = 0.90

$$2\sqrt{d}\sqrt{f}\,\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)\left(a^2+2ab\left(\log\left(cx^n\right)-n\log(x)\right)+2abn+b^2\left(\log\left(cx^n\right)-n\log(x)\right)^2+2b^2n\left(\log\left(cx^n\right)-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 2*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*
a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + b^2*(
-(n*Log[x]) + Log[c*x^n])^2) - ((a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*
Log[c*x^n] + b^2*Log[c*x^n]^2)*Log[1 + d*f*x^2])/x + (2*I)*b*Sqrt[d]*Sqrt[f
]*n*(a + b*n - b*n*Log[x] + b*Log[c*x^n])*(Log[x]*(Log[1 - I*Sqrt[d]*Sqrt[f
]*x] - Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] +
 PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) + I*b^2*Sqrt[d]*Sqrt[f]*n^2*(Log[x]^2*Log
[1 - I*Sqrt[d]*Sqrt[f]*x] - Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] - 2*Log[x
]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f
]*x] + 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*Sqrt[f
]*x])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(dfx^2+1\right)\log\left(cx^n\right)^2+2\,ab\log\left(dfx^2+1\right)\log\left(cx^n\right)+a^2\log\left(dfx^2+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*
x^n) + a^2*log(d*f*x^2 + 1))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{2}\log\left(\left(fx^{2}+\frac{1}{d}\right)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x^2, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^{n}\right)+a\right)^{2}\ln\left(\left(f\,x^{2}+\frac{1}{d}\right)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d)/x^2,x)
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^{2}\log\left(x^{n}\right)^{2}+\left(2\,n^{2}+2\,n\log(c)+\log(c)^{2}\right)b^{2}+2\,ab\left(n+\log(c)\right)+a^{2}+2\left(b^{2}\left(n+\log(c)\right)+ab\right)\log\left(x^{n}\right)\right)\log}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^2,x, algorithm="maxima")
[Out] -(b^2*log(x^n)^2 + (2*n^2 + 2*n*log(c) + log(c)^2)*b^2 + 2*a*b*(n + log(c))
 + a^2 + 2*(b^2*(n + log(c)) + a*b)*log(x^n))*log(d*f*x^2 + 1)/x + integrat
e(2*(b^2*d*f*log(x^n)^2 + a^2*d*f + 2*(d*f*n + d*f*log(c))*a*b + (2*d*f*n^2
 + 2*d*f*n*log(c) + d*f*log(c)^2)*b^2 + 2*(a*b*d*f + (d*f*n + d*f*log(c))*b
^2)*log(x^n))/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(f\,x^{2}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^2,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x**2,x)
[Out] Timed out
```

3.39 $\int \frac{(a+b\log(cx^n))^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4} dx$

Optimal. Leaf size=543

$$-\frac{4}{9}bd^{3/2}f^{3/2}n\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)-\frac{2}{3}b(-d)^{3/2}f^{3/2}n\text{Li}_2\left(-\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)+\frac{2}{3}b(-d)^{3/2}f^{3/2}$$

```
[Out] -52/27*b^2*d*f*n^2/x-4/27*b^2*d^(3/2)*f^(3/2)*n^2*arctan(x*d^(1/2)*f^(1/2))
-16/9*b*d*f*n*(a+b*ln(c*x^n))/x-4/9*b*d^(3/2)*f^(3/2)*n*arctan(x*d^(1/2)*f^
(1/2))*(a+b*ln(c*x^n))-2/3*d*f*(a+b*ln(c*x^n))^2/x-2/27*b^2*n^2*ln(d*f*x^2+
1)/x^3-2/9*b*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^3-1/3*(a+b*ln(c*x^n))^2*ln(d
*f*x^2+1)/x^3+1/3*(-d)^(3/2)*f^(3/2)*(a+b*ln(c*x^n))^2*ln(1-x*(-d)^(1/2)*f^
(1/2))-1/3*(-d)^(3/2)*f^(3/2)*(a+b*ln(c*x^n))^2*ln(1+x*(-d)^(1/2)*f^(1/2))-
2/3*b*(-d)^(3/2)*f^(3/2)*n*(a+b*ln(c*x^n))*polylog(2,-x*(-d)^(1/2)*f^(1/2))
+2/3*b*(-d)^(3/2)*f^(3/2)*n*(a+b*ln(c*x^n))*polylog(2,x*(-d)^(1/2)*f^(1/2))
-2/9*I*b^2*d^(3/2)*f^(3/2)*n^2*polylog(2,I*x*d^(1/2)*f^(1/2))+2/9*I*b^2*d^(
3/2)*f^(3/2)*n^2*polylog(2,-I*x*d^(1/2)*f^(1/2))+2/3*b^2*(-d)^(3/2)*f^(3/2)
*n^2*polylog(3,-x*(-d)^(1/2)*f^(1/2))-2/3*b^2*(-d)^(3/2)*f^(3/2)*n^2*polylo
g(3,x*(-d)^(1/2)*f^(1/2))
```

Rubi [A] time = 0.87, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {2305, 2304, 2378, 325, 203, 2351, 2324, 12, 4848, 2391, 2353, 2330, 2317, 2374, 6589}

$$-\frac{2}{3}b(-d)^{3/2}f^{3/2}n\text{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(cx^n\right)\right)+\frac{2}{3}b(-d)^{3/2}f^{3/2}n\text{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right)\left(a+b\log\left(cx\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

```
[Out] (-52*b^2*d*f*n^2)/(27*x) - (4*b^2*d^(3/2)*f^(3/2)*n^2*ArcTan[Sqrt[d]*Sqrt[f
]*x])/27 - (16*b*d*f*n*(a + b*Log[c*x^n]))/(9*x) - (4*b*d^(3/2)*f^(3/2)*n*A
rcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]))/9 - (2*d*f*(a + b*Log[c*x^n])^
2)/(3*x) + ((-d)^(3/2)*f^(3/2)*(a + b*Log[c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f
]*x])/3 - ((-d)^(3/2)*f^(3/2)*(a + b*Log[c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]
*x])/3 - (2*b^2*n^2*Log[1 + d*f*x^2])/(27*x^3) - (2*b*n*(a + b*Log[c*x^n])*
Log[1 + d*f*x^2])/(9*x^3) - ((a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(3*x^3)
 - (2*b*(-d)^(3/2)*f^(3/2)*n*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sqrt[
f]*x)])/3 + (2*b*(-d)^(3/2)*f^(3/2)*n*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d
]*Sqrt[f]*x])/3 + ((2*I)/9)*b^2*d^(3/2)*f^(3/2)*n^2*PolyLog[2, (-I)*Sqrt[d]
*Sqrt[f]*x] - ((2*I)/9)*b^2*d^(3/2)*f^(3/2)*n^2*PolyLog[2, I*Sqrt[d]*Sqrt[f
]*x] + (2*b^2*(-d)^(3/2)*f^(3/2)*n^2*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/3 -
 (2*b^2*(-d)^(3/2)*f^(3/2)*n^2*PolyLog[3, Sqrt[-d]*Sqrt[f]*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = -\frac{2b^2n^2\log\left(1+dfx^2\right)}{27x^3} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)}{9x^3} - $$

$$= -\frac{2b^2n^2\log\left(1+dfx^2\right)}{27x^3} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)}{9x^3} - $$

$$= -\frac{4b^2dfn^2}{27x} - \frac{2b^2n^2\log\left(1+dfx^2\right)}{27x^3} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(1\right.}{9x^3}$$

$$= -\frac{4b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right) - \frac{2b^2n^2\log\left(1+d\right.}{27x^3}$$

$$= -\frac{16b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right) - \frac{4bdfn\left(a+b\,\mathrm{lo}\right.}{9x}$$

$$= -\frac{52b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right) - \frac{16bdfn\left(a+b\,\mathrm{l}\right.}{9x}$$

$$= -\frac{52b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right) - \frac{16bdfn\left(a+b\,\mathrm{l}\right.}{9x}$$

$$= -\frac{52b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right) - \frac{16bdfn\left(a+b\,\mathrm{l}\right.}{9x}$$

$$= -\frac{52b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right) - \frac{16bdfn\left(a+b\,\mathrm{l}\right.}{9x}$$

Mathematica [A] time = 0.55, size = 585, normalized size = 1.08

$$\frac{1}{27}\left(-2d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)\left(9a^2+18ab\left(\log\left(cx^n\right)-n\log(x)\right)+6abn+9b^2\left(\log\left(cx^n\right)-n\log(x)\right)^2+6b^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

```
[Out] (-2*d^(3/2)*f^(3/2)*ArcTan[Sqrt[d]*Sqrt[f]*x]*(9*a^2 + 6*a*b*n + 2*b^2*n^2
+ 18*a*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^2*n*(-(n*Log[x]) + Log[c*x^n]) +
9*b^2*(-(n*Log[x]) + Log[c*x^n])^2) - (2*d*f*(9*a^2 + 6*a*b*n + 2*b^2*n^2 +
 9*b^2*n^2*Log[x]^2 + 6*b*(3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2 - 6*b
*n*Log[x]*(3*a + b*n + 3*b*Log[c*x^n])))/x - ((9*a^2 + 6*a*b*n + 2*b^2*n^2
+ 6*b*(3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2)*Log[1 + d*f*x^2])/x^3 +
((6*I)*b*d*f*n*(3*a + b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n])*(2*I + (2*I)*Log
[x] + Sqrt[d]*Sqrt[f]*x*(Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (
-I)*Sqrt[d]*Sqrt[f]*x]) - Sqrt[d]*Sqrt[f]*x*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[
f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])))/x + ((9*I)*b^2*d*f*n^2*(4*I + (4
*I)*Log[x] + (2*I)*Log[x]^2 + Sqrt[d]*Sqrt[f]*x*(Log[x]^2*Log[1 + I*Sqrt[d]
*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, (-
I)*Sqrt[d]*Sqrt[f]*x]) - Sqrt[d]*Sqrt[f]*x*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt
[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*
Sqrt[f]*x])))/x)/27
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2\log\left(dfx^2+1\right)\log\left(cx^n\right)^2+2\,ab\log\left(dfx^2+1\right)\log\left(cx^n\right)+a^2\log\left(dfx^2+1\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*
x^n) + a^2*log(d*f*x^2 + 1))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right)^{2} \log\left(\left(fx^{2} + \frac{1}{d}\right)d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x^4, x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^{n}\right) + a\right)^{2} \ln\left(\left(f\,x^{2} + \frac{1}{d}\right)d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d)/x^4,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^2+1/d)*d)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(9\,b^{2} \log\left(x^{n}\right)^{2} + \left(2\,n^{2} + 6\,n \log(c) + 9\,\log(c)^{2}\right)b^{2} + 6\,ab\left(n + 3\,\log(c)\right) + 9\,a^{2} + 6\,\left(b^{2}\left(n + 3\,\log(c)\right) + 3\,ab\right)\log}{27\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(9*b^2*log(x^n)^2 + (2*n^2 + 6*n*log(c) + 9*log(c)^2)*b^2 + 6*a*b*(n
+ 3*log(c)) + 9*a^2 + 6*(b^2*(n + 3*log(c)) + 3*a*b)*log(x^n))*log(d*f*x^2
+ 1)/x^3 + integrate(2/27*(9*b^2*d*f*log(x^n)^2 + 9*a^2*d*f + 6*(d*f*n + 3*
d*f*log(c))*a*b + (2*d*f*n^2 + 6*d*f*n*log(c) + 9*d*f*log(c)^2)*b^2 + 6*(3*
a*b*d*f + (d*f*n + 3*d*f*log(c))*b^2)*log(x^n))/(d*f*x^4 + x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(f\,x^{2} + \frac{1}{d}\right)\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^4,x)
```

```
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x**4,x)
```

```
[Out] Timed out
```

3.40 $\int x^3 \left(a+b\log\left(cx^n\right)\right)^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=591

$$\frac{3b^2n^2\text{Li}_2\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{16d^2f^2}+\frac{3b^2n^2\text{Li}_3\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{8d^2f^2}-\frac{3b^2n^2\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right.\right.}{32d^2f^2}$$

```
[Out] -45/128*b^3*n^3*x^2/d/f+3/64*b^3*n^3*x^4+21/32*b^2*n^2*x^2*(a+b*ln(c*x^n))/
d/f-9/64*b^2*n^2*x^4*(a+b*ln(c*x^n))-9/16*b*n*x^2*(a+b*ln(c*x^n))^2/d/f+3/1
6*b*n*x^4*(a+b*ln(c*x^n))^2+1/4*x^2*(a+b*ln(c*x^n))^3/d/f-1/8*x^4*(a+b*ln(c
*x^n))^3+3/128*b^3*n^3*ln(d*f*x^2+1)/d^2/f^2-3/128*b^3*n^3*x^4*ln(d*f*x^2+1
)-3/32*b^2*n^2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d^2/f^2+3/32*b^2*n^2*x^4*(a+b*
ln(c*x^n))*ln(d*f*x^2+1)+3/16*b*n*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d^2/f^2-3
/16*b*n*x^4*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)-1/4*(a+b*ln(c*x^n))^3*ln(d*f*x^
2+1)/d^2/f^2+1/4*x^4*(a+b*ln(c*x^n))^3*ln(d*f*x^2+1)-3/64*b^3*n^3*polylog(2
,-d*f*x^2)/d^2/f^2+3/16*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^2)/d^2/f^2
-3/8*b*n*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^2)/d^2/f^2-3/32*b^3*n^3*polylog
(3,-d*f*x^2)/d^2/f^2+3/8*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^2)/d^2/f^
2-3/16*b^3*n^3*polylog(4,-d*f*x^2)/d^2/f^2
```

Rubi [A] time = 0.73, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2454, 2395, 43, 2377, 2305, 2304, 2374, 2383, 6589, 2376, 2391}

$$\frac{3b^2n^2\text{PolyLog}\left(2,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{16d^2f^2}+\frac{3b^2n^2\text{PolyLog}\left(3,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{8d^2f^2}-\frac{3bn\text{PolyLog}\left(2,-d\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-45*b^3*n^3*x^2)/(128*d*f) + (3*b^3*n^3*x^4)/64 + (21*b^2*n^2*x^2*(a + b*L
og[c*x^n]))/(32*d*f) - (9*b^2*n^2*x^4*(a + b*Log[c*x^n]))/64 - (9*b*n*x^2*(
a + b*Log[c*x^n])^2)/(16*d*f) + (3*b*n*x^4*(a + b*Log[c*x^n])^2)/16 + (x^2*
(a + b*Log[c*x^n])^3)/(4*d*f) - (x^4*(a + b*Log[c*x^n])^3)/8 + (3*b^3*n^3*L
og[1 + d*f*x^2])/(128*d^2*f^2) - (3*b^3*n^3*x^4*Log[1 + d*f*x^2])/128 - (3*
b^2*n^2*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(32*d^2*f^2) + (3*b^2*n^2*x^4*
(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/32 + (3*b*n*(a + b*Log[c*x^n])^2*Log[1
 + d*f*x^2])/(16*d^2*f^2) - (3*b*n*x^4*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2
])/16 - ((a + b*Log[c*x^n])^3*Log[1 + d*f*x^2])/(4*d^2*f^2) + (x^4*(a + b*L
og[c*x^n])^3*Log[1 + d*f*x^2])/4 - (3*b^3*n^3*PolyLog[2, -(d*f*x^2)])/(64*d
^2*f^2) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^2)])/(16*d^2*f^2
) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*x^2)])/(8*d^2*f^2) - (3*b^
3*n^3*PolyLog[3, -(d*f*x^2)])/(32*d^2*f^2) + (3*b^2*n^2*(a + b*Log[c*x^n])*
PolyLog[3, -(d*f*x^2)])/(8*d^2*f^2) - (3*b^3*n^3*PolyLog[4, -(d*f*x^2)])/(1
6*d^2*f^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \left(a+b\log\left(cx^n\right)\right)^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right) dx = \frac{x^2\left(a+b\log\left(cx^n\right)\right)^3}{4df} - \frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right)^3 - \frac{\left(a+b\log\left(cx\right.\right.}{4}$$

$$= \frac{x^2\left(a+b\log\left(cx^n\right)\right)^3}{4df} - \frac{1}{8}x^4\left(a+b\log\left(cx^n\right)\right)^3 - \frac{\left(a+b\log\left(cx\right.\right.}{4}$$

$$= -\frac{9bnx^2\left(a+b\log\left(cx^n\right)\right)^2}{16df} + \frac{3}{16}bnx^4\left(a+b\log\left(cx^n\right)\right)^2 + \frac{x^2\left(a\right.}{}$$

$$= -\frac{3b^3n^3x^2}{16df} + \frac{3}{256}b^3n^3x^4 + \frac{3b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}{8df} - \frac{3}{64}b^2n^2$$

$$= -\frac{9b^3n^3x^2}{32df} + \frac{3}{128}b^3n^3x^4 + \frac{21b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}{32df} - \frac{9}{64}b^2n$$

$$= -\frac{21b^3n^3x^2}{64df} + \frac{9}{256}b^3n^3x^4 + \frac{21b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}{32df} - \frac{9}{64}b^2$$

$$= -\frac{21b^3n^3x^2}{64df} + \frac{9}{256}b^3n^3x^4 + \frac{21b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}{32df} - \frac{9}{64}b^2$$

$$= -\frac{21b^3n^3x^2}{64df} + \frac{9}{256}b^3n^3x^4 + \frac{21b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}{32df} - \frac{9}{64}b^2$$

$$= -\frac{21b^3n^3x^2}{64df} + \frac{9}{256}b^3n^3x^4 + \frac{21b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}{32df} - \frac{9}{64}b^2$$

$$= -\frac{45b^3n^3x^2}{128df} + \frac{3}{64}b^3n^3x^4 + \frac{21b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}{32df} - \frac{9}{64}b^2n$$

Mathematica [C] time = 1.09, size = 1234, normalized size = 2.09

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] -1/256*(-2*d*f*x^2*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3*b^3*n^3 + 48*a*b
^2*n*(n*Log[x] - Log[c*x^n]) + 96*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*b^3
*n^2*(-(n*Log[x]) + Log[c*x^n]) + 96*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 2
4*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 32*b^3*(-(n*Log[x]) + Log[c*x^n])^3)
 + d^2*f^2*x^4*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3*b^3*n^3 + 48*a*b^2*n
```

```
*(n*Log[x] - Log[c*x^n]) + 96*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*b^3*n^2
*(-(n*Log[x]) + Log[c*x^n]) + 96*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 24*b^
3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 32*b^3*(-(n*Log[x]) + Log[c*x^n])^3) - 2
*d^2*f^2*x^4*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3*b^3*n^3 + 12*b*(8*a^2
- 4*a*b*n + b^2*n^2)*Log[c*x^n] - 24*b^2*(-4*a + b*n)*Log[c*x^n]^2 + 32*b^3
*Log[c*x^n]^3)*Log[1 + d*f*x^2] + 2*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3
*b^3*n^3 + 48*a*b^2*n*(n*Log[x] - Log[c*x^n]) + 96*a^2*b*(-(n*Log[x]) + Log
[c*x^n]) + 12*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 96*a*b^2*(-(n*Log[x]) +
Log[c*x^n])^2 - 24*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 32*b^3*(-(n*Log[x])
 + Log[c*x^n])^3)*Log[1 + d*f*x^2] + 24*b*n*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*
b^2*n*(n*Log[x] - Log[c*x^n]) + 16*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(
-(n*Log[x]) + Log[c*x^n])^2)*((d*f*x^2)/2 - (d^2*f^2*x^4)/8 - d*f*x^2*Log[x
] + (d^2*f^2*x^4*Log[x])/2 + Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]*L
og[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + PolyLog[
2, I*Sqrt[d]*Sqrt[f]*x]) - 96*b^2*n^2*(4*a - b*n - 4*b*n*Log[x] + 4*b*Log[c
*x^n])*((d*f*x^2*(1 - 2*Log[x] + 2*Log[x]^2))/4 - (d^2*f^2*x^4*(1 - 4*Log[x
] + 8*Log[x]^2))/32 - (Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x])/2 - (Log[x]^2
*Log[1 + I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x
] - Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, (-I)*Sqrt[d]*Sqrt[f
]*x] + PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) + b^3*n^3*(-16*d*f*x^2*(-3 + 6*Log[
x] - 6*Log[x]^2 + 4*Log[x]^3) + d^2*f^2*x^4*(-3 + 12*Log[x] - 24*Log[x]^2 +
 32*Log[x]^3) + 64*(Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*Poly
Log[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x
] + 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x]) + 64*(Log[x]^3*Log[1 - I*Sqrt[d]*
Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[
3, I*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, I*Sqrt[d]*Sqrt[f]*x])))/(d^2*f^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\log\left(dfx^2+1\right)\log\left(cx^n\right)^3+3\,ab^2x^3\log\left(dfx^2+1\right)\log\left(cx^n\right)^2+3\,a^2bx^3\log\left(dfx^2+1\right)\log\left(cx^n\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*x^3*log(d*f*x^2 +
1)*log(c*x^n)^2 + 3*a^2*b*x^3*log(d*f*x^2 + 1)*log(c*x^n) + a^3*x^3*log(d*f
*x^2 + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)^3 x^3 \ln\left(\left(f\,x^2+\frac{1}{d}\right)d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d),x)
```

```
[Out] int(x^3*(b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{128}\left(32\,b^3x^4\log\left(x^n\right)^3 - 24\left(b^3\left(n - 4\,\log(c)\right) - 4\,ab^2\right)x^4\log\left(x^n\right)^2 + 12\left(\left(n^2 - 4\,n\log(c) + 8\,\log(c)^2\right)b^3 - 4\,ab^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/128*(32*b^3*x^4*log(x^n)^3 - 24*(b^3*(n - 4*log(c)) - 4*a*b^2)*x^4*log(x^
n)^2 + 12*((n^2 - 4*n*log(c) + 8*log(c)^2)*b^3 - 4*a*b^2*(n - 4*log(c)) + 8
*a^2*b)*x^4*log(x^n) + (12*(n^2 - 4*n*log(c) + 8*log(c)^2)*a*b^2 - (3*n^3 -
 12*n^2*log(c) + 24*n*log(c)^2 - 32*log(c)^3)*b^3 - 24*a^2*b*(n - 4*log(c))
 + 32*a^3)*x^4)*log(d*f*x^2 + 1) - integrate(1/64*(32*b^3*d*f*x^5*log(x^n)^
3 + 24*(4*a*b^2*d*f - (d*f*n - 4*d*f*log(c))*b^3)*x^5*log(x^n)^2 + 12*(8*a^
2*b*d*f - 4*(d*f*n - 4*d*f*log(c))*a*b^2 + (d*f*n^2 - 4*d*f*n*log(c) + 8*d*
f*log(c)^2)*b^3)*x^5*log(x^n) + (32*a^3*d*f - 24*(d*f*n - 4*d*f*log(c))*a^2
*b + 12*(d*f*n^2 - 4*d*f*n*log(c) + 8*d*f*log(c)^2)*a*b^2 - (3*d*f*n^3 - 12
*d*f*n^2*log(c) + 24*d*f*n*log(c)^2 - 32*d*f*log(c)^3)*b^3)*x^5)/(d*f*x^2 +
 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

3.41 $\int x \left(a+b\log\left(cx^n\right)\right)^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right) dx$

Optimal. Leaf size=411

$$-\frac{3b^2n^2\text{Li}_2\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{4df}-\frac{3b^2n^2\text{Li}_3\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{4df}+\frac{3b^2n^2\left(dfx^2+1\right)\log\left(dfx^2+1\right)\left(a+\right.}{4df}$$

```
[Out] 3/2*b^3*n^3*x^2-9/4*b^2*n^2*x^2*(a+b*ln(c*x^n))+3/2*b*n*x^2*(a+b*ln(c*x^n))
^2-1/2*x^2*(a+b*ln(c*x^n))^3-3/8*b^3*n^3*(d*f*x^2+1)*ln(d*f*x^2+1)/d/f+3/4*
b^2*n^2*(d*f*x^2+1)*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d/f-3/4*b*n*(d*f*x^2+1)*(
a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d/f+1/2*(d*f*x^2+1)*(a+b*ln(c*x^n))^3*ln(d*f
*x^2+1)/d/f+3/8*b^3*n^3*polylog(2,-d*f*x^2)/d/f-3/4*b^2*n^2*(a+b*ln(c*x^n))
*polylog(2,-d*f*x^2)/d/f+3/4*b*n*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^2)/d/f+
3/8*b^3*n^3*polylog(3,-d*f*x^2)/d/f-3/4*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-
d*f*x^2)/d/f+3/8*b^3*n^3*polylog(4,-d*f*x^2)/d/f
```

Rubi [A] time = 1.04, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 21, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.808, Rules used = {2454, 2389, 2295, 2377, 2305, 2304, 2353, 2302, 30, 6742, 2374, 2383, 6589, 14, 2351, 2301, 2376, 2475, 2411, 43, 2315}

$$-\frac{3b^2n^2\text{PolyLog}\left(2,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{4df}-\frac{3b^2n^2\text{PolyLog}\left(3,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)}{4df}+\frac{3bn\text{PolyLog}\left(2,-df\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (3*b^3*n^3*x^2)/2 - (9*b^2*n^2*x^2*(a + b*Log[c*x^n]))/4 + (3*b*n*x^2*(a +
b*Log[c*x^n])^2)/2 - (x^2*(a + b*Log[c*x^n])^3)/2 - (3*b^3*n^3*(1 + d*f*x^2
)*Log[1 + d*f*x^2])/(8*d*f) + (3*b^2*n^2*(1 + d*f*x^2)*(a + b*Log[c*x^n])*L
og[1 + d*f*x^2])/(4*d*f) - (3*b*n*(1 + d*f*x^2)*(a + b*Log[c*x^n])^2*Log[1
+ d*f*x^2])/(4*d*f) + ((1 + d*f*x^2)*(a + b*Log[c*x^n])^3*Log[1 + d*f*x^2])
/(2*d*f) + (3*b^3*n^3*PolyLog[2, -(d*f*x^2)])/(8*d*f) - (3*b^2*n^2*(a + b*L
og[c*x^n])*PolyLog[2, -(d*f*x^2)])/(4*d*f) + (3*b*n*(a + b*Log[c*x^n])^2*Po
lyLog[2, -(d*f*x^2)])/(4*d*f) + (3*b^3*n^3*PolyLog[3, -(d*f*x^2)])/(8*d*f)
- (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(d*f*x^2)])/(4*d*f) + (3*b^3*n^
3*PolyLog[4, -(d*f*x^2)])/(8*d*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx=-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^3+\frac{\left(1+dfx^2\right)\left(a+b\log\left(cx^n\right)\right)^3\log\left(1+d\right.}{2df}$$

$$=-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^3+\frac{\left(1+dfx^2\right)\left(a+b\log\left(cx^n\right)\right)^3\log\left(1+d\right.}{2df}$$

$$=\frac{3}{4}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^3+\frac{\left(1+dfx^2\right)\left(a\right.}{}$$

$$=\frac{3}{8}b^3n^3x^2-\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{4}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{3}{8}b^3n^3x^2-\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{3}{8}b^3n^3x^2-\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{3}{4}b^3n^3x^2-\frac{3}{2}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{3}{4}b^3n^3x^2-\frac{3}{2}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{3}{4}b^3n^3x^2-\frac{9}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{9}{8}b^3n^3x^2-\frac{9}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{9}{8}b^3n^3x^2-\frac{9}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{9}{8}b^3n^3x^2-\frac{9}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{9}{8}b^3n^3x^2-\frac{9}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{9}{8}b^3n^3x^2-\frac{9}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

$$=\frac{3}{2}b^3n^3x^2-\frac{9}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)^2-$$

Mathematica [C] time = 0.60, size = 1004, normalized size = 2.44

$$\frac{-b^3\left(4dfx^2\log^3(x)-4\log\left(1-i\sqrt{d}\sqrt{f}x\right)\log^3(x)-4\log\left(i\sqrt{d}\sqrt{f}x+1\right)\log^3(x)-6dfx^2\log^2(x)-12\mathrm{Li}_2\left(-i\sqrt{d}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-(d*f*x^2*(4*a^3 - 6*a^2*b*n + 6*a*b^2*n^2 - 3*b^3*n^3 + 12*a*b^2*n*(n*Log
[x] - Log[c*x^n]) + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Lo
g[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 6*b^3*n*(-(n*
Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3)) + d*f*x^2*(4
*a^3 - 6*a^2*b*n + 6*a*b^2*n^2 - 3*b^3*n^3 + 6*b*(2*a^2 - 2*a*b*n + b^2*n^2
)*Log[c*x^n] - 6*b^2*(-2*a + b*n)*Log[c*x^n]^2 + 4*b^3*Log[c*x^n]^3)*Log[1
+ d*f*x^2] + (4*a^3 - 6*a^2*b*n + 6*a*b^2*n^2 - 3*b^3*n^3 + 12*a*b^2*n*(n*L
og[x] - Log[c*x^n]) + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*
Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 6*b^3*n*(-(
n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3)*Log[1 + d*f
*x^2] + 6*b*n*(2*a^2 - 2*a*b*n + b^2*n^2 + 2*b^2*n*(n*Log[x] - Log[c*x^n])
+ 4*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Log[c*x^n])^2)*((
d*f*x^2)/2 - d*f*x^2*Log[x] + Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]*
Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + PolyLog
[2, I*Sqrt[d]*Sqrt[f]*x]) + 3*b^2*n^2*(-2*a + b*n + 2*b*n*Log[x] - 2*b*Log[
c*x^n])*(d*f*x^2 - 2*d*f*x^2*Log[x] + 2*d*f*x^2*Log[x]^2 - 2*Log[x]^2*Log[1
 - I*Sqrt[d]*Sqrt[f]*x] - 2*Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] - 4*Log[x
]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 4*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f
]*x] + 4*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] + 4*PolyLog[3, I*Sqrt[d]*Sqrt[f
]*x]) - b^3*n^3*(-3*d*f*x^2 + 6*d*f*x^2*Log[x] - 6*d*f*x^2*Log[x]^2 + 4*d*f
*x^2*Log[x]^3 - 4*Log[x]^3*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - 4*Log[x]^3*Log[1
+ I*Sqrt[d]*Sqrt[f]*x] - 12*Log[x]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 1
2*Log[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + 24*Log[x]*PolyLog[3, (-I)*Sqrt
[d]*Sqrt[f]*x] + 24*Log[x]*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x] - 24*PolyLog[4,
(-I)*Sqrt[d]*Sqrt[f]*x] - 24*PolyLog[4, I*Sqrt[d]*Sqrt[f]*x]))/(8*d*f)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

integral $\left(b^3 x \log\left(dfx^2+1\right) \log\left(cx^n\right)^3 + 3\,ab^2 x \log\left(dfx^2+1\right) \log\left(cx^n\right)^2 + 3\,a^2 bx \log\left(dfx^2+1\right) \log\left(cx^n\right) + \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*x*log(d*f*x^2 + 1)*l
og(c*x^n)^2 + 3*a^2*b*x*log(d*f*x^2 + 1)*log(c*x^n) + a^3*x*log(d*f*x^2 + 1
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^n\right) + a\right)^3 x \log\left(\left(fx^2 + \frac{1}{d}\right)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*x^2 + 1/d)*d), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\,x^n\right) + a\right)^3 x \ln\left(\left(f\,x^2 + \frac{1}{d}\right) d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(4\,b^3x^2\log\left(x^n\right)^3 - 6\left(b^3\left(n-2\,\log(c)\right) - 2\,ab^2\right)x^2\log\left(x^n\right)^2 + 6\left(\left(n^2-2\,n\log(c)+2\,\log(c)^2\right)b^3 - 2\,ab^2\left(n-2\,\mathrm{lo}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*b^3*x^2*log(x^n)^3 - 6*(b^3*(n - 2*log(c)) - 2*a*b^2)*x^2*log(x^n)^2
 + 6*((n^2 - 2*n*log(c) + 2*log(c)^2)*b^3 - 2*a*b^2*(n - 2*log(c)) + 2*a^2*
b)*x^2*log(x^n) + (6*(n^2 - 2*n*log(c) + 2*log(c)^2)*a*b^2 - (3*n^3 - 6*n^2
*log(c) + 6*n*log(c)^2 - 4*log(c)^3)*b^3 - 6*a^2*b*(n - 2*log(c)) + 4*a^3)*
x^2)*log(d*f*x^2 + 1) - integrate(1/4*(4*b^3*d*f*x^3*log(x^n)^3 + 6*(2*a*b^
2*d*f - (d*f*n - 2*d*f*log(c))*b^3)*x^3*log(x^n)^2 + 6*(2*a^2*b*d*f - 2*(d*
f*n - 2*d*f*log(c))*a*b^2 + (d*f*n^2 - 2*d*f*n*log(c) + 2*d*f*log(c)^2)*b^3
)*x^3*log(x^n) + (4*a^3*d*f - 6*(d*f*n - 2*d*f*log(c))*a^2*b + 6*(d*f*n^2 -
 2*d*f*n*log(c) + 2*d*f*log(c)^2)*a*b^2 - (3*d*f*n^3 - 6*d*f*n^2*log(c) + 6
*d*f*n*log(c)^2 - 4*d*f*log(c)^3)*b^3)*x^3)/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

3.42 $\int \frac{\left(a+b\log(cx^n)\right)^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x} dx$

Optimal. Leaf size=101

$$-\frac{3}{4}b^2n^2\text{Li}_4\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)+\frac{3}{4}bn\text{Li}_3\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{2}\text{Li}_2\left(-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)^3+\frac{3}{8}b$$

```
[Out] -1/2*(a+b*ln(c*x^n))^3*polylog(2,-d*f*x^2)+3/4*b*n*(a+b*ln(c*x^n))^2*polylo
g(3,-d*f*x^2)-3/4*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-d*f*x^2)+3/8*b^3*n^3*p
olylog(5,-d*f*x^2)
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2374, 2383, 6589}

$$-\frac{3}{4}b^2n^2\text{PolyLog}\left(4,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)+\frac{3}{4}bn\text{PolyLog}\left(3,-dfx^2\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{2}\text{PolyLog}\left(2,-dfx\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*x^2)])/2 + (3*b*n*(a + b*Log[c*x^n]
)^2*PolyLog[3, -(d*f*x^2)])/4 - (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[4, -(
d*f*x^2)])/4 + (3*b^3*n^3*PolyLog[5, -(d*f*x^2)])/8
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx &= -\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2\left(-dfx^2\right)+\frac{1}{2}(3bn)\int\frac{\left(a+b\log\left(cx^n\right)\right)^2}{x} \\
&= -\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2\left(-dfx^2\right)+\frac{3}{4}bn\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_3\left(-\right. \\
&= -\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2\left(-dfx^2\right)+\frac{3}{4}bn\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_3\left(-\right. \\
&= -\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2\left(-dfx^2\right)+\frac{3}{4}bn\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_3\left(-\right.
\end{aligned}$$

Mathematica [C] time = 0.32, size = 754, normalized size = 7.47

$$\frac{1}{4}\left(4b^2n^2\left(6\text{Li}_4\left(-i\sqrt{d}\sqrt{f}x\right)+6\text{Li}_4\left(i\sqrt{d}\sqrt{f}x\right)+3\log^2(x)\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)+3\log^2(x)\text{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)-6\log(x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] (-(Log[x]*(b^3*n^3*Log[x]^3 - 4*b^2*n^2*Log[x]^2*(a + b*Log[c*x^n]) + 6*b*n
*Log[x]*(a + b*Log[c*x^n])^2 - 4*(a + b*Log[c*x^n])^3)*Log[1 + d*f*x^2]) -
4*(a - b*n*Log[x] + b*Log[c*x^n])^3*(Log[x]*(Log[1 - I*Sqrt[d]*Sqrt[f]*x] +
 Log[1 + I*Sqrt[d]*Sqrt[f]*x]) + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + PolyL
og[2, I*Sqrt[d]*Sqrt[f]*x]) - 6*b*n*(a - b*n*Log[x] + b*Log[c*x^n])^2*(Log[
x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] +
 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d
]*Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d
]*Sqrt[f]*x]) + 4*b^2*n^2*(-a + b*n*Log[x] - b*Log[c*x^n])*(Log[x]^3*Log[1
- I*Sqrt[d]*Sqrt[f]*x] + Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2
*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[
f]*x] - 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[3, I
*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, I
*Sqrt[d]*Sqrt[f]*x]) - b^3*n^3*(Log[x]^4*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log
[x]^4*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 4*Log[x]^3*PolyLog[2, (-I)*Sqrt[d]*Sqr
t[f]*x] + 4*Log[x]^3*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 12*Log[x]^2*PolyLog[
3, (-I)*Sqrt[d]*Sqrt[f]*x] - 12*Log[x]^2*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x] +
24*Log[x]*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] + 24*Log[x]*PolyLog[4, I*Sqrt[
d]*Sqrt[f]*x] - 24*PolyLog[5, (-I)*Sqrt[d]*Sqrt[f]*x] - 24*PolyLog[5, I*Sqr
t[d]*Sqrt[f]*x]))/4
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(dfx^2+1\right)\log\left(cx^n\right)^3+3\,ab^2\log\left(dfx^2+1\right)\log\left(cx^n\right)^2+3\,a^2b\log\left(dfx^2+1\right)\log\left(cx^n\right)+a^3\log}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log(
c*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1))/x, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d)/x, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^n\right)+a\right)^3\ln\left(\left(f\,x^2+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d)/x,x)
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\left(b^3n^3\log(x)^4 - 4\,b^3\log(x)\log\left(x^n\right)^3 - 4\left(b^3n^2\log(c) + ab^2n^2\right)\log(x)^3 + 6\left(b^3n\log(c)^2 + 2\,ab^2n\log(c) + a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x,x, algorithm="maxima")
[Out] -1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b^
2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^2
+ 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3*n^
2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*a*b^
2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3
*a^2*b*log(c) + a^3)*log(x))*log(d*f*x^2 + 1) - integrate(-1/2*(b^3*d*f*n^3
*x*log(x)^4 - 4*b^3*d*f*x*log(x)*log(x^n)^3 - 4*(b^3*d*f*n^2*log(c) + a*b^2
*d*f*n^2)*x*log(x)^3 + 6*(b^3*d*f*n*log(c)^2 + 2*a*b^2*d*f*n*log(c) + a^2*b
*d*f*n)*x*log(x)^2 - 4*(b^3*d*f*log(c)^3 + 3*a*b^2*d*f*log(c)^2 + 3*a^2*b*d
*f*log(c) + a^3*d*f)*x*log(x) + 6*(b^3*d*f*n*x*log(x)^2 - 2*(b^3*d*f*log(c)
 + a*b^2*d*f)*x*log(x))*log(x^n)^2 - 4*(b^3*d*f*n^2*x*log(x)^3 - 3*(b^3*d*f
*n*log(c) + a*b^2*d*f*n)*x*log(x)^2 + 3*(b^3*d*f*log(c)^2 + 2*a*b^2*d*f*log
(c) + a^2*b*d*f)*x*log(x))*log(x^n))/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2))/x,x)
[Out] Timed out
```

3.43 $\int \frac{\left(a+b\log(cx^n)\right)^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=425

$$\frac{3}{4}b^2dfn^2\text{Li}_2\left(-\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{3}{4}b^2dfn^2\text{Li}_3\left(-\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{3}{4}b^2dfn^2\log\left(\frac{1}{dfx^2}+1\right)\left(a+b\right.$$

```
[Out] 3/4*b^3*d*f*n^3*ln(x)-3/4*b^2*d*f*n^2*ln(1+1/d/f/x^2)*(a+b*ln(c*x^n))-3/4*b
*d*f*n*ln(1+1/d/f/x^2)*(a+b*ln(c*x^n))^2-1/2*d*f*ln(1+1/d/f/x^2)*(a+b*ln(c*
x^n))^3-3/8*b^3*d*f*n^3*ln(d*f*x^2+1)-3/8*b^3*n^3*ln(d*f*x^2+1)/x^2-3/4*b^2
*n^2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln(d*f*x^2
+1)/x^2-1/2*(a+b*ln(c*x^n))^3*ln(d*f*x^2+1)/x^2+3/8*b^3*d*f*n^3*polylog(2,-
1/d/f/x^2)+3/4*b^2*d*f*n^2*(a+b*ln(c*x^n))*polylog(2,-1/d/f/x^2)+3/4*b*d*f*
n*(a+b*ln(c*x^n))^2*polylog(2,-1/d/f/x^2)+3/8*b^3*d*f*n^3*polylog(3,-1/d/f/
x^2)+3/4*b^2*d*f*n^2*(a+b*ln(c*x^n))*polylog(3,-1/d/f/x^2)+3/8*b^3*d*f*n^3*
polylog(4,-1/d/f/x^2)
```

Rubi [A] time = 0.58, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2305, 2304, 2378, 266, 36, 29, 31, 2345, 2391, 2374, 6589, 2383}

$$\frac{3}{4}b^2dfn^2\text{PolyLog}\left(2,-\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{3}{4}b^2dfn^2\text{PolyLog}\left(3,-\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{3}{4}bdfn\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] (3*b^3*d*f*n^3*Log[x])/4 - (3*b^2*d*f*n^2*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c
*x^n]))/4 - (3*b*d*f*n*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n])^2)/4 - (d*f*
Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n])^3)/2 - (3*b^3*d*f*n^3*Log[1 + d*f*x
^2])/8 - (3*b^3*n^3*Log[1 + d*f*x^2])/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n
])*Log[1 + d*f*x^2])/(4*x^2) - (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2]
)/(4*x^2) - ((a + b*Log[c*x^n])^3*Log[1 + d*f*x^2])/(2*x^2) + (3*b^3*d*f*n^
3*PolyLog[2, -(1/(d*f*x^2))])/8 + (3*b^2*d*f*n^2*(a + b*Log[c*x^n])*PolyLog
[2, -(1/(d*f*x^2))])/4 + (3*b*d*f*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(1/(d*
f*x^2))])/4 + (3*b^3*d*f*n^3*PolyLog[3, -(1/(d*f*x^2))])/8 + (3*b^2*d*f*n^2
*(a + b*Log[c*x^n])*PolyLog[3, -(1/(d*f*x^2))])/4 + (3*b^3*d*f*n^3*PolyLog[
4, -(1/(d*f*x^2))])/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2345

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> -Simp[(Log[1 + d/(e*x^r)]*(a + b*Log[c*x^n])^p)/(d*r),
x] + Dist[(b*n*p)/(d*r), Int[(Log[1 + d/(e*x^r)]*(a + b*Log[c*x^n])^(p - 1)
)/x, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = -\frac{3b^3n^3\log\left(1+dfx^2\right)}{8x^2} - \frac{3b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)}{4x^2} - 3$$

$$= -\frac{3b^3n^3\log\left(1+dfx^2\right)}{8x^2} - \frac{3b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)}{4x^2} - 3$$

$$= -\frac{3}{4}b^2dfn^2\log\left(1+\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right) - \frac{3}{4}bdfn\log\left(1+\frac{1}{dfx^2}\right.$$

$$= -\frac{3}{4}b^2dfn^2\log\left(1+\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right) - \frac{3}{4}bdfn\log\left(1+\frac{1}{dfx^2}\right.$$

$$= \frac{3}{4}b^3dfn^3\log(x) - \frac{3}{4}b^2dfn^2\log\left(1+\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right) - \frac{3}{4}bdf$$

$$= \frac{3}{4}b^3dfn^3\log(x) - \frac{3}{4}b^2dfn^2\log\left(1+\frac{1}{dfx^2}\right)\left(a+b\log\left(cx^n\right)\right) - \frac{3}{4}bdf$$

Mathematica [C] time = 0.39, size = 940, normalized size = 2.21

$$\frac{1}{8}\left(2b^3df\left(\log^4(x) - 2\log\left(1-i\sqrt{d}\sqrt{f}x\right)\log^3(x) - 2\log\left(i\sqrt{d}\sqrt{f}x+1\right)\log^3(x) - 6\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)\log^2(x) - 6\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] (2*d*f*Log[x]*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*
Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(
-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n
*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3) - ((4*a
^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 6*b*(2*a^2 + 2*a*b*n + b^2*n^2)*
Log[c*x^n] + 6*b^2*(2*a + b*n)*Log[c*x^n]^2 + 4*b^3*Log[c*x^n]^3)*Log[1 + d
*f*x^2])/x^2 - d*f*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*
(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*
n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*
b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3)*Lo
g[1 + d*f*x^2] + 6*b*d*f*n*(2*a^2 + 2*a*b*n + b^2*n^2 + 4*a*b*(-(n*Log[x])
+ Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + L
og[c*x^n])^2)*(Log[x]*(Log[x] - Log[1 - I*Sqrt[d]*Sqrt[f]*x] - Log[1 + I*Sq
rt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - PolyLog[2, I*Sqrt[
d]*Sqrt[f]*x]) + 12*b^2*d*f*n^2*(2*a + b*n - 2*b*n*Log[x] + 2*b*Log[c*x^n])
*(Log[x]^3/3 - (Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x])/2 - (Log[x]^2*Log[1
+ I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - Log
[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] +
PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) + 2*b^3*d*f*n^3*(Log[x]^4 - 2*Log[x]^3*Log
[1 - I*Sqrt[d]*Sqrt[f]*x] - 2*Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x] - 6*Log
[x]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]^2*PolyLog[2, I*Sqrt[d]*
Sqrt[f]*x] + 12*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] + 12*Log[x]*PolyL
og[3, I*Sqrt[d]*Sqrt[f]*x] - 12*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] - 12*Pol
yLog[4, I*Sqrt[d]*Sqrt[f]*x]))/8
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(dfx^2+1\right)\log\left(cx^n\right)^3 + 3\,ab^2\log\left(dfx^2+1\right)\log\left(cx^n\right)^2 + 3\,a^2b\log\left(dfx^2+1\right)\log\left(cx^n\right) + a^3\log}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log(
c*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1))/x^3,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(cx^{n}\right) + a\right)^{3} \log \left(\left(fx^{2} + \frac{1}{d}\right)d\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d)/x^3, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c\,x^{n}\right)+a \right)^{3} \ln \left(\left(f\,x^{2}+\frac{1}{d}\right) d \right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(4\,b^{3} \log \left(x^{n}\right)^{3} + 6\left(n^{2} + 2\,n \log(c) + 2\,\log(c)^{2}\right)ab^{2} + \left(3\,n^{3} + 6\,n^{2} \log(c) + 6\,n \log(c)^{2} + 4\,\log(c)^{3}\right)b^{3} + 6\,a^{2}b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*log(x^n)^3 + 6*(n^2 + 2*n*log(c) + 2*log(c)^2)*a*b^2 + (3*n^3 +
 6*n^2*log(c) + 6*n*log(c)^2 + 4*log(c)^3)*b^3 + 6*a^2*b*(n + 2*log(c)) + 4
*a^3 + 6*(b^3*(n + 2*log(c)) + 2*a*b^2)*log(x^n)^2 + 6*((n^2 + 2*n*log(c) +
 2*log(c)^2)*b^3 + 2*a*b^2*(n + 2*log(c)) + 2*a^2*b)*log(x^n))*log(d*f*x^2
+ 1)/x^2 + integrate(1/4*(4*b^3*d*f*log(x^n)^3 + 4*a^3*d*f + 6*(d*f*n + 2*d
*f*log(c))*a^2*b + 6*(d*f*n^2 + 2*d*f*n*log(c) + 2*d*f*log(c)^2)*a*b^2 + (3
*d*f*n^3 + 6*d*f*n^2*log(c) + 6*d*f*n*log(c)^2 + 4*d*f*log(c)^3)*b^3 + 6*(2
*a*b^2*d*f + (d*f*n + 2*d*f*log(c))*b^3)*log(x^n)^2 + 6*(2*a^2*b*d*f + 2*(d
*f*n + 2*d*f*log(c))*a*b^2 + (d*f*n^2 + 2*d*f*n*log(c) + 2*d*f*log(c)^2)*b^
3)*log(x^n))/(d*f*x^3 + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^3,x)
```

```
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2))/x**3,x)
[Out] Timed out
```

3.44 $\int \left(a+b\log\left(cx^n\right)\right)^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=938

$$36n^3xb^3-36n^2x\log\left(cx^n\right)b^3+\frac{12n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)\log\left(cx^n\right)b^3}{\sqrt{d}\sqrt{f}}-6n^3x\log\left(dfx^2+1\right)b^3+6n^2x\log\left(cx^n\right)\log\left(d\right.$$

```
[Out] -3*b*n*(a+b*ln(c*x^n))^2*ln(1+x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-6*b^
2*n^2*(a+b*ln(c*x^n))*polylog(2,-x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+3
*b*n*(a+b*ln(c*x^n))^2*polylog(2,-x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+
6*I*b^3*n^3*polylog(2,I*x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)+6*b^2*n^2*(a+b*l
n(c*x^n))*polylog(2,x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-3*b*n*(a+b*ln(
c*x^n))^2*polylog(2,x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-6*b^2*n^2*(a+b
*ln(c*x^n))*polylog(3,-x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+6*b^2*n^2*(
a+b*ln(c*x^n))*polylog(3,x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+12*b^2*n^
2*(-b*n+a)*arctan(x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)+12*b^3*n^2*arctan(x*d^
(1/2)*f^(1/2))*ln(c*x^n)/d^(1/2)/f^(1/2)-6*I*b^3*n^3*polylog(2,-I*x*d^(1/2)
*f^(1/2))/d^(1/2)/f^(1/2)-12*b^2*n^2*(-b*n+a)*x-36*b^3*n^2*x*ln(c*x^n)+12*b
*n*x*(a+b*ln(c*x^n))^2-24*a*b^2*n^2*x+6*b^3*n^3*polylog(3,-x*(-d)^(1/2)*f^(
1/2))/(-d)^(1/2)/f^(1/2)-6*b^3*n^3*polylog(3,x*(-d)^(1/2)*f^(1/2))/(-d)^(1/
2)/f^(1/2)+6*b^3*n^3*polylog(4,-x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-6*
b^3*n^3*polylog(4,x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-2*x*(a+b*ln(c*x^
n))^3+36*b^3*n^3*x+x*(a+b*ln(c*x^n))^3*ln(d*f*x^2+1)+6*a*b^2*n^2*x*ln(d*f*x
^2+1)+6*b^3*n^2*x*ln(c*x^n)*ln(d*f*x^2+1)-3*b*n*x*(a+b*ln(c*x^n))^2*ln(d*f*
x^2+1)-6*b^3*n^3*x*ln(d*f*x^2+1)-(a+b*ln(c*x^n))^3*ln(1-x*(-d)^(1/2)*f^(1/2
))/(-d)^(1/2)/f^(1/2)+(a+b*ln(c*x^n))^3*ln(1+x*(-d)^(1/2)*f^(1/2))/(-d)^(1/
2)/f^(1/2)+3*b*n*(a+b*ln(c*x^n))^2*ln(1-x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^
(1/2)
```

Rubi [A] time = 1.55, antiderivative size = 938, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {2296, 2295, 2371, 6, 321, 203, 2351, 2324, 12, 4848, 2391, 2353, 2330, 2317, 2374, 6589, 2383}

$$36n^3xb^3-36n^2x\log\left(cx^n\right)b^3+\frac{12n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)\log\left(cx^n\right)b^3}{\sqrt{d}\sqrt{f}}-6n^3x\log\left(dfx^2+1\right)b^3+6n^2x\log\left(cx^n\right)\log\left(d\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] -24*a*b^2*n^2*x + 36*b^3*n^3*x - 12*b^2*n^2*(a - b*n)*x + (12*b^2*n^2*(a -
b*n)*ArcTan[Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - 36*b^3*n^2*x*Log[c*x^n]
 + (12*b^3*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x]*Log[c*x^n])/(Sqrt[d]*Sqrt[f]) + 12
*b*n*x*(a + b*Log[c*x^n])^2 - 2*x*(a + b*Log[c*x^n])^3 + (3*b*n*(a + b*Log[
c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) - ((a + b*Log[c*x
^n])^3*Log[1 - Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) - (3*b*n*(a + b*Log[
c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + ((a + b*Log[c*x
^n])^3*Log[1 + Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + 6*a*b^2*n^2*x*Log[
1 + d*f*x^2] - 6*b^3*n^3*x*Log[1 + d*f*x^2] + 6*b^3*n^2*x*Log[c*x^n]*Log[1
+ d*f*x^2] - 3*b*n*x*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2] + x*(a + b*Log[c
*x^n])^3*Log[1 + d*f*x^2] - (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt
[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) + (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[
2, -(Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) + (6*b^2*n^2*(a + b*Log[c*x^n
])*PolyLog[2, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) - (3*b*n*(a + b*Log[c
*x^n])^2*PolyLog[2, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) - ((6*I)*b^3*n^
3*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) + ((6*I)*b^3*n^3*Po
lyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) + (6*b^3*n^3*PolyLog[3, -(
```

```
Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) - (6*b^2*n^2*(a + b*Log[c*x^n])*Po
lyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) - (6*b^3*n^3*PolyLog[3,
 Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + (6*b^2*n^2*(a + b*Log[c*x^n])*Po
lyLog[3, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + (6*b^3*n^3*PolyLog[4, -(
Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) - (6*b^3*n^3*PolyLog[4, Sqrt[-d]*S
qrt[f]*x])/(Sqrt[-d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

```
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2371

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx = 6ab^2n^2x\log\left(1+dfx^2\right)-6b^3n^3x\log\left(1+dfx^2\right)+6b^3n^2x\log\left(cx^n\right)$$

$$= 6ab^2n^2x\log\left(1+dfx^2\right)-6b^3n^3x\log\left(1+dfx^2\right)+6b^3n^2x\log\left(cx^n\right)$$

$$= 6ab^2n^2x\log\left(1+dfx^2\right)-6b^3n^3x\log\left(1+dfx^2\right)+6b^3n^2x\log\left(cx^n\right)$$

$$= -12b^2n^2(a-bn)x+6ab^2n^2x\log\left(1+dfx^2\right)-6b^3n^3x\log\left(1+dfx^2\right.$$

$$= -12b^2n^2(a-bn)x+\frac{12b^2n^2(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}+6ab^2n^2x\log$$

$$= 12b^3n^3x-12b^2n^2(a-bn)x+\frac{12b^2n^2(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}-12$$

$$= -12ab^2n^2x+12b^3n^3x-12b^2n^2(a-bn)x+\frac{12b^2n^2(a-bn)\tan^{-1}\left(\right.}{\sqrt{d}\sqrt{f}}$$

$$= -24ab^2n^2x+24b^3n^3x-12b^2n^2(a-bn)x+\frac{12b^2n^2(a-bn)\tan^{-1}\left(\right.}{\sqrt{d}\sqrt{f}}$$

$$= -24ab^2n^2x+36b^3n^3x-12b^2n^2(a-bn)x+\frac{12b^2n^2(a-bn)\tan^{-1}\left(\right.}{\sqrt{d}\sqrt{f}}$$

$$= -24ab^2n^2x+36b^3n^3x-12b^2n^2(a-bn)x+\frac{12b^2n^2(a-bn)\tan^{-1}\left(\right.}{\sqrt{d}\sqrt{f}}$$

$$= -24ab^2n^2x+36b^3n^3x-12b^2n^2(a-bn)x+\frac{12b^2n^2(a-bn)\tan^{-1}\left(\right.}{\sqrt{d}\sqrt{f}}$$

Mathematica [A] time = 0.72, size = 1027, normalized size = 1.09

$$\frac{2b^3\left(-\sqrt{d}\sqrt{f}x\left(\log^3(x)-3\log^2(x)+6\log(x)-6\right)-\frac{1}{2}i\left(\log\left(i\sqrt{d}\sqrt{f}x+1\right)\log^3(x)+3\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)\log^2(x)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[f]*x*(a^3 - 3*a^2*b*n + 6*a*b^2*n^2 - 6*b^3*n^3 + 6*a*b^2*
n*(n*Log[x] - Log[c*x^n]) + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*
(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 3*b^3*n
*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3) + 2*ArcTa
n[Sqrt[d]*Sqrt[f]*x]*(a^3 - 3*a^2*b*n + 6*a*b^2*n^2 - 6*b^3*n^3 + 6*a*b^2*n
*(n*Log[x] - Log[c*x^n]) + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(
-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 3*b^3*n*
(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3) + Sqrt[d]*
Sqrt[f]*x*(a^3 - 3*a^2*b*n + 6*a*b^2*n^2 - 6*b^3*n^3 + 3*b*(a^2 - 2*a*b*n +
 2*b^2*n^2)*Log[c*x^n] + 3*b^2*(a - b*n)*Log[c*x^n]^2 + b^3*Log[c*x^n]^3)*L
```

```
og[1 + d*f*x^2] + 3*b*n*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b^2*n*(n*Log[x] - Lo
g[c*x^n]) + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x]) + Log[c*x^n
])^2)*(-2*Sqrt[d]*Sqrt[f]*x*(-1 + Log[x]) - I*(Log[x]*Log[1 + I*Sqrt[d]*Sqr
t[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]) + I*(Log[x]*Log[1 - I*Sqrt[d]
*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])) - 6*b^2*n^2*(a - b*n - b*n*
Log[x] + b*Log[c*x^n])*(Sqrt[d]*Sqrt[f]*x*(2 - 2*Log[x] + Log[x]^2) + (I/2)
*(Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*
Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x]) - (I/2)*(Log[x]^2*Log[1
- I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyL
og[3, I*Sqrt[d]*Sqrt[f]*x])) + 2*b^3*n^3*(-(Sqrt[d]*Sqrt[f]*x*(-6 + 6*Log[x
] - 3*Log[x]^2 + Log[x]^3)) - (I/2)*(Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x]
+ 3*Log[x]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[3, (-I)*
Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x]) + (I/2)*(Log[x]^
3*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]
 - 6*Log[x]*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, I*Sqrt[d]*Sqrt[f
]*x])))/(Sqrt[d]*Sqrt[f])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\log\left(dfx^2+1\right)\log\left(cx^n\right)^3+3\,ab^2\log\left(dfx^2+1\right)\log\left(cx^n\right)^2+3\,a^2b\log\left(dfx^2+1\right)\log\left(cx^n\right)+a^3\,\text{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log(c
*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)^3\ln\left(\left(f\,x^2+\frac{1}{d}\right)d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d),x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(b^3x\log\left(x^n\right)^3-3\left(b^3\left(n-\log(c)\right)-ab^2\right)x\log\left(x^n\right)^2+3\left(\left(2\,n^2-2\,n\log(c)+\log(c)^2\right)b^3-2\,ab^2\left(n-\log(c)\right)+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] (b^3*x*log(x^n)^3 - 3*(b^3*(n - log(c)) - a*b^2)*x*log(x^n)^2 + 3*((2*n^2 -
 2*n*log(c) + log(c)^2)*b^3 - 2*a*b^2*(n - log(c)) + a^2*b)*x*log(x^n) + (3
*(2*n^2 - 2*n*log(c) + log(c)^2)*a*b^2 - (6*n^3 - 6*n^2*log(c) + 3*n*log(c)
```

```
^2 - log(c)^3)*b^3 - 3*a^2*b*(n - log(c)) + a^3)*x)*log(d*f*x^2 + 1) - inte
grate(2*(b^3*d*f*x^2*log(x^n)^3 + 3*(a*b^2*d*f - (d*f*n - d*f*log(c))*b^3)*
x^2*log(x^n)^2 + 3*(a^2*b*d*f - 2*(d*f*n - d*f*log(c))*a*b^2 + (2*d*f*n^2 -
 2*d*f*n*log(c) + d*f*log(c)^2)*b^3)*x^2*log(x^n) + (a^3*d*f - 3*(d*f*n - d
*f*log(c))*a^2*b + 3*(2*d*f*n^2 - 2*d*f*n*log(c) + d*f*log(c)^2)*a*b^2 - (6
*d*f*n^3 - 6*d*f*n^2*log(c) + 3*d*f*n*log(c)^2 - d*f*log(c)^3)*b^3)*x^2)/(d
*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

3.45 $\int \frac{(a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2} dx$

Optimal. Leaf size=849

$$12b^3\sqrt{d}\sqrt{f}\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)n^3-\frac{6b^3\log\left(dfx^2+1\right)n^3}{x}-6ib^3\sqrt{d}\sqrt{f}\,\mathrm{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)n^3+6ib^3\sqrt{d}\sqrt{f}\,\mathrm{Li}_2\left(i\sqrt{d}\right.$$

```
[Out] -6*b^3*n^3*ln(d*f*x^2+1)/x-6*b^2*n^2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x-3*b*n*
(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/x-(a+b*ln(c*x^n))^3*ln(d*f*x^2+1)/x+3*b*n*(
a+b*ln(c*x^n))^2*ln(1-x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+(a+b*ln(c*x^
n))^3*ln(1-x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-3*b*n*(a+b*ln(c*x^n))^2
*ln(1+x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-(a+b*ln(c*x^n))^3*ln(1+x*(-d
)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-x*
(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-3*b*n*(a+b*ln(c*x^n))^2*polylog(2,-x
*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2
,x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+3*b*n*(a+b*ln(c*x^n))^2*polylog(2
,x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+6*b^3*n^3*polylog(3,-x*(-d)^(1/2)
*f^(1/2))*(-d)^(1/2)*f^(1/2)+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-x*(-d)^(1
/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-6*b^3*n^3*polylog(3,x*(-d)^(1/2)*f^(1/2))*(
-d)^(1/2)*f^(1/2)-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,x*(-d)^(1/2)*f^(1/2))
*(-d)^(1/2)*f^(1/2)-6*b^3*n^3*polylog(4,-x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f
^(1/2)+6*b^3*n^3*polylog(4,x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+12*b^3*
n^3*arctan(x*d^(1/2)*f^(1/2))*d^(1/2)*f^(1/2)+12*b^2*n^2*arctan(x*d^(1/2)*f
^(1/2))*(a+b*ln(c*x^n))*d^(1/2)*f^(1/2)+6*I*b^3*n^3*polylog(2,I*x*d^(1/2)*f
^(1/2))*d^(1/2)*f^(1/2)-6*I*b^3*n^3*polylog(2,-I*x*d^(1/2)*f^(1/2))*d^(1/2)
*f^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 849, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {2305, 2304, 2378, 203, 2324, 12, 4848, 2391, 2330, 2317, 2374, 6589, 2383}

$$12b^3\sqrt{d}\sqrt{f}\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)n^3-\frac{6b^3\log\left(dfx^2+1\right)n^3}{x}-6ib^3\sqrt{d}\sqrt{f}\,\mathrm{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)n^3+6ib^3\sqrt{d}\sqrt{f}\,\mathrm{P}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 12*b^3*Sqrt[d]*Sqrt[f]*n^3*ArcTan[Sqrt[d]*Sqrt[f]*x] + 12*b^2*Sqrt[d]*Sqrt[
f]*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]) + 3*b*Sqrt[-d]*Sqrt[f]*
n*(a + b*Log[c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x] + Sqrt[-d]*Sqrt[f]*(a +
b*Log[c*x^n])^3*Log[1 - Sqrt[-d]*Sqrt[f]*x] - 3*b*Sqrt[-d]*Sqrt[f]*n*(a + b
*Log[c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]*x] - Sqrt[-d]*Sqrt[f]*(a + b*Log[c*
x^n])^3*Log[1 + Sqrt[-d]*Sqrt[f]*x] - (6*b^3*n^3*Log[1 + d*f*x^2])/x - (6*b
^2*n^2*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/x - (3*b*n*(a + b*Log[c*x^n])^2
*Log[1 + d*f*x^2])/x - ((a + b*Log[c*x^n])^3*Log[1 + d*f*x^2])/x - 6*b^2*Sq
rt[-d]*Sqrt[f]*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sqrt[f]*x)] - 3
*b*Sqrt[-d]*Sqrt[f]*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(Sqrt[-d]*Sqrt[f]*x)
] + 6*b^2*Sqrt[-d]*Sqrt[f]*n^2*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d]*Sqrt[
f]*x] + 3*b*Sqrt[-d]*Sqrt[f]*n*(a + b*Log[c*x^n])^2*PolyLog[2, Sqrt[-d]*Sqr
t[f]*x] - (6*I)*b^3*Sqrt[d]*Sqrt[f]*n^3*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]
+ (6*I)*b^3*Sqrt[d]*Sqrt[f]*n^3*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + 6*b^3*Sqr
t[-d]*Sqrt[f]*n^3*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)] + 6*b^2*Sqrt[-d]*Sqrt[f
]*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)] - 6*b^3*Sqrt[-d]
*Sqrt[f]*n^3*PolyLog[3, Sqrt[-d]*Sqrt[f]*x] - 6*b^2*Sqrt[-d]*Sqrt[f]*n^2*(a
 + b*Log[c*x^n])*PolyLog[3, Sqrt[-d]*Sqrt[f]*x] - 6*b^3*Sqrt[-d]*Sqrt[f]*n^
```

```
3*PolyLog[4, -(Sqrt[-d]*Sqrt[f]*x)] + 6*b^3*Sqrt[-d]*Sqrt[f]*n^3*PolyLog[4,
 Sqrt[-d]*Sqrt[f]*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
```

```
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = -\frac{6b^3n^3\log\left(1+dfx^2\right)}{x} - \frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)}{x}$$

$$= -\frac{6b^3n^3\log\left(1+dfx^2\right)}{x} - \frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(1+dfx^2\right)}{x}$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)$$

Mathematica [A] time = 0.43, size = 794, normalized size = 0.94

$$3ib\sqrt{d}\sqrt{f}\,n\left(-\text{Li}_2\left(-i\sqrt{d}\sqrt{f}\,x\right)+\text{Li}_2\left(i\sqrt{d}\sqrt{f}\,x\right)+\log(x)\left(\log\left(1-i\sqrt{d}\sqrt{f}\,x\right)-\log\left(1+i\sqrt{d}\sqrt{f}\,x\right)\right)\right)\left(a^2+2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 2*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2
+ 6*b^3*n^3 + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) +
 Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x])
+ Log[c*x^n])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) +
 Log[c*x^n])^3) - ((a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3*b*(a^2 +
2*a*b*n + 2*b^2*n^2)*Log[c*x^n] + 3*b^2*(a + b*n)*Log[c*x^n]^2 + b^3*Log[c*
x^n]^3)*Log[1 + d*f*x^2])/x + (3*I)*b*Sqrt[d]*Sqrt[f]*n*(a^2 + 2*a*b*n + 2*
b^2*n^2 + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x
^n]) + b^2*(-(n*Log[x]) + Log[c*x^n])^2)*(Log[x]*(Log[1 - I*Sqrt[d]*Sqrt[f]
*x] - Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] +
PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) + (6*I)*b^2*Sqrt[d]*Sqrt[f]*n^2*(a + b*n -
 b*n*Log[x] + b*Log[c*x^n])*((Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x])/2 - (L
og[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sq
rt[f]*x] + Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, (-I)*Sqrt[d]
*Sqrt[f]*x] - PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) + I*b^3*Sqrt[d]*Sqrt[f]*n^3*
(Log[x]^3*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]
*x] - 3*Log[x]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2,
 I*Sqrt[d]*Sqrt[f]*x] + 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log
[x]*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x] - 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x]
+ 6*PolyLog[4, I*Sqrt[d]*Sqrt[f]*x])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \log\left(dfx^2+1\right)\log\left(cx^n\right)^3 + 3\,ab^2\log\left(dfx^2+1\right)\log\left(cx^n\right)^2 + 3\,a^2b\log\left(dfx^2+1\right)\log\left(cx^n\right) + a^3\log}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log(
c*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1))/x^2,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(fx^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d)/x^2, x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right)+a\right)^3\ln\left(\left(f\,x^2+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^2+1/d)*d)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^3\log\left(x^n\right)^3+3\left(2\,n^2+2\,n\log(c)+\log(c)^2\right)ab^2+\left(6\,n^3+6\,n^2\log(c)+3\,n\log(c)^2+\log(c)^3\right)b^3+3\,a^2b\left(n+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] -(b^3*log(x^n)^3 + 3*(2*n^2 + 2*n*log(c) + log(c)^2)*a*b^2 + (6*n^3 + 6*n^2
*log(c) + 3*n*log(c)^2 + log(c)^3)*b^3 + 3*a^2*b*(n + log(c)) + a^3 + 3*(b^
3*(n + log(c)) + a*b^2)*log(x^n)^2 + 3*((2*n^2 + 2*n*log(c) + log(c)^2)*b^3
 + 2*a*b^2*(n + log(c)) + a^2*b)*log(x^n))*log(d*f*x^2 + 1)/x + integrate(2
*(b^3*d*f*log(x^n)^3 + a^3*d*f + 3*(d*f*n + d*f*log(c))*a^2*b + 3*(2*d*f*n^
2 + 2*d*f*n*log(c) + d*f*log(c)^2)*a*b^2 + (6*d*f*n^3 + 6*d*f*n^2*log(c) +
3*d*f*n*log(c)^2 + d*f*log(c)^3)*b^3 + 3*(a*b^2*d*f + (d*f*n + d*f*log(c))*
b^3)*log(x^n)^2 + 3*(a^2*b*d*f + 2*(d*f*n + d*f*log(c))*a*b^2 + (2*d*f*n^2
+ 2*d*f*n*log(c) + d*f*log(c)^2)*b^3)*log(x^n))/(d*f*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^2,x)
```

```
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2))/x**2,x)
```

```
[Out] Timed out
```

3.46 $\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$

Optimal. Leaf size=350

$$-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{3d^6f^6}+\frac{\sqrt{x}\,(a+b\log(cx^n))}{3d^5f^5}-\frac{x\,(a+b\log(cx^n))}{6d^4f^4}+\frac{x^{3/2}\,(a+b\log(cx^n))}{9d^3f^3}-\frac{x^2\,(a+b\,\mathrm{lo}}{12d^2}$$

```
[Out] 2/9*b*n*x/d^4/f^4-1/9*b*n*x^(3/2)/d^3/f^3+5/72*b*n*x^2/d^2/f^2-11/225*b*n*x
^(5/2)/d/f+1/27*b*n*x^3-1/6*x*(a+b*ln(c*x^n))/d^4/f^4+1/9*x^(3/2)*(a+b*ln(c
*x^n))/d^3/f^3-1/12*x^2*(a+b*ln(c*x^n))/d^2/f^2+1/15*x^(5/2)*(a+b*ln(c*x^n)
)/d/f-1/18*x^3*(a+b*ln(c*x^n))+1/9*b*n*ln(1+d*f*x^(1/2))/d^6/f^6-1/9*b*n*x^
3*ln(1+d*f*x^(1/2))-1/3*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/d^6/f^6+1/3*x^3*(
a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-2/3*b*n*polylog(2,-d*f*x^(1/2))/d^6/f^6-7/
9*b*n*x^(1/2)/d^5/f^5+1/3*(a+b*ln(c*x^n))*x^(1/2)/d^5/f^5
```

Rubi [A] time = 0.28, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2454, 2395, 43, 2376, 2391}

$$-\frac{2bn\mathrm{PolyLog}\left(2,-df\sqrt{x}\right)}{3d^6f^6}-\frac{x^2\,(a+b\log(cx^n))}{12d^2f^2}+\frac{x^{3/2}\,(a+b\log(cx^n))}{9d^3f^3}-\frac{x\,(a+b\log(cx^n))}{6d^4f^4}+\frac{\sqrt{x}\,(a+b\log(cx^n)}{3d^5f^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-7*b*n*Sqrt[x])/(9*d^5*f^5) + (2*b*n*x)/(9*d^4*f^4) - (b*n*x^(3/2))/(9*d^3
*f^3) + (5*b*n*x^2)/(72*d^2*f^2) - (11*b*n*x^(5/2))/(225*d*f) + (b*n*x^3)/2
7 + (b*n*Log[1 + d*f*Sqrt[x]])/(9*d^6*f^6) - (b*n*x^3*Log[1 + d*f*Sqrt[x]])
/9 + (Sqrt[x]*(a + b*Log[c*x^n]))/(3*d^5*f^5) - (x*(a + b*Log[c*x^n]))/(6*d
^4*f^4) + (x^(3/2)*(a + b*Log[c*x^n]))/(9*d^3*f^3) - (x^2*(a + b*Log[c*x^n]
))/(12*d^2*f^2) + (x^(5/2)*(a + b*Log[c*x^n]))/(15*d*f) - (x^3*(a + b*Log[c
*x^n]))/18 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(3*d^6*f^6) + (x^3*L
og[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/3 - (2*b*n*PolyLog[2, -(d*f*Sqrt[x]
)])/(3*d^6*f^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

```
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{3d^5f^5}-\frac{x\left(a+b\log\left(cx^n\right)\right)}{6d^4f^4}+\frac{x^{3/2}\left(a+b\log\left(c\right.\right.}{9d^3f^3}$$

$$= -\frac{2bn\sqrt{x}}{3d^5f^5}+\frac{bnx}{6d^4f^4}-\frac{2bnx^{3/2}}{27d^3f^3}+\frac{bnx^2}{24d^2f^2}-\frac{2bnx^{5/2}}{75df}+\frac{1}{54}bnx^3$$

$$= -\frac{2bn\sqrt{x}}{3d^5f^5}+\frac{bnx}{6d^4f^4}-\frac{2bnx^{3/2}}{27d^3f^3}+\frac{bnx^2}{24d^2f^2}-\frac{2bnx^{5/2}}{75df}+\frac{1}{54}bnx^3$$

$$= -\frac{2bn\sqrt{x}}{3d^5f^5}+\frac{bnx}{6d^4f^4}-\frac{2bnx^{3/2}}{27d^3f^3}+\frac{bnx^2}{24d^2f^2}-\frac{2bnx^{5/2}}{75df}+\frac{1}{54}bnx^3$$

$$= -\frac{2bn\sqrt{x}}{3d^5f^5}+\frac{bnx}{6d^4f^4}-\frac{2bnx^{3/2}}{27d^3f^3}+\frac{bnx^2}{24d^2f^2}-\frac{2bnx^{5/2}}{75df}+\frac{1}{54}bnx^3$$

$$= -\frac{7bn\sqrt{x}}{9d^5f^5}+\frac{2bnx}{9d^4f^4}-\frac{bnx^{3/2}}{9d^3f^3}+\frac{5bnx^2}{72d^2f^2}-\frac{11bnx^{5/2}}{225df}+\frac{1}{27}bnx^3$$

Mathematica [A] time = 0.31, size = 263, normalized size = 0.75

$$\frac{600\left(d^6f^6x^3-1\right)\log\left(df\sqrt{x}+1\right)\left(3a+3b\log\left(cx^n\right)-bn\right)+df\sqrt{x}\left(-30a\left(10d^5f^5x^{5/2}-12d^4f^4x^2+15d^3f^3x^{3/}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] (600*(-1 + d^6*f^6*x^3)*Log[1 + d*f*Sqrt[x]]*(3*a - b*n + 3*b*Log[c*x^n]) +
 d*f*Sqrt[x]*(-30*a*(-60 + 30*d*f*Sqrt[x] - 20*d^2*f^2*x + 15*d^3*f^3*x^(3/
2) - 12*d^4*f^4*x^2 + 10*d^5*f^5*x^(5/2)) + b*n*(-4200 + 1200*d*f*Sqrt[x] -
 600*d^2*f^2*x + 375*d^3*f^3*x^(3/2) - 264*d^4*f^4*x^2 + 200*d^5*f^5*x^(5/2
)) - 30*b*(-60 + 30*d*f*Sqrt[x] - 20*d^2*f^2*x + 15*d^3*f^3*x^(3/2) - 12*d^
4*f^4*x^2 + 10*d^5*f^5*x^(5/2))*Log[c*x^n]) - 3600*b*n*PolyLog[2, -(d*f*Sqr
t[x])])/(5400*d^6*f^6)
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2\log\left(cx^n\right)+ax^2\right)\log\left(df\sqrt{x}+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)*log(d*f*sqrt(x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^{n}\right) + a\right)x^{2} \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\,x^{n}\right) + a\right) x^{2} \ln\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)*ln(d*(1/d+f*x^(1/2))),x)
[Out] int(x^2*(b*ln(c*x^n)+a)*ln(d*(1/d+f*x^(1/2))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^{n}\right) + a\right)x^{2} \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima"
)
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{2}\,\ln\left(d\,\left(f\,\sqrt{x} + \frac{1}{d}\right)\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)),x)
[Out] int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2))),x)
[Out] Timed out
```

3.47 $\int x \log\left(d\left(\frac{1}{d} + f\sqrt{x}\right)\right)\left(a + b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=268

$$-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{2d^4f^4}+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{2d^3f^3}-\frac{x\left(a+b\log\left(cx^n\right)\right)}{4d^2f^2}+\frac{x^{3/2}\left(a+b\log\left(cx^n\right)\right)}{6df}+\frac{1}{2}x^2\log$$

```
[Out] 3/8*b*n*x/d^2/f^2-7/36*b*n*x^(3/2)/d/f+1/8*b*n*x^2-1/4*x*(a+b*ln(c*x^n))/d^
2/f^2+1/6*x^(3/2)*(a+b*ln(c*x^n))/d/f-1/8*x^2*(a+b*ln(c*x^n))+1/4*b*n*ln(1+
d*f*x^(1/2))/d^4/f^4-1/4*b*n*x^2*ln(1+d*f*x^(1/2))-1/2*(a+b*ln(c*x^n))*ln(1
+d*f*x^(1/2))/d^4/f^4+1/2*x^2*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-b*n*polylog
(2,-d*f*x^(1/2))/d^4/f^4-5/4*b*n*x^(1/2)/d^3/f^3+1/2*(a+b*ln(c*x^n))*x^(1/2
)/d^3/f^3
```

Rubi [A] time = 0.19, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2454, 2395, 43, 2376, 2391}

$$-\frac{bn\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^4f^4}-\frac{x\left(a+b\log\left(cx^n\right)\right)}{4d^2f^2}+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{2d^3f^3}-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{2d^4f^4}+\frac{1}{2}x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-5*b*n*Sqrt[x])/(4*d^3*f^3) + (3*b*n*x)/(8*d^2*f^2) - (7*b*n*x^(3/2))/(36*
d*f) + (b*n*x^2)/8 + (b*n*Log[1 + d*f*Sqrt[x]])/(4*d^4*f^4) - (b*n*x^2*Log[
1 + d*f*Sqrt[x]])/4 + (Sqrt[x]*(a + b*Log[c*x^n]))/(2*d^3*f^3) - (x*(a + b*
Log[c*x^n]))/(4*d^2*f^2) + (x^(3/2)*(a + b*Log[c*x^n]))/(6*d*f) - (x^2*(a +
 b*Log[c*x^n]))/8 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*d^4*f^4) +
 (x^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/2 - (b*n*PolyLog[2, -(d*f*Sq
rt[x])])/(d^4*f^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)dx=\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{2d^3f^3}-\frac{x\left(a+b\log\left(cx^n\right)\right)}{4d^2f^2}+\frac{x^{3/2}\left(a+b\log\left(cx^n\right)\right)}{6df}$$

$$=-\frac{bn\sqrt{x}}{d^3f^3}+\frac{bnx}{4d^2f^2}-\frac{bnx^{3/2}}{9df}+\frac{1}{16}bnx^2+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{2d^3f^3}-\frac{x}{}$$

$$=-\frac{bn\sqrt{x}}{d^3f^3}+\frac{bnx}{4d^2f^2}-\frac{bnx^{3/2}}{9df}+\frac{1}{16}bnx^2+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{2d^3f^3}-\frac{x}{}$$

$$=-\frac{bn\sqrt{x}}{d^3f^3}+\frac{bnx}{4d^2f^2}-\frac{bnx^{3/2}}{9df}+\frac{1}{16}bnx^2-\frac{1}{4}bnx^2\log\left(1+df\sqrt{x}\right)+$$

$$=-\frac{bn\sqrt{x}}{d^3f^3}+\frac{bnx}{4d^2f^2}-\frac{bnx^{3/2}}{9df}+\frac{1}{16}bnx^2-\frac{1}{4}bnx^2\log\left(1+df\sqrt{x}\right)+$$

$$=-\frac{5bn\sqrt{x}}{4d^3f^3}+\frac{3bnx}{8d^2f^2}-\frac{7bnx^{3/2}}{36df}+\frac{1}{8}bnx^2+\frac{bn\log\left(1+df\sqrt{x}\right)}{4d^4f^4}-\frac{1}{4}$$

Mathematica [A] time = 0.22, size = 191, normalized size = 0.71

$$\frac{18\left(d^4f^4x^2-1\right)\log\left(df\sqrt{x}+1\right)\left(2a+2b\log\left(cx^n\right)-bn\right)+df\sqrt{x}\left(-3a\left(3d^3f^3x^{3/2}-4d^2f^2x+6df\sqrt{x}-12\right)-3b\right.}{72d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] (18*(-1 + d^4*f^4*x^2)*Log[1 + d*f*Sqrt[x]]*(2*a - b*n + 2*b*Log[c*x^n]) +
d*f*Sqrt[x]*(-3*a*(-12 + 6*d*f*Sqrt[x] - 4*d^2*f^2*x + 3*d^3*f^3*x^(3/2)) +
 b*n*(-90 + 27*d*f*Sqrt[x] - 14*d^2*f^2*x + 9*d^3*f^3*x^(3/2)) - 3*b*(-12 +
 6*d*f*Sqrt[x] - 4*d^2*f^2*x + 3*d^3*f^3*x^(3/2))*Log[c*x^n]) - 72*b*n*Poly
Log[2, -(d*f*Sqrt[x])])/(72*d^4*f^4)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx\log\left(cx^n\right)+ax\right)\log\left(df\sqrt{x}+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)*log(d*f*sqrt(x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x*log((f*sqrt(x) + 1/d)*d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (b\ln (c\,x^{n})+a)\,x\ln \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d),x)
[Out] int(x*(b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log (cx^{n})+a\right)x\log \left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*x*log((f*sqrt(x) + 1/d)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)),x)
[Out] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2))),x)
[Out] Timed out
```

3.48 $\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=172

$$-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{df}-\frac{1}{2}x\left(a+b\log\right.$$

```
[Out] b*n*x-1/2*x*(a+b*ln(c*x^n))-b*n*x*ln(d*(1/d+f*x^(1/2)))+x*(a+b*ln(c*x^n))*l
n(d*(1/d+f*x^(1/2)))+b*n*ln(1+d*f*x^(1/2))/d^2/f^2-(a+b*ln(c*x^n))*ln(1+d*f
*x^(1/2))/d^2/f^2-2*b*n*polylog(2,-d*f*x^(1/2))/d^2/f^2-3*b*n*x^(1/2)/d/f+(
a+b*ln(c*x^n))*x^(1/2)/d/f
```

Rubi [A] time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2448, 266, 43, 2370, 2391}

$$-\frac{2bn\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2}-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)+\frac{\sqrt{x}\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-3*b*n*Sqrt[x])/(d*f) + b*n*x - b*n*x*Log[d*(d^(-1) + f*Sqrt[x])] + (b*n*L
og[1 + d*f*Sqrt[x]])/(d^2*f^2) + (Sqrt[x]*(a + b*Log[c*x^n]))/(d*f) - (x*(a
 + b*Log[c*x^n]))/2 + x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]) - (L
og[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(d^2*f^2) - (2*b*n*PolyLog[2, -(d*f
*Sqrt[x])])/(d^2*f^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rubi steps

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{df}-\frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right.\right.$$

$$= -\frac{2bn\sqrt{x}}{df}+\frac{bnx}{2}+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{df}-\frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)+$$

$$= -\frac{2bn\sqrt{x}}{df}+\frac{bnx}{2}-bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right.\right.}{df}$$

$$= -\frac{2bn\sqrt{x}}{df}+\frac{bnx}{2}-bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right.\right.}{df}$$

$$= -\frac{2bn\sqrt{x}}{df}+\frac{bnx}{2}-bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right.\right.}{df}$$

$$= -\frac{3bn\sqrt{x}}{df}+bnx-bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)+\frac{bn\log\left(1+df\sqrt{x}\right)}{d^2f^2}$$

Mathematica [A] time = 0.16, size = 117, normalized size = 0.68

$$-\frac{-2\left(d^2f^2x-1\right)\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)-bn\right)+df\sqrt{x}\left(adf\sqrt{x}-2a+b\left(df\sqrt{x}-2\right)\log\left(cx^n\right)-2bd\right.}{2d^2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
[Out] -1/2*(-2*(-1 + d^2*f^2*x)*Log[1 + d*f*Sqrt[x]]*(a - b*n + b*Log[c*x^n]) + d
*f*Sqrt[x]*(-2*a + 6*b*n + a*d*f*Sqrt[x] - 2*b*d*f*n*Sqrt[x] + b*(-2 + d*f*
Sqrt[x])*Log[c*x^n]) + 4*b*n*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(cx^n\right)+a\right)\log\left(df\sqrt{x}+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)\ln\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d),x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(bx\log\left(x^n\right)-\left(b\left(n-\log(c)\right)-a\right)x\right)\log\left(df\sqrt{x}+1\right)-\frac{3\,bdfx^2\log\left(x^n\right)+\left(3\,adf-\left(5\,dfn-3\,df\log(c)\right)b\right)x^2}{9\,\sqrt{x}}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima")
```

```
[Out] (b*x*log(x^n) - (b*(n - log(c)) - a)*x)*log(d*f*sqrt(x) + 1) - 1/9*(3*b*d*f
*x^2*log(x^n) + (3*a*d*f - (5*d*f*n - 3*d*f*log(c))*b)*x^2)/sqrt(x) + integ
rate(1/2*(b*d^2*f^2*x*log(x^n) + (a*d^2*f^2 - (d^2*f^2*n - d^2*f^2*log(c))*
b)*x)/(d*f*sqrt(x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2))),x)
```

```
[Out] Timed out
```

3.49 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=39

$$4bn\text{Li}_3\left(-df\sqrt{x}\right)-2\text{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] -2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))+4*b*n*polylog(3,-d*f*x^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2374, 6589}

$$4bn\text{PolyLog}\left(3,-df\sqrt{x}\right)-2\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] -2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] + 4*b*n*PolyLog[3, -(d*f*S
qrt[x])]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= -2\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(-df\sqrt{x}\right)+(2bn)\int \frac{\text{Li}_2\left(-df\sqrt{x}\right)}{x}\,dx\\ &= -2\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(-df\sqrt{x}\right)+4bn\text{Li}_3\left(-df\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.28

$$-2a\text{Li}_2\left(-df\sqrt{x}\right)-2b\log\left(cx^n\right)\text{Li}_2\left(-df\sqrt{x}\right)+4bn\text{Li}_3\left(-df\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] -2*a*PolyLog[2, -(d*f*Sqrt[x])] - 2*b*Log[c*x^n]*PolyLog[2, -(d*f*Sqrt[x])]
 + 4*b*n*PolyLog[3, -(d*f*Sqrt[x])]
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(df\sqrt{x}+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right) \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^{n}\right) + a\right) \ln\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d)/x,x)
[Out] int((b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right) \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\,\left(f\,\sqrt{x} + \frac{1}{d}\right)\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x,x)
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2)))/x,x)
[Out] Timed out
```

3.50 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=196

$$d^2f^2\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}d^2f^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{df\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}}-\frac{\log\left(df\sqrt{x}+1\right)(}{x}$$

```
[Out] -1/2*b*d^2*f^2*n*ln(x)+1/4*b*d^2*f^2*n*ln(x)^2-1/2*d^2*f^2*ln(x)*(a+b*ln(c*
x^n))+b*d^2*f^2*n*ln(1+d*f*x^(1/2))-b*n*ln(1+d*f*x^(1/2))/x+d^2*f^2*(a+b*ln
(c*x^n))*ln(1+d*f*x^(1/2))-(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/x+2*b*d^2*f^2*
n*polylog(2,-d*f*x^(1/2))-3*b*d*f*n/x^(1/2)-d*f*(a+b*ln(c*x^n))/x^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2395, 44, 2376, 2391, 2301}

$$2bd^2f^2n\text{PolyLog}\left(2,-df\sqrt{x}\right)+d^2f^2\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}d^2f^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{df\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] (-3*b*d*f*n)/Sqrt[x] + b*d^2*f^2*n*Log[1 + d*f*Sqrt[x]] - (b*n*Log[1 + d*f*
Sqrt[x]])/x - (b*d^2*f^2*n*Log[x])/2 + (b*d^2*f^2*n*Log[x]^2)/4 - (d*f*(a +
 b*Log[c*x^n]))/Sqrt[x] + d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]) -
 (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/x - (d^2*f^2*Log[x]*(a + b*Log[c
*x^n]))/2 + 2*b*d^2*f^2*n*PolyLog[2, -(d*f*Sqrt[x])]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

```
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = -\frac{df\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + d^2f^2\log\left(1+df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right) - \frac{\log}{}$$

$$= -\frac{2bdfn}{\sqrt{x}} - \frac{df\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + d^2f^2\log\left(1+df\sqrt{x}\right)\left(a+b\log(c\right.$$

$$= -\frac{2bdfn}{\sqrt{x}} + \frac{1}{4}bd^2f^2n\log^2(x) - \frac{df\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + d^2f^2\log\left(1+\right.$$

$$= -\frac{2bdfn}{\sqrt{x}} - \frac{bn\log\left(1+df\sqrt{x}\right)}{x} + \frac{1}{4}bd^2f^2n\log^2(x) - \frac{df\left(a+b\log(\right.}{\sqrt{x}}$$

$$= -\frac{2bdfn}{\sqrt{x}} - \frac{bn\log\left(1+df\sqrt{x}\right)}{x} + \frac{1}{4}bd^2f^2n\log^2(x) - \frac{df\left(a+b\log(\right.}{\sqrt{x}}$$

$$= -\frac{3bdfn}{\sqrt{x}} + bd^2f^2n\log\left(1+df\sqrt{x}\right) - \frac{bn\log\left(1+df\sqrt{x}\right)}{x} - \frac{1}{2}bd^2f^2$$

Mathematica [A] time = 0.20, size = 124, normalized size = 0.63

$$-\frac{1}{2}d^2f^2\log(x)\left(a+b\log\left(cx^n\right)+bn\right)+\frac{\left(d^2f^2x-1\right)\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)+bn\right)}{x}-\frac{df\left(a+b\log\left(cx^n\right)+3\right.}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] (b*d^2*f^2*n*Log[x]^2)/4 + ((-1 + d^2*f^2*x)*Log[1 + d*f*Sqrt[x]]*(a + b*n
+ b*Log[c*x^n]))/x - (d^2*f^2*Log[x]*(a + b*n + b*Log[c*x^n]))/2 - (d*f*(a
+ 3*b*n + b*Log[c*x^n]))/Sqrt[x] + 2*b*d^2*f^2*n*PolyLog[2, -(d*f*Sqrt[x])]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(df\sqrt{x}+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right) \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\, x^{n}\right) + a\right) \ln\left(\left(f\sqrt{x} + \frac{1}{d}\right) d\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right) \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,\left(f\,\sqrt{x} + \frac{1}{d}\right)\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2)))/x**2,x)
```

```
[Out] Timed out
```

3.51 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)}{x^3}\,dx$

Optimal. Leaf size=289

$$\frac{1}{2}d^4f^4\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}d^4f^4\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{d^3f^3\left(a+b\log\left(cx^n\right)\right)}{2\sqrt{x}}+\frac{d^2f^2\left(a+b\log\right.}{4x}$$

```
[Out] -7/36*b*d*f*n/x^(3/2)+3/8*b*d^2*f^2*n/x-1/8*b*d^4*f^4*n*ln(x)+1/8*b*d^4*f^4
*n*ln(x)^2-1/6*d*f*(a+b*ln(c*x^n))/x^(3/2)+1/4*d^2*f^2*(a+b*ln(c*x^n))/x-1/
4*d^4*f^4*ln(x)*(a+b*ln(c*x^n))+1/4*b*d^4*f^4*n*ln(1+d*f*x^(1/2))-1/4*b*n*l
n(1+d*f*x^(1/2))/x^2+1/2*d^4*f^4*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-1/2*(a+b
*ln(c*x^n))*ln(1+d*f*x^(1/2))/x^2+b*d^4*f^4*n*polylog(2,-d*f*x^(1/2))-5/4*b
*d^3*f^3*n/x^(1/2)-1/2*d^3*f^3*(a+b*ln(c*x^n))/x^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2395, 44, 2376, 2391, 2301}

$$bd^4f^4n\text{PolyLog}\left(2,-df\sqrt{x}\right)+\frac{1}{2}d^4f^4\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}d^4f^4\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{d^3f^3\left(a+\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] (-7*b*d*f*n)/(36*x^(3/2)) + (3*b*d^2*f^2*n)/(8*x) - (5*b*d^3*f^3*n)/(4*Sqrt
[x]) + (b*d^4*f^4*n*Log[1 + d*f*Sqrt[x]])/4 - (b*n*Log[1 + d*f*Sqrt[x]])/(4
*x^2) - (b*d^4*f^4*n*Log[x])/8 + (b*d^4*f^4*n*Log[x]^2)/8 - (d*f*(a + b*Log
[c*x^n]))/(6*x^(3/2)) + (d^2*f^2*(a + b*Log[c*x^n]))/(4*x) - (d^3*f^3*(a +
b*Log[c*x^n]))/(2*Sqrt[x]) + (d^4*f^4*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n
]))/2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*x^2) - (d^4*f^4*Log[x]
*(a + b*Log[c*x^n]))/4 + b*d^4*f^4*n*PolyLog[2, -(d*f*Sqrt[x])]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx = -\frac{df\left(a+b\log\left(cx^n\right)\right)}{6x^{3/2}}+\frac{d^2f^2\left(a+b\log\left(cx^n\right)\right)}{4x}-\frac{d^3f^3\left(a+b\log\right.}{2\sqrt{x}}$$

$$= -\frac{bdfn}{9x^{3/2}}+\frac{bd^2f^2n}{4x}-\frac{bd^3f^3n}{\sqrt{x}}-\frac{df\left(a+b\log\left(cx^n\right)\right)}{6x^{3/2}}+\frac{d^2f^2\left(a+b\right.}{4}$$

$$= -\frac{bdfn}{9x^{3/2}}+\frac{bd^2f^2n}{4x}-\frac{bd^3f^3n}{\sqrt{x}}+\frac{1}{8}bd^4f^4n\log^2(x)-\frac{df\left(a+b\log\right.}{6x^{3/2}}$$

$$= -\frac{bdfn}{9x^{3/2}}+\frac{bd^2f^2n}{4x}-\frac{bd^3f^3n}{\sqrt{x}}-\frac{bn\log\left(1+df\sqrt{x}\right)}{4x^2}+\frac{1}{8}bd^4f^4n\log$$

$$= -\frac{bdfn}{9x^{3/2}}+\frac{bd^2f^2n}{4x}-\frac{bd^3f^3n}{\sqrt{x}}-\frac{bn\log\left(1+df\sqrt{x}\right)}{4x^2}+\frac{1}{8}bd^4f^4n\log$$

$$= -\frac{7bdfn}{36x^{3/2}}+\frac{3bd^2f^2n}{8x}-\frac{5bd^3f^3n}{4\sqrt{x}}+\frac{1}{4}bd^4f^4n\log\left(1+df\sqrt{x}\right)-\frac{bn}{}$$

Mathematica [A] time = 0.25, size = 207, normalized size = 0.72

$$\frac{\left(d^4f^4x^2-1\right)\log\left(df\sqrt{x}+1\right)\left(2a+2b\log\left(cx^n\right)+bn\right)}{4x^2}-\frac{df\left(9d^3f^3x^{3/2}\log(x)\left(2a+2b\log\left(cx^n\right)+bn\right)+36ad^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] ((-1 + d^4*f^4*x^2)*Log[1 + d*f*Sqrt[x]]*(2*a + b*n + 2*b*Log[c*x^n]))/(4*x
^2) - (d*f*(12*a + 14*b*n - 18*a*d*f*Sqrt[x] - 27*b*d*f*n*Sqrt[x] + 36*a*d^
2*f^2*x + 90*b*d^2*f^2*n*x - 9*b*d^3*f^3*n*x^(3/2)*Log[x]^2 + 6*b*(2 - 3*d*
f*Sqrt[x] + 6*d^2*f^2*x)*Log[c*x^n] + 9*d^3*f^3*x^(3/2)*Log[x]*(2*a + b*n +
 2*b*Log[c*x^n])))/(72*x^(3/2)) + b*d^4*f^4*n*PolyLog[2, -(d*f*Sqrt[x])]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(df\sqrt{x}+1\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right) \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\, x^{n}\right) + a\right) \ln\left(\left(f\sqrt{x} + \frac{1}{d}\right) d\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right) \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(f\,\sqrt{x} + \frac{1}{d}\right)\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2)))/x**3,x)
```

```
[Out] Timed out
```

3.52 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)}{x^4}\,dx$

Optimal. Leaf size=372

$$\frac{1}{3}d^6f^6\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{6}d^6f^6\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{d^5f^5\left(a+b\log\left(cx^n\right)\right)}{3\sqrt{x}}+\frac{d^4f^4\left(a+b\log\right.}{6x}$$

```
[Out] -11/225*b*d*f*n/x^(5/2)+5/72*b*d^2*f^2*n/x^2-1/9*b*d^3*f^3*n/x^(3/2)+2/9*b*
d^4*f^4*n/x-1/18*b*d^6*f^6*n*ln(x)+1/12*b*d^6*f^6*n*ln(x)^2-1/15*d*f*(a+b*l
n(c*x^n))/x^(5/2)+1/12*d^2*f^2*(a+b*ln(c*x^n))/x^2-1/9*d^3*f^3*(a+b*ln(c*x^
n))/x^(3/2)+1/6*d^4*f^4*(a+b*ln(c*x^n))/x-1/6*d^6*f^6*ln(x)*(a+b*ln(c*x^n))
+1/9*b*d^6*f^6*n*ln(1+d*f*x^(1/2))-1/9*b*n*ln(1+d*f*x^(1/2))/x^3+1/3*d^6*f^
6*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-1/3*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/x
^3+2/3*b*d^6*f^6*n*polylog(2,-d*f*x^(1/2))-7/9*b*d^5*f^5*n/x^(1/2)-1/3*d^5*
f^5*(a+b*ln(c*x^n))/x^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2395, 44, 2376, 2391, 2301}

$$\frac{2}{3}bd^6f^6n\text{PolyLog}\left(2,-df\sqrt{x}\right)+\frac{1}{3}d^6f^6\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{6}d^6f^6\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{d^5f^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] (-11*b*d*f*n)/(225*x^(5/2)) + (5*b*d^2*f^2*n)/(72*x^2) - (b*d^3*f^3*n)/(9*x
^(3/2)) + (2*b*d^4*f^4*n)/(9*x) - (7*b*d^5*f^5*n)/(9*Sqrt[x]) + (b*d^6*f^6*
n*Log[1 + d*f*Sqrt[x]])/9 - (b*n*Log[1 + d*f*Sqrt[x]])/(9*x^3) - (b*d^6*f^6
*n*Log[x])/18 + (b*d^6*f^6*n*Log[x]^2)/12 - (d*f*(a + b*Log[c*x^n]))/(15*x^
(5/2)) + (d^2*f^2*(a + b*Log[c*x^n]))/(12*x^2) - (d^3*f^3*(a + b*Log[c*x^n]
))/(9*x^(3/2)) + (d^4*f^4*(a + b*Log[c*x^n]))/(6*x) - (d^5*f^5*(a + b*Log[c
*x^n]))/(3*Sqrt[x]) + (d^6*f^6*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/3 -
 (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(3*x^3) - (d^6*f^6*Log[x]*(a + b
*Log[c*x^n]))/6 + (2*b*d^6*f^6*n*PolyLog[2, -(d*f*Sqrt[x])])/3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^4}\,dx = -\frac{df\left(a+b\log\left(cx^n\right)\right)}{15x^{5/2}}+\frac{d^2f^2\left(a+b\log\left(cx^n\right)\right)}{12x^2}-\frac{d^3f^3\left(a+b\log\left(cx\right.\right.}{9x^{3/2}}$$

$$= -\frac{2bdfn}{75x^{5/2}}+\frac{bd^2f^2n}{24x^2}-\frac{2bd^3f^3n}{27x^{3/2}}+\frac{bd^4f^4n}{6x}-\frac{2bd^5f^5n}{3\sqrt{x}}-\frac{df\left(a+b\log\right.}{15x^{5/2}}$$

$$= -\frac{2bdfn}{75x^{5/2}}+\frac{bd^2f^2n}{24x^2}-\frac{2bd^3f^3n}{27x^{3/2}}+\frac{bd^4f^4n}{6x}-\frac{2bd^5f^5n}{3\sqrt{x}}+\frac{1}{12}bd^6f^6n\,\mathrm{lc}$$

$$= -\frac{2bdfn}{75x^{5/2}}+\frac{bd^2f^2n}{24x^2}-\frac{2bd^3f^3n}{27x^{3/2}}+\frac{bd^4f^4n}{6x}-\frac{2bd^5f^5n}{3\sqrt{x}}-\frac{bn\log\left(1+\right.}{9x^3}$$

$$= -\frac{2bdfn}{75x^{5/2}}+\frac{bd^2f^2n}{24x^2}-\frac{2bd^3f^3n}{27x^{3/2}}+\frac{bd^4f^4n}{6x}-\frac{2bd^5f^5n}{3\sqrt{x}}-\frac{bn\log\left(1+\right.}{9x^3}$$

$$= -\frac{11bdfn}{225x^{5/2}}+\frac{5bd^2f^2n}{72x^2}-\frac{bd^3f^3n}{9x^{3/2}}+\frac{2bd^4f^4n}{9x}-\frac{7bd^5f^5n}{9\sqrt{x}}+\frac{1}{9}bd^6f^6n\,\mathrm{l}$$

Mathematica [A] time = 0.36, size = 288, normalized size = 0.77

$$\frac{\left(d^6f^6x^3-1\right)\log\left(df\sqrt{x}+1\right)\left(3a+3b\log\left(cx^n\right)+bn\right)}{9x^3}-\frac{df\left(100d^5f^5x^{5/2}\log(x)\left(3a+3b\log\left(cx^n\right)+bn\right)+600ad^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] ((-1 + d^6*f^6*x^3)*Log[1 + d*f*Sqrt[x]]*(3*a + b*n + 3*b*Log[c*x^n]))/(9*x
^3) - (d*f*(120*a + 88*b*n - 150*a*d*f*Sqrt[x] - 125*b*d*f*n*Sqrt[x] + 200*
a*d^2*f^2*x + 200*b*d^2*f^2*n*x - 300*a*d^3*f^3*x^(3/2) - 400*b*d^3*f^3*n*x
^(3/2) + 600*a*d^4*f^4*x^2 + 1400*b*d^4*f^4*n*x^2 - 150*b*d^5*f^5*n*x^(5/2)
*Log[x]^2 + 10*b*(12 - 15*d*f*Sqrt[x] + 20*d^2*f^2*x - 30*d^3*f^3*x^(3/2) +
 60*d^4*f^4*x^2)*Log[c*x^n] + 100*d^5*f^5*x^(5/2)*Log[x]*(3*a + b*n + 3*b*L
og[c*x^n])))/(1800*x^(5/2)) + (2*b*d^6*f^6*n*PolyLog[2, -(d*f*Sqrt[x])])/3
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(df\sqrt{x}+1\right)}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^4,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^4, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^{n}\right)+a\right)\ln\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d)/x^4,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln((f*x^(1/2)+1/d)*d)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2)))/x**4,x)
```

```
[Out] Timed out
```

3.53 $\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=708

$$-\frac{4bn\text{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{3d^6f^6}-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{3d^6f^6}+\frac{2bn\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{9d^6f^6}+$$

```
[Out] 1/3*b^2*n*x*ln(c*x^n)/d^4/f^4+1/9*b*n*x*(a+b*ln(c*x^n))/d^4/f^4-2/9*b*n*x^(
3/2)*(a+b*ln(c*x^n))/d^3/f^3+5/36*b*n*x^2*(a+b*ln(c*x^n))/d^2/f^2-22/225*b*
n*x^(5/2)*(a+b*ln(c*x^n))/d/f+2/9*b*n*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/d^6
/f^6-4/3*b*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^6/f^6-14/9*b*n*(a+b*
ln(c*x^n))*x^(1/2)/d^5/f^5+2/27*b*n*x^3*(a+b*ln(c*x^n))-2/27*b^2*n^2*ln(1+d
*f*x^(1/2))/d^6/f^6-2/9*b*n*x^3*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))+4/9*b^2*n
^2*polylog(2,-d*f*x^(1/2))/d^6/f^6+8/3*b^2*n^2*polylog(3,-d*f*x^(1/2))/d^6/
f^6+86/27*b^2*n^2*x^(1/2)/d^5/f^5-1/27*b^2*n^2*x^3-1/18*x^3*(a+b*ln(c*x^n))
^2+1/3*x^3*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))-1/6*x*(a+b*ln(c*x^n))^2/d^4/
f^4+1/9*x^(3/2)*(a+b*ln(c*x^n))^2/d^3/f^3-1/12*x^2*(a+b*ln(c*x^n))^2/d^2/f^
2+1/15*x^(5/2)*(a+b*ln(c*x^n))^2/d/f+2/27*b^2*n^2*x^3*ln(1+d*f*x^(1/2))-1/3
*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/d^6/f^6+1/3*(a+b*ln(c*x^n))^2*x^(1/2)/
d^5/f^5-13/27*b^2*n^2*x/d^4/f^4+14/81*b^2*n^2*x^(3/2)/d^3/f^3-19/216*b^2*n^
2*x^2/d^2/f^2+182/3375*b^2*n^2*x^(5/2)/d/f+1/3*a*b*n*x/d^4/f^4
```

Rubi [A] time = 0.64, antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2395, 43, 2377, 2295, 2304, 2374, 6589, 2376, 2391}

$$-\frac{4bn\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{3d^6f^6}+\frac{4b^2n^2\text{PolyLog}\left(2,-df\sqrt{x}\right)}{9d^6f^6}+\frac{8b^2n^2\text{PolyLog}\left(3,-df\sqrt{x}\right)}{3d^6f^6}-\frac{x^2\left(a+b\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (86*b^2*n^2*Sqrt[x])/(27*d^5*f^5) + (a*b*n*x)/(3*d^4*f^4) - (13*b^2*n^2*x)/
(27*d^4*f^4) + (14*b^2*n^2*x^(3/2))/(81*d^3*f^3) - (19*b^2*n^2*x^2)/(216*d^
2*f^2) + (182*b^2*n^2*x^(5/2))/(3375*d*f) - (b^2*n^2*x^3)/27 - (2*b^2*n^2*L
og[1 + d*f*Sqrt[x]])/(27*d^6*f^6) + (2*b^2*n^2*x^3*Log[1 + d*f*Sqrt[x]])/27
 + (b^2*n*x*Log[c*x^n])/(3*d^4*f^4) - (14*b*n*Sqrt[x]*(a + b*Log[c*x^n]))/(
9*d^5*f^5) + (b*n*x*(a + b*Log[c*x^n]))/(9*d^4*f^4) - (2*b*n*x^(3/2)*(a + b
*Log[c*x^n]))/(9*d^3*f^3) + (5*b*n*x^2*(a + b*Log[c*x^n]))/(36*d^2*f^2) - (
22*b*n*x^(5/2)*(a + b*Log[c*x^n]))/(225*d*f) + (2*b*n*x^3*(a + b*Log[c*x^n]
))/27 + (2*b*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(9*d^6*f^6) - (2*b*
n*x^3*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/9 + (Sqrt[x]*(a + b*Log[c*x^
n])^2)/(3*d^5*f^5) - (x*(a + b*Log[c*x^n])^2)/(6*d^4*f^4) + (x^(3/2)*(a + b
*Log[c*x^n])^2)/(9*d^3*f^3) - (x^2*(a + b*Log[c*x^n])^2)/(12*d^2*f^2) + (x^
(5/2)*(a + b*Log[c*x^n])^2)/(15*d*f) - (x^3*(a + b*Log[c*x^n])^2)/18 - (Log
[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(3*d^6*f^6) + (x^3*Log[1 + d*f*Sqrt
[x]]*(a + b*Log[c*x^n])^2)/3 + (4*b^2*n^2*PolyLog[2, -(d*f*Sqrt[x])])/(9*d^
6*f^6) - (4*b*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])])/(3*d^6*f^6)
+ (8*b^2*n^2*PolyLog[3, -(d*f*Sqrt[x])])/(3*d^6*f^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{3d^5f^5}-\frac{x\left(a+b\log\left(cx^n\right)\right)^2}{6d^4f^4}+\frac{x^{3/2}\left(a+b\log\left(c\right.\right.}{9d^3f^3}$$

$$= \frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{3d^5f^5}-\frac{x\left(a+b\log\left(cx^n\right)\right)^2}{6d^4f^4}+\frac{x^{3/2}\left(a+b\log\left(c\right.\right.}{9d^3f^3}$$

$$= \frac{8b^2n^2\sqrt{x}}{3d^5f^5}+\frac{abnx}{3d^4f^4}+\frac{8b^2n^2x^{3/2}}{81d^3f^3}-\frac{b^2n^2x^2}{24d^2f^2}+\frac{8b^2n^2x^{5/2}}{375df}-\frac{1}{81}b^2n$$

$$= \frac{28b^2n^2\sqrt{x}}{9d^5f^5}+\frac{abnx}{3d^4f^4}-\frac{4b^2n^2x}{9d^4f^4}+\frac{4b^2n^2x^{3/2}}{27d^3f^3}-\frac{5b^2n^2x^2}{72d^2f^2}+\frac{44b^2n}{1125}$$

$$= \frac{28b^2n^2\sqrt{x}}{9d^5f^5}+\frac{abnx}{3d^4f^4}-\frac{4b^2n^2x}{9d^4f^4}+\frac{4b^2n^2x^{3/2}}{27d^3f^3}-\frac{5b^2n^2x^2}{72d^2f^2}+\frac{44b^2n}{1125}$$

$$= \frac{28b^2n^2\sqrt{x}}{9d^5f^5}+\frac{abnx}{3d^4f^4}-\frac{4b^2n^2x}{9d^4f^4}+\frac{4b^2n^2x^{3/2}}{27d^3f^3}-\frac{5b^2n^2x^2}{72d^2f^2}+\frac{44b^2n}{1125}$$

$$= \frac{28b^2n^2\sqrt{x}}{9d^5f^5}+\frac{abnx}{3d^4f^4}-\frac{4b^2n^2x}{9d^4f^4}+\frac{4b^2n^2x^{3/2}}{27d^3f^3}-\frac{5b^2n^2x^2}{72d^2f^2}+\frac{44b^2n}{1125}$$

$$= \frac{86b^2n^2\sqrt{x}}{27d^5f^5}+\frac{abnx}{3d^4f^4}-\frac{13b^2n^2x}{27d^4f^4}+\frac{14b^2n^2x^{3/2}}{81d^3f^3}-\frac{19b^2n^2x^2}{216d^2f^2}+\frac{18}{3}$$

Mathematica [A] time = 0.59, size = 995, normalized size = 1.41

$$\frac{-4500a^2d^6x^3f^6-3000b^2d^6n^2x^3f^6+6000abd^6nx^3f^6-4500b^2d^6x^3\log^2\left(cx^n\right)f^6+27000b^2d^6x^3\log\left(d\sqrt{x}f+1\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (27000*a^2*d*f*Sqrt[x] - 126000*a*b*d*f*n*Sqrt[x] + 258000*b^2*d*f*n^2*Sqrt
[x] - 13500*a^2*d^2*f^2*x + 36000*a*b*d^2*f^2*n*x - 39000*b^2*d^2*f^2*n^2*x
 + 9000*a^2*d^3*f^3*x^(3/2) - 18000*a*b*d^3*f^3*n*x^(3/2) + 14000*b^2*d^3*f
^3*n^2*x^(3/2) - 6750*a^2*d^4*f^4*x^2 + 11250*a*b*d^4*f^4*n*x^2 - 7125*b^2*
d^4*f^4*n^2*x^2 + 5400*a^2*d^5*f^5*x^(5/2) - 7920*a*b*d^5*f^5*n*x^(5/2) + 4
368*b^2*d^5*f^5*n^2*x^(5/2) - 4500*a^2*d^6*f^6*x^3 + 6000*a*b*d^6*f^6*n*x^3
 - 3000*b^2*d^6*f^6*n^2*x^3 - 27000*a^2*Log[1 + d*f*Sqrt[x]] + 18000*a*b*n*
Log[1 + d*f*Sqrt[x]] - 6000*b^2*n^2*Log[1 + d*f*Sqrt[x]] + 27000*a^2*d^6*f^
6*x^3*Log[1 + d*f*Sqrt[x]] - 18000*a*b*d^6*f^6*n*x^3*Log[1 + d*f*Sqrt[x]] +
 6000*b^2*d^6*f^6*n^2*x^3*Log[1 + d*f*Sqrt[x]] + 54000*a*b*d*f*Sqrt[x]*Log[
c*x^n] - 126000*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 27000*a*b*d^2*f^2*x*Log[c*x^
n] + 36000*b^2*d^2*f^2*n*x*Log[c*x^n] + 18000*a*b*d^3*f^3*x^(3/2)*Log[c*x^n
] - 18000*b^2*d^3*f^3*n*x^(3/2)*Log[c*x^n] - 13500*a*b*d^4*f^4*x^2*Log[c*x^
n] + 11250*b^2*d^4*f^4*n*x^2*Log[c*x^n] + 10800*a*b*d^5*f^5*x^(5/2)*Log[c*x
^n] - 7920*b^2*d^5*f^5*n*x^(5/2)*Log[c*x^n] - 9000*a*b*d^6*f^6*x^3*Log[c*x^
n] + 6000*b^2*d^6*f^6*n*x^3*Log[c*x^n] - 54000*a*b*Log[1 + d*f*Sqrt[x]]*Log
[c*x^n] + 18000*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 54000*a*b*d^6*f^6*x
```

```
^3*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 18000*b^2*d^6*f^6*n*x^3*Log[1 + d*f*Sq
rt[x]]*Log[c*x^n] + 27000*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 - 13500*b^2*d^2*f^2*
x*Log[c*x^n]^2 + 9000*b^2*d^3*f^3*x^(3/2)*Log[c*x^n]^2 - 6750*b^2*d^4*f^4*x
^2*Log[c*x^n]^2 + 5400*b^2*d^5*f^5*x^(5/2)*Log[c*x^n]^2 - 4500*b^2*d^6*f^6*
x^3*Log[c*x^n]^2 - 27000*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 27000*b^2*
d^6*f^6*x^3*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 36000*b*n*(-3*a + b*n - 3*b
*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] + 216000*b^2*n^2*PolyLog[3, -(d*f*S
qrt[x])])/(81000*d^6*f^6)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^2\log\left(cx^n\right)^2 + 2\,abx^2\log\left(cx^n\right) + a^2x^2\right)\log\left(df\sqrt{x}+1\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log(d*f*sq
rt(x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right) + a\right)^2 x^2 \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right) + a\right)^2 x^2 \ln\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right) + a\right)^2 x^2 \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="maxim
a")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,\left(f\,\sqrt{x} + \frac{1}{d}\right)\right)\,{\left(a + b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2,x)
[Out] int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2))),x)
[Out] Timed out
```

3.54 $\int x \log \left(d \left(\frac{1}{d}+f \sqrt{x}\right)\right) \left(a+b \log \left(c x^n\right)\right)^2 \, dx$

Optimal. Leaf size=557

$$-\frac{2bn\text{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{d^4f^4}+\frac{bn\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{2d^4f^4}-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{2d^4f^4}$$

```
[Out] 1/2*a*b*n*x/d^2/f^2-7/8*b^2*n^2*x/d^2/f^2+37/108*b^2*n^2*x^(3/2)/d/f-3/16*b
^2*n^2*x^2+1/2*b^2*n*x*ln(c*x^n)/d^2/f^2+1/4*b*n*x*(a+b*ln(c*x^n))/d^2/f^2-
7/18*b*n*x^(3/2)*(a+b*ln(c*x^n))/d/f+1/4*b*n*x^2*(a+b*ln(c*x^n))-1/4*x*(a+b
*ln(c*x^n))^2/d^2/f^2+1/6*x^(3/2)*(a+b*ln(c*x^n))^2/d/f-1/8*x^2*(a+b*ln(c*x
^n))^2-1/4*b^2*n^2*ln(1+d*f*x^(1/2))/d^4/f^4+1/4*b^2*n^2*x^2*ln(1+d*f*x^(1/
2))+1/2*b*n*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/d^4/f^4-1/2*b*n*x^2*(a+b*ln(c
*x^n))*ln(1+d*f*x^(1/2))-1/2*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/d^4/f^4+1/
2*x^2*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))+b^2*n^2*polylog(2,-d*f*x^(1/2))/d
^4/f^4-2*b*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^4/f^4+4*b^2*n^2*poly
log(3,-d*f*x^(1/2))/d^4/f^4+21/4*b^2*n^2*x^(1/2)/d^3/f^3-5/2*b*n*(a+b*ln(c*
x^n))*x^(1/2)/d^3/f^3+1/2*(a+b*ln(c*x^n))^2*x^(1/2)/d^3/f^3
```

Rubi [A] time = 0.46, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2454, 2395, 43, 2377, 2295, 2304, 2374, 6589, 2376, 2391}

$$-\frac{2bn\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{d^4f^4}+\frac{b^2n^2\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^4f^4}+\frac{4b^2n^2\text{PolyLog}\left(3,-df\sqrt{x}\right)}{d^4f^4}+\frac{bnx\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (21*b^2*n^2*Sqrt[x])/(4*d^3*f^3) + (a*b*n*x)/(2*d^2*f^2) - (7*b^2*n^2*x)/(8
*d^2*f^2) + (37*b^2*n^2*x^(3/2))/(108*d*f) - (3*b^2*n^2*x^2)/16 - (b^2*n^2*
Log[1 + d*f*Sqrt[x]])/(4*d^4*f^4) + (b^2*n^2*x^2*Log[1 + d*f*Sqrt[x]])/4 +
(b^2*n*x*Log[c*x^n])/(2*d^2*f^2) - (5*b*n*Sqrt[x]*(a + b*Log[c*x^n]))/(2*d^
3*f^3) + (b*n*x*(a + b*Log[c*x^n]))/(4*d^2*f^2) - (7*b*n*x^(3/2)*(a + b*Log
[c*x^n]))/(18*d*f) + (b*n*x^2*(a + b*Log[c*x^n]))/4 + (b*n*Log[1 + d*f*Sqrt
[x]]*(a + b*Log[c*x^n]))/(2*d^4*f^4) - (b*n*x^2*Log[1 + d*f*Sqrt[x]]*(a + b
*Log[c*x^n]))/2 + (Sqrt[x]*(a + b*Log[c*x^n])^2)/(2*d^3*f^3) - (x*(a + b*Lo
g[c*x^n])^2)/(4*d^2*f^2) + (x^(3/2)*(a + b*Log[c*x^n])^2)/(6*d*f) - (x^2*(a
 + b*Log[c*x^n])^2)/8 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(2*d^4*
f^4) + (x^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/2 + (b^2*n^2*PolyLog
[2, -(d*f*Sqrt[x])])/(d^4*f^4) - (2*b*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f
*Sqrt[x])])/(d^4*f^4) + (4*b^2*n^2*PolyLog[3, -(d*f*Sqrt[x])])/(d^4*f^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{\sqrt{x}\,\left(a+b\log\left(cx^n\right)\right)^2}{2d^3f^3}-\frac{x\left(a+b\log\left(cx^n\right)\right)^2}{4d^2f^2}+\frac{x^{3/2}\left(a+b\log\right.}{6df}$$

$$=\frac{\sqrt{x}\,\left(a+b\log\left(cx^n\right)\right)^2}{2d^3f^3}-\frac{x\left(a+b\log\left(cx^n\right)\right)^2}{4d^2f^2}+\frac{x^{3/2}\left(a+b\log\right.}{6df}$$

$$=\frac{4b^2n^2\sqrt{x}}{d^3f^3}+\frac{abnx}{2d^2f^2}+\frac{4b^2n^2x^{3/2}}{27df}-\frac{1}{16}b^2n^2x^2-\frac{5bn\sqrt{x}\,\left(a+b\right.}{2d^3f^3}$$

$$=\frac{5b^2n^2\sqrt{x}}{d^3f^3}+\frac{abnx}{2d^2f^2}-\frac{3b^2n^2x}{4d^2f^2}+\frac{7b^2n^2x^{3/2}}{27df}-\frac{1}{8}b^2n^2x^2+\frac{b^2nx}{2}$$

$$=\frac{5b^2n^2\sqrt{x}}{d^3f^3}+\frac{abnx}{2d^2f^2}-\frac{3b^2n^2x}{4d^2f^2}+\frac{7b^2n^2x^{3/2}}{27df}-\frac{1}{8}b^2n^2x^2+\frac{b^2nx}{2}$$

$$=\frac{5b^2n^2\sqrt{x}}{d^3f^3}+\frac{abnx}{2d^2f^2}-\frac{3b^2n^2x}{4d^2f^2}+\frac{7b^2n^2x^{3/2}}{27df}-\frac{1}{8}b^2n^2x^2+\frac{1}{4}b^2n^2$$

$$=\frac{5b^2n^2\sqrt{x}}{d^3f^3}+\frac{abnx}{2d^2f^2}-\frac{3b^2n^2x}{4d^2f^2}+\frac{7b^2n^2x^{3/2}}{27df}-\frac{1}{8}b^2n^2x^2+\frac{1}{4}b^2n^2$$

$$=\frac{21b^2n^2\sqrt{x}}{4d^3f^3}+\frac{abnx}{2d^2f^2}-\frac{7b^2n^2x}{8d^2f^2}+\frac{37b^2n^2x^{3/2}}{108df}-\frac{3}{16}b^2n^2x^2-\frac{b^2}{}$$

Mathematica [A] time = 0.41, size = 769, normalized size = 1.38

$$\frac{-54a^2d^4f^4x^2+216a^2d^4f^4x^2\log\left(df\sqrt{x}+1\right)+72a^2d^3f^3x^{3/2}-108a^2d^2f^2x+216a^2df\sqrt{x}-216a^2\log\left(df\sqrt{x}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (216*a^2*d*f*Sqrt[x] - 1080*a*b*d*f*n*Sqrt[x] + 2268*b^2*d*f*n^2*Sqrt[x] -
108*a^2*d^2*f^2*x + 324*a*b*d^2*f^2*n*x - 378*b^2*d^2*f^2*n^2*x + 72*a^2*d^
3*f^3*x^(3/2) - 168*a*b*d^3*f^3*n*x^(3/2) + 148*b^2*d^3*f^3*n^2*x^(3/2) - 5
4*a^2*d^4*f^4*x^2 + 108*a*b*d^4*f^4*n*x^2 - 81*b^2*d^4*f^4*n^2*x^2 - 216*a^
2*Log[1 + d*f*Sqrt[x]] + 216*a*b*n*Log[1 + d*f*Sqrt[x]] - 108*b^2*n^2*Log[1
 + d*f*Sqrt[x]] + 216*a^2*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]] - 216*a*b*d^4*f^
4*n*x^2*Log[1 + d*f*Sqrt[x]] + 108*b^2*d^4*f^4*n^2*x^2*Log[1 + d*f*Sqrt[x]]
 + 432*a*b*d*f*Sqrt[x]*Log[c*x^n] - 1080*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 216
*a*b*d^2*f^2*x*Log[c*x^n] + 324*b^2*d^2*f^2*n*x*Log[c*x^n] + 144*a*b*d^3*f^
3*x^(3/2)*Log[c*x^n] - 168*b^2*d^3*f^3*n*x^(3/2)*Log[c*x^n] - 108*a*b*d^4*f
^4*x^2*Log[c*x^n] + 108*b^2*d^4*f^4*n*x^2*Log[c*x^n] - 432*a*b*Log[1 + d*f*
Sqrt[x]]*Log[c*x^n] + 216*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 432*a*b*d
^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 216*b^2*d^4*f^4*n*x^2*Log[1 +
d*f*Sqrt[x]]*Log[c*x^n] + 216*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 - 108*b^2*d^2*f^
2*x*Log[c*x^n]^2 + 72*b^2*d^3*f^3*x^(3/2)*Log[c*x^n]^2 - 54*b^2*d^4*f^4*x^2
*Log[c*x^n]^2 - 216*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 216*b^2*d^4*f^4
*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 432*b*n*(-2*a + b*n - 2*b*Log[c*x^
n])*PolyLog[2, -(d*f*Sqrt[x])] + 1728*b^2*n^2*PolyLog[3, -(d*f*Sqrt[x])])/(
432*d^4*f^4)
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2x\log\left(cx^n\right)^2+2\,abx\log\left(cx^n\right)+a^2x\right)\log\left(df\sqrt{x}+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas"
)
```

```
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log(d*f*sqrt(x)
+ 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(cx^{n}\right) + a\right)^{2} x \log \left(\left(f \sqrt{x} + \frac{1}{d}\right) d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + 1/d)*d), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c\,x^{n}\right) + a\right)^{2} x \ln \left(\left(f \sqrt{x} + \frac{1}{d}\right) d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(cx^{n}\right) + a\right)^{2} x \log \left(\left(f \sqrt{x} + \frac{1}{d}\right) d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + 1/d)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2))),x)
```

```
[Out] Timed out
```

3.55 $\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^2 \, dx$

Optimal. Leaf size=374

$$-\frac{4bn\text{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}+\frac{2bn\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}$$

```
[Out] a*b*n*x-3*b^2*n^2*x+b^2*n*x*ln(c*x^n)+b*n*x*(a+b*ln(c*x^n))-1/2*x*(a+b*ln(c
*x^n))^2+2*b^2*n^2*x*ln(d*(1/d+f*x^(1/2)))-2*b*n*x*(a+b*ln(c*x^n))*ln(d*(1/
d+f*x^(1/2)))+x*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2)))-2*b^2*n^2*ln(1+d*f*
x^(1/2))/d^2/f^2+2*b*n*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/d^2/f^2-(a+b*ln(c*
x^n))^2*ln(1+d*f*x^(1/2))/d^2/f^2+4*b^2*n^2*polylog(2,-d*f*x^(1/2))/d^2/f^2
-4*b*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^2/f^2+8*b^2*n^2*polylog(3,
-d*f*x^(1/2))/d^2/f^2+14*b^2*n^2*x^(1/2)/d/f-6*b*n*(a+b*ln(c*x^n))*x^(1/2)/
d/f+(a+b*ln(c*x^n))^2*x^(1/2)/d/f
```

Rubi [A] time = 0.27, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2448, 266, 43, 2370, 2295, 2304, 2391, 2374, 6589}

$$-\frac{4bn\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}+\frac{4b^2n^2\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2}+\frac{8b^2n^2\text{PolyLog}\left(3,-df\sqrt{x}\right)}{d^2f^2}+\frac{2bn\,l}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (14*b^2*n^2*Sqrt[x])/(d*f) + a*b*n*x - 3*b^2*n^2*x + 2*b^2*n^2*x*Log[d*(d^(
-1) + f*Sqrt[x])] - (2*b^2*n^2*Log[1 + d*f*Sqrt[x]])/(d^2*f^2) + b^2*n*x*Lo
g[c*x^n] - (6*b*n*Sqrt[x]*(a + b*Log[c*x^n]))/(d*f) + b*n*x*(a + b*Log[c*x^
n]) - 2*b*n*x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]) + (2*b*n*Log[1
 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(d^2*f^2) + (Sqrt[x]*(a + b*Log[c*x^n])
^2)/(d*f) - (x*(a + b*Log[c*x^n])^2)/2 + x*Log[d*(d^(-1) + f*Sqrt[x])]*(a +
 b*Log[c*x^n])^2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(d^2*f^2) +
(4*b^2*n^2*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2) - (4*b*n*(a + b*Log[c*x^n]
)*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2) + (8*b^2*n^2*PolyLog[3, -(d*f*Sqrt[
x])])/(d^2*f^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{df} - \frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)^2 + x\log\left(d\left(\frac{1}{d}+f\right.\right.$$

$$= \frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{df} - \frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)^2 + x\log\left(d\left(\frac{1}{d}+f\right.\right.$$

$$= \frac{8b^2n^2\sqrt{x}}{df} + abnx - \frac{6bn\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{df} + bnx\left(a+b\log\right($$

$$= \frac{12b^2n^2\sqrt{x}}{df} + abnx - 2b^2n^2x + b^2nx\log\left(cx^n\right) - \frac{6bn\sqrt{x}\left(a+b\,l\right.}{df}$$

$$= \frac{12b^2n^2\sqrt{x}}{df} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) + b^2$$

$$= \frac{12b^2n^2\sqrt{x}}{df} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) + b^2$$

$$= \frac{12b^2n^2\sqrt{x}}{df} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) + b^2$$

$$= \frac{14b^2n^2\sqrt{x}}{df} + abnx - 3b^2n^2x + 2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) - \frac{2b}{}$$

Mathematica [A] time = 0.33, size = 527, normalized size = 1.41

$$-\frac{a^2d^2f^2x - 2a^2d^2f^2x\log\left(df\sqrt{x}+1\right) - 2a^2df\sqrt{x} + 2a^2\log\left(df\sqrt{x}+1\right) + 2abd^2f^2x\log\left(cx^n\right) - 4abd^2f^2x\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] -1/2*(-2*a^2*d*f*Sqrt[x] + 12*a*b*d*f*n*Sqrt[x] - 28*b^2*d*f*n^2*Sqrt[x] +
a^2*d^2*f^2*x - 4*a*b*d^2*f^2*n*x + 6*b^2*d^2*f^2*n^2*x + 2*a^2*Log[1 + d*f
*Sqrt[x]] - 4*a*b*n*Log[1 + d*f*Sqrt[x]] + 4*b^2*n^2*Log[1 + d*f*Sqrt[x]] -
 2*a^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]] + 4*a*b*d^2*f^2*n*x*Log[1 + d*f*Sqrt[
x]] - 4*b^2*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt[x]] - 4*a*b*d*f*Sqrt[x]*Log[c*x^
n] + 12*b^2*d*f*n*Sqrt[x]*Log[c*x^n] + 2*a*b*d^2*f^2*x*Log[c*x^n] - 4*b^2*d
^2*f^2*n*x*Log[c*x^n] + 4*a*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 4*b^2*n*Log
[1 + d*f*Sqrt[x]]*Log[c*x^n] - 4*a*b*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x
^n] + 4*b^2*d^2*f^2*n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 2*b^2*d*f*Sqrt[x]
*Log[c*x^n]^2 + b^2*d^2*f^2*x*Log[c*x^n]^2 + 2*b^2*Log[1 + d*f*Sqrt[x]]*Log
[c*x^n]^2 - 2*b^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 8*b*n*(a -
b*n + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] - 16*b^2*n^2*PolyLog[3, -(d*
f*Sqrt[x])])/(d^2*f^2)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\log\left(cx^n\right)^2 + 2\,ab\log\left(cx^n\right) + a^2\right)\log\left(df\sqrt{x}+1\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)^2\ln\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d),x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(b^2x\log\left(x^n\right)^2-2\left(b^2\left(n-\log(c)\right)-ab\right)x\log\left(x^n\right)+\left(\left(2\,n^2-2\,n\log(c)+\log(c)^2\right)b^2-2\,ab\left(n-\log(c)\right)+a^2\right)x\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima")
```

```
[Out] (b^2*x*log(x^n)^2 - 2*(b^2*(n - log(c)) - a*b)*x*log(x^n) + ((2*n^2 - 2*n*l
og(c) + log(c)^2)*b^2 - 2*a*b*(n - log(c)) + a^2)*x)*log(d*f*sqrt(x) + 1) -
 1/27*(9*b^2*d*f*x^2*log(x^n)^2 + 6*(3*a*b*d*f - (5*d*f*n - 3*d*f*log(c))*b
^2)*x^2*log(x^n) + (9*a^2*d*f - 6*(5*d*f*n - 3*d*f*log(c))*a*b + (38*d*f*n^
2 - 30*d*f*n*log(c) + 9*d*f*log(c)^2)*b^2)*x^2)/sqrt(x) + integrate(1/2*(b^
2*d^2*f^2*x*log(x^n)^2 + 2*(a*b*d^2*f^2 - (d^2*f^2*n - d^2*f^2*log(c))*b^2)
*x*log(x^n) + (a^2*d^2*f^2 - 2*(d^2*f^2*n - d^2*f^2*log(c))*a*b + (2*d^2*f^
2*n^2 - 2*d^2*f^2*n*log(c) + d^2*f^2*log(c)^2)*b^2)*x)/(d*f*sqrt(x) + 1), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\,\ln\left(c\,x^n\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2))),x)
```

```
[Out] Timed out
```

3.56 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx$

Optimal. Leaf size=70

$$8bn\text{Li}_3\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)-2\text{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2-16b^2n^2\text{Li}_4\left(-df\sqrt{x}\right)$$

```
[Out] -2*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^(1/2))+8*b*n*(a+b*ln(c*x^n))*polylog(
3,-d*f*x^(1/2))-16*b^2*n^2*polylog(4,-d*f*x^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2374, 2383, 6589}

$$8bn\text{PolyLog}\left(3,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)-2\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2-16b^2n^2\text{PolyLog}\left(4,-df\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] -2*(a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])] + 8*b*n*(a + b*Log[c*x^n
])*PolyLog[3, -(d*f*Sqrt[x])] - 16*b^2*n^2*PolyLog[4, -(d*f*Sqrt[x])]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx &= -2\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2\left(-df\sqrt{x}\right)+(4bn)\int \frac{\left(a+b\log\left(cx^n\right)\right)\text{I}}{x}\\ &= -2\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2\left(-df\sqrt{x}\right)+8bn\left(a+b\log\left(cx^n\right)\right)\text{Li}_3\left(-\right.\\ &= -2\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2\left(-df\sqrt{x}\right)+8bn\left(a+b\log\left(cx^n\right)\right)\text{Li}_3\left(-\right.\end{aligned}$$

Mathematica [A] time = 0.15, size = 70, normalized size = 1.00

$$-2\left(\text{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2+4bn\left(2bn\text{Li}_4\left(-df\sqrt{x}\right)-\text{Li}_3\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] -2*((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])] + 4*b*n*(-((a + b*Log[c
*x^n])*PolyLog[3, -(d*f*Sqrt[x])]) + 2*b*n*PolyLog[4, -(d*f*Sqrt[x])]))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^{2}\log\left(cx^{n}\right)^{2}+2\,ab\log\left(cx^{n}\right)+a^{2}\right)\log\left(df\sqrt{x}+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="fricas"
)
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1)/x
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^{n}\right)+a\right)^{2}\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^{n}\right)+a\right)^{2}\ln\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^{n}\right)+a\right)^{2}\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^{n}\right)\right)}^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x,x)
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2)))/x,x)
[Out] Timed out
```

3.57 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^2}{x^2}\,dx$

Optimal. Leaf size=389

$$4bd^2f^2n\text{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{d^2f^2\left(a+b\log\left(cx^n\right)\right)^3}{6bn}+d^2f^2\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)^2+2bd^2f^2$$

```
[Out] -b^2*d^2*f^2*n^2*ln(x)+1/2*b^2*d^2*f^2*n^2*ln(x)^2-b*d^2*f^2*n*ln(x)*(a+b*l
n(c*x^n))-1/6*d^2*f^2*(a+b*ln(c*x^n))^3/b/n+2*b^2*d^2*f^2*n^2*ln(1+d*f*x^(1
/2))-2*b^2*n^2*ln(1+d*f*x^(1/2))/x+2*b*d^2*f^2*n*(a+b*ln(c*x^n))*ln(1+d*f*x
^(1/2))-2*b*n*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/x+d^2*f^2*(a+b*ln(c*x^n))^2
*ln(1+d*f*x^(1/2))-(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/x+4*b^2*d^2*f^2*n^2*
polylog(2,-d*f*x^(1/2))+4*b*d^2*f^2*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2
))-8*b^2*d^2*f^2*n^2*polylog(3,-d*f*x^(1/2))-14*b^2*d*f*n^2/x^(1/2)-6*b*d*f
*n*(a+b*ln(c*x^n))/x^(1/2)-d*f*(a+b*ln(c*x^n))^2/x^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2454, 2395, 44, 2377, 2304, 2376, 2391, 2301, 2374, 6589, 2366, 12, 2302, 30}

$$4bd^2f^2n\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)+4b^2d^2f^2n^2\text{PolyLog}\left(2,-df\sqrt{x}\right)-8b^2d^2f^2n^2\text{PolyLog}\left(3,-df\sqrt{x}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

```
[Out] (-14*b^2*d*f*n^2)/Sqrt[x] + 2*b^2*d^2*f^2*n^2*Log[1 + d*f*Sqrt[x]] - (2*b^2
*n^2*Log[1 + d*f*Sqrt[x]])/x - b^2*d^2*f^2*n^2*Log[x] + (b^2*d^2*f^2*n^2*Lo
g[x]^2)/2 - (6*b*d*f*n*(a + b*Log[c*x^n]))/Sqrt[x] + 2*b*d^2*f^2*n*Log[1 +
d*f*Sqrt[x]]*(a + b*Log[c*x^n]) - (2*b*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*
x^n]))/x - b*d^2*f^2*n*Log[x]*(a + b*Log[c*x^n]) - (d*f*(a + b*Log[c*x^n])^
2)/Sqrt[x] + d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2 - (Log[1 + d
*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/x - (d^2*f^2*(a + b*Log[c*x^n])^3)/(6*b*n
) + 4*b^2*d^2*f^2*n^2*PolyLog[2, -(d*f*Sqrt[x])] + 4*b*d^2*f^2*n*(a + b*Log
[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] - 8*b^2*d^2*f^2*n^2*PolyLog[3, -(d*f*Sq
rt[x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx = -\frac{df\left(a+b\log\left(cx^n\right)\right)^2}{\sqrt{x}} + d^2f^2\log\left(1+df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2 -$$

$$= -\frac{df\left(a+b\log\left(cx^n\right)\right)^2}{\sqrt{x}} + d^2f^2\log\left(1+df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2 -$$

$$= -\frac{8b^2dfn^2}{\sqrt{x}} - \frac{6bdfn\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + 2bd^2f^2n\log\left(1+df\sqrt{x}\right)($$

$$= -\frac{12b^2dfn^2}{\sqrt{x}} - \frac{6bdfn\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + 2bd^2f^2n\log\left(1+df\sqrt{x}\right)($$

$$= -\frac{12b^2dfn^2}{\sqrt{x}} + \frac{1}{2}b^2d^2f^2n^2\log^2(x) - \frac{6bdfn\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + 2bd^2$$

$$= -\frac{12b^2dfn^2}{\sqrt{x}} - \frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{x} + \frac{1}{2}b^2d^2f^2n^2\log^2(x) - \frac{6bdf}{}$$

$$= -\frac{12b^2dfn^2}{\sqrt{x}} - \frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{x} + \frac{1}{2}b^2d^2f^2n^2\log^2(x) - \frac{6bdf}{}$$

$$= -\frac{14b^2dfn^2}{\sqrt{x}} + 2b^2d^2f^2n^2\log\left(1+df\sqrt{x}\right) - \frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{x}$$

Mathematica [A] time = 0.41, size = 627, normalized size = 1.61

$$-\frac{3a^2d^2f^2x\log(x) - 6a^2d^2f^2x\log\left(df\sqrt{x}+1\right) + 6a^2df\sqrt{x} + 6a^2\log\left(df\sqrt{x}+1\right) - 24bd^2f^2nx\text{Li}_2\left(-df\sqrt{x}\right)(a+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

```
[Out] -1/6*(6*a^2*d*f*Sqrt[x] + 36*a*b*d*f*n*Sqrt[x] + 84*b^2*d*f*n^2*Sqrt[x] + 6
*a^2*Log[1 + d*f*Sqrt[x]] + 12*a*b*n*Log[1 + d*f*Sqrt[x]] + 12*b^2*n^2*Log[
1 + d*f*Sqrt[x]] - 6*a^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]] - 12*a*b*d^2*f^2*n*
x*Log[1 + d*f*Sqrt[x]] - 12*b^2*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt[x]] + 3*a^2*
```

```
d^2*f^2*x*Log[x] + 6*a*b*d^2*f^2*n*x*Log[x] + 6*b^2*d^2*f^2*n^2*x*Log[x] -
3*a*b*d^2*f^2*n*x*Log[x]^2 - 3*b^2*d^2*f^2*n^2*x*Log[x]^2 + b^2*d^2*f^2*n^2
*x*Log[x]^3 + 12*a*b*d*f*Sqrt[x]*Log[c*x^n] + 36*b^2*d*f*n*Sqrt[x]*Log[c*x^
n] + 12*a*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 12*b^2*n*Log[1 + d*f*Sqrt[x]]
*Log[c*x^n] - 12*a*b*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 12*b^2*d^2
*f^2*n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 6*a*b*d^2*f^2*x*Log[x]*Log[c*x^n
] + 6*b^2*d^2*f^2*n*x*Log[x]*Log[c*x^n] - 3*b^2*d^2*f^2*n*x*Log[x]^2*Log[c*
x^n] + 6*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 + 6*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^
n]^2 - 6*b^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 3*b^2*d^2*f^2*x*
Log[x]*Log[c*x^n]^2 - 24*b*d^2*f^2*n*x*(a + b*n + b*Log[c*x^n])*PolyLog[2,
-(d*f*Sqrt[x])] + 48*b^2*d^2*f^2*n^2*x*PolyLog[3, -(d*f*Sqrt[x])])/x
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\log\left(cx^n\right)^2+2\,ab\log\left(cx^n\right)+a^2\right)\log\left(df\sqrt{x}+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="frica
s")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1)/x
^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^n\right)+a\right)^2\ln\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^2,x)
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2)))/x**2,x)
[Out] Timed out
```

3.58 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^2}{x^3}\,dx$

Optimal. Leaf size=555

$$2bd^4f^4n\mathrm{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{d^4f^4\left(a+b\log\left(cx^n\right)\right)^3}{12bn}+\frac{1}{2}d^4f^4\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)^2+\frac{1}{2}$$

```
[Out] -37/108*b^2*d*f*n^2/x^(3/2)+7/8*b^2*d^2*f^2*n^2/x-1/8*b^2*d^4*f^4*n^2*ln(x)
+1/8*b^2*d^4*f^4*n^2*ln(x)^2-7/18*b*d*f*n*(a+b*ln(c*x^n))/x^(3/2)+3/4*b*d^2
*f^2*n*(a+b*ln(c*x^n))/x-1/4*b*d^4*f^4*n*ln(x)*(a+b*ln(c*x^n))-1/6*d*f*(a+b
*ln(c*x^n))^2/x^(3/2)+1/4*d^2*f^2*(a+b*ln(c*x^n))^2/x-1/12*d^4*f^4*(a+b*ln(
c*x^n))^3/b/n+1/4*b^2*d^4*f^4*n^2*ln(1+d*f*x^(1/2))-1/4*b^2*n^2*ln(1+d*f*x^
(1/2))/x^2+1/2*b*d^4*f^4*n*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-1/2*b*n*(a+b*l
n(c*x^n))*ln(1+d*f*x^(1/2))/x^2+1/2*d^4*f^4*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1
/2))-1/2*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/x^2+b^2*d^4*f^4*n^2*polylog(2,
-d*f*x^(1/2))+2*b*d^4*f^4*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))-4*b^2*d
^4*f^4*n^2*polylog(3,-d*f*x^(1/2))-21/4*b^2*d^3*f^3*n^2/x^(1/2)-5/2*b*d^3*f
^3*n*(a+b*ln(c*x^n))/x^(1/2)-1/2*d^3*f^3*(a+b*ln(c*x^n))^2/x^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2454, 2395, 44, 2377, 2304, 2376, 2391, 2301, 2374, 6589, 2366, 12, 2302, 30}

$$2bd^4f^4n\,\mathrm{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)+b^2d^4f^4n^2\,\mathrm{PolyLog}\left(2,-df\sqrt{x}\right)-4b^2d^4f^4n^2\,\mathrm{PolyLog}\left(3,-df\sqrt{\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] (-37*b^2*d*f*n^2)/(108*x^(3/2)) + (7*b^2*d^2*f^2*n^2)/(8*x) - (21*b^2*d^3*f
^3*n^2)/(4*Sqrt[x]) + (b^2*d^4*f^4*n^2*Log[1 + d*f*Sqrt[x]])/4 - (b^2*n^2*L
og[1 + d*f*Sqrt[x]])/(4*x^2) - (b^2*d^4*f^4*n^2*Log[x])/8 + (b^2*d^4*f^4*n^
2*Log[x]^2)/8 - (7*b*d*f*n*(a + b*Log[c*x^n]))/(18*x^(3/2)) + (3*b*d^2*f^2*
n*(a + b*Log[c*x^n]))/(4*x) - (5*b*d^3*f^3*n*(a + b*Log[c*x^n]))/(2*Sqrt[x]
) + (b*d^4*f^4*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/2 - (b*n*Log[1 +
d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*x^2) - (b*d^4*f^4*n*Log[x]*(a + b*Log[c
*x^n]))/4 - (d*f*(a + b*Log[c*x^n])^2)/(6*x^(3/2)) + (d^2*f^2*(a + b*Log[c*
x^n])^2)/(4*x) - (d^3*f^3*(a + b*Log[c*x^n])^2)/(2*Sqrt[x]) + (d^4*f^4*Log[
1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log
[c*x^n])^2)/(2*x^2) - (d^4*f^4*(a + b*Log[c*x^n])^3)/(12*b*n) + b^2*d^4*f^4
*n^2*PolyLog[2, -(d*f*Sqrt[x])] + 2*b*d^4*f^4*n*(a + b*Log[c*x^n])*PolyLog[
2, -(d*f*Sqrt[x])] - 4*b^2*d^4*f^4*n^2*PolyLog[3, -(d*f*Sqrt[x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx = -\frac{df\left(a+b\log\left(cx^n\right)\right)^2}{6x^{3/2}}+\frac{d^2f^2\left(a+b\log\left(cx^n\right)\right)^2}{4x}-\frac{d^3f^3\left(a+b\mathrm{l}\right.}{2\sqrt{\cdot}}$$

$$= -\frac{df\left(a+b\log\left(cx^n\right)\right)^2}{6x^{3/2}}+\frac{d^2f^2\left(a+b\log\left(cx^n\right)\right)^2}{4x}-\frac{d^3f^3\left(a+b\mathrm{l}\right.}{2\sqrt{\cdot}}$$

$$= -\frac{4b^2dfn^2}{27x^{3/2}}+\frac{b^2d^2f^2n^2}{2x}-\frac{4b^2d^3f^3n^2}{\sqrt{x}}-\frac{7bdfn\left(a+b\log\left(cx^n\right)\right)}{18x^{3/2}}$$

$$= -\frac{7b^2dfn^2}{27x^{3/2}}+\frac{3b^2d^2f^2n^2}{4x}-\frac{5b^2d^3f^3n^2}{\sqrt{x}}-\frac{7bdfn\left(a+b\log\left(cx^n\right)\right)}{18x^{3/2}}$$

$$= -\frac{7b^2dfn^2}{27x^{3/2}}+\frac{3b^2d^2f^2n^2}{4x}-\frac{5b^2d^3f^3n^2}{\sqrt{x}}+\frac{1}{8}b^2d^4f^4n^2\log^2(x)-\frac{7}{}$$

$$= -\frac{7b^2dfn^2}{27x^{3/2}}+\frac{3b^2d^2f^2n^2}{4x}-\frac{5b^2d^3f^3n^2}{\sqrt{x}}-\frac{b^2n^2\log\left(1+df\sqrt{x}\right)}{4x^2}+$$

$$= -\frac{7b^2dfn^2}{27x^{3/2}}+\frac{3b^2d^2f^2n^2}{4x}-\frac{5b^2d^3f^3n^2}{\sqrt{x}}-\frac{b^2n^2\log\left(1+df\sqrt{x}\right)}{4x^2}+$$

$$= -\frac{37b^2dfn^2}{108x^{3/2}}+\frac{7b^2d^2f^2n^2}{8x}-\frac{21b^2d^3f^3n^2}{4\sqrt{x}}+\frac{1}{4}b^2d^4f^4n^2\log\left(1+\right.$$

Mathematica [A] time = 0.54, size = 881, normalized size = 1.59

$$-\frac{18b^2d^4n^2x^2\log^3(x)f^4-27b^2d^4n^2x^2\log^2(x)f^4-54abd^4nx^2\log^2(x)f^4-108b^2d^4x^2\log\left(d\sqrt{x}f+1\right)\log^2\left(cx^n\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] -1/216*(36*a^2*d*f*Sqrt[x] + 84*a*b*d*f*n*Sqrt[x] + 74*b^2*d*f*n^2*Sqrt[x]
- 54*a^2*d^2*f^2*x - 162*a*b*d^2*f^2*n*x - 189*b^2*d^2*f^2*n^2*x + 108*a^2*
d^3*f^3*x^(3/2) + 540*a*b*d^3*f^3*n*x^(3/2) + 1134*b^2*d^3*f^3*n^2*x^(3/2)
+ 108*a^2*Log[1 + d*f*Sqrt[x]] + 108*a*b*n*Log[1 + d*f*Sqrt[x]] + 54*b^2*n^
2*Log[1 + d*f*Sqrt[x]] - 108*a^2*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]] - 108*a*b
*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]] - 54*b^2*d^4*f^4*n^2*x^2*Log[1 + d*f*Sq
rt[x]] + 54*a^2*d^4*f^4*x^2*Log[x] + 54*a*b*d^4*f^4*n*x^2*Log[x] + 27*b^2*d
^4*f^4*n^2*x^2*Log[x] - 54*a*b*d^4*f^4*n*x^2*Log[x]^2 - 27*b^2*d^4*f^4*n^2*
x^2*Log[x]^2 + 18*b^2*d^4*f^4*n^2*x^2*Log[x]^3 + 72*a*b*d*f*Sqrt[x]*Log[c*x
^n] + 84*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 108*a*b*d^2*f^2*x*Log[c*x^n] - 162*
b^2*d^2*f^2*n*x*Log[c*x^n] + 216*a*b*d^3*f^3*x^(3/2)*Log[c*x^n] + 540*b^2*d
^3*f^3*n*x^(3/2)*Log[c*x^n] + 216*a*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 108
*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 216*a*b*d^4*f^4*x^2*Log[1 + d*f*Sq
rt[x]]*Log[c*x^n] - 108*b^2*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] +
 108*a*b*d^4*f^4*x^2*Log[x]*Log[c*x^n] + 54*b^2*d^4*f^4*n*x^2*Log[x]*Log[c*
x^n] - 54*b^2*d^4*f^4*n*x^2*Log[x]^2*Log[c*x^n] + 36*b^2*d*f*Sqrt[x]*Log[c*
x^n]^2 - 54*b^2*d^2*f^2*x*Log[c*x^n]^2 + 108*b^2*d^3*f^3*x^(3/2)*Log[c*x^n]
^2 + 108*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 108*b^2*d^4*f^4*x^2*Log[1
+ d*f*Sqrt[x]]*Log[c*x^n]^2 + 54*b^2*d^4*f^4*x^2*Log[x]*Log[c*x^n]^2 - 216*
b*d^4*f^4*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] + 8
64*b^2*d^4*f^4*n^2*x^2*PolyLog[3, -(d*f*Sqrt[x])])/x^2
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2 \log\left(cx^n\right)^2 + 2\,ab \log\left(cx^n\right) + a^2\right) \log\left(df\sqrt{x} + 1\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="frica
s")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1)/x
^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^n\right) + a\right)^2 \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^n\right) + a\right)^2 \ln\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+1/d)*d)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^n\right) + a\right)^2 \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2)))/x**3,x)
```

```
[Out] Timed out
```

3.59 $\int x \log \left(d \left(\frac{1}{d}+f \sqrt{x}\right)\right) \left(a+b \log \left(c x^n\right)\right)^3 \, dx$

Optimal. Leaf size=858

$$\frac{3}{8} n^3 x^2 b^3 - \frac{175 n^3 x^{3/2} b^3}{216 d f} + \frac{45 n^3 x b^3}{16 d^2 f^2} + \frac{3 n^3 \log\left(d\sqrt{x} f+1\right) b^3}{8 d^4 f^4} - \frac{3}{8} n^3 x^2 \log\left(d\sqrt{x} f+1\right) b^3 - \frac{9 n^2 x \log\left(c x^n\right) b^3}{4 d^2 f^2} - \frac{3 n^3 \mathrm{Li}_2 (}{2}$$

```
[Out] -9/16*b^2*n^2*x^2*(a+b*ln(c*x^n))+3/8*b*n*x^2*(a+b*ln(c*x^n))^2-1/8*x^2*(a+
b*ln(c*x^n))^3+3/8*b^3*n^3*x^2+1/2*x^2*(a+b*ln(c*x^n))^3*ln(1+d*f*x^(1/2))-
9/4*a*b^2*n^2*x/d^2/f^2-1/4*x*(a+b*ln(c*x^n))^3/d^2/f^2+1/6*x^(3/2)*(a+b*ln
(c*x^n))^3/d/f-3/8*b^3*n^3*x^2*ln(1+d*f*x^(1/2))-1/2*(a+b*ln(c*x^n))^3*ln(1
+d*f*x^(1/2))/d^4/f^4+1/2*(a+b*ln(c*x^n))^3*x^(1/2)/d^3/f^3+45/16*b^3*n^3*x
/d^2/f^2-175/216*b^3*n^3*x^(3/2)/d/f+3/8*b^3*n^3*ln(1+d*f*x^(1/2))/d^4/f^4+
3/4*b^2*n^2*x^2*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-3/4*b*n*x^2*(a+b*ln(c*x^n
))^2*ln(1+d*f*x^(1/2))-3/2*b^3*n^3*polylog(2,-d*f*x^(1/2))/d^4/f^4-6*b^3*n^
3*polylog(3,-d*f*x^(1/2))/d^4/f^4-24*b^3*n^3*polylog(4,-d*f*x^(1/2))/d^4/f^
4-255/8*b^3*n^3*x^(1/2)/d^3/f^3-9/4*b^3*n^2*x*ln(c*x^n)/d^2/f^2-3/8*b^2*n^2
*x*(a+b*ln(c*x^n))/d^2/f^2+37/36*b^2*n^2*x^(3/2)*(a+b*ln(c*x^n))/d/f+9/8*b*
n*x*(a+b*ln(c*x^n))^2/d^2/f^2-7/12*b*n*x^(3/2)*(a+b*ln(c*x^n))^2/d/f-3/4*b^
2*n^2*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/d^4/f^4+3/4*b*n*(a+b*ln(c*x^n))^2*l
n(1+d*f*x^(1/2))/d^4/f^4+3*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/
d^4/f^4-3*b*n*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^(1/2))/d^4/f^4+12*b^2*n^2*
(a+b*ln(c*x^n))*polylog(3,-d*f*x^(1/2))/d^4/f^4+63/4*b^2*n^2*(a+b*ln(c*x^n)
)*x^(1/2)/d^3/f^3-15/4*b*n*(a+b*ln(c*x^n))^2*x^(1/2)/d^3/f^3
```

Rubi [A] time = 0.94, antiderivative size = 858, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {2454, 2395, 43, 2377, 2296, 2295, 2305, 2304, 2374, 2383, 6589, 2376, 2391}

$$\frac{3}{8} n^3 x^2 b^3 - \frac{175 n^3 x^{3/2} b^3}{216 d f} + \frac{45 n^3 x b^3}{16 d^2 f^2} + \frac{3 n^3 \log\left(d\sqrt{x} f+1\right) b^3}{8 d^4 f^4} - \frac{3}{8} n^3 x^2 \log\left(d\sqrt{x} f+1\right) b^3 - \frac{9 n^2 x \log\left(c x^n\right) b^3}{4 d^2 f^2} - \frac{3 n^3 \mathrm{Pol}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-255*b^3*n^3*Sqrt[x])/(8*d^3*f^3) - (9*a*b^2*n^2*x)/(4*d^2*f^2) + (45*b^3*
n^3*x)/(16*d^2*f^2) - (175*b^3*n^3*x^(3/2))/(216*d*f) + (3*b^3*n^3*x^2)/8 +
 (3*b^3*n^3*Log[1 + d*f*Sqrt[x]])/(8*d^4*f^4) - (3*b^3*n^3*x^2*Log[1 + d*f*
Sqrt[x]])/8 - (9*b^3*n^2*x*Log[c*x^n])/(4*d^2*f^2) + (63*b^2*n^2*Sqrt[x]*(a
 + b*Log[c*x^n]))/(4*d^3*f^3) - (3*b^2*n^2*x*(a + b*Log[c*x^n]))/(8*d^2*f^2
) + (37*b^2*n^2*x^(3/2)*(a + b*Log[c*x^n]))/(36*d*f) - (9*b^2*n^2*x^2*(a +
b*Log[c*x^n]))/16 - (3*b^2*n^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(4*
d^4*f^4) + (3*b^2*n^2*x^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/4 - (15*
b*n*Sqrt[x]*(a + b*Log[c*x^n])^2)/(4*d^3*f^3) + (9*b*n*x*(a + b*Log[c*x^n])
^2)/(8*d^2*f^2) - (7*b*n*x^(3/2)*(a + b*Log[c*x^n])^2)/(12*d*f) + (3*b*n*x^
2*(a + b*Log[c*x^n])^2)/8 + (3*b*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^
2)/(4*d^4*f^4) - (3*b*n*x^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/4 +
(Sqrt[x]*(a + b*Log[c*x^n])^3)/(2*d^3*f^3) - (x*(a + b*Log[c*x^n])^3)/(4*d^
2*f^2) + (x^(3/2)*(a + b*Log[c*x^n])^3)/(6*d*f) - (x^2*(a + b*Log[c*x^n])^3
)/8 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(2*d^4*f^4) + (x^2*Log[1
+ d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/2 - (3*b^3*n^3*PolyLog[2, -(d*f*Sqrt[x
])])/(2*d^4*f^4) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])]
)/(d^4*f^4) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])])/(d^4*
f^4) - (6*b^3*n^3*PolyLog[3, -(d*f*Sqrt[x])])/(d^4*f^4) + (12*b^2*n^2*(a +
b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[x])])/(d^4*f^4) - (24*b^3*n^3*PolyLog[4
, -(d*f*Sqrt[x])])/(d^4*f^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3\,dx = \frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^3}{2d^3f^3} - \frac{x\left(a+b\log\left(cx^n\right)\right)^3}{4d^2f^2} + \frac{x^{3/2}\left(a+b\log\right.}{6df}$$

$$= \frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^3}{2d^3f^3} - \frac{x\left(a+b\log\left(cx^n\right)\right)^3}{4d^2f^2} + \frac{x^{3/2}\left(a+b\log\right.}{6df}$$

$$= -\frac{15bn\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{4d^3f^3} + \frac{9bnx\left(a+b\log\left(cx^n\right)\right)^2}{8d^2f^2} - \frac{7bn}{}$$

$$= -\frac{24b^3n^3\sqrt{x}}{d^3f^3} - \frac{3ab^2n^2x}{2d^2f^2} - \frac{8b^3n^3x^{3/2}}{27df} + \frac{3}{32}b^3n^3x^2 + \frac{12b^2n^2\sqrt{}}{}$$

$$= -\frac{30b^3n^3\sqrt{x}}{d^3f^3} - \frac{9ab^2n^2x}{4d^2f^2} + \frac{3b^3n^3x}{2d^2f^2} - \frac{14b^3n^3x^{3/2}}{27df} + \frac{3}{16}b^3n^3x^2$$

$$= -\frac{63b^3n^3\sqrt{x}}{2d^3f^3} - \frac{9ab^2n^2x}{4d^2f^2} + \frac{21b^3n^3x}{8d^2f^2} - \frac{37b^3n^3x^{3/2}}{54df} + \frac{9}{32}b^3n^3x^2$$

$$= -\frac{63b^3n^3\sqrt{x}}{2d^3f^3} - \frac{9ab^2n^2x}{4d^2f^2} + \frac{21b^3n^3x}{8d^2f^2} - \frac{37b^3n^3x^{3/2}}{54df} + \frac{9}{32}b^3n^3x^2$$

$$= -\frac{63b^3n^3\sqrt{x}}{2d^3f^3} - \frac{9ab^2n^2x}{4d^2f^2} + \frac{21b^3n^3x}{8d^2f^2} - \frac{37b^3n^3x^{3/2}}{54df} + \frac{9}{32}b^3n^3x^2$$

$$= -\frac{63b^3n^3\sqrt{x}}{2d^3f^3} - \frac{9ab^2n^2x}{4d^2f^2} + \frac{21b^3n^3x}{8d^2f^2} - \frac{37b^3n^3x^{3/2}}{54df} + \frac{9}{32}b^3n^3x^2$$

$$= -\frac{255b^3n^3\sqrt{x}}{8d^3f^3} - \frac{9ab^2n^2x}{4d^2f^2} + \frac{45b^3n^3x}{16d^2f^2} - \frac{175b^3n^3x^{3/2}}{216df} + \frac{3}{8}b^3n^3x$$

Mathematica [A] time = 0.64, size = 1432, normalized size = 1.67

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (216*a^3*d*f*Sqrt[x] - 1620*a^2*b*d*f*n*Sqrt[x] + 6804*a*b^2*d*f*n^2*Sqrt[x
] - 13770*b^3*d*f*n^3*Sqrt[x] - 108*a^3*d^2*f^2*x + 486*a^2*b*d^2*f^2*n*x -
 1134*a*b^2*d^2*f^2*n^2*x + 1215*b^3*d^2*f^2*n^3*x + 72*a^3*d^3*f^3*x^(3/2)
 - 252*a^2*b*d^3*f^3*n*x^(3/2) + 444*a*b^2*d^3*f^3*n^2*x^(3/2) - 350*b^3*d^
3*f^3*n^3*x^(3/2) - 54*a^3*d^4*f^4*x^2 + 162*a^2*b*d^4*f^4*n*x^2 - 243*a*b^
2*d^4*f^4*n^2*x^2 + 162*b^3*d^4*f^4*n^3*x^2 - 216*a^3*Log[1 + d*f*Sqrt[x]]
+ 324*a^2*b*n*Log[1 + d*f*Sqrt[x]] - 324*a*b^2*n^2*Log[1 + d*f*Sqrt[x]] + 1
62*b^3*n^3*Log[1 + d*f*Sqrt[x]] + 216*a^3*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]
- 324*a^2*b*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]] + 324*a*b^2*d^4*f^4*n^2*x^2*
Log[1 + d*f*Sqrt[x]] - 162*b^3*d^4*f^4*n^3*x^2*Log[1 + d*f*Sqrt[x]] + 648*a
^2*b*d*f*Sqrt[x]*Log[c*x^n] - 3240*a*b^2*d*f*n*Sqrt[x]*Log[c*x^n] + 6804*b^
3*d*f*n^2*Sqrt[x]*Log[c*x^n] - 324*a^2*b*d^2*f^2*x*Log[c*x^n] + 972*a*b^2*d
^2*f^2*n*x*Log[c*x^n] - 1134*b^3*d^2*f^2*n^2*x*Log[c*x^n] + 216*a^2*b*d^3*f
^3*x^(3/2)*Log[c*x^n] - 504*a*b^2*d^3*f^3*n*x^(3/2)*Log[c*x^n] + 444*b^3*d^
3*f^3*n^2*x^(3/2)*Log[c*x^n] - 162*a^2*b*d^4*f^4*x^2*Log[c*x^n] + 324*a*b^2
*d^4*f^4*n*x^2*Log[c*x^n] - 243*b^3*d^4*f^4*n^2*x^2*Log[c*x^n] - 648*a^2*b*
Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 648*a*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^
```

```
n] - 324*b^3*n^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 648*a^2*b*d^4*f^4*x^2*Lo
g[1 + d*f*Sqrt[x]]*Log[c*x^n] - 648*a*b^2*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]
]*Log[c*x^n] + 324*b^3*d^4*f^4*n^2*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 64
8*a*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 - 1620*b^3*d*f*n*Sqrt[x]*Log[c*x^n]^2 - 32
4*a*b^2*d^2*f^2*x*Log[c*x^n]^2 + 486*b^3*d^2*f^2*n*x*Log[c*x^n]^2 + 216*a*b
^2*d^3*f^3*x^(3/2)*Log[c*x^n]^2 - 252*b^3*d^3*f^3*n*x^(3/2)*Log[c*x^n]^2 -
162*a*b^2*d^4*f^4*x^2*Log[c*x^n]^2 + 162*b^3*d^4*f^4*n*x^2*Log[c*x^n]^2 - 6
48*a*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 324*b^3*n*Log[1 + d*f*Sqrt[x]]
*Log[c*x^n]^2 + 648*a*b^2*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 3
24*b^3*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 216*b^3*d*f*Sqrt[x
]*Log[c*x^n]^3 - 108*b^3*d^2*f^2*x*Log[c*x^n]^3 + 72*b^3*d^3*f^3*x^(3/2)*Lo
g[c*x^n]^3 - 54*b^3*d^4*f^4*x^2*Log[c*x^n]^3 - 216*b^3*Log[1 + d*f*Sqrt[x]]
*Log[c*x^n]^3 + 216*b^3*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^3 - 648
*b*n*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c
*x^n]^2)*PolyLog[2, -(d*f*Sqrt[x])] + 2592*b^2*n^2*(2*a - b*n + 2*b*Log[c*x
^n])*PolyLog[3, -(d*f*Sqrt[x])] - 10368*b^3*n^3*PolyLog[4, -(d*f*Sqrt[x])])
/(432*d^4*f^4)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^3 x \log\left(cx^n\right)^3 + 3\,ab^2 x \log\left(cx^n\right)^2 + 3\,a^2 bx \log\left(cx^n\right) + a^3 x\right) \log\left(df\sqrt{x} + 1\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas"
)
```

```
[Out] integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^n
) + a^3*x)*log(d*f*sqrt(x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^n\right) + a\right)^3 x \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + 1/d)*d), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\,x^n\right) + a\right)^3 x \ln\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+1/d)*d),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^n\right) + a\right)^3 x \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + 1/d)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3,x)
[Out] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**(1/2))),x)
[Out] Timed out
```

3.60 $\int \log \left(d \left(\frac{1}{d}+f \sqrt{x}\right)\right) \left(a+b \log \left(c x^n\right)\right)^3 \, dx$

Optimal. Leaf size=604

$$\frac{12b^2n^2\text{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}+\frac{24b^2n^2\text{Li}_3\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}-\frac{6b^2n^2\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(\right.\right.}{d^2f^2}$$

```
[Out] -6*a*b^2*n^2*x+12*b^3*n^3*x-6*b^3*n^2*x*ln(c*x^n)-3*b^2*n^2*x*(a+b*ln(c*x^n
))+3*b*n*x*(a+b*ln(c*x^n))^2-1/2*x*(a+b*ln(c*x^n))^3-6*b^3*n^3*x*ln(d*(1/d+
f*x^(1/2)))+6*b^2*n^2*x*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2)))-3*b*n*x*(a+b*
ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2)))+x*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2))
)+6*b^3*n^3*ln(1+d*f*x^(1/2))/d^2/f^2-6*b^2*n^2*(a+b*ln(c*x^n))*ln(1+d*f*x^
(1/2))/d^2/f^2+3*b*n*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/d^2/f^2-(a+b*ln(c*
x^n))^3*ln(1+d*f*x^(1/2))/d^2/f^2-12*b^3*n^3*polylog(2,-d*f*x^(1/2))/d^2/f^
2+12*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^2/f^2-6*b*n*(a+b*ln(
c*x^n))^2*polylog(2,-d*f*x^(1/2))/d^2/f^2-24*b^3*n^3*polylog(3,-d*f*x^(1/2)
)/d^2/f^2+24*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^(1/2))/d^2/f^2-48*b^3
*n^3*polylog(4,-d*f*x^(1/2))/d^2/f^2-90*b^3*n^3*x^(1/2)/d/f+42*b^2*n^2*(a+b
*ln(c*x^n))*x^(1/2)/d/f-9*b*n*(a+b*ln(c*x^n))^2*x^(1/2)/d/f+(a+b*ln(c*x^n))
^3*x^(1/2)/d/f
```

Rubi [A] time = 0.52, antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2448, 266, 43, 2370, 2296, 2295, 2305, 2304, 2391, 2374, 6589, 2383}

$$\frac{12b^2n^2\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}+\frac{24b^2n^2\text{PolyLog}\left(3,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}-\frac{6bn\text{PolyLog}\left(2,-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-90*b^3*n^3*Sqrt[x])/(d*f) - 6*a*b^2*n^2*x + 12*b^3*n^3*x - 6*b^3*n^3*x*Lo
g[d*(d^(-1) + f*Sqrt[x])] + (6*b^3*n^3*Log[1 + d*f*Sqrt[x]])/(d^2*f^2) - 6*
b^3*n^2*x*Log[c*x^n] + (42*b^2*n^2*Sqrt[x]*(a + b*Log[c*x^n]))/(d*f) - 3*b^
2*n^2*x*(a + b*Log[c*x^n]) + 6*b^2*n^2*x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b
*Log[c*x^n]) - (6*b^2*n^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(d^2*f^2
) - (9*b*n*Sqrt[x]*(a + b*Log[c*x^n])^2)/(d*f) + 3*b*n*x*(a + b*Log[c*x^n])
^2 - 3*b*n*x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2 + (3*b*n*Log[
1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(d^2*f^2) + (Sqrt[x]*(a + b*Log[c*x^
n])^3)/(d*f) - (x*(a + b*Log[c*x^n])^3)/2 + x*Log[d*(d^(-1) + f*Sqrt[x])]*(
a + b*Log[c*x^n])^3 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(d^2*f^2)
 - (12*b^3*n^3*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2) + (12*b^2*n^2*(a + b*L
og[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2) - (6*b*n*(a + b*Log[c*x^n]
)^2*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2) - (24*b^3*n^3*PolyLog[3, -(d*f*Sq
rt[x])])/(d^2*f^2) + (24*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[x
])])/(d^2*f^2) - (48*b^3*n^3*PolyLog[4, -(d*f*Sqrt[x])])/(d^2*f^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3\,dx = \frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^3}{df}-\frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)^3+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right.\right.$$

$$=\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^3}{df}-\frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)^3+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right.\right.$$

$$=-\frac{9bn\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{df}+3bnx\left(a+b\log\left(cx^n\right)\right)^2-3bnx\log\left(\right.$$

$$=-\frac{48b^3n^3\sqrt{x}}{df}-3ab^2n^2x+\frac{24b^2n^2\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{df}-\frac{9bn\sqrt{x}\left(\right.}{}$$

$$=-\frac{72b^3n^3\sqrt{x}}{df}-6ab^2n^2x+3b^3n^3x-3b^3n^2x\log\left(cx^n\right)+\frac{42b^2n^2\sqrt{x}}{}$$

$$=-\frac{84b^3n^3\sqrt{x}}{df}-6ab^2n^2x+9b^3n^3x-6b^3n^2x\log\left(cx^n\right)+\frac{42b^2n^2\sqrt{x}}{}$$

$$=-\frac{84b^3n^3\sqrt{x}}{df}-6ab^2n^2x+9b^3n^3x-6b^3n^3x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)-6$$

$$=-\frac{84b^3n^3\sqrt{x}}{df}-6ab^2n^2x+9b^3n^3x-6b^3n^3x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)-6$$

$$=-\frac{84b^3n^3\sqrt{x}}{df}-6ab^2n^2x+9b^3n^3x-6b^3n^3x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)-6$$

$$=-\frac{90b^3n^3\sqrt{x}}{df}-6ab^2n^2x+12b^3n^3x-6b^3n^3x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)+$$

Mathematica [A] time = 0.51, size = 986, normalized size = 1.63

$$-\frac{d^2f^2xa^3-2d^2f^2x\log\left(d\sqrt{x}f+1\right)a^3+2\log\left(d\sqrt{x}f+1\right)a^3-2df\sqrt{x}a^3-6bd^2f^2nxa^2-6bn\log\left(d\sqrt{x}f+1\right)a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] -1/2*(-2*a^3*d*f*Sqrt[x] + 18*a^2*b*d*f*n*Sqrt[x] - 84*a*b^2*d*f*n^2*Sqrt[x
] + 180*b^3*d*f*n^3*Sqrt[x] + a^3*d^2*f^2*x - 6*a^2*b*d^2*f^2*n*x + 18*a*b^
2*d^2*f^2*n^2*x - 24*b^3*d^2*f^2*n^3*x + 2*a^3*Log[1 + d*f*Sqrt[x]] - 6*a^2
*b*n*Log[1 + d*f*Sqrt[x]] + 12*a*b^2*n^2*Log[1 + d*f*Sqrt[x]] - 12*b^3*n^3*
Log[1 + d*f*Sqrt[x]] - 2*a^3*d^2*f^2*x*Log[1 + d*f*Sqrt[x]] + 6*a^2*b*d^2*f
```

```
^2*n*x*Log[1 + d*f*Sqrt[x]] - 12*a*b^2*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt[x]] +
 12*b^3*d^2*f^2*n^3*x*Log[1 + d*f*Sqrt[x]] - 6*a^2*b*d*f*Sqrt[x]*Log[c*x^n]
 + 36*a*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 84*b^3*d*f*n^2*Sqrt[x]*Log[c*x^n] +
3*a^2*b*d^2*f^2*x*Log[c*x^n] - 12*a*b^2*d^2*f^2*n*x*Log[c*x^n] + 18*b^3*d^2
*f^2*n^2*x*Log[c*x^n] + 6*a^2*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 12*a*b^2*
n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 12*b^3*n^2*Log[1 + d*f*Sqrt[x]]*Log[c*x
^n] - 6*a^2*b*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 12*a*b^2*d^2*f^2*
n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 12*b^3*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt
[x]]*Log[c*x^n] - 6*a*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 + 18*b^3*d*f*n*Sqrt[x]*L
og[c*x^n]^2 + 3*a*b^2*d^2*f^2*x*Log[c*x^n]^2 - 6*b^3*d^2*f^2*n*x*Log[c*x^n]
^2 + 6*a*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 6*b^3*n*Log[1 + d*f*Sqrt[x
]]*Log[c*x^n]^2 - 6*a*b^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 6*b
^3*d^2*f^2*n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 2*b^3*d*f*Sqrt[x]*Log[c*
x^n]^3 + b^3*d^2*f^2*x*Log[c*x^n]^3 + 2*b^3*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]
^3 - 2*b^3*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^3 + 12*b*n*(a^2 - 2*a*
b*n + 2*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -
(d*f*Sqrt[x])] - 48*b^2*n^2*(a - b*n + b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[
x])] + 96*b^3*n^3*PolyLog[4, -(d*f*Sqrt[x])])/(d^2*f^2)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^{3} \log\left(cx^{n}\right)^{3} + 3\,ab^{2} \log\left(cx^{n}\right)^{2} + 3\,a^{2}b \log\left(cx^{n}\right) + a^{3}\right) \log\left(df\sqrt{x} + 1\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^{n}\right) + a\right)^{3} \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\,x^{n}\right) + a\right)^{3} \ln\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+1/d)*d),x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+1/d)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(b^{3}x \log\left(x^{n}\right)^{3} - 3\left(b^{3}\left(n - \log(c)\right) - ab^{2}\right)x \log\left(x^{n}\right)^{2} + 3\left(\left(2\,n^{2} - 2\,n \log(c) + \log(c)^{2}\right)b^{3} - 2\,ab^{2}\left(n - \log(c)\right) + \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima")
```

```
[Out] (b^3*x*log(x^n)^3 - 3*(b^3*(n - log(c)) - a*b^2)*x*log(x^n)^2 + 3*((2*n^2 -
 2*n*log(c) + log(c)^2)*b^3 - 2*a*b^2*(n - log(c)) + a^2*b)*x*log(x^n) + (3
```

```
*(2*n^2 - 2*n*log(c) + log(c)^2)*a*b^2 - (6*n^3 - 6*n^2*log(c) + 3*n*log(c)
^2 - log(c)^3)*b^3 - 3*a^2*b*(n - log(c)) + a^3)*x)*log(d*f*sqrt(x) + 1) -
1/27*(9*b^3*d*f*x^2*log(x^n)^3 + 9*(3*a*b^2*d*f - (5*d*f*n - 3*d*f*log(c))*
b^3)*x^2*log(x^n)^2 + 3*(9*a^2*b*d*f - 6*(5*d*f*n - 3*d*f*log(c))*a*b^2 + (
38*d*f*n^2 - 30*d*f*n*log(c) + 9*d*f*log(c)^2)*b^3)*x^2*log(x^n) + (9*a^3*d
*f - 9*(5*d*f*n - 3*d*f*log(c))*a^2*b + 3*(38*d*f*n^2 - 30*d*f*n*log(c) + 9
*d*f*log(c)^2)*a*b^2 - (130*d*f*n^3 - 114*d*f*n^2*log(c) + 45*d*f*n*log(c)^
2 - 9*d*f*log(c)^3)*b^3)*x^2)/sqrt(x) + integrate(1/2*(b^3*d^2*f^2*x*log(x^
n)^3 + 3*(a*b^2*d^2*f^2 - (d^2*f^2*n - d^2*f^2*log(c))*b^3)*x*log(x^n)^2 +
3*(a^2*b*d^2*f^2 - 2*(d^2*f^2*n - d^2*f^2*log(c))*a*b^2 + (2*d^2*f^2*n^2 -
2*d^2*f^2*n*log(c) + d^2*f^2*log(c)^2)*b^3)*x*log(x^n) + (a^3*d^2*f^2 - 3*(
d^2*f^2*n - d^2*f^2*log(c))*a^2*b + 3*(2*d^2*f^2*n^2 - 2*d^2*f^2*n*log(c) +
 d^2*f^2*log(c)^2)*a*b^2 - (6*d^2*f^2*n^3 - 6*d^2*f^2*n^2*log(c) + 3*d^2*f^
2*n*log(c)^2 - d^2*f^2*log(c)^3)*b^3)*x)/(d*f*sqrt(x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**(1/2))),x)
```

```
[Out] Timed out
```

3.61 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^3}{x}\,dx$

Optimal. Leaf size=101

$$-48b^2n^2\text{Li}_4\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)+12bn\text{Li}_3\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2-2\text{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] -2*(a+b*ln(c*x^n))^3*polylog(2,-d*f*x^(1/2))+12*b*n*(a+b*ln(c*x^n))^2*polyl
og(3,-d*f*x^(1/2))-48*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-d*f*x^(1/2))+96*b^
3*n^3*polylog(5,-d*f*x^(1/2))
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2374, 2383, 6589}

$$-48b^2n^2\text{PolyLog}\left(4,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)+12bn\text{PolyLog}\left(3,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2-2\text{PolyLog}\left(2,-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

```
[Out] -2*(a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*Sqrt[x])] + 12*b*n*(a + b*Log[c*x^
n])^2*PolyLog[3, -(d*f*Sqrt[x])] - 48*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[4,
 -(d*f*Sqrt[x])] + 96*b^3*n^3*PolyLog[5, -(d*f*Sqrt[x])]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x}\,dx=-2\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2\left(-df\sqrt{x}\right)+(6bn)\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{x}$$

$$=-2\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2\left(-df\sqrt{x}\right)+12bn\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_3$$

$$=-2\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2\left(-df\sqrt{x}\right)+12bn\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_3$$

$$=-2\left(a+b\log\left(cx^n\right)\right)^3\text{Li}_2\left(-df\sqrt{x}\right)+12bn\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_3$$

Mathematica [A] time = 0.21, size = 98, normalized size = 0.97

$$12bn\left(\mathrm{Li}_3\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2+4bn\left(2bn\mathrm{Li}_5\left(-df\sqrt{x}\right)-\mathrm{Li}_4\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)\right)\right)-2\mathrm{Li}_2\left(-df\sqrt{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

```
[Out] -2*(a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*Sqrt[x])] + 12*b*n*((a + b*Log[c*x
^n])^2*PolyLog[3, -(d*f*Sqrt[x])] + 4*b*n*(-((a + b*Log[c*x^n])*PolyLog[4,
-(d*f*Sqrt[x])]) + 2*b*n*PolyLog[5, -(d*f*Sqrt[x])]))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^3\log\left(cx^n\right)^3+3\,ab^2\log\left(cx^n\right)^2+3\,a^2b\log\left(cx^n\right)+a^3\right)\log\left(df\sqrt{x}+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="fricas"
)
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + 1)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^n\right)+a\right)^3\ln\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+1/d)*d)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+1/d)*d)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x,x)
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**(1/2)))/x,x)
[Out] Timed out
```

3.62 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^3}{x^2}\,dx$

Optimal. Leaf size=610

$$12b^2d^2f^2n^2\mathrm{Li}_2\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)-24b^2d^2f^2n^2\mathrm{Li}_3\left(-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)+6b^2d^2f^2n^2\log\left(df\sqrt{x}+1\right.$$

```
[Out] -3*b^3*d^2*f^2*n^3*ln(x)+3/2*b^3*d^2*f^2*n^3*ln(x)^2-3*b^2*d^2*f^2*n^2*ln(x
)*(a+b*ln(c*x^n))-1/2*d^2*f^2*(a+b*ln(c*x^n))^3-1/8*d^2*f^2*(a+b*ln(c*x^n))
^4/b/n+6*b^3*d^2*f^2*n^3*ln(1+d*f*x^(1/2))-6*b^3*n^3*ln(1+d*f*x^(1/2))/x+6*
b^2*d^2*f^2*n^2*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-6*b^2*n^2*(a+b*ln(c*x^n))
*ln(1+d*f*x^(1/2))/x+3*b*d^2*f^2*n*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))-3*b*
n*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/x+d^2*f^2*(a+b*ln(c*x^n))^3*ln(1+d*f*
x^(1/2))-(a+b*ln(c*x^n))^3*ln(1+d*f*x^(1/2))/x+12*b^3*d^2*f^2*n^3*polylog(2
,-d*f*x^(1/2))+12*b^2*d^2*f^2*n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))+6
*b*d^2*f^2*n*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^(1/2))-24*b^3*d^2*f^2*n^3*p
olylog(3,-d*f*x^(1/2))-24*b^2*d^2*f^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^
(1/2))+48*b^3*d^2*f^2*n^3*polylog(4,-d*f*x^(1/2))-90*b^3*d*f*n^3/x^(1/2)-42
*b^2*d*f*n^2*(a+b*ln(c*x^n))/x^(1/2)-9*b*d*f*n*(a+b*ln(c*x^n))^2/x^(1/2)-d*
f*(a+b*ln(c*x^n))^3/x^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 16, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2454, 2395, 44, 2377, 2305, 2304, 2376, 2391, 2301, 2374, 6589, 2366, 12, 2302, 30, 2383}

$$12b^2d^2f^2n^2\mathrm{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)-24b^2d^2f^2n^2\mathrm{PolyLog}\left(3,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)+6bd^2f^2n\mathrm{P}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

```
[Out] (-90*b^3*d*f*n^3)/Sqrt[x] + 6*b^3*d^2*f^2*n^3*Log[1 + d*f*Sqrt[x]] - (6*b^3
*n^3*Log[1 + d*f*Sqrt[x]])/x - 3*b^3*d^2*f^2*n^3*Log[x] + (3*b^3*d^2*f^2*n^
3*Log[x]^2)/2 - (42*b^2*d*f*n^2*(a + b*Log[c*x^n]))/Sqrt[x] + 6*b^2*d^2*f^2
*n^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]) - (6*b^2*n^2*Log[1 + d*f*Sqrt[
x]]*(a + b*Log[c*x^n]))/x - 3*b^2*d^2*f^2*n^2*Log[x]*(a + b*Log[c*x^n]) - (
9*b*d*f*n*(a + b*Log[c*x^n])^2)/Sqrt[x] + 3*b*d^2*f^2*n*Log[1 + d*f*Sqrt[x]
]*(a + b*Log[c*x^n])^2 - (3*b*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/
x - (d^2*f^2*(a + b*Log[c*x^n])^3)/2 - (d*f*(a + b*Log[c*x^n])^3)/Sqrt[x] +
 d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3 - (Log[1 + d*f*Sqrt[x]]*
(a + b*Log[c*x^n])^3)/x - (d^2*f^2*(a + b*Log[c*x^n])^4)/(8*b*n) + 12*b^3*d
^2*f^2*n^3*PolyLog[2, -(d*f*Sqrt[x])] + 12*b^2*d^2*f^2*n^2*(a + b*Log[c*x^n
])*PolyLog[2, -(d*f*Sqrt[x])] + 6*b*d^2*f^2*n*(a + b*Log[c*x^n])^2*PolyLog[
2, -(d*f*Sqrt[x])] - 24*b^3*d^2*f^2*n^3*PolyLog[3, -(d*f*Sqrt[x])] - 24*b^2
*d^2*f^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[x])] + 48*b^3*d^2*f^2
*n^3*PolyLog[4, -(d*f*Sqrt[x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
```

```
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^2}\,dx = -\frac{df\left(a+b\log\left(cx^n\right)\right)^3}{\sqrt{x}} + d^2f^2\log\left(1+df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3$$

$$= -\frac{df\left(a+b\log\left(cx^n\right)\right)^3}{\sqrt{x}} + d^2f^2\log\left(1+df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3$$

$$= -\frac{9bdfn\left(a+b\log\left(cx^n\right)\right)^2}{\sqrt{x}} + 3bd^2f^2n\log\left(1+df\sqrt{x}\right)\left(a+b\log\right.$$

$$= -\frac{48b^3dfn^3}{\sqrt{x}} - \frac{24b^2dfn^2\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} - \frac{9bdfn\left(a+b\log\left(c\right.\right.}{\sqrt{x}}$$

$$= -\frac{72b^3dfn^3}{\sqrt{x}} - \frac{42b^2dfn^2\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + 6b^2d^2f^2n^2\log\left(1+\right.$$

$$= -\frac{84b^3dfn^3}{\sqrt{x}} - \frac{42b^2dfn^2\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + 6b^2d^2f^2n^2\log\left(1+\right.$$

$$= -\frac{84b^3dfn^3}{\sqrt{x}} + \frac{3}{2}b^3d^2f^2n^3\log^2(x) - \frac{42b^2dfn^2\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}}$$

$$= -\frac{84b^3dfn^3}{\sqrt{x}} - \frac{6b^3n^3\log\left(1+df\sqrt{x}\right)}{x} + \frac{3}{2}b^3d^2f^2n^3\log^2(x) - \frac{42}{}$$

$$= -\frac{84b^3dfn^3}{\sqrt{x}} - \frac{6b^3n^3\log\left(1+df\sqrt{x}\right)}{x} + \frac{3}{2}b^3d^2f^2n^3\log^2(x) - \frac{42}{}$$

$$= -\frac{90b^3dfn^3}{\sqrt{x}} + 6b^3d^2f^2n^3\log\left(1+df\sqrt{x}\right) - \frac{6b^3n^3\log\left(1+df\sqrt{\right.}{x}$$

Mathematica [B] time = 0.85, size = 1455, normalized size = 2.39

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

```
[Out] d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3
*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) +
6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2
 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3)
 - d^2*f^2*Log[Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3*a^2*
b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3
*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 3*
b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3) - (L
og[1 + d*f*Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3*a^2*b*n*
Log[x] + 6*a*b^2*n^2*Log[x] + 6*b^3*n^3*Log[x] + 3*a*b^2*n^2*Log[x]^2 + 3*b
^3*n^3*Log[x]^2 + b^3*n^3*Log[x]^3 + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6
*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n])
+ 6*a*b^2*n*Log[x]*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*Log[x]*(-(n*Log[x
]) + Log[c*x^n]) + 3*b^3*n^2*Log[x]^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*
(-(n*Log[x]) + Log[c*x^n])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 3*b^3
*n*Log[x]*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3))
/x + (-(a^3*d*f) - 3*a^2*b*d*f*n - 6*a*b^2*d*f*n^2 - 6*b^3*d*f*n^3 - 3*a^2*
b*d*f*(-(n*Log[x]) + Log[c*x^n]) - 6*a*b^2*d*f*n*(-(n*Log[x]) + Log[c*x^n])
```

```
 - 6*b^3*d*f*n^2*(-(n*Log[x]) + Log[c*x^n]) - 3*a*b^2*d*f*(-(n*Log[x]) + Lo
g[c*x^n])^2 - 3*b^3*d*f*n*(-(n*Log[x]) + Log[c*x^n])^2 - b^3*d*f*(-(n*Log[x
]) + Log[c*x^n])^3)/Sqrt[x] + 3*b*d*f*n*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*a*b*
(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*
Log[x]) + Log[c*x^n])^2)*((-(1/Sqrt[x]) + d*f*Log[1 + d*f*Sqrt[x]] - d*f*Lo
g[Sqrt[x]])*(-2*Log[Sqrt[x]] + Log[x]) + 2*(-(1/Sqrt[x]) - Log[Sqrt[x]]/Sqr
t[x] + d*f*Log[1 + d*f*Sqrt[x]]*Log[Sqrt[x]] - (d*f*Log[Sqrt[x]]^2)/2 + d*f
*PolyLog[2, -(d*f*Sqrt[x])])) + 3*b^2*d*f*n^2*(a + b*n + b*(-(n*Log[x]) + L
og[c*x^n]))*((-(1/Sqrt[x]) + d*f*Log[1 + d*f*Sqrt[x]] - d*f*Log[Sqrt[x]])*(
-2*Log[Sqrt[x]] + Log[x])^2 + 4*(-2*Log[Sqrt[x]] + Log[x])*(-(1/Sqrt[x]) -
Log[Sqrt[x]]/Sqrt[x] + d*f*Log[1 + d*f*Sqrt[x]]*Log[Sqrt[x]] - (d*f*Log[Sqr
t[x]]^2)/2 + d*f*PolyLog[2, -(d*f*Sqrt[x])]) + 4*(-2/Sqrt[x] - (2*Log[Sqrt[
x]])/Sqrt[x] - Log[Sqrt[x]]^2/Sqrt[x] + d*f*Log[1 + d*f*Sqrt[x]]*Log[Sqrt[x
]]^2 - (d*f*Log[Sqrt[x]]^3)/3 + 2*d*f*Log[Sqrt[x]]*PolyLog[2, -(d*f*Sqrt[x]
)] - 2*d*f*PolyLog[3, -(d*f*Sqrt[x])])) + (b^3*d*f*n^3*(1 + 1/(d*f*Sqrt[x])
)*(2*(-(d*f*Sqrt[x]) + d^2*f^2*x*Log[1 + 1/(d*f*Sqrt[x])])*Log[x]^3 - 12*d*
f*Sqrt[x]*Log[x]^2*(1 + d*f*Sqrt[x]*PolyLog[2, -(1/(d*f*Sqrt[x]))]) - 48*d*
f*Sqrt[x]*Log[x]*(1 + d*f*Sqrt[x]*PolyLog[3, -(1/(d*f*Sqrt[x]))]) - 96*d*f*
Sqrt[x]*(1 + d*f*Sqrt[x]*PolyLog[4, -(1/(d*f*Sqrt[x]))])))/(2*(1 + d*f*Sqrt
[x])*Sqrt[x])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\log\left(cx^n\right)^3+3\,ab^2\log\left(cx^n\right)^2+3\,a^2b\log\left(cx^n\right)+a^3\right)\log\left(df\sqrt{x}+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="frica
s")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + 1)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right)+a\right)^3\ln\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+1/d)*d)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+1/d)*d)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^2,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**(1/2)))/x**2,x)
```

```
[Out] Timed out
```

3.63 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^3}{x^3}\,dx$

Optimal. Leaf size=849

$$-\frac{d^4\left(a+b\log\left(cx^n\right)\right)^4 f^4}{16bn}-\frac{1}{8}d^4\left(a+b\log\left(cx^n\right)\right)^3 f^4+\frac{1}{2}d^4\log\left(d\sqrt{x}f+1\right)\left(a+b\log\left(cx^n\right)\right)^3 f^4+\frac{3}{16}b^3d^4n^3\log^2(x)$$

```
[Out] -1/8*d^4*f^4*(a+b*ln(c*x^n))^3-1/2*(a+b*ln(c*x^n))^3*ln(1+d*f*x^(1/2))/x^2-
1/6*d*f*(a+b*ln(c*x^n))^3/x^(3/2)+1/4*d^2*f^2*(a+b*ln(c*x^n))^3/x-3/8*b^3*n
^3*ln(1+d*f*x^(1/2))/x^2+1/2*d^4*f^4*(a+b*ln(c*x^n))^3*ln(1+d*f*x^(1/2))-1/
2*d^3*f^3*(a+b*ln(c*x^n))^3/x^(1/2)-175/216*b^3*d*f*n^3/x^(3/2)+45/16*b^3*d
^2*f^2*n^3/x-3/8*b^2*d^4*f^4*n^2*ln(x)*(a+b*ln(c*x^n))-7/12*b*d*f*n*(a+b*ln
(c*x^n))^2/x^(3/2)+9/8*b*d^2*f^2*n*(a+b*ln(c*x^n))^2/x+3/4*b^2*d^4*f^4*n^2*
(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))+3/4*b*d^4*f^4*n*(a+b*ln(c*x^n))^2*ln(1+d*
f*x^(1/2))+3*b^2*d^4*f^4*n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))+3*b*d^
4*f^4*n*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^(1/2))-12*b^2*d^4*f^4*n^2*(a+b*l
n(c*x^n))*polylog(3,-d*f*x^(1/2))-63/4*b^2*d^3*f^3*n^2*(a+b*ln(c*x^n))/x^(1
/2)-15/4*b*d^3*f^3*n*(a+b*ln(c*x^n))^2/x^(1/2)-3/16*b^3*d^4*f^4*n^3*ln(x)+3
/16*b^3*d^4*f^4*n^3*ln(x)^2-1/16*d^4*f^4*(a+b*ln(c*x^n))^4/b/n+3/8*b^3*d^4*
f^4*n^3*ln(1+d*f*x^(1/2))-3/4*b^2*n^2*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/x^2
-3/4*b*n*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/x^2+3/2*b^3*d^4*f^4*n^3*polylo
g(2,-d*f*x^(1/2))-6*b^3*d^4*f^4*n^3*polylog(3,-d*f*x^(1/2))+24*b^3*d^4*f^4*
n^3*polylog(4,-d*f*x^(1/2))-255/8*b^3*d^3*f^3*n^3/x^(1/2)-37/36*b^2*d*f*n^2
*(a+b*ln(c*x^n))/x^(3/2)+21/8*b^2*d^2*f^2*n^2*(a+b*ln(c*x^n))/x
```

Rubi [A] time = 1.14, antiderivative size = 849, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 16, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2454, 2395, 44, 2377, 2305, 2304, 2376, 2391, 2301, 2374, 6589, 2366, 12, 2302, 30, 2383}

$$-\frac{d^4\left(a+b\log\left(cx^n\right)\right)^4 f^4}{16bn}-\frac{1}{8}d^4\left(a+b\log\left(cx^n\right)\right)^3 f^4+\frac{1}{2}d^4\log\left(d\sqrt{x}f+1\right)\left(a+b\log\left(cx^n\right)\right)^3 f^4+\frac{3}{16}b^3d^4n^3\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

```
[Out] (-175*b^3*d*f*n^3)/(216*x^(3/2)) + (45*b^3*d^2*f^2*n^3)/(16*x) - (255*b^3*d
^3*f^3*n^3)/(8*Sqrt[x]) + (3*b^3*d^4*f^4*n^3*Log[1 + d*f*Sqrt[x]])/8 - (3*b
^3*n^3*Log[1 + d*f*Sqrt[x]])/(8*x^2) - (3*b^3*d^4*f^4*n^3*Log[x])/16 + (3*b
^3*d^4*f^4*n^3*Log[x]^2)/16 - (37*b^2*d*f*n^2*(a + b*Log[c*x^n]))/(36*x^(3/
2)) + (21*b^2*d^2*f^2*n^2*(a + b*Log[c*x^n]))/(8*x) - (63*b^2*d^3*f^3*n^2*(
a + b*Log[c*x^n]))/(4*Sqrt[x]) + (3*b^2*d^4*f^4*n^2*Log[1 + d*f*Sqrt[x]]*(a
 + b*Log[c*x^n]))/4 - (3*b^2*n^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(
4*x^2) - (3*b^2*d^4*f^4*n^2*Log[x]*(a + b*Log[c*x^n]))/8 - (7*b*d*f*n*(a +
b*Log[c*x^n])^2)/(12*x^(3/2)) + (9*b*d^2*f^2*n*(a + b*Log[c*x^n])^2)/(8*x)
- (15*b*d^3*f^3*n*(a + b*Log[c*x^n])^2)/(4*Sqrt[x]) + (3*b*d^4*f^4*n*Log[1
+ d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/4 - (3*b*n*Log[1 + d*f*Sqrt[x]]*(a + b
*Log[c*x^n])^2)/(4*x^2) - (d^4*f^4*(a + b*Log[c*x^n])^3)/8 - (d*f*(a + b*Lo
g[c*x^n])^3)/(6*x^(3/2)) + (d^2*f^2*(a + b*Log[c*x^n])^3)/(4*x) - (d^3*f^3*
(a + b*Log[c*x^n])^3)/(2*Sqrt[x]) + (d^4*f^4*Log[1 + d*f*Sqrt[x]]*(a + b*Lo
g[c*x^n])^3)/2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(2*x^2) - (d^4
*f^4*(a + b*Log[c*x^n])^4)/(16*b*n) + (3*b^3*d^4*f^4*n^3*PolyLog[2, -(d*f*S
qrt[x])])/2 + 3*b^2*d^4*f^4*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x]
)] + 3*b*d^4*f^4*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])] - 6*b^3*
d^4*f^4*n^3*PolyLog[3, -(d*f*Sqrt[x])] - 12*b^2*d^4*f^4*n^2*(a + b*Log[c*x^
n])*PolyLog[3, -(d*f*Sqrt[x])] + 24*b^3*d^4*f^4*n^3*PolyLog[4, -(d*f*Sqrt[x
])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}\,dx = -\frac{df\left(a+b\log\left(cx^n\right)\right)^3}{6x^{3/2}} + \frac{d^2f^2\left(a+b\log\left(cx^n\right)\right)^3}{4x} - \frac{d^3f^3\left(a+b\,\text{l}\cdots\right.}{2\sqrt{\cdots}}$$

$$= -\frac{df\left(a+b\log\left(cx^n\right)\right)^3}{6x^{3/2}} + \frac{d^2f^2\left(a+b\log\left(cx^n\right)\right)^3}{4x} - \frac{d^3f^3\left(a+b\,\text{l}\cdots\right.}{2\sqrt{\cdots}}$$

$$= -\frac{7bdfn\left(a+b\log\left(cx^n\right)\right)^2}{12x^{3/2}} + \frac{9bd^2f^2n\left(a+b\log\left(cx^n\right)\right)^2}{8x} - \frac{15bd\cdots}{\cdots}$$

$$= -\frac{8b^3dfn^3}{27x^{3/2}} + \frac{3b^3d^2f^2n^3}{2x} - \frac{24b^3d^3f^3n^3}{\sqrt{x}} - \frac{4b^2dfn^2\left(a+b\log(c\cdots\right.}{9x^{3/2}}$$

$$= -\frac{14b^3dfn^3}{27x^{3/2}} + \frac{9b^3d^2f^2n^3}{4x} - \frac{30b^3d^3f^3n^3}{\sqrt{x}} - \frac{37b^2dfn^2\left(a+b\log\cdots\right.}{36x^{3/2}}$$

$$= -\frac{37b^3dfn^3}{54x^{3/2}} + \frac{21b^3d^2f^2n^3}{8x} - \frac{63b^3d^3f^3n^3}{2\sqrt{x}} - \frac{37b^2dfn^2\left(a+b\log\cdots\right.}{36x^{3/2}}$$

$$= -\frac{37b^3dfn^3}{54x^{3/2}} + \frac{21b^3d^2f^2n^3}{8x} - \frac{63b^3d^3f^3n^3}{2\sqrt{x}} + \frac{3}{16}b^3d^4f^4n^3\log^2(x\cdots$$

$$= -\frac{37b^3dfn^3}{54x^{3/2}} + \frac{21b^3d^2f^2n^3}{8x} - \frac{63b^3d^3f^3n^3}{2\sqrt{x}} - \frac{3b^3n^3\log\left(1+df\sqrt{\cdots}\right.}{8x^2}$$

$$= -\frac{37b^3dfn^3}{54x^{3/2}} + \frac{21b^3d^2f^2n^3}{8x} - \frac{63b^3d^3f^3n^3}{2\sqrt{x}} - \frac{3b^3n^3\log\left(1+df\sqrt{\cdots}\right.}{8x^2}$$

$$= -\frac{175b^3dfn^3}{216x^{3/2}} + \frac{45b^3d^2f^2n^3}{16x} - \frac{255b^3d^3f^3n^3}{8\sqrt{x}} + \frac{3}{8}b^3d^4f^4n^3\log\left(1\cdots\right.$$

Mathematica [B] time = 1.07, size = 2009, normalized size = 2.37

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

```
[Out] -1/6*(a^3*d*f)/x^(3/2) - (7*a^2*b*d*f*n)/(12*x^(3/2)) - (37*a*b^2*d*f*n^2)/
(36*x^(3/2)) - (175*b^3*d*f*n^3)/(216*x^(3/2)) + (a^3*d^2*f^2)/(4*x) + (9*a
^2*b*d^2*f^2*n)/(8*x) + (21*a*b^2*d^2*f^2*n^2)/(8*x) + (45*b^3*d^2*f^2*n^3)
/(16*x) - (a^3*d^3*f^3)/(2*Sqrt[x]) - (15*a^2*b*d^3*f^3*n)/(4*Sqrt[x]) - (6
3*a*b^2*d^3*f^3*n^2)/(4*Sqrt[x]) - (255*b^3*d^3*f^3*n^3)/(8*Sqrt[x]) + (a^3
*d^4*f^4*Log[1 + d*f*Sqrt[x]])/2 + (3*a^2*b*d^4*f^4*n*Log[1 + d*f*Sqrt[x]])
/4 + (3*a*b^2*d^4*f^4*n^2*Log[1 + d*f*Sqrt[x]])/4 + (3*b^3*d^4*f^4*n^3*Log[
1 + d*f*Sqrt[x]])/8 - (a^3*Log[1 + d*f*Sqrt[x]])/(2*x^2) - (3*a^2*b*n*Log[1
 + d*f*Sqrt[x]])/(4*x^2) - (3*a*b^2*n^2*Log[1 + d*f*Sqrt[x]])/(4*x^2) - (3*
b^3*n^3*Log[1 + d*f*Sqrt[x]])/(8*x^2) - (a^3*d^4*f^4*Log[x])/4 - (3*a^2*b*d
^4*f^4*n*Log[x])/8 - (3*a*b^2*d^4*f^4*n^2*Log[x])/8 - (3*b^3*d^4*f^4*n^3*Lo
g[x])/16 + (3*a^2*b*d^4*f^4*n*Log[x]^2)/8 + (3*a*b^2*d^4*f^4*n^2*Log[x]^2)/
8 + (3*b^3*d^4*f^4*n^3*Log[x]^2)/16 - (a*b^2*d^4*f^4*n^2*Log[x]^3)/4 - (b^3
*d^4*f^4*n^3*Log[x]^3)/8 + (b^3*d^4*f^4*n^3*Log[1 + 1/(d*f*Sqrt[x])]*Log[x]
^3)/2 - (b^3*d^4*f^4*n^3*Log[1 + d*f*Sqrt[x]]*Log[x]^3)/2 + (b^3*d^4*f^4*n^
3*Log[x]^4)/8 - (a^2*b*d*f*Log[c*x^n])/(2*x^(3/2)) - (7*a*b^2*d*f*n*Log[c*x
^n])/(6*x^(3/2)) - (37*b^3*d*f*n^2*Log[c*x^n])/(36*x^(3/2)) + (3*a^2*b*d^2*
f^2*Log[c*x^n])/(4*x) + (9*a*b^2*d^2*f^2*n*Log[c*x^n])/(4*x) + (21*b^3*d^2*
```

```
f^2*n^2*Log[c*x^n])/(8*x) - (3*a^2*b*d^3*f^3*Log[c*x^n])/(2*Sqrt[x]) - (15*
a*b^2*d^3*f^3*n*Log[c*x^n])/(2*Sqrt[x]) - (63*b^3*d^3*f^3*n^2*Log[c*x^n])/(
4*Sqrt[x]) + (3*a^2*b*d^4*f^4*Log[1 + d*f*Sqrt[x]]*Log[c*x^n])/2 + (3*a*b^2
*d^4*f^4*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n])/2 + (3*b^3*d^4*f^4*n^2*Log[1 +
d*f*Sqrt[x]]*Log[c*x^n])/4 - (3*a^2*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n])/(2*x
^2) - (3*a*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n])/(2*x^2) - (3*b^3*n^2*Log[
1 + d*f*Sqrt[x]]*Log[c*x^n])/(4*x^2) - (3*a^2*b*d^4*f^4*Log[x]*Log[c*x^n])/
4 - (3*a*b^2*d^4*f^4*n*Log[x]*Log[c*x^n])/4 - (3*b^3*d^4*f^4*n^2*Log[x]*Log
[c*x^n])/8 + (3*a*b^2*d^4*f^4*n*Log[x]^2*Log[c*x^n])/4 + (3*b^3*d^4*f^4*n^2
*Log[x]^2*Log[c*x^n])/8 - (b^3*d^4*f^4*n^2*Log[x]^3*Log[c*x^n])/4 - (a*b^2*
d*f*Log[c*x^n]^2)/(2*x^(3/2)) - (7*b^3*d*f*n*Log[c*x^n]^2)/(12*x^(3/2)) + (
3*a*b^2*d^2*f^2*Log[c*x^n]^2)/(4*x) + (9*b^3*d^2*f^2*n*Log[c*x^n]^2)/(8*x)
- (3*a*b^2*d^3*f^3*Log[c*x^n]^2)/(2*Sqrt[x]) - (15*b^3*d^3*f^3*n*Log[c*x^n]
^2)/(4*Sqrt[x]) + (3*a*b^2*d^4*f^4*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2)/2 + (
3*b^3*d^4*f^4*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2)/4 - (3*a*b^2*Log[1 + d*f
*Sqrt[x]]*Log[c*x^n]^2)/(2*x^2) - (3*b^3*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^
2)/(4*x^2) - (3*a*b^2*d^4*f^4*Log[x]*Log[c*x^n]^2)/4 - (3*b^3*d^4*f^4*n*Log
[x]*Log[c*x^n]^2)/8 + (3*b^3*d^4*f^4*n*Log[x]^2*Log[c*x^n]^2)/8 - (b^3*d*f*
Log[c*x^n]^3)/(6*x^(3/2)) + (b^3*d^2*f^2*Log[c*x^n]^3)/(4*x) - (b^3*d^3*f^3
*Log[c*x^n]^3)/(2*Sqrt[x]) + (b^3*d^4*f^4*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^3
)/2 - (b^3*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^3)/(2*x^2) - (b^3*d^4*f^4*Log[x]
*Log[c*x^n]^3)/4 - 3*b^3*d^4*f^4*n^3*Log[x]^2*PolyLog[2, -(1/(d*f*Sqrt[x]))
] + (3*b*d^4*f^4*n*(2*a^2 + 2*a*b*n + b^2*n^2 - 2*b^2*n^2*Log[x]^2 + 2*b*(2
*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, -(d*f*Sqrt[x])])/2 -
12*b^3*d^4*f^4*n^3*Log[x]*PolyLog[3, -(1/(d*f*Sqrt[x]))] - 12*a*b^2*d^4*f^4
*n^2*PolyLog[3, -(d*f*Sqrt[x])] - 6*b^3*d^4*f^4*n^3*PolyLog[3, -(d*f*Sqrt[x
])] + 12*b^3*d^4*f^4*n^3*Log[x]*PolyLog[3, -(d*f*Sqrt[x])] - 12*b^3*d^4*f^4
*n^2*Log[c*x^n]*PolyLog[3, -(d*f*Sqrt[x])] - 24*b^3*d^4*f^4*n^3*PolyLog[4,
-(1/(d*f*Sqrt[x]))]
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3 \log\left(cx^n\right)^3 + 3\,ab^2 \log\left(cx^n\right)^2 + 3\,a^2 b \log\left(cx^n\right) + a^3\right) \log\left(df\sqrt{x} + 1\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="frica
s")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + 1)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^n\right) + a\right)^3 \log\left(\left(f\sqrt{x} + \frac{1}{d}\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^n\right) + a\right)^3 \ln\left(\left(f\sqrt{x} + \frac{1}{d}\right) d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+1/d)*d)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+1/d)*d)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{3}\log\left(\left(f\sqrt{x}+\frac{1}{d}\right)d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^3,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**(1/2)))/x**3,x)
```

```
[Out] Timed out
```

3.64 $\int \frac{\left(a+b\log(cx^n)\right)^4 \log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$

Optimal. Leaf size=137

$$\frac{24b^3n^3\text{Li}_5\left(-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)}{m^4}-\frac{12b^2n^2\text{Li}_4\left(-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^2}{m^3}+\frac{4bn\text{Li}_3\left(-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^3}{m^2}$$

```
[Out] -(a+b*ln(c*x^n))^4*polylog(2,-d*f*x^m)/m+4*b*n*(a+b*ln(c*x^n))^3*polylog(3,
-d*f*x^m)/m^2-12*b^2*n^2*(a+b*ln(c*x^n))^2*polylog(4,-d*f*x^m)/m^3+24*b^3*n
^3*(a+b*ln(c*x^n))*polylog(5,-d*f*x^m)/m^4-24*b^4*n^4*polylog(6,-d*f*x^m)/m
^5
```

Rubi [A] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2374, 2383, 6589}

$$\frac{24b^3n^3\text{PolyLog}\left(5,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)}{m^4}-\frac{12b^2n^2\text{PolyLog}\left(4,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^2}{m^3}+\frac{4bn\text{PolyLog}\left(3,-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^4*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -(((a + b*Log[c*x^n])^4*PolyLog[2, -(d*f*x^m)])/m) + (4*b*n*(a + b*Log[c*x^
n])^3*PolyLog[3, -(d*f*x^m)])/m^2 - (12*b^2*n^2*(a + b*Log[c*x^n])^2*PolyLo
g[4, -(d*f*x^m)])/m^3 + (24*b^3*n^3*(a + b*Log[c*x^n])*PolyLog[5, -(d*f*x^m
)])/m^4 - (24*b^4*n^4*PolyLog[6, -(d*f*x^m)])/m^5
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^4 \log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{\left(a+b\log\left(cx^n\right)\right)^4 \operatorname{Li}_2\left(-dfx^m\right)}{m} + \frac{(4bn)\int \frac{\left(a+b\log\left(cx^n\right)\right)^3 \operatorname{Li}_2\left(-dfx^m\right.}{x}}{m}$$

$$= -\frac{\left(a+b\log\left(cx^n\right)\right)^4 \operatorname{Li}_2\left(-dfx^m\right)}{m} + \frac{4bn\left(a+b\log\left(cx^n\right)\right)^3 \operatorname{Li}_3\left(-\right.}{m^2}$$

$$= -\frac{\left(a+b\log\left(cx^n\right)\right)^4 \operatorname{Li}_2\left(-dfx^m\right)}{m} + \frac{4bn\left(a+b\log\left(cx^n\right)\right)^3 \operatorname{Li}_3\left(-\right.}{m^2}$$

$$= -\frac{\left(a+b\log\left(cx^n\right)\right)^4 \operatorname{Li}_2\left(-dfx^m\right)}{m} + \frac{4bn\left(a+b\log\left(cx^n\right)\right)^3 \operatorname{Li}_3\left(-\right.}{m^2}$$

$$= -\frac{\left(a+b\log\left(cx^n\right)\right)^4 \operatorname{Li}_2\left(-dfx^m\right)}{m} + \frac{4bn\left(a+b\log\left(cx^n\right)\right)^3 \operatorname{Li}_3\left(-\right.}{m^2}$$

Mathematica [B] time = 0.70, size = 1700, normalized size = 12.41

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^4*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] (-2*a^3*b*m*n*Log[x]^3)/3 + (3*a^2*b^2*m*n^2*Log[x]^4)/2 - (6*a*b^3*m*n^3*L
og[x]^5)/5 + (b^4*m*n^4*Log[x]^6)/3 - 2*a^2*b^2*m*n*Log[x]^3*Log[c*x^n] + 3
*a*b^3*m*n^2*Log[x]^4*Log[c*x^n] - (6*b^4*m*n^3*Log[x]^5*Log[c*x^n])/5 - 2*
a*b^3*m*n*Log[x]^3*Log[c*x^n]^2 + (3*b^4*m*n^2*Log[x]^4*Log[c*x^n]^2)/2 - (
2*b^4*m*n*Log[x]^3*Log[c*x^n]^3)/3 - 2*a^3*b*n*Log[x]^2*Log[1 + 1/(d*f*x^m)
] + 4*a^2*b^2*n^2*Log[x]^3*Log[1 + 1/(d*f*x^m)] - 3*a*b^3*n^3*Log[x]^4*Log[
1 + 1/(d*f*x^m)] + (4*b^4*n^4*Log[x]^5*Log[1 + 1/(d*f*x^m)])/5 - 6*a^2*b^2*
n*Log[x]^2*Log[c*x^n]*Log[1 + 1/(d*f*x^m)] + 8*a*b^3*n^2*Log[x]^3*Log[c*x^n
]*Log[1 + 1/(d*f*x^m)] - 3*b^4*n^3*Log[x]^4*Log[c*x^n]*Log[1 + 1/(d*f*x^m)]
 - 6*a*b^3*n*Log[x]^2*Log[c*x^n]^2*Log[1 + 1/(d*f*x^m)] + 4*b^4*n^2*Log[x]^
3*Log[c*x^n]^2*Log[1 + 1/(d*f*x^m)] - 2*b^4*n*Log[x]^2*Log[c*x^n]^3*Log[1 +
 1/(d*f*x^m)] + 2*a^3*b*n*Log[x]^2*Log[1 + d*f*x^m] - 4*a^2*b^2*n^2*Log[x]^
3*Log[1 + d*f*x^m] + 3*a*b^3*n^3*Log[x]^4*Log[1 + d*f*x^m] - (4*b^4*n^4*Log
[x]^5*Log[1 + d*f*x^m])/5 + (a^4*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (4*a
^3*b*n*Log[x]*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m + (6*a^2*b^2*n^2*Log[x]^2
*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (4*a*b^3*n^3*Log[x]^3*Log[-(d*f*x^m)
]*Log[1 + d*f*x^m])/m + (b^4*n^4*Log[x]^4*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])
/m + 6*a^2*b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + d*f*x^m] - 8*a*b^3*n^2*Log[x]^
3*Log[c*x^n]*Log[1 + d*f*x^m] + 3*b^4*n^3*Log[x]^4*Log[c*x^n]*Log[1 + d*f*x
^m] + (4*a^3*b*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m - (12*a^2*b^2
*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m + (12*a*b^3*n^2*Lo
g[x]^2*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m - (4*b^4*n^3*Log[x]^3
*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m + 6*a*b^3*n*Log[x]^2*Log[c*
x^n]^2*Log[1 + d*f*x^m] - 4*b^4*n^2*Log[x]^3*Log[c*x^n]^2*Log[1 + d*f*x^m]
+ (6*a^2*b^2*Log[-(d*f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m - (12*a*b^3*n
*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m + (6*b^4*n^2*Log[x
]^2*Log[-(d*f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m + 2*b^4*n*Log[x]^2*Log
[c*x^n]^3*Log[1 + d*f*x^m] + (4*a*b^3*Log[-(d*f*x^m)]*Log[c*x^n]^3*Log[1 +
d*f*x^m])/m - (4*b^4*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]^3*Log[1 + d*f*x^m]
)/m + (b^4*Log[-(d*f*x^m)]*Log[c*x^n]^4*Log[1 + d*f*x^m])/m + (b*n*Log[x]*(
-(b^3*n^3*Log[x]^3) + 4*b^2*n^2*Log[x]^2*(a + b*Log[c*x^n]) - 6*b*n*Log[x]*
(a + b*Log[c*x^n])^2 + 4*(a + b*Log[c*x^n])^3)*PolyLog[2, -(1/(d*f*x^m))])/
m + ((a - b*n*Log[x] + b*Log[c*x^n])^4*PolyLog[2, 1 + d*f*x^m])/m + (4*a^3*
b*n*PolyLog[3, -(1/(d*f*x^m))])/m^2 + (12*a^2*b^2*n*Log[c*x^n]*PolyLog[3, -
```

```
(1/(d*f*x^m))])/m^2 + (12*a*b^3*n*Log[c*x^n]^2*PolyLog[3, -(1/(d*f*x^m))])/
m^2 + (4*b^4*n*Log[c*x^n]^3*PolyLog[3, -(1/(d*f*x^m))])/m^2 + (12*a^2*b^2*n
^2*PolyLog[4, -(1/(d*f*x^m))])/m^3 + (24*a*b^3*n^2*Log[c*x^n]*PolyLog[4, -(
1/(d*f*x^m))])/m^3 + (12*b^4*n^2*Log[c*x^n]^2*PolyLog[4, -(1/(d*f*x^m))])/m
^3 + (24*a*b^3*n^3*PolyLog[5, -(1/(d*f*x^m))])/m^4 + (24*b^4*n^3*Log[c*x^n]
*PolyLog[5, -(1/(d*f*x^m))])/m^4 + (24*b^4*n^4*PolyLog[6, -(1/(d*f*x^m))])/
m^5
```

fricas [C] time = 0.89, size = 523, normalized size = 3.82

$$-\frac{24\,b^4n^4\text{polylog}\left(6,-dfx^m\right)+\left(b^4m^4n^4\log(x)^4+b^4m^4\log(c)^4+4\,ab^3m^4\log(c)^3+6\,a^2b^2m^4\log(c)^2+4\,a^3bm^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^4*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

```
[Out] -(24*b^4*n^4*polylog(6, -d*f*x^m) + (b^4*m^4*n^4*log(x)^4 + b^4*m^4*log(c)^
4 + 4*a*b^3*m^4*log(c)^3 + 6*a^2*b^2*m^4*log(c)^2 + 4*a^3*b*m^4*log(c) + a^
4*m^4 + 4*(b^4*m^4*n^3*log(c) + a*b^3*m^4*n^3)*log(x)^3 + 6*(b^4*m^4*n^2*lo
g(c)^2 + 2*a*b^3*m^4*n^2*log(c) + a^2*b^2*m^4*n^2)*log(x)^2 + 4*(b^4*m^4*n*
log(c)^3 + 3*a*b^3*m^4*n*log(c)^2 + 3*a^2*b^2*m^4*n*log(c) + a^3*b*m^4*n)*l
og(x))*dilog(-d*f*x^m) - 24*(b^4*m*n^4*log(x) + b^4*m*n^3*log(c) + a*b^3*m*
n^3)*polylog(5, -d*f*x^m) + 12*(b^4*m^2*n^4*log(x)^2 + b^4*m^2*n^2*log(c)^2
 + 2*a*b^3*m^2*n^2*log(c) + a^2*b^2*m^2*n^2 + 2*(b^4*m^2*n^3*log(c) + a*b^3
*m^2*n^3)*log(x))*polylog(4, -d*f*x^m) - 4*(b^4*m^3*n^4*log(x)^3 + b^4*m^3*
n*log(c)^3 + 3*a*b^3*m^3*n*log(c)^2 + 3*a^2*b^2*m^3*n*log(c) + a^3*b*m^3*n
+ 3*(b^4*m^3*n^3*log(c) + a*b^3*m^3*n^3)*log(x)^2 + 3*(b^4*m^3*n^2*log(c)^2
 + 2*a*b^3*m^3*n^2*log(c) + a^2*b^2*m^3*n^2)*log(x))*polylog(3, -d*f*x^m))/
m^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^4\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^4*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^4*log((f*x^m + 1/d)*d)/x, x)
```

maple [C] time = 1.89, size = 38574, normalized size = 281.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^4*ln(d*(1/d+f*x^m))/x,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^4*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*n^4*log(x)^5 + 5*b^4*log(x)*log(x^n)^4 - 5*(b^4*n^3*log(c) + a*b^3
*n^3)*log(x)^4 + 10*(b^4*n^2*log(c)^2 + 2*a*b^3*n^2*log(c) + a^2*b^2*n^2)*l
```

```
og(x)^3 - 10*(b^4*n*log(x)^2 - 2*(b^4*log(c) + a*b^3)*log(x))*log(x^n)^3 +
10*(b^4*n^2*log(x)^3 - 3*(b^4*n*log(c) + a*b^3*n)*log(x)^2 + 3*(b^4*log(c)^
2 + 2*a*b^3*log(c) + a^2*b^2)*log(x))*log(x^n)^2 - 10*(b^4*n*log(c)^3 + 3*a
*b^3*n*log(c)^2 + 3*a^2*b^2*n*log(c) + a^3*b*n)*log(x)^2 - 5*(b^4*n^3*log(x
)^4 - 4*(b^4*n^2*log(c) + a*b^3*n^2)*log(x)^3 + 6*(b^4*n*log(c)^2 + 2*a*b^3
*n*log(c) + a^2*b^2*n)*log(x)^2 - 4*(b^4*log(c)^3 + 3*a*b^3*log(c)^2 + 3*a^
2*b^2*log(c) + a^3*b)*log(x))*log(x^n) + 5*(b^4*log(c)^4 + 4*a*b^3*log(c)^3
 + 6*a^2*b^2*log(c)^2 + 4*a^3*b*log(c) + a^4)*log(x))*log(d*f*x^m + 1) - in
tegrate(1/5*(5*b^4*d*f*m*x^m*log(x)*log(x^n)^4 - 10*(b^4*d*f*m*n*log(x)^2 -
 2*(b^4*d*f*m*log(c) + a*b^3*d*f*m)*log(x))*x^m*log(x^n)^3 + 10*(b^4*d*f*m*
n^2*log(x)^3 - 3*(b^4*d*f*m*n*log(c) + a*b^3*d*f*m*n)*log(x)^2 + 3*(b^4*d*f
*m*log(c)^2 + 2*a*b^3*d*f*m*log(c) + a^2*b^2*d*f*m)*log(x))*x^m*log(x^n)^2
- 5*(b^4*d*f*m*n^3*log(x)^4 - 4*(b^4*d*f*m*n^2*log(c) + a*b^3*d*f*m*n^2)*lo
g(x)^3 + 6*(b^4*d*f*m*n*log(c)^2 + 2*a*b^3*d*f*m*n*log(c) + a^2*b^2*d*f*m*n
)*log(x)^2 - 4*(b^4*d*f*m*log(c)^3 + 3*a*b^3*d*f*m*log(c)^2 + 3*a^2*b^2*d*f
*m*log(c) + a^3*b*d*f*m)*log(x))*x^m*log(x^n) + (b^4*d*f*m*n^4*log(x)^5 - 5
*(b^4*d*f*m*n^3*log(c) + a*b^3*d*f*m*n^3)*log(x)^4 + 10*(b^4*d*f*m*n^2*log(
c)^2 + 2*a*b^3*d*f*m*n^2*log(c) + a^2*b^2*d*f*m*n^2)*log(x)^3 - 10*(b^4*d*f
*m*n*log(c)^3 + 3*a*b^3*d*f*m*n*log(c)^2 + 3*a^2*b^2*d*f*m*n*log(c) + a^3*b
*d*f*m*n)*log(x)^2 + 5*(b^4*d*f*m*log(c)^4 + 4*a*b^3*d*f*m*log(c)^3 + 6*a^2
*b^2*d*f*m*log(c)^2 + 4*a^3*b*d*f*m*log(c) + a^4*d*f*m)*log(x))*x^m)/(d*f*x
*x^m + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)\left(a+b\,\ln\left(c\,x^n\right)\right)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^4)/x,x)
[Out] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^4)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**4*ln(d*(1/d+f*x**m))/x,x)
[Out] Timed out
```

3.65 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$

Optimal. Leaf size=105

$$-\frac{6b^2n^2\text{Li}_4\left(-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)}{m^3}+\frac{3bn\text{Li}_3\left(-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^2}{m^2}-\frac{\text{Li}_2\left(-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^3}{m}+\frac{6b^3n}{}$$

```
[Out] -(a+b*ln(c*x^n))^3*polylog(2,-d*f*x^m)/m+3*b*n*(a+b*ln(c*x^n))^2*polylog(3,
-d*f*x^m)/m^2-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-d*f*x^m)/m^3+6*b^3*n^3*p
olylog(5,-d*f*x^m)/m^4
```

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2374, 2383, 6589}

$$-\frac{6b^2n^2\text{PolyLog}\left(4,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)}{m^3}+\frac{3bn\text{PolyLog}\left(3,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^2}{m^2}-\frac{\text{PolyLog}\left(2,-dfx^m\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -(((a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*x^m)])/m) + (3*b*n*(a + b*Log[c*x^
n])^2*PolyLog[3, -(d*f*x^m)])/m^2 - (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[4
, -(d*f*x^m)])/m^3 + (6*b^3*n^3*PolyLog[5, -(d*f*x^m)])/m^4
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{\left(a+b\log\left(cx^n\right)\right)^3\operatorname{Li}_2\left(-dfx^m\right)}{m} + \frac{(3bn)\int\frac{\left(a+b\log(cx^n)\right)^2\operatorname{Li}_2\left(-dfx^m\right.}{x}}{m}$$

$$= -\frac{\left(a+b\log\left(cx^n\right)\right)^3\operatorname{Li}_2\left(-dfx^m\right)}{m} + \frac{3bn\left(a+b\log\left(cx^n\right)\right)^2\operatorname{Li}_3\left(-\right.}{m^2}$$

$$= -\frac{\left(a+b\log\left(cx^n\right)\right)^3\operatorname{Li}_2\left(-dfx^m\right)}{m} + \frac{3bn\left(a+b\log\left(cx^n\right)\right)^2\operatorname{Li}_3\left(-\right.}{m^2}$$

$$= -\frac{\left(a+b\log\left(cx^n\right)\right)^3\operatorname{Li}_2\left(-dfx^m\right)}{m} + \frac{3bn\left(a+b\log\left(cx^n\right)\right)^2\operatorname{Li}_3\left(-\right.}{m^2}$$

Mathematica [B] time = 0.41, size = 1035, normalized size = 9.86

$$-\frac{3}{10}b^3mn^3\log^5(x)+\frac{3}{4}ab^2mn^2\log^4(x)+\frac{3}{4}b^3mn^2\log\left(cx^n\right)\log^4(x)-\frac{3}{4}b^3n^3\log\left(\frac{x^{-m}}{df}+1\right)\log^4(x)+\frac{3}{4}b^3n^3\log\left(d\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -1/2*(a^2*b*m*n*Log[x]^3) + (3*a*b^2*m*n^2*Log[x]^4)/4 - (3*b^3*m*n^3*Log[x
]^5)/10 - a*b^2*m*n*Log[x]^3*Log[c*x^n] + (3*b^3*m*n^2*Log[x]^4*Log[c*x^n])
/4 - (b^3*m*n*Log[x]^3*Log[c*x^n]^2)/2 - (3*a^2*b*n*Log[x]^2*Log[1 + 1/(d*f
*x^m)])/2 + 2*a*b^2*n^2*Log[x]^3*Log[1 + 1/(d*f*x^m)] - (3*b^3*n^3*Log[x]^4
*Log[1 + 1/(d*f*x^m)])/4 - 3*a*b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + 1/(d*f*x^m
)] + 2*b^3*n^2*Log[x]^3*Log[c*x^n]*Log[1 + 1/(d*f*x^m)] - (3*b^3*n*Log[x]^2
*Log[c*x^n]^2*Log[1 + 1/(d*f*x^m)])/2 + (3*a^2*b*n*Log[x]^2*Log[1 + d*f*x^m
])/2 - 2*a*b^2*n^2*Log[x]^3*Log[1 + d*f*x^m] + (3*b^3*n^3*Log[x]^4*Log[1 +
d*f*x^m])/4 + (a^3*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (3*a^2*b*n*Log[x]*
Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m + (3*a*b^2*n^2*Log[x]^2*Log[-(d*f*x^m)]
*Log[1 + d*f*x^m])/m - (b^3*n^3*Log[x]^3*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/
m + 3*a*b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + d*f*x^m] - 2*b^3*n^2*Log[x]^3*Log
[c*x^n]*Log[1 + d*f*x^m] + (3*a^2*b*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*
x^m])/m - (6*a*b^2*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m
+ (3*b^3*n^2*Log[x]^2*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m + (3*b
^3*n*Log[x]^2*Log[c*x^n]^2*Log[1 + d*f*x^m])/2 + (3*a*b^2*Log[-(d*f*x^m)]*L
og[c*x^n]^2*Log[1 + d*f*x^m])/m - (3*b^3*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n
]^2*Log[1 + d*f*x^m])/m + (b^3*Log[-(d*f*x^m)]*Log[c*x^n]^3*Log[1 + d*f*x^m
])/m + (b*n*Log[x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a + b*Log[c*x^n]) + 3*
(a + b*Log[c*x^n])^2)*PolyLog[2, -(1/(d*f*x^m))])/m + ((a - b*n*Log[x] + b*
Log[c*x^n])^3*PolyLog[2, 1 + d*f*x^m])/m + (3*a^2*b*n*PolyLog[3, -(1/(d*f*x
^m))])/m^2 + (6*a*b^2*n*Log[c*x^n]*PolyLog[3, -(1/(d*f*x^m))])/m^2 + (3*b^3
*n*Log[c*x^n]^2*PolyLog[3, -(1/(d*f*x^m))])/m^2 + (6*a*b^2*n^2*PolyLog[4, -
(1/(d*f*x^m))])/m^3 + (6*b^3*n^2*Log[c*x^n]*PolyLog[4, -(1/(d*f*x^m))])/m^3
 + (6*b^3*n^3*PolyLog[5, -(1/(d*f*x^m))])/m^4
```

fricas [C] time = 0.74, size = 285, normalized size = 2.71

$$\frac{6\,b^3n^3\operatorname{polylog}\left(5,-dfx^m\right)-\left(b^3m^3n^3\log(x)^3+b^3m^3\log(c)^3+3\,ab^2m^3\log(c)^2+3\,a^2bm^3\log(c)+a^3m^3+3\,\right(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

```
[Out] (6*b^3*n^3*polylog(5, -d*f*x^m) - (b^3*m^3*n^3*log(x)^3 + b^3*m^3*log(c)^3
+ 3*a*b^2*m^3*log(c)^2 + 3*a^2*b*m^3*log(c) + a^3*m^3 + 3*(b^3*m^3*n^2*log(
```

```
c) + a*b^2*m^3*n^2)*log(x)^2 + 3*(b^3*m^3*n*log(c)^2 + 2*a*b^2*m^3*n*log(c)
 + a^2*b*m^3*n)*log(x))*dilog(-d*f*x^m) - 6*(b^3*m*n^3*log(x) + b^3*m*n^2*l
og(c) + a*b^2*m*n^2)*polylog(4, -d*f*x^m) + 3*(b^3*m^2*n^3*log(x)^2 + b^3*m
^2*n*log(c)^2 + 2*a*b^2*m^2*n*log(c) + a^2*b*m^2*n + 2*(b^3*m^2*n^2*log(c)
+ a*b^2*m^2*n^2)*log(x))*polylog(3, -d*f*x^m))/m^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right)^{3} \log\left(\left(fx^{m} + \frac{1}{d}\right)d\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^m + 1/d)*d)/x, x)
```

maple [C] time = 1.27, size = 11734, normalized size = 111.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(1/d+f*x^m))/x,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\left(b^{3}n^{3}\log(x)^{4} - 4\,b^{3}\log(x)\log\left(x^{n}\right)^{3} - 4\left(b^{3}n^{2}\log(c) + ab^{2}n^{2}\right)\log(x)^{3} + 6\left(b^{3}n\log(c)^{2} + 2\,ab^{2}n\log(c) + a^{2}bn\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b^
2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^2
+ 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3*n^
2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*a*b^
2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3
*a^2*b*log(c) + a^3)*log(x))*log(d*f*x^m + 1) - integrate(1/4*(4*b^3*d*f*m*
x^m*log(x)*log(x^n)^3 - 6*(b^3*d*f*m*n*log(x)^2 - 2*(b^3*d*f*m*log(c) + a*b
^2*d*f*m)*log(x))*x^m*log(x^n)^2 + 4*(b^3*d*f*m*n^2*log(x)^3 - 3*(b^3*d*f*m
*n*log(c) + a*b^2*d*f*m*n)*log(x)^2 + 3*(b^3*d*f*m*log(c)^2 + 2*a*b^2*d*f*m
*log(c) + a^2*b*d*f*m)*log(x))*x^m*log(x^n) - (b^3*d*f*m*n^3*log(x)^4 - 4*(
b^3*d*f*m*n^2*log(c) + a*b^2*d*f*m*n^2)*log(x)^3 + 6*(b^3*d*f*m*n*log(c)^2
+ 2*a*b^2*d*f*m*n*log(c) + a^2*b*d*f*m*n)*log(x)^2 - 4*(b^3*d*f*m*log(c)^3
+ 3*a*b^2*d*f*m*log(c)^2 + 3*a^2*b*d*f*m*log(c) + a^3*d*f*m)*log(x))*x^m)/(
d*f*x*x^m + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,\left(f\,x^m+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^3)/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**m))/x,x)
```

```
[Out] Timed out
```

3.66 $\int \frac{\left(a+b\log(cx^n)\right)^2 \log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x} dx$

Optimal. Leaf size=73

$$\frac{2bn\text{Li}_3\left(-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)}{m^2}-\frac{\text{Li}_2\left(-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^2}{m}-\frac{2b^2n^2\text{Li}_4\left(-dfx^m\right)}{m^3}$$

```
[Out] -(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^m)/m+2*b*n*(a+b*ln(c*x^n))*polylog(3,-d
*f*x^m)/m^2-2*b^2*n^2*polylog(4,-d*f*x^m)/m^3
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2374, 2383, 6589}

$$\frac{2bn\text{PolyLog}\left(3,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)}{m^2}-\frac{\text{PolyLog}\left(2,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)^2}{m}-\frac{2b^2n^2\text{PolyLog}\left(4,-dfx^m\right)}{m^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -(((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*x^m)])/m) + (2*b*n*(a + b*Log[c*x^
n])*PolyLog[3, -(d*f*x^m)])/m^2 - (2*b^2*n^2*PolyLog[4, -(d*f*x^m)])/m^3
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx &= -\frac{\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2\left(-dfx^m\right)}{m}+\frac{(2bn)\int\frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(-dfx^m\right)}{x}\,dx}{m}\\ &= -\frac{\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2\left(-dfx^m\right)}{m}+\frac{2bn\left(a+b\log\left(cx^n\right)\right)\text{Li}_3\left(-dfx^{m}\right)}{m^2}\\ &= -\frac{\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2\left(-dfx^m\right)}{m}+\frac{2bn\left(a+b\log\left(cx^n\right)\right)\text{Li}_3\left(-dfx^{m}\right)}{m^2}\end{aligned}$$

Mathematica [B] time = 0.24, size = 526, normalized size = 7.21

$$\frac{a^2\log\left(-dfx^m\right)\log\left(dfx^m+1\right)}{m}+\frac{bn\log(x)\text{Li}_2\left(-\frac{x^{-m}}{df}\right)\left(2\left(a+b\log\left(cx^n\right)\right)-bn\log(x)\right)}{m}+\frac{\text{Li}_2\left(dfx^m+1\right)\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -1/3*(a*b*m*n*Log[x]^3) + (b^2*m*n^2*Log[x]^4)/4 - (b^2*m*n*Log[x]^3*Log[c*
x^n])/3 - a*b*n*Log[x]^2*Log[1 + 1/(d*f*x^m)] + (2*b^2*n^2*Log[x]^3*Log[1 +
 1/(d*f*x^m)])/3 - b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + 1/(d*f*x^m)] + a*b*n*L
og[x]^2*Log[1 + d*f*x^m] - (2*b^2*n^2*Log[x]^3*Log[1 + d*f*x^m])/3 + (a^2*L
og[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (2*a*b*n*Log[x]*Log[-(d*f*x^m)]*Log[1
+ d*f*x^m])/m + (b^2*n^2*Log[x]^2*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m + b^2
*n*Log[x]^2*Log[c*x^n]*Log[1 + d*f*x^m] + (2*a*b*Log[-(d*f*x^m)]*Log[c*x^n]
*Log[1 + d*f*x^m])/m - (2*b^2*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d
*f*x^m])/m + (b^2*Log[-(d*f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m + (b*n*L
og[x]*(-(b*n*Log[x]) + 2*(a + b*Log[c*x^n]))*PolyLog[2, -(1/(d*f*x^m))])/m
+ ((a - b*n*Log[x] + b*Log[c*x^n])^2*PolyLog[2, 1 + d*f*x^m])/m + (2*a*b*n*
PolyLog[3, -(1/(d*f*x^m))])/m^2 + (2*b^2*n*Log[c*x^n]*PolyLog[3, -(1/(d*f*x
^m))])/m^2 + (2*b^2*n^2*PolyLog[4, -(1/(d*f*x^m))])/m^3
```

fricas [C] time = 0.86, size = 131, normalized size = 1.79

$$-\frac{2\,b^2n^2\text{polylog}\left(4,-dfx^m\right)+\left(b^2m^2n^2\log(x)^2+b^2m^2\log(c)^2+2\,abm^2\log(c)+a^2m^2+2\left(b^2m^2n\log(c)+al\right.\right.}{m^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

```
[Out] -(2*b^2*n^2*polylog(4, -d*f*x^m) + (b^2*m^2*n^2*log(x)^2 + b^2*m^2*log(c)^2
 + 2*a*b*m^2*log(c) + a^2*m^2 + 2*(b^2*m^2*n*log(c) + a*b*m^2*n)*log(x))*di
log(-d*f*x^m) - 2*(b^2*m*n^2*log(x) + b^2*m*n*log(c) + a*b*m*n)*polylog(3,
-d*f*x^m))/m^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^m + 1/d)*d)/x, x)
```

maple [C] time = 1.08, size = 2578, normalized size = 35.32

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(1/d+f*x^m))/x,x)
```

```
[Out] -I/m*dilog(d*f*x^m+1)*n*ln(x)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/
3*b^2*n^2*ln(x)^3*ln(d*f*x^m+1)+1/3*b^2/n*ln(d*(1/d+f*x^m))*ln(x^n)^3+1/3*b
^2*n^2*ln(1/d+f*x^m)*ln(x)^3-1/3*b^2/n*ln(1/d+f*x^m)*ln(x^n)^3-b^2/m*dilog(
d*f*x^m+1)*ln(x^n)^2-b^2*ln(x)*ln(d*f*x^m+1)*ln(x^n)^2-1/m*dilog(d*f*x^m+1)
*ln(c)^2*b^2+b^2*ln(1/d+f*x^m)*ln(x)*ln(x^n)^2-1/m*dilog(d*f*x^m+1)*a^2+I*n
```

```
/m*ln(x)*polylog(2,-d*f*x^m)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*l
n(x)*ln(d*f*x^m+1)*ln(x^n)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*ln(x)*ln(d*
f*x^m+1)*ln(x^n)*b^2*Pi*csgn(I*c*x^n)^2*csgn(I*c)-I/m*dilog(d*f*x^m+1)*ln(c
)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-I/m*dilog(d*f*x^m+1)*ln(x^n)*b^2*Pi*cs
gn(I*c*x^n)^2*csgn(I*c)-I/m*dilog(d*f*x^m+1)*ln(x^n)*b^2*Pi*csgn(I*x^n)*csg
n(I*c*x^n)^2-I*ln(x)*ln(x^n)*ln(d*(1/d+f*x^m))*b^2*Pi*csgn(I*x^n)*csgn(I*c*
x^n)*csgn(I*c)+I*n/m^2*polylog(3,-d*f*x^m)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)
^2+I*n/m^2*polylog(3,-d*f*x^m)*b^2*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/2*I*n*ln(
x)^2*ln(d*f*x^m+1)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/2*I*n*ln(d*
(1/d+f*x^m))*ln(x)^2*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I/m*dilog(d
*f*x^m+1)*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+I/m*dilog(d*f*x^m+1)*ln(x^n)*b
^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I/m*dilog(d*f*x^m+1)*Pi*a*b*csgn(
I*c*x^n)^2*csgn(I*c)-I/m*dilog(d*f*x^m+1)*n*ln(x)*b^2*Pi*csgn(I*c*x^n)^3-I/
m*dilog(d*f*x^m+1)*ln(c)*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)-1/2*I*n*ln(d*(1/d
+f*x^m))*ln(x)^2*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*n*ln(x)^2*ln(d*f*
x^m+1)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-2*n/m*ln(x)*polylog(2,-d*f*x^m)*b
^2*ln(c)+2/m*dilog(d*f*x^m+1)*n*ln(x)*b^2*ln(c)-2*b^2*n/m*ln(x)*polylog(2,-
d*f*x^m)*ln(x^n)-I*n/m*ln(x)*polylog(2,-d*f*x^m)*b^2*Pi*csgn(I*x^n)*csgn(I*
c*x^n)^2-2*b/m*dilog(d*f*x^m+1)*ln(x^n)*a-2*b*ln(x)*ln(d*f*x^m+1)*ln(x^n)*a
-n*ln(d*(1/d+f*x^m))*ln(x)^2*b^2*ln(c)-2/m*dilog(d*f*x^m+1)*ln(c)*a*b+2*ln(
x)*ln(x^n)*ln(d*(1/d+f*x^m))*b^2*ln(c)+n*ln(x)^2*ln(d*f*x^m+1)*b^2*ln(c)+2*
n/m^2*polylog(3,-d*f*x^m)*b^2*ln(c)+1/4/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*
c*x^n)^6+I*ln(x)*ln(x^n)*ln(d*(1/d+f*x^m))*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)
^2+I*ln(x)*ln(x^n)*ln(d*(1/d+f*x^m))*b^2*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/2*I
*n*ln(x)^2*ln(d*f*x^m+1)*b^2*Pi*csgn(I*c*x^n)^2*csgn(I*c)+I*n/m*ln(x)*polyl
og(2,-d*f*x^m)*b^2*Pi*csgn(I*c*x^n)^3+2*b^2/m*dilog(d*f*x^m+1)*ln(x)*ln(x^n
)*n-2*b*n/m*ln(x)*polylog(2,-d*f*x^m)*a+2*b/m*dilog(d*f*x^m+1)*n*ln(x)*a-I*
n/m*ln(x)*polylog(2,-d*f*x^m)*b^2*Pi*csgn(I*c*x^n)^2*csgn(I*c)-I*n/m^2*poly
log(3,-d*f*x^m)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2/m*dilog(d*f*
x^m+1)*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5+b^2*ln(x)^2*ln(d*f*x^m+1)*ln(x^
n)*n+b^2*n^2/m*ln(x)^2*polylog(2,-d*f*x^m)-b^2*n*ln(1/d+f*x^m)*ln(x)^2*ln(x
^n)-b^2/m*dilog(d*f*x^m+1)*ln(x)^2*n^2-1/2*I*n*ln(d*(1/d+f*x^m))*ln(x)^2*b^
2*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/2/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*x^n)^
2*csgn(I*c*x^n)^3*csgn(I*c)+1/4/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*x^n)^2*c
sgn(I*c*x^n)^2*csgn(I*c)^2+I/m*dilog(d*f*x^m+1)*ln(x^n)*b^2*Pi*csgn(I*c*x^n
)^3-b*n*ln(d*(1/d+f*x^m))*ln(x)^2*a+2*b*ln(x)*ln(x^n)*ln(d*(1/d+f*x^m))*a+b
*n*ln(x)^2*ln(d*f*x^m+1)*a+2*b*n/m^2*polylog(3,-d*f*x^m)*a-2/m*dilog(d*f*x^
m+1)*ln(x^n)*b^2*ln(c)-2*ln(x)*ln(d*f*x^m+1)*ln(x^n)*b^2*ln(c)+2*b^2*n/m^2*
polylog(3,-d*f*x^m)*ln(x^n)+I*ln(x)*ln(d*f*x^m+1)*ln(x^n)*b^2*Pi*csgn(I*c*x
^n)^3-1/2/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)
^2+I/m*dilog(d*f*x^m+1)*ln(c)*Pi*b^2*csgn(I*c*x^n)^3+I/m*dilog(d*f*x^m+1)*P
i*a*b*csgn(I*c*x^n)^3+1/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^
n)^4*csgn(I*c)-I*n/m^2*polylog(3,-d*f*x^m)*b^2*Pi*csgn(I*c*x^n)^3+1/2*I*n*l
n(d*(1/d+f*x^m))*ln(x)^2*b^2*Pi*csgn(I*c*x^n)^3-I*ln(x)*ln(x^n)*ln(d*(1/d+f
*x^m))*b^2*Pi*csgn(I*c*x^n)^3-1/2*I*n*ln(x)^2*ln(d*f*x^m+1)*b^2*Pi*csgn(I*c
*x^n)^3+1/4/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4-1/2/m
*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)+1/4/m*dilog(d*f*x^m+1)
*Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-2*b^2*n^2*polylog(4,-d*f*x^m)/m^3+I/m
*dilog(d*f*x^m+1)*n*ln(x)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+I*ln(x)*ln(d*f
*x^m+1)*ln(x^n)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+I/m*dilog(d*f*x^
m+1)*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+I/m*dilog(d*f*x^m+1)*
n*ln(x)*b^2*Pi*csgn(I*c*x^n)^2*csgn(I*c)+I/m*dilog(d*f*x^m+1)*Pi*a*b*csgn(I
*x^n)*csgn(I*c*x^n)*csgn(I*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(b^2n^2\log(x)^3 + 3\,b^2\log(x)\log\left(x^n\right)^2 - 3\left(b^2n\log(c) + abn\right)\log(x)^2 - 3\left(b^2n\log(x)^2 - 2\left(b^2\log(c) + ab\right)\log(x)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)*
log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*(b
^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log(d*f*x^m + 1) - integrate(1/3*
(3*b^2*d*f*m*x^m*log(x)*log(x^n)^2 - 3*(b^2*d*f*m*n*log(x)^2 - 2*(b^2*d*f*m
*log(c) + a*b*d*f*m)*log(x))*x^m*log(x^n) + (b^2*d*f*m*n^2*log(x)^3 - 3*(b^
2*d*f*m*n*log(c) + a*b*d*f*m*n)*log(x)^2 + 3*(b^2*d*f*m*log(c)^2 + 2*a*b*d*
f*m*log(c) + a^2*d*f*m)*log(x))*x^m)/(d*f*x*x^m + x), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,\left(f\,x^m+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^2)/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**m))/x,x)
```

```
[Out] Timed out
```

3.67 $\int \frac{\left(a+b\log(cx^n)\right)\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$

Optimal. Leaf size=40

$$\frac{bn\text{Li}_3\left(-dfx^m\right)}{m^2}-\frac{\text{Li}_2\left(-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)}{m}$$

```
[Out] -(a+b*ln(c*x^n))*polylog(2,-d*f*x^m)/m+b*n*polylog(3,-d*f*x^m)/m^2
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2374, 6589}

$$\frac{bn\text{PolyLog}\left(3,-dfx^m\right)}{m^2}-\frac{\text{PolyLog}\left(2,-dfx^m\right)\left(a+b\log\left(cx^n\right)\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -(((a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^m)])/m) + (b*n*PolyLog[3, -(d*f*x^
m)])/m^2
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx&=-\frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(-dfx^m\right)}{m}+\frac{(bn)\int \frac{\text{Li}_2\left(-dfx^m\right)}{x}\,dx}{m}\\&=-\frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(-dfx^m\right)}{m}+\frac{bn\text{Li}_3\left(-dfx^m\right)}{m^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.30

$$-\frac{a\text{Li}_2\left(-dfx^m\right)}{m}-\frac{b\log\left(cx^n\right)\text{Li}_2\left(-dfx^m\right)}{m}+\frac{bn\text{Li}_3\left(-dfx^m\right)}{m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -((a*PolyLog[2, -(d*f*x^m)])/m) - (b*Log[c*x^n]*PolyLog[2, -(d*f*x^m)])/m +
 (b*n*PolyLog[3, -(d*f*x^m)])/m^2
```

fricas [C] time = 0.86, size = 42, normalized size = 1.05

$$\frac{bn\text{polylog}\left(3, -dfx^{m}\right) - \left(bmn\log(x) + bm\log(c) + am\right)\text{Li}_2\left(-dfx^{m}\right)}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

```
[Out] (b*n*polylog(3, -d*f*x^m) - (b*m*n*log(x) + b*m*log(c) + a*m)*dilog(-d*f*x^
m))/m^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right) + a\right)\log\left(\left(fx^{m} + \frac{1}{d}\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + 1/d)*d)/x, x)
```

maple [C] time = 0.94, size = 308, normalized size = 7.70

$$-\frac{bn\ln(x)^2\ln\left(\left(f\,x^{m} + \frac{1}{d}\right)d\right)}{2} + \frac{bn\ln(x)^2\ln\left(d f\,x^{m} + 1\right)}{2} + \frac{i\pi b\,\text{dilog}\left(d f\,x^{m} + 1\right)\text{csgn}\left(ic\right)\text{csgn}\left(ix^{n}\right)\text{csgn}\left(ic\,x^{n}\right)}{2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln((f*x^m+1/d)*d)/x,x)
```

```
[Out] -1/2*b*ln((f*x^m+1/d)*d)*n*ln(x)^2+b*ln(x)*ln((f*x^m+1/d)*d)*ln(x^n)+1/2*b*
n*ln(x)^2*ln(d*f*x^m+1)-b*n/m*ln(x)*polylog(2,-d*f*x^m)+b*n*polylog(3,-d*f*
x^m)/m^2+b/m*dilog(d*f*x^m+1)*n*ln(x)-b/m*dilog(d*f*x^m+1)*ln(x^n)-b*ln(x)*
ln(d*f*x^m+1)*ln(x^n)-1/2*I/m*dilog(d*f*x^m+1)*b*Pi*csgn(I*x^n)*csgn(I*c*x^
n)^2+1/2*I/m*dilog(d*f*x^m+1)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/2*
I/m*dilog(d*f*x^m+1)*b*Pi*csgn(I*c*x^n)^3-1/2*I/m*dilog(d*f*x^m+1)*b*Pi*csg
n(I*c*x^n)^2*csgn(I*c)-1/m*dilog(d*f*x^m+1)*b*ln(c)-1/m*dilog(d*f*x^m+1)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(bn\log(x)^2 - 2\,b\log(x)\log\left(x^{n}\right) - 2\left(b\log(c) + a\right)\log(x)\right)\log\left(dfx^{m} + 1\right) - \int \frac{2\,bdfmx^{m}\log(x)\log\left(x^{n}\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log(d*f
*x^m + 1) - integrate(1/2*(2*b*d*f*m*x^m*log(x)*log(x^n) - (b*d*f*m*n*log(x
)^2 - 2*(b*d*f*m*log(c) + a*d*f*m)*log(x))*x^m)/(d*f*x*x^m + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(d\left(f\,x^{m} + \frac{1}{d}\right)\right)\left(a + b\,\ln\left(c\,x^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**m))/x,x)
```

```
[Out] Timed out
```

3.68 $\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)},x\right)$$

[Out] Unintegrable(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n)),x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])),x]

[Out] Defer[Int][Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])), x]

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Mathematica [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])),x]

[Out] Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])), x]

fricas [A] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(dfx^m+1\right)}{bx\log\left(cx^n\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n)),x, algorithm="fricas")

[Out] integral(log(d*f*x^m + 1)/(b*x*log(c*x^n) + a*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{\left(b\log\left(cx^n\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((f*x^m + 1/d)*d)/((b*log(c*x^n) + a)*x), x)
```

maple [A] time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(\left(f\,x^{m}+\frac{1}{d}\right) d\right)}{\left(b \ln \left(c\,x^{n}\right)+a\right) x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((f*x^m+1/d)*d)/x/(b*ln(c*x^n)+a),x)
```

```
[Out] int(ln((f*x^m+1/d)*d)/x/(b*ln(c*x^n)+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(\left(f x^{m}+\frac{1}{d}\right) d\right)}{\left(b \log \left(c x^{n}\right)+a\right) x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] integrate(log((f*x^m + 1/d)*d)/((b*log(c*x^n) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln \left(d\,\left(f\,x^{m}+\frac{1}{d}\right)\right)}{x\,\left(a+b\,\ln \left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))),x)
```

```
[Out] int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(1/d+f*x**m))/x/(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

3.69 $\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x(a+b\log(cx^n))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n))^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2),x]
```

```
[Out] Defer[Int][Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2), x]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx = \int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Mathematica [A] time = 1.99, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2),x]
```

```
[Out] Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2), x]
```

fricas [A] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(dfx^m+1\right)}{b^2x\log\left(cx^n\right)^2+2\,abx\log\left(cx^n\right)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(log(d*f*x^m + 1)/(b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)
, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(fx^m+\frac{1}{d}\right)d\right)}{\left(b\log\left(cx^n\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(log((f*x^m + 1/d)*d)/((b*log(c*x^n) + a)^2*x), x)
```

maple [A] time = 4.70, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\left(f\,x^{m}+\frac{1}{d}\right)d\right)}{\left(b\ln\left(c\,x^{n}\right)+a\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((f*x^m+1/d)*d)/x/(b*ln(c*x^n)+a)^2,x)
```

```
[Out] int(ln((f*x^m+1/d)*d)/x/(b*ln(c*x^n)+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$dfm\int \frac{x^{m}}{\left(b^{2}dfn\log(c)+abdfn\right)xx^{m}+\left(b^{2}n\log(c)+abn\right)x+\left(b^{2}dfnxx^{m}+b^{2}nx\right)\log\left(x^{n}\right)}\,dx-\frac{\log\left(dfx^{m}\right.}{b^{2}n\log(c)+b^{2}n\,\mathrm{l}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] d*f*m*integrate(x^m/((b^2*d*f*n*log(c) + a*b*d*f*n)*x*x^m + (b^2*n*log(c) +
 a*b*n)*x + (b^2*d*f*n*x*x^m + b^2*n*x)*log(x^n)), x) - log(d*f*x^m + 1)/(b
^2*n*log(c) + b^2*n*log(x^n) + a*b*n)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\,\left(f\,x^{m}+\frac{1}{d}\right)\right)}{x\,{\left(a+b\,\ln\left(c\,x^{n}\right)\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))^2),x)
```

```
[Out] int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(1/d+f*x**m))/x/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Timed out
```

3.70 $\int x^3 \left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)\,dx$

Optimal. Leaf size=283

$$\frac{1}{4}x^4\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)-\frac{e^4m\log(e+fx)\left(a+b\log\left(cx^n\right)\right)}{4f^4}+\frac{e^3mx\left(a+b\log\left(cx^n\right)\right)}{4f^3}-\frac{e^2mx^2\left(a+\right.}{8\ }$$

```
[Out] -5/16*b*e^3*m*n*x/f^3+3/32*b*e^2*m*n*x^2/f^2-7/144*b*e*m*n*x^3/f+1/32*b*m*n
*x^4+1/4*e^3*m*x*(a+b*ln(c*x^n))/f^3-1/8*e^2*m*x^2*(a+b*ln(c*x^n))/f^2+1/12
*e*m*x^3*(a+b*ln(c*x^n))/f-1/16*m*x^4*(a+b*ln(c*x^n))+1/16*b*e^4*m*n*ln(f*x
+e)/f^4+1/4*b*e^4*m*n*ln(-f*x/e)*ln(f*x+e)/f^4-1/4*e^4*m*(a+b*ln(c*x^n))*ln
(f*x+e)/f^4-1/16*b*n*x^4*ln(d*(f*x+e)^m)+1/4*x^4*(a+b*ln(c*x^n))*ln(d*(f*x+
e)^m)+1/4*b*e^4*m*n*polylog(2,1+f*x/e)/f^4
```

Rubi [A] time = 0.21, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2395, 43, 2376, 2394, 2315}

$$\frac{be^4mn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{4f^4}+\frac{1}{4}x^4\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)-\frac{e^4m\log(e+fx)\left(a+b\log\left(cx^n\right)\right)}{4f^4}+\frac{e^3mx\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (-5*b*e^3*m*n*x)/(16*f^3) + (3*b*e^2*m*n*x^2)/(32*f^2) - (7*b*e*m*n*x^3)/(1
44*f) + (b*m*n*x^4)/32 + (e^3*m*x*(a + b*Log[c*x^n]))/(4*f^3) - (e^2*m*x^2*
(a + b*Log[c*x^n]))/(8*f^2) + (e*m*x^3*(a + b*Log[c*x^n]))/(12*f) - (m*x^4*
(a + b*Log[c*x^n]))/16 + (b*e^4*m*n*Log[e + f*x])/(16*f^4) + (b*e^4*m*n*Log
[-((f*x)/e)]*Log[e + f*x])/(4*f^4) - (e^4*m*(a + b*Log[c*x^n])*Log[e + f*x]
)/(4*f^4) - (b*n*x^4*Log[d*(e + f*x)^m])/16 + (x^4*(a + b*Log[c*x^n])*Log[d
*(e + f*x)^m])/4 + (b*e^4*m*n*PolyLog[2, 1 + (f*x)/e])/(4*f^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x^3 \left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)\,dx = \frac{e^3mx\left(a+b\log\left(cx^n\right)\right)}{4f^3} - \frac{e^2mx^2\left(a+b\log\left(cx^n\right)\right)}{8f^2} + \frac{emx^3\left(a+b\log\right.}{12f}$$

$$= -\frac{be^3mnx}{4f^3} + \frac{be^2mnx^2}{16f^2} - \frac{bemnx^3}{36f} + \frac{1}{64}bmnx^4 + \frac{e^3mx\left(a+b\log\left(cx\right.\right.}{4f^3}$$

$$= -\frac{be^3mnx}{4f^3} + \frac{be^2mnx^2}{16f^2} - \frac{bemnx^3}{36f} + \frac{1}{64}bmnx^4 + \frac{e^3mx\left(a+b\log\left(cx\right.\right.}{4f^3}$$

$$= -\frac{be^3mnx}{4f^3} + \frac{be^2mnx^2}{16f^2} - \frac{bemnx^3}{36f} + \frac{1}{64}bmnx^4 + \frac{e^3mx\left(a+b\log\left(cx\right.\right.}{4f^3}$$

$$= -\frac{5be^3mnx}{16f^3} + \frac{3be^2mnx^2}{32f^2} - \frac{7bemnx^3}{144f} + \frac{1}{32}bmnx^4 + \frac{e^3mx\left(a+b\log\right.}{4f^3}$$

Mathematica [A] time = 0.22, size = 290, normalized size = 1.02

$$-\frac{-72af^4x^4\log\left(d(e+fx)^m\right)+72ae^4m\log(e+fx)-72ae^3fmx+36ae^2f^2mx^2-24aef^3mx^3+18af^4mx^4+6b\,l}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] -1/288*(-72*a*e^3*f*m*x + 90*b*e^3*f*m*n*x + 36*a*e^2*f^2*m*x^2 - 27*b*e^2*
f^2*m*n*x^2 - 24*a*e*f^3*m*x^3 + 14*b*e*f^3*m*n*x^3 + 18*a*f^4*m*x^4 - 9*b*
f^4*m*n*x^4 + 72*a*e^4*m*Log[e + f*x] - 18*b*e^4*m*n*Log[e + f*x] - 72*b*e^
4*m*n*Log[x]*Log[e + f*x] - 72*a*f^4*x^4*Log[d*(e + f*x)^m] + 18*b*f^4*n*x^
4*Log[d*(e + f*x)^m] + 6*b*Log[c*x^n]*(f*m*x*(-12*e^3 + 6*e^2*f*x - 4*e*f^2
*x^2 + 3*f^3*x^3) + 12*e^4*m*Log[e + f*x] - 12*f^4*x^4*Log[d*(e + f*x)^m])
+ 72*b*e^4*m*n*Log[x]*Log[1 + (f*x)/e] + 72*b*e^4*m*n*PolyLog[2, -((f*x)/e)
])/f^4
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3\log\left(cx^n\right)+ax^3\right)\log\left(\left(fx+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*log(c*x^n) + a*x^3)*log((f*x + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^{n}\right)+a\right)x^{3}\log\left(\left(fx+e\right)^{m}d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3*log((f*x + e)^m*d), x)
```

maple [C] time = 0.87, size = 2403, normalized size = 8.49

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*x^n)+a)*ln(d*(f*x+e)^m),x)
```

```
[Out] -1/8*I/f^3*Pi*b*e^3*m*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*x-5/16*b*e^3*m*n*
x/f^3+3/32*b*e^2*m*n*x^2/f^2-7/144*b*e*m*n*x^3/f-1/24*I/f*Pi*x^3*b*e*m*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/16*x^4*a*m+1/4*x^4*ln(d)*a-1/16*m*b*ln(x^
n)*x^4+1/4*ln(d)*b*x^4*ln(x^n)+1/32*I*x^4*Pi*b*m*csgn(I*x^n)*csgn(I*c*x^n)*
csgn(I*c)+(1/4*b*x^4*ln(x^n)+1/16*x^4*(2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2
-2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-2*I*b*Pi*csgn(I*c*x^n)^3+2*I*
b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+4*b*ln(c)-b*n+4*a))*ln((f*x+e)^m)+1/16*I/f^2
*Pi*x^2*b*e^2*m*csgn(I*c*x^n)^3-1/8*I/f^3*Pi*b*e^3*m*csgn(I*c*x^n)^3*x-1/8*
I*x^4*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/16*Pi^2*csgn(I*d)*cs
gn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^4*b*csgn(I*c*x^n)^2*csgn(I*c)+1/16*Pi
^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c
)-1/16*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^4*b*csgn(I*c*
x^n)^3-1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^4*b*csgn(I*x^n)*csgn(I*c
*x^n)^2+1/4*x^4*ln(c)*ln(d)*b-1/16*x^4*ln(c)*b*m-1/16*ln(d)*b*n*x^4+1/8*I/f
^3*Pi*b*e^3*m*csgn(I*c*x^n)^2*csgn(I*c)*x-1/8*I/f^4*e^4*m*ln(f*x+e)*Pi*b*cs
gn(I*x^n)*csgn(I*c*x^n)^2-1/8*I/f^4*e^4*m*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^2*cs
gn(I*c)-1/16*I/f^2*Pi*x^2*b*e^2*m*csgn(I*c*x^n)^2*csgn(I*c)+1/8*I/f^3*Pi*b*
e^3*m*csgn(I*x^n)*csgn(I*c*x^n)^2*x+1/16*b*e^4*m*n*ln(f*x+e)/f^4-1/8*I*x^4*
ln(c)*Pi*b*csgn(I*d*(f*x+e)^m)^3-1/8*I*x^4*Pi*ln(d)*b*csgn(I*c*x^n)^3+1/8*I
*x^4*Pi*a*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/16*Pi^2*csgn(I*(f*x+e)^m)*csgn(
I*d*(f*x+e)^m)^2*x^4*b*csgn(I*c*x^n)^3+1/16*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^4*
b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I*Pi*csgn(I*d*(f*x+e)^m)^3*b*x^4*ln(x^n)-
1/32*I*Pi*b*n*x^4*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/8*I*Pi*csgn(I*(
f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b*x^4*ln(x^n)+1/8*I*Pi*csgn(I*d)*csgn(I*d*(
f*x+e)^m)^2*b*x^4*ln(x^n)-1/32*I*x^4*Pi*b*m*csgn(I*x^n)*csgn(I*c*x^n)^2-1/1
6*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^4*b*csgn(I*c*x^n)^3-1/8*I*x^4*Pi*a*csgn(I*d*
(f*x+e)^m)^3+1/12*m/f*b*ln(x^n)*e*x^3+1/8*I*x^4*Pi*a*csgn(I*(f*x+e)^m)*csgn
(I*d*(f*x+e)^m)^2-1/32*I*x^4*Pi*b*m*csgn(I*c*x^n)^2*csgn(I*c)+1/8*I*x^4*ln(
c)*Pi*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/8*I*x^4*ln(c)*Pi*b*csgn(I*(f*x+e)
^m)*csgn(I*d*(f*x+e)^m)^2+1/8*I*x^4*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-
1/8*I*x^4*ln(c)*Pi*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/8*I*
Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b*x^4*ln(x^n)+1/8*I/f^4*
e^4*m*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^3-1/24*I/f*Pi*x^3*b*e*m*csgn(I*c*x^n)^3-
205/576*b*e^4*m*n/f^4+1/4/f^3*a*e^3*m*x+1/12/f*x^3*a*e*m-1/8/f^2*x^2*a*e^2*
m-1/4/f^4*e^4*m*ln(f*x+e)*a+1/32*I*Pi*b*n*x^4*csgn(I*d*(f*x+e)^m)^3+1/4*b*e
^4*m*n*ln(-f*x/e)*ln(f*x+e)/f^4+1/16*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^4*b*csgn(
I*c*x^n)^2*csgn(I*c)+1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^4*b*csgn(I
*c*x^n)^3-1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^4*b*csgn(I*c*x^n)^2*c
sgn(I*c)-1/16*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x^4*b*csgn(I*x^n
)*csgn(I*c*x^n)^2-1/16*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x^4*b*c
sgn(I*c*x^n)^2*csgn(I*c)-1/16*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^4*b*csgn(I*x^n)*
csgn(I*c*x^n)*csgn(I*c)+1/8*I*x^4*Pi*ln(d)*b*csgn(I*c*x^n)^2*csgn(I*c)-1/8*
```

```
I*x^4*Pi*a*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/32*I*Pi*b*n*x^
4*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/32*b*m*n*x^4+1/16*I/f^2*Pi*x^2*b*e^2*m*
csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*I/f^4*e^4*m*ln(f*x+e)*Pi*b*csgn(I*x
^n)*csgn(I*c*x^n)*csgn(I*c)+1/16*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*
(f*x+e)^m)*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/32*I*Pi*b*n*x^4*csgn(I*d)*cs
gn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/4*n*b/f^4*e^4*m*dilog(-f*x/e)-1/8*m/f
^2*b*ln(x^n)*x^2*e^2+1/4*m/f^3*b*ln(x^n)*x*e^3-1/4*m/f^4*b*ln(x^n)*e^4*ln(f
*x+e)-1/4/f^4*e^4*m*ln(f*x+e)*b*ln(c)+1/4/f^3*ln(c)*b*e^3*m*x+1/12/f*ln(c)*
x^3*b*e*m-1/8/f^2*ln(c)*x^2*b*e^2*m+1/16*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f
*x+e)^m)^2*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/16*Pi^2*csgn(I*d)*cs
gn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*
c)+1/24*I/f*Pi*x^3*b*e*m*csgn(I*x^n)*csgn(I*c*x^n)^2+1/24*I/f*Pi*x^3*b*e*m*
csgn(I*c*x^n)^2*csgn(I*c)-1/16*I/f^2*Pi*x^2*b*e^2*m*csgn(I*x^n)*csgn(I*c*x^
n)^2+1/32*I*x^4*Pi*b*m*csgn(I*c*x^n)^3
```

maxima [A] time = 2.30, size = 381, normalized size = 1.35

$$-\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)be^4mn}{4\,f^4}-\frac{\left(4\,ae^4m-\left(e^4mn-4\,e^4m\log(c)\right)b\right)\log\left(fx+e\right)}{16\,f^4}+\frac{72\,be^4mn\log\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] -1/4*(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*e^4*m*n/f^4 - 1/16*(4*a*e^4*
m - (e^4*m*n - 4*e^4*m*log(c))*b)*log(f*x + e)/f^4 + 1/288*(72*b*e^4*m*n*lo
g(f*x + e)*log(x) - 9*(2*(f^4*m - 4*f^4*log(d))*a - (f^4*m*n - 2*f^4*n*log(
d) - 2*(f^4*m - 4*f^4*log(d))*log(c))*b)*x^4 + 2*(12*a*e*f^3*m - (7*e*f^3*m
*n - 12*e*f^3*m*log(c))*b)*x^3 - 9*(4*a*e^2*f^2*m - (3*e^2*f^2*m*n - 4*e^2*
f^2*m*log(c))*b)*x^2 + 18*(4*a*e^3*f*m - (5*e^3*f*m*n - 4*e^3*f*m*log(c))*b
)*x + 18*(4*b*f^4*x^4*log(x^n) + (4*a*f^4 - (f^4*n - 4*f^4*log(c))*b)*x^4)*
log((f*x + e)^m) + 6*(4*b*e*f^3*m*x^3 - 6*b*e^2*f^2*m*x^2 + 12*b*e^3*f*m*x
- 12*b*e^4*m*log(f*x + e) - 3*(f^4*m - 4*f^4*log(d))*b*x^4)*log(x^n))/f^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^3*log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

3.71 $\int x^2 \left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)\,dx$

Optimal. Leaf size=243

$$\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)+\frac{e^3m\log(e+fx)\left(a+b\log\left(cx^n\right)\right)}{3f^3}-\frac{e^2mx\left(a+b\log\left(cx^n\right)\right)}{3f^2}+\frac{emx^2\left(a+\right.}{6}$$

```
[Out] 4/9*b*e^2*m*n*x/f^2-5/36*b*e*m*n*x^2/f+2/27*b*m*n*x^3-1/3*e^2*m*x*(a+b*ln(c
*x^n))/f^2+1/6*e*m*x^2*(a+b*ln(c*x^n))/f-1/9*m*x^3*(a+b*ln(c*x^n))-1/9*b*e^
3*m*n*ln(f*x+e)/f^3-1/3*b*e^3*m*n*ln(-f*x/e)*ln(f*x+e)/f^3+1/3*e^3*m*(a+b*l
n(c*x^n))*ln(f*x+e)/f^3-1/9*b*n*x^3*ln(d*(f*x+e)^m)+1/3*x^3*(a+b*ln(c*x^n))
*ln(d*(f*x+e)^m)-1/3*b*e^3*m*n*polylog(2,1+f*x/e)/f^3
```

Rubi [A] time = 0.17, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2395, 43, 2376, 2394, 2315}

$$-\frac{be^3mn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{3f^3}+\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)+\frac{e^3m\log(e+fx)\left(a+b\log\left(cx^n\right)\right)}{3f^3}-\frac{e^2mx}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (4*b*e^2*m*n*x)/(9*f^2) - (5*b*e*m*n*x^2)/(36*f) + (2*b*m*n*x^3)/27 - (e^2*
m*x*(a + b*Log[c*x^n]))/(3*f^2) + (e*m*x^2*(a + b*Log[c*x^n]))/(6*f) - (m*x
^3*(a + b*Log[c*x^n]))/9 - (b*e^3*m*n*Log[e + f*x])/(9*f^3) - (b*e^3*m*n*Lo
g[-((f*x)/e)]*Log[e + f*x])/(3*f^3) + (e^3*m*(a + b*Log[c*x^n])*Log[e + f*x
])/(3*f^3) - (b*n*x^3*Log[d*(e + f*x)^m])/9 + (x^3*(a + b*Log[c*x^n])*Log[d
*(e + f*x)^m])/3 - (b*e^3*m*n*PolyLog[2, 1 + (f*x)/e])/(3*f^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)\,dx=-\frac{e^2mx\left(a+b\log\left(cx^n\right)\right)}{3f^2}+\frac{emx^2\left(a+b\log\left(cx^n\right)\right)}{6f}-\frac{1}{9}mx^3\left(a+b\log\right.$$

$$=\frac{be^2mnx}{3f^2}-\frac{bemnx^2}{12f}+\frac{1}{27}bmnx^3-\frac{e^2mx\left(a+b\log\left(cx^n\right)\right)}{3f^2}+\frac{emx^2\left(a\right.}{}$$

$$=\frac{be^2mnx}{3f^2}-\frac{bemnx^2}{12f}+\frac{1}{27}bmnx^3-\frac{e^2mx\left(a+b\log\left(cx^n\right)\right)}{3f^2}+\frac{emx^2\left(a\right.}{}$$

$$=\frac{be^2mnx}{3f^2}-\frac{bemnx^2}{12f}+\frac{1}{27}bmnx^3-\frac{e^2mx\left(a+b\log\left(cx^n\right)\right)}{3f^2}+\frac{emx^2\left(a\right.}{}$$

$$=\frac{4be^2mnx}{9f^2}-\frac{5bemnx^2}{36f}+\frac{2}{27}bmnx^3-\frac{e^2mx\left(a+b\log\left(cx^n\right)\right)}{3f^2}+\frac{emx^2}{}$$

Mathematica [A] time = 0.15, size = 252, normalized size = 1.04

$$\frac{36af^3x^3\log\left(d(e+fx)^m\right)+36ae^3m\log(e+fx)-36ae^2fmx+18aef^2mx^2-12af^3mx^3-6b\log\left(cx^n\right)\left(-6f^3x^3\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (-36*a*e^2*f*m*x + 48*b*e^2*f*m*n*x + 18*a*e*f^2*m*x^2 - 15*b*e*f^2*m*n*x^2
 - 12*a*f^3*m*x^3 + 8*b*f^3*m*n*x^3 + 36*a*e^3*m*Log[e + f*x] - 12*b*e^3*m*
n*Log[e + f*x] - 36*b*e^3*m*n*Log[x]*Log[e + f*x] + 36*a*f^3*x^3*Log[d*(e +
 f*x)^m] - 12*b*f^3*n*x^3*Log[d*(e + f*x)^m] - 6*b*Log[c*x^n]*(f*m*x*(6*e^2
 - 3*e*f*x + 2*f^2*x^2) - 6*e^3*m*Log[e + f*x] - 6*f^3*x^3*Log[d*(e + f*x)^
m]) + 36*b*e^3*m*n*Log[x]*Log[1 + (f*x)/e] + 36*b*e^3*m*n*PolyLog[2, -((f*x
)/e)])/(108*f^3)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2\log\left(cx^n\right)+ax^2\right)\log\left(\left(fx+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)*log((f*x + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x^2\log\left(\left(fx+e\right)^m d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*x + e)^m*d), x)
```

maple [C] time = 0.75, size = 2222, normalized size = 9.14

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)*ln(d*(f*x+e)^m),x)
```

```
[Out] -1/3*n*b/f^3*e^3*m*dilog(-1/e*f*x)-1/9*ln(d)*b*n*x^3+1/3*x^3*ln(c)*ln(d)*b-
1/9*x^3*ln(c)*b*m+1/3*x^3*ln(d)*a-1/9*m*b*ln(x^n)*x^3+1/3*ln(d)*b*x^3*ln(x^
n)-1/6*I/f^3*e^3*m*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^3-1/12*I/f*Pi*x^2*b*e*m*csg
n(I*c*x^n)^3+1/18*I*x^3*Pi*b*m*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/6*I*x^
3*ln(c)*Pi*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/6*I*x^3*Pi*l
n(d)*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/9*x^3*a*m+(1/3*b*x^3*ln(x^n)+1
/18*x^3*(3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*x^n)*csgn(I*c
*x^n)*csgn(I*c)-3*I*b*Pi*csgn(I*c*x^n)^3+3*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)
+6*b*ln(c)-2*b*n+6*a))*ln((f*x+e)^m)-1/12*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^3*b*
csgn(I*c*x^n)^3-1/6*I*x^3*Pi*a*csgn(I*d*(f*x+e)^m)^3-1/3/f^2*ln(c)*b*e^2*m*
x+1/6/f*ln(c)*x^2*b*e*m+1/3/f^3*e^3*m*ln(f*x+e)*b*ln(c)+1/12*Pi^2*csgn(I*d)
*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^3*b*csgn(I*c*x^n)^2*csgn(I*c)+1/6/
f*x^2*a*e*m+4/9*b*e^2*m*n*x/f^2-5/36*b*e*m*n*x^2/f-1/6*I/f^2*Pi*b*e^2*m*csg
n(I*c*x^n)^2*csgn(I*c)*x+1/6*I/f^3*e^3*m*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*
c*x^n)^2+1/6*I/f^3*e^3*m*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+1/12*I/f*
Pi*x^2*b*e*m*csgn(I*x^n)*csgn(I*c*x^n)^2-1/3/f^2*a*e^2*m*x+1/3/f^3*e^3*m*ln
(f*x+e)*a-1/9*b*e^3*m*n*ln(f*x+e)/f^3-1/6*I*Pi*csgn(I*d*(f*x+e)^m)^3*b*x^3*
ln(x^n)+1/12*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1
/12*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^3*b*csgn(I*c*x^n)^2*csgn(I*c)+1/12*Pi^2*cs
gn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^3*b*csgn(I*c*x^n)^3+1/12*Pi^2*csgn(I*(f*x+e
)^m)*csgn(I*d*(f*x+e)^m)^2*x^3*b*csgn(I*c*x^n)^3+1/18*I*x^3*Pi*b*m*csgn(I*c
*x^n)^3+1/18*I*Pi*b*n*x^3*csgn(I*d*(f*x+e)^m)^3-1/6*I*x^3*ln(c)*Pi*b*csgn(I
*d*(f*x+e)^m)^3-1/12*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x
^3*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/6*I/f^2*Pi*b*e^2*m*csgn(I*x^n)*c
sgn(I*c*x^n)^2*x+1/6*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b*x^3*ln(
x^n)+1/6*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b*x^3*ln(x^n)-1/18*I*x^3*Pi*b
*m*csgn(I*x^n)*csgn(I*c*x^n)^2-1/18*I*Pi*b*n*x^3*csgn(I*d)*csgn(I*d*(f*x+e)
^m)^2-1/6*I*x^3*Pi*ln(d)*b*csgn(I*c*x^n)^3+1/12*I/f*Pi*x^2*b*e*m*csgn(I*c*x
^n)^2*csgn(I*c)+49/108*b*e^3*n*m/f^3+1/6*I*x^3*Pi*a*csgn(I*d)*csgn(I*d*(f*x
+e)^m)^2-1/3*b*e^3*m*n*ln(-1/e*f*x)*ln(f*x+e)/f^3+2/27*b*m*n*x^3-1/18*I*Pi*
b*n*x^3*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/18*I*x^3*Pi*b*m*csgn(I*c*
x^n)^2*csgn(I*c)+1/6*I*x^3*ln(c)*Pi*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/12*
Pi^2*csgn(I*d*(f*x+e)^m)^3*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/6*I*
x^3*Pi*ln(d)*b*csgn(I*c*x^n)^2*csgn(I*c)-1/6*I*x^3*Pi*a*csgn(I*d)*csgn(I*(f
*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^3*
b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/12*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*
(f*x+e)^m)^2*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/12*Pi^2*csgn(I*d)*
csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/6
*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b*x^3*ln(x^n)+1/6*I/f
^2*Pi*b*e^2*m*csgn(I*c*x^n)^3*x-1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x
^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*
x^3*b*csgn(I*c*x^n)^2*csgn(I*c)-1/12*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e
)^m)^2*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/18*I*Pi*b*n*x^3*csgn(I*d)*csgn(I
*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/12*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(
I*d*(f*x+e)^m)*x^3*b*csgn(I*c*x^n)^3-1/12*I/f*Pi*x^2*b*e*m*csgn(I*x^n)*csgn
(I*c*x^n)*csgn(I*c)+1/6*I/f^2*Pi*b*e^2*m*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c
)*x-1/6*I/f^3*e^3*m*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/12
*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x^3*b*csgn(I*c*x^n)^2*csgn(I*
c)+1/6*I*x^3*ln(c)*Pi*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/6*I*x^3*P
```

```
i*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/6*m/f*b*ln(x^n)*e*x^2-1/3*m/f^2*b*l
n(x^n)*x*e^2+1/3*m/f^3*b*ln(x^n)*e^3*ln(f*x+e)+1/6*I*x^3*Pi*a*csgn(I*(f*x+e
)^m)*csgn(I*d*(f*x+e)^m)^2
```

maxima [A] time = 2.39, size = 328, normalized size = 1.35

$$\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\text{Li}_2\left(-\frac{fx}{e}\right)\right)be^3mn}{3\,f^3}+\frac{\left(3\,ae^3m-\left(e^3mn-3\,e^3m\log(c)\right)b\right)\log\left(fx+e\right)}{9\,f^3}-\frac{36\,be^3mn\log\left(fx+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] 1/3*(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*e^3*m*n/f^3 + 1/9*(3*a*e^3*m
- (e^3*m*n - 3*e^3*m*log(c))*b)*log(f*x + e)/f^3 - 1/108*(36*b*e^3*m*n*log(
f*x + e)*log(x) + 4*(3*(f^3*m - 3*f^3*log(d))*a - (2*f^3*m*n - 3*f^3*n*log(
d) - 3*(f^3*m - 3*f^3*log(d))*log(c))*b)*x^3 - 3*(6*a*e*f^2*m - (5*e*f^2*m*
n - 6*e*f^2*m*log(c))*b)*x^2 + 12*(3*a*e^2*f*m - (4*e^2*f*m*n - 3*e^2*f*m*l
og(c))*b)*x - 12*(3*b*f^3*x^3*log(x^n) + (3*a*f^3 - (f^3*n - 3*f^3*log(c))*
b)*x^3)*log((f*x + e)^m) - 6*(3*b*e*f^2*m*x^2 - 6*b*e^2*f*m*x + 6*b*e^3*m*l
og(f*x + e) - 2*(f^3*m - 3*f^3*log(d))*b*x^3)*log(x^n))/f^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

3.72 $\int x \left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)\,dx$

Optimal. Leaf size=203

$$\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)-\frac{e^2m\log(e+fx)\left(a+b\log\left(cx^n\right)\right)}{2f^2}+\frac{emx\left(a+b\log\left(cx^n\right)\right)}{2f}-\frac{1}{4}mx^2\left(a+b\right.$$

```
[Out] -3/4*b*e*m*n*x/f+1/4*b*m*n*x^2+1/2*e*m*x*(a+b*ln(c*x^n))/f-1/4*m*x^2*(a+b*l
n(c*x^n))+1/4*b*e^2*m*n*ln(f*x+e)/f^2+1/2*b*e^2*m*n*ln(-f*x/e)*ln(f*x+e)/f^
2-1/2*e^2*m*(a+b*ln(c*x^n))*ln(f*x+e)/f^2-1/4*b*n*x^2*ln(d*(f*x+e)^m)+1/2*x
^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)+1/2*b*e^2*m*n*polylog(2,1+f*x/e)/f^2
```

Rubi [A] time = 0.13, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2395, 43, 2376, 2394, 2315}

$$\frac{be^2mn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{2f^2}+\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)-\frac{e^2m\log(e+fx)\left(a+b\log\left(cx^n\right)\right)}{2f^2}+\frac{emx\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (-3*b*e*m*n*x)/(4*f) + (b*m*n*x^2)/4 + (e*m*x*(a + b*Log[c*x^n]))/(2*f) - (
m*x^2*(a + b*Log[c*x^n]))/4 + (b*e^2*m*n*Log[e + f*x])/(4*f^2) + (b*e^2*m*n
*Log[-((f*x)/e)]*Log[e + f*x])/(2*f^2) - (e^2*m*(a + b*Log[c*x^n])*Log[e +
f*x])/(2*f^2) - (b*n*x^2*Log[d*(e + f*x)^m])/4 + (x^2*(a + b*Log[c*x^n])*Lo
g[d*(e + f*x)^m])/2 + (b*e^2*m*n*PolyLog[2, 1 + (f*x)/e])/(2*f^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)\,dx=\frac{emx\left(a+b\log\left(cx^n\right)\right)}{2f}-\frac{1}{4}mx^2\left(a+b\log\left(cx^n\right)\right)-\frac{e^2m\left(a+b\log\left(cx\right.\right.}{2f^2}$$

$$=-\frac{bemnx}{2f}+\frac{1}{8}bmnx^2+\frac{emx\left(a+b\log\left(cx^n\right)\right)}{2f}-\frac{1}{4}mx^2\left(a+b\log\left(cx^{n}\right.\right.$$

$$=-\frac{bemnx}{2f}+\frac{1}{8}bmnx^2+\frac{emx\left(a+b\log\left(cx^n\right)\right)}{2f}-\frac{1}{4}mx^2\left(a+b\log\left(cx^{n}\right.\right.$$

$$=-\frac{bemnx}{2f}+\frac{1}{8}bmnx^2+\frac{emx\left(a+b\log\left(cx^n\right)\right)}{2f}-\frac{1}{4}mx^2\left(a+b\log\left(cx^{n}\right.\right.$$

$$=-\frac{3bemnx}{4f}+\frac{1}{4}bmnx^2+\frac{emx\left(a+b\log\left(cx^n\right)\right)}{2f}-\frac{1}{4}mx^2\left(a+b\log\left(cx\right.\right.$$

Mathematica [A] time = 0.12, size = 208, normalized size = 1.02

$$2af^2x^2\log\left(d(e+fx)^m\right)-2ae^2m\log(e+fx)+2aefmx-af^2mx^2+b\log\left(cx^n\right)\left(fx\left(2fx\log\left(d(e+fx)^m\right)+2em\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (2*a*e*f*m*x - 3*b*e*f*m*n*x - a*f^2*m*x^2 + b*f^2*m*n*x^2 - 2*a*e^2*m*Log[
e + f*x] + b*e^2*m*n*Log[e + f*x] + 2*b*e^2*m*n*Log[x]*Log[e + f*x] + 2*a*f
^2*x^2*Log[d*(e + f*x)^m] - b*f^2*n*x^2*Log[d*(e + f*x)^m] + b*Log[c*x^n]*(
-2*e^2*m*Log[e + f*x] + f*x*(2*e*m - f*m*x + 2*f*x*Log[d*(e + f*x)^m])) - 2
*b*e^2*m*n*Log[x]*Log[1 + (f*x)/e] - 2*b*e^2*m*n*PolyLog[2, -((f*x)/e)])/(4
*f^2)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx\log\left(cx^n\right)+ax\right)\log\left(\left(fx+e\right)^md\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)*log((f*x + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x\log\left(\left(fx+e\right)^md\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*log((f*x + e)^m*d), x)
```

maple [C] time = 0.73, size = 2041, normalized size = 10.05

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)*ln(d*(f*x+e)^m),x)
```

```
[Out] 1/2*n*b/f^2*e^2*m*dilog(-1/e*f*x)+1/4*I/f^2*e^2*m*ln(f*x+e)*Pi*b*csgn(I*x^n
)*csgn(I*c*x^n)*csgn(I*c)+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^2*b*cs
gn(I*c*x^n)^3-1/4*x^2*a*m+1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*
x^2*b*csgn(I*c*x^n)^3+1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^2*b*csgn(I*x^n)*csgn
(I*c*x^n)^2+1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^2*b*csgn(I*c*x^n)^2*csgn(I*c)-
1/4*I*Pi*csgn(I*d*(f*x+e)^m)^3*b*x^2*ln(x^n)+1/2*x^2*ln(d)*a-1/4*m*b*ln(x^n
)*x^2+1/2*ln(d)*b*x^2*ln(x^n)-1/4*ln(d)*b*n*x^2+1/2*x^2*ln(c)*ln(d)*b-1/4*x
^2*ln(c)*b*m+(1/2*b*x^2*ln(x^n)+1/4*x^2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2
-I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*c
sgn(I*c)*csgn(I*c*x^n)^2+2*b*ln(c)-b*n+2*a))*ln((f*x+e)^m)-3/4*b*e*m*n*x/f+
1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b*x^2*ln(x^n)+1/4*I*Pi*csgn(I*(f*x
+e)^m)*csgn(I*d*(f*x+e)^m)^2*b*x^2*ln(x^n)+1/4*I*x^2*ln(c)*Pi*b*csgn(I*d)*c
sgn(I*d*(f*x+e)^m)^2+1/4*I*x^2*ln(c)*Pi*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e
)^m)^2-1/8*I*Pi*b*n*x^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/8*I*Pi*b*n*x^2*cs
gn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/4*I*x^2*Pi*a*csgn(I*d)*csgn(I*(f*x+
e)^m)*csgn(I*d*(f*x+e)^m)+1/4*b*e^2*m*n*ln(f*x+e)/f^2-1/8*Pi^2*csgn(I*d*(f*
x+e)^m)^3*x^2*b*csgn(I*c*x^n)^3-1/4*I*x^2*Pi*a*csgn(I*d*(f*x+e)^m)^3-1/8*Pi
^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x^2*b*csgn(I*c*x^n)^2*csgn(I*c)-
1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/
8*I*x^2*Pi*b*m*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I*x^2*Pi*b*m*csgn(I*c*x^n)^2
*csgn(I*c)+1/4*I*x^2*Pi*a*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/8*I*Pi*
b*n*x^2*csgn(I*d*(f*x+e)^m)^3+1/4*I/f*Pi*b*e*m*csgn(I*x^n)*csgn(I*c*x^n)^2*
x+1/4*I/f*Pi*b*e*m*csgn(I*c*x^n)^2*csgn(I*c)*x-1/2*m*a*e^2/f^2*ln(f*x+e)+1/
2*e*a*m/f*x-5/8*b*e^2*n*m/f^2+1/2*b*e^2*m*n*ln(-1/e*f*x)*ln(f*x+e)/f^2-1/4*
I*x^2*ln(c)*Pi*b*csgn(I*d*(f*x+e)^m)^3-1/4*I*x^2*Pi*ln(d)*b*csgn(I*c*x^n)^3
+1/8*I*x^2*Pi*b*m*csgn(I*c*x^n)^3+1/4*I*x^2*Pi*a*csgn(I*d)*csgn(I*d*(f*x+e)
^m)^2-1/4*I/f^2*e^2*m*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I/f^2*
e^2*m*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/8*Pi^2*csgn(I*d)*csgn(I*(f
*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/4*
b*m*n*x^2+1/8*I*x^2*Pi*b*m*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*I*x^2*ln
(c)*Pi*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/4*I*x^2*Pi*ln(d)
*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*I*Pi*b*n*x^2*csgn(I*d)*csgn(I*(f
*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/2/f^2*e^2*m*ln(f*x+e)*b*ln(c)+1/2/f*ln(c)*b*
e*m*x+1/4*I/f^2*e^2*m*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^3+1/8*Pi^2*csgn(I*d)*csg
n(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi
^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^2*b*csgn(I*c*x^n)^2*cs
gn(I*c)+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^2*b*csgn(I*x^n)*csgn(I*c
*x^n)*csgn(I*c)+1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x^2*b*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I
*d*(f*x+e)^m)*b*x^2*ln(x^n)-1/4*I/f*Pi*b*e*m*csgn(I*c*x^n)^3*x-1/4*I/f*Pi*b
*e*m*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*x-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x+e
)^m)*csgn(I*d*(f*x+e)^m)*x^2*b*csgn(I*c*x^n)^3-1/8*Pi^2*csgn(I*d)*csgn(I*d*
(f*x+e)^m)^2*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*d)*csgn(I*d*
(f*x+e)^m)^2*x^2*b*csgn(I*c*x^n)^2*csgn(I*c)-1/8*Pi^2*csgn(I*(f*x+e)^m)*csg
n(I*d*(f*x+e)^m)^2*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2/f*m*b*ln(x^n)*x*e-
1/2/f^2*m*b*ln(x^n)*e^2*ln(f*x+e)+1/4*I*x^2*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c
*x^n)^2+1/4*I*x^2*Pi*ln(d)*b*csgn(I*c*x^n)^2*csgn(I*c)
```

maxima [A] time = 2.28, size = 269, normalized size = 1.33

$$-\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)be^2mn}{2\,f^2}-\frac{\left(2\,ae^2m-\left(e^2mn-2\,e^2m\log(c)\right)b\right)\log\left(fx+e\right)}{4\,f^2}+\frac{2\,be^2mn\log\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] -1/2*(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*e^2*m*n/f^2 - 1/4*(2*a*e^2*m
 - (e^2*m*n - 2*e^2*m*log(c))*b)*log(f*x + e)/f^2 + 1/4*(2*b*e^2*m*n*log(f*
x + e)*log(x) - ((f^2*m - 2*f^2*log(d))*a - (f^2*m*n - f^2*n*log(d) - (f^2*
m - 2*f^2*log(d))*log(c))*b)*x^2 + (2*a*e*f*m - (3*e*f*m*n - 2*e*f*m*log(c)
)*b)*x + (2*b*f^2*x^2*log(x^n) + (2*a*f^2 - (f^2*n - 2*f^2*log(c))*b)*x^2)*
log((f*x + e)^m) + (2*b*e*f*m*x - 2*b*e^2*m*log(f*x + e) - (f^2*m - 2*f^2*l
og(d))*b*x^2)*log(x^n))/f^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

3.73 $\int \left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)\,dx$

Optimal. Leaf size=117

$$\frac{(e+fx)\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{f}-mx\left(a+b\log\left(cx^n\right)\right)-\frac{bn(e+fx)\log\left(d(e+fx)^m\right)}{f}-\frac{ben\log\left(-\frac{fx}{e}\right)}{}$$

```
[Out] 2*b*m*n*x-m*x*(a+b*ln(c*x^n))-b*n*(f*x+e)*ln(d*(f*x+e)^m)/f-b*e*n*ln(-f*x/e
)*ln(d*(f*x+e)^m)/f+(f*x+e)*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/f-b*e*m*n*polyl
og(2,1+f*x/e)/f
```

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2389, 2295, 2370, 2411, 43, 2351, 2317, 2391}

$$-\frac{bemn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{f}+\frac{(e+fx)\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{f}-mx\left(a+b\log\left(cx^n\right)\right)-\frac{bn(e+fx)\log}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] 2*b*m*n*x - m*x*(a + b*Log[c*x^n]) - (b*n*(e + f*x)*Log[d*(e + f*x)^m])/f -
 (b*e*n*Log[-((f*x)/e)]*Log[d*(e + f*x)^m])/f + ((e + f*x)*(a + b*Log[c*x^n
])*Log[d*(e + f*x)^m])/f - (b*e*m*n*PolyLog[2, 1 + (f*x)/e])/f
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
```

```
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)\,dx = -mx\left(a+b\log\left(cx^n\right)\right)+\frac{(e+fx)\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{f}-($$

$$= bmnx-mx\left(a+b\log\left(cx^n\right)\right)+\frac{(e+fx)\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx\right.}{f}$$

$$= bmnx-mx\left(a+b\log\left(cx^n\right)\right)+\frac{(e+fx)\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx\right.}{f}$$

$$= bmnx-mx\left(a+b\log\left(cx^n\right)\right)+\frac{(e+fx)\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx\right.}{f}$$

$$= bmnx-mx\left(a+b\log\left(cx^n\right)\right)+\frac{(e+fx)\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx\right.}{f}$$

$$= 2bmnx-mx\left(a+b\log\left(cx^n\right)\right)-\frac{bn(e+fx)\log\left(d(e+fx)^m\right)}{f}-\frac{ben\log}{}$$

$$= 2bmnx-mx\left(a+b\log\left(cx^n\right)\right)-\frac{bn(e+fx)\log\left(d(e+fx)^m\right)}{f}-\frac{ben\log}{}$$

Mathematica [A] time = 0.07, size = 152, normalized size = 1.30

$$\frac{afx\log\left(d(e+fx)^m\right)+ae\log\left(d(e+fx)^m\right)-afmx+b\log\left(cx^n\right)\left(fx\left(\log\left(d(e+fx)^m\right)-m\right)+em\log(e+fx)\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (-(a*f*m*x) + 2*b*f*m*n*x - b*e*m*n*Log[e + f*x] - b*e*m*n*Log[x]*Log[e + f
*x] + a*e*Log[d*(e + f*x)^m] + a*f*x*Log[d*(e + f*x)^m] - b*f*n*x*Log[d*(e
+ f*x)^m] + b*Log[c*x^n]*(e*m*Log[e + f*x] + f*x*(-m + Log[d*(e + f*x)^m]))
 + b*e*m*n*Log[x]*Log[1 + (f*x)/e] + b*e*m*n*PolyLog[2, -((f*x)/e)])/f
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(fx+e\right)^{m}d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(fx+e\right)^{m}d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d), x)
```

maple [C] time = 0.64, size = 1762, normalized size = 15.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x+e)^m),x)
```

```
[Out] -m*ln(c)*b*x+ln(c)*ln(d)*b*x-ln(d)*b*n*x+2*b*m*n*x-a*m*x+ln(d)*a*x+(b*x*ln(
x^n)+1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*c)*csgn(I*x^n)*c
sgn(I*c*x^n)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+2*b*ln
(c)-2*b*n+2*a)*x)*ln((f*x+e)^m)+1/2*I*ln(c)*Pi*b*x*csgn(I*(f*x+e)^m)*csgn(I
*d*(f*x+e)^m)^2+1/2*I*Pi*ln(d)*b*x*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*Pi*ln(
x^n)*x*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/2*I/f*m*e*ln(f*x+e)*Pi*b*csgn(I*
c*x^n)^3-1/2*I*Pi*ln(x^n)*x*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^
m)+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*d*(f*x+e)^m)^2*c
sgn(I*c)+1/4*Pi^2*x*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*(f*x+e)^m)*csgn(I*d*
(f*x+e)^m)^2*csgn(I*c)+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*x^n)*csgn(I*c*x^n)^2*c
sgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*c*x^n)^2
*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*csgn(I*c)-m*b*ln(x^n)*x+ln(d)*ln(x^n
)*x*b+1/2*I*Pi*b*n*x*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/2*I*
Pi*ln(d)*b*x*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/2*I*m*Pi*b*x*csgn(I*x^n)
*csgn(I*c*x^n)*csgn(I*c)-1/2*I*ln(c)*Pi*b*x*csgn(I*d)*csgn(I*(f*x+e)^m)*csg
n(I*d*(f*x+e)^m)-1/2*I*Pi*a*x*csgn(I*d*(f*x+e)^m)^3-1/4*Pi^2*x*b*csgn(I*c*x
^n)^3*csgn(I*d*(f*x+e)^m)^3-n*b*e*m/f*dilog(-1/e*f*x)-1/4*Pi^2*x*b*csgn(I*d
)*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*csgn(I*c)
+1/2*I/f*m*e*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I/f*m*e*ln(f*x+
e)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*c*x^n)^3*cs
gn(I*d*(f*x+e)^m)^2+1/4*Pi^2*x*b*csgn(I*c*x^n)^3*csgn(I*(f*x+e)^m)*csgn(I*d
*(f*x+e)^m)^2+1/f*m*b*ln(x^n)*e*ln(f*x+e)+b*e*n*m/f-1/2*I*m*Pi*b*x*csgn(I*c
*x^n)^2*csgn(I*c)-1/2*I*Pi*a*x*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)
^m)-1/2*I*Pi*ln(x^n)*x*b*csgn(I*d*(f*x+e)^m)^3+1/2*I*Pi*b*n*x*csgn(I*d*(f*x
+e)^m)^3+1/4*Pi^2*x*b*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*d*(f*x+e)^m)^3+1/4
*Pi^2*x*b*csgn(I*c*x^n)^2*csgn(I*d*(f*x+e)^m)^3*csgn(I*c)+1/2*I*ln(c)*Pi*b*
x*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/2*I*Pi*b*n*x*csgn(I*d)*csgn(I*d*(f*x+e)
^m)^2-1/2*I*Pi*b*n*x*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/2*I*m*Pi*b*x
```

```
*csgn(I*x^n)*csgn(I*c*x^n)^2-n*b*e*m/f*ln(f*x+e)*ln(-1/e*f*x)+a/f*m*e*ln(f*
x+e)-b*e*n*m/f*ln(f*x+e)-1/4*Pi^2*x*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*d*(f
*x+e)^m)^3*csgn(I*c)-1/4*Pi^2*x*b*csgn(I*d)*csgn(I*c*x^n)^3*csgn(I*(f*x+e)^
m)*csgn(I*d*(f*x+e)^m)-1/4*Pi^2*x*b*csgn(I*d)*csgn(I*x^n)*csgn(I*c*x^n)^2*c
sgn(I*d*(f*x+e)^m)^2+1/2*I*Pi*ln(x^n)*x*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e
)^m)^2+1/2*I*Pi*ln(d)*b*x*csgn(I*c*x^n)^2*csgn(I*c)+1/2*I*m*Pi*b*x*csgn(I*c
*x^n)^3+1/f*m*e*ln(f*x+e)*b*ln(c)-1/4*Pi^2*x*b*csgn(I*d)*csgn(I*c*x^n)^2*cs
gn(I*d*(f*x+e)^m)^2*csgn(I*c)-1/4*Pi^2*x*b*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn
(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/4*Pi^2*x*b*csgn(I*c*x^n)^2*csgn(I*(f*
x+e)^m)*csgn(I*d*(f*x+e)^m)^2*csgn(I*c)-1/2*I/f*m*e*ln(f*x+e)*Pi*b*csgn(I*x
^n)*csgn(I*c*x^n)*csgn(I*c)+1/2*I*Pi*a*x*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/
2*I*Pi*a*x*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/2*I*ln(c)*Pi*b*x*csgn(
I*d*(f*x+e)^m)^3-1/2*I*Pi*ln(d)*b*x*csgn(I*c*x^n)^3
```

maxima [**A**] time = 2.09, size = 188, normalized size = 1.61

$$\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)bemn}{f}+\frac{\left(aem-\left(emn-em\log(c)\right)b\right)\log\left(fx+e\right)}{f}-\frac{bemn\log\left(fx+e\right)\log(x)+(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] (log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*e*m*n/f + (a*e*m - (e*m*n - e*m*l
og(c))*b)*log(f*x + e)/f - (b*e*m*n*log(f*x + e)*log(x) + ((f*m - f*log(d))
*a - (2*f*m*n - f*n*log(d) - (f*m - f*log(d))*log(c))*b)*x - (b*f*x*log(x^n
) - ((f*n - f*log(c))*b - a*f)*x)*log((f*x + e)^m) - (b*e*m*log(f*x + e) -
(f*m - f*log(d))*b*x)*log(x^n))/f
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

3.74 $\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x}\,dx$

Optimal. Leaf size=100

$$\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{2bn}-m\text{Li}_2\left(-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{m\log\left(\frac{fx}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{2bn}+bmn\text{Li}_3$$

```
[Out] 1/2*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/b/n-1/2*m*(a+b*ln(c*x^n))^2*ln(1+f*x/
e)/b/n-m*(a+b*ln(c*x^n))*polylog(2,-f*x/e)+b*m*n*polylog(3,-f*x/e)
```

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2375, 2317, 2374, 6589}

$$-m\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)+bmn\text{PolyLog}\left(3,-\frac{fx}{e}\right)+\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{2bn}-\frac{m\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/(2*b*n) - (m*(a + b*Log[c*x^n])^2
*Log[1 + (f*x)/e])/(2*b*n) - m*(a + b*Log[c*x^n])*PolyLog[2, -((f*x)/e)] +
b*m*n*PolyLog[3, -((f*x)/e)]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x}\,dx = \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{2bn} - \frac{(fm)\int \frac{\left(a+b\log(cx^n)\right)^2}{e+fx}\,dx}{2bn}$$
$$= \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{2bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^2\log\left(1+\frac{fx}{e}\right.}{2bn}$$
$$= \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{2bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^2\log\left(1+\frac{fx}{e}\right.}{2bn}$$
$$= \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{2bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^2\log\left(1+\frac{fx}{e}\right.}{2bn}$$

Mathematica [A] time = 0.07, size = 147, normalized size = 1.47

$$a\log\left(-\frac{fx}{e}\right)\log\left(d(e+fx)^m\right)+am\mathrm{Li}_2\left(\frac{fx}{e}+1\right)+b\log(x)\log\left(cx^n\right)\log\left(d(e+fx)^m\right)-bm\log\left(cx^n\right)\mathrm{Li}_2\left(-\frac{fx}{e}\right)-bm$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] -1/2*(b*n*Log[x]^2*Log[d*(e + f*x)^m]) + a*Log[-((f*x)/e)]*Log[d*(e + f*x)^
m] + b*Log[x]*Log[c*x^n]*Log[d*(e + f*x)^m] + (b*m*n*Log[x]^2*Log[1 + (f*x)
/e])/2 - b*m*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] - b*m*Log[c*x^n]*PolyLog[2,
 -((f*x)/e)] + a*m*PolyLog[2, 1 + (f*x)/e] + b*m*n*PolyLog[3, -((f*x)/e)]
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx+e\right)^m d\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx+e\right)^m d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x, x)
```

maple [C] time = 0.47, size = 1795, normalized size = 17.95

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x+e)^m)/x,x)
```

```
[Out] (b*ln(x)*ln(x^n)-1/2*b*n*ln(x)^2+1/2*I*Pi*ln(x)*b*csgn(I*x^n)*csgn(I*c*x^n)
^2-1/2*I*Pi*ln(x)*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*ln(x)*b*cs
gn(I*c*x^n)^3+1/2*I*Pi*ln(x)*b*csgn(I*c*x^n)^2*csgn(I*c)+ln(c)*ln(x)*b+ln(x
)*a)*ln((f*x+e)^m)-m*ln(x)*ln((f*x+e)/e)*a-m*dilog((f*x+e)/e)*b*ln(c)+ln(d)
*b*ln(c)*ln(x)-m*dilog((f*x+e)/e)*a+ln(d)*a*ln(x)-m*dilog((f*x+e)/e)*b*ln(x
^n)+1/2*ln(d)*b/n*ln(x^n)^2+b*m*n*polylog(3,-1/e*f*x)-1/4*Pi^2*csgn(I*d)*cs
gn(I*d*(f*x+e)^m)^2*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(x)-1/4*Pi^2*csgn(I*(f*x+
e)^m)*csgn(I*d*(f*x+e)^m)^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(x)-1/4*Pi^2*cs
gn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(x)+1/4
*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b*csgn(I*c*x^n)^3*ln(x)+1/2*I*Pi*csgn
(I*d)*csgn(I*d*(f*x+e)^m)^2*a*ln(x)+1/2*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*
x+e)^m)^2*a*ln(x)-1/2*I*Pi*csgn(I*d*(f*x+e)^m)^3*b*ln(c)*ln(x)-1/4*I*Pi*csg
n(I*d*(f*x+e)^m)^3*b/n*ln(x^n)^2-1/4*Pi^2*csgn(I*d*(f*x+e)^m)^3*b*csgn(I*x^
n)*csgn(I*c*x^n)*csgn(I*c)*ln(x)-1/2*I*m*ln(x)*ln((f*x+e)/e)*b*Pi*csgn(I*x^
n)*csgn(I*c*x^n)^2-1/2*I*m*ln(x)*ln((f*x+e)/e)*b*Pi*csgn(I*c*x^n)^2*csgn(I*
c)-1/2*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*a*ln(x)+1/2*I*P
i*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b*ln(c)*ln(x)+1/4*I*Pi*csgn(I*d)*csgn(I*d
*(f*x+e)^m)^2*b/n*ln(x^n)^2+1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^
2*b*csgn(I*c*x^n)^3*ln(x)+1/4*Pi^2*csgn(I*d*(f*x+e)^m)^3*b*csgn(I*x^n)*csgn
(I*c*x^n)^2*ln(x)+1/4*Pi^2*csgn(I*d*(f*x+e)^m)^3*b*csgn(I*c*x^n)^2*csgn(I*c
)*ln(x)+1/4*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b/n*ln(x^n)^2+1/2*
I*ln(d)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(x)+1/2*I*ln(d)*b*Pi*csgn(I*c*x^
n)^2*csgn(I*c)*ln(x)-m*ln(x)*ln((f*x+e)/e)*b*ln(x^n)-1/2*I*Pi*csgn(I*d*(f*x
+e)^m)^3*a*ln(x)-1/4*Pi^2*csgn(I*d*(f*x+e)^m)^3*b*csgn(I*c*x^n)^3*ln(x)-1/2
*m*b*n*ln(x)^2*ln(1+1/e*f*x)+1/2*I*m*ln(x)*ln((f*x+e)/e)*b*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)*csgn(I*c)+m*ln(x)^2*ln((f*x+e)/e)*b*n-m*b*n*ln(x)*polylog(2,-1
/e*f*x)+m*dilog((f*x+e)/e)*b*n*ln(x)-m*ln(x)*ln((f*x+e)/e)*b*ln(c)-1/2*I*Pi
*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b*ln(c)*ln(x)-1/2*I*ln(d)*
b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(x)+1/2*I*m*ln(x)*ln((f*x+e)/e)*
b*Pi*csgn(I*c*x^n)^3-1/2*I*m*dilog((f*x+e)/e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n
)^2-1/2*I*m*dilog((f*x+e)/e)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/2*I*Pi*csgn(I
*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b*ln(c)*ln(x)+1/4*Pi^2*csgn(I*d)*csgn(I*(
f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(x)+1/4*Pi^2*cs
gn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b*csgn(I*x^n)*csgn(I*c*x^n)^2
*ln(x)-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b/n*ln(x^n)
^2+1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b*csgn(I*x^n)*csgn(I*c*
x^n)*csgn(I*c)*ln(x)+1/2*I*m*dilog((f*x+e)/e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n
)*csgn(I*c)+1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b*csgn(I*x^n)*csgn(I*c
*x^n)*csgn(I*c)*ln(x)-1/4*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)
^m)*b*csgn(I*c*x^n)^3*ln(x)-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b*csgn
(I*x^n)*csgn(I*c*x^n)^2*ln(x)-1/2*I*ln(d)*b*Pi*csgn(I*c*x^n)^3*ln(x)+1/2*I*
m*dilog((f*x+e)/e)*b*Pi*csgn(I*c*x^n)^3-1/4*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m
)*csgn(I*d*(f*x+e)^m)*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(bn\log(x)^2 - 2\,b\log(x)\log\left(x^n\right) - 2\left(b\log(c) + a\right)\log(x)\right)\log\left(\left(fx + e\right)^m\right) - \int -\frac{bfmnx\log(x)^2 + 2\,be\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log((f*
x + e)^m) - integrate(-1/2*(b*f*m*n*x*log(x)^2 + 2*b*e*log(c)*log(d) + 2*a*
e*log(d) - 2*(b*f*m*log(c) + a*f*m)*x*log(x) + 2*(b*f*log(c)*log(d) + a*f*l
og(d))*x - 2*(b*f*m*x*log(x) - b*f*x*log(d) - b*e*log(d))*log(x^n))/(f*x^2
+ e*x), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x,x)
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x,x)
[Out] Timed out
```

3.75 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx$

Optimal. Leaf size=164

$$-\frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}+\frac{fm\log(x)(a+b\log(cx^n))}{e}-\frac{fm\log(e+fx)(a+b\log(cx^n))}{e}-\frac{bn\log(d}{}$$

```
[Out] b*f*m*n*ln(x)/e-1/2*b*f*m*n*ln(x)^2/e+f*m*ln(x)*(a+b*ln(c*x^n))/e-b*f*m*n*l
n(f*x+e)/e+b*f*m*n*ln(-f*x/e)*ln(f*x+e)/e-f*m*(a+b*ln(c*x^n))*ln(f*x+e)/e-b
*n*ln(d*(f*x+e)^m)/x-(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x+b*f*m*n*polylog(2,1+
f*x/e)/e
```

Rubi [A] time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2395, 36, 29, 31, 2376, 2301, 2394, 2315}

$$\frac{bfmn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{e}-\frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}+\frac{fm\log(x)(a+b\log(cx^n))}{e}-\frac{fm\log(e+fx)(}{e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] (b*f*m*n*Log[x])/e - (b*f*m*n*Log[x]^2)/(2*e) + (f*m*Log[x]*(a + b*Log[c*x^
n]))/e - (b*f*m*n*Log[e + f*x])/e + (b*f*m*n*Log[-((f*x)/e)]*Log[e + f*x])/
e - (f*m*(a + b*Log[c*x^n])*Log[e + f*x])/e - (b*n*Log[d*(e + f*x)^m])/x -
((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x + (b*f*m*n*PolyLog[2, 1 + (f*x)/e
])/e
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
```

```
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x^2}\,dx = \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{fm\left(a+b\log\left(cx^n\right)\right)\log(e+fx)}{e} - \frac{(a+$$

$$= \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{fm\left(a+b\log\left(cx^n\right)\right)\log(e+fx)}{e} - \frac{(a+$$

$$= -\frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} + \frac{bfmn\log\left(-\frac{fx}{e}\right)\log($$

$$= -\frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} + \frac{bfmn\log\left(-\frac{fx}{e}\right)\log($$

$$= \frac{bfmn\log(x)}{e} - \frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{bfmn$$

Mathematica [A] time = 0.11, size = 117, normalized size = 0.71

$$-\frac{2\left(a+b\log\left(cx^n\right)+bn\right)\left(e\log\left(d(e+fx)^m\right)+fmx\log(e+fx)\right)-2fmx\log(x)\left(a+b\log\left(cx^n\right)+bn\log(e+fx\right.}{2ex}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] -1/2*(b*f*m*n*x*Log[x]^2 + 2*(a + b*n + b*Log[c*x^n])*(f*m*x*Log[e + f*x] +
 e*Log[d*(e + f*x)^m]) - 2*f*m*x*Log[x]*(a + b*n + b*Log[c*x^n] + b*n*Log[e
 + f*x] - b*n*Log[1 + (f*x)/e]) + 2*b*f*m*n*x*PolyLog[2, -((f*x)/e)])/(e*x)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx+e\right)^m d\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(fx+e\right)^{m}d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^2, x)
```

maple [C] time = 0.63, size = 1892, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x+e)^m)/x^2,x)
```

```
[Out] -1/x*ln(d)*a+1/2*I/e*f*m*ln(f*x+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)
-1/2*I/e*f*m*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/x*ln(c)*ln(d)
*b-1/x*ln(d)*b*n+1/2*I/e*f*m*ln(x)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/4*Pi^2*
csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x*b*csgn(I*x^n)*csgn(I*c*x^
n)*csgn(I*c)-1/2*I/e*f*m*ln(f*x+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*Pi^
2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x*b*csgn(I*x^n)*csgn(I*c*
x^n)^2-1/4*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x*b*csgn(I*
c*x^n)^2*csgn(I*c)-ln(d)*b/x*ln(x^n)+(-b/x*ln(x^n)-1/2*(I*Pi*b*csgn(I*x^n)*
csgn(I*c*x^n)^2-I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*Pi*b*csgn(I*c*
x^n)^3+I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+2*b*ln(c)+2*b*n+2*a)/x)*ln((f*x+e)^
m)+1/4*Pi^2*csgn(I*d*(f*x+e)^m)^3/x*b*csgn(I*c*x^n)^3+1/2*I/x*Pi*a*csgn(I*d
*(f*x+e)^m)^3+1/2*I/x*Pi*a*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-
1/e*f*m*ln(f*x+e)*a+b*f*m*n*ln(x)/e-1/2*b*f*m*n*ln(x)^2/e-b*f*m*n*ln(f*x+e)
/e+1/e*f*m*ln(x)*a-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*c*x^
n)^3-1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*c*x^n)^3-1
/4*Pi^2*csgn(I*d*(f*x+e)^m)^3/x*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*Pi^2*csgn
(I*d*(f*x+e)^m)^3/x*b*csgn(I*c*x^n)^2*csgn(I*c)-1/2*I/x*Pi*ln(d)*b*csgn(I*c
*x^n)^2*csgn(I*c)-1/2*I/x*Pi*b*n*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/2*I/x*Pi
*b*n*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-m*f*b*ln(x^n)/e*ln(f*x+e)+m*f*
b*ln(x^n)*ln(x)/e+b*f*m*n*ln(-1/e*f*x)*ln(f*x+e)/e-1/e*f*m*ln(f*x+e)*b*ln(c
)+1/e*f*m*ln(x)*b*ln(c)-1/2*I/e*f*m*ln(x)*b*Pi*csgn(I*c*x^n)^3+1/2*I/e*f*m*
ln(f*x+e)*b*Pi*csgn(I*c*x^n)^3+1/2*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*
d*(f*x+e)^m)*b/x*ln(x^n)-1/2*I/x*ln(c)*Pi*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2
-1/2*I/x*ln(c)*Pi*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/4*Pi^2*csgn(I
*d)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*c*x^n)^2*csgn(I*c)+1/4*Pi^2*csgn(I*(f*
x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*x^n)*csgn(I*c*x^n)^2+n*b*f*m/e*dil
og(-1/e*f*x)+1/2*I/x*ln(c)*Pi*b*csgn(I*d*(f*x+e)^m)^3+1/2*I/x*Pi*ln(d)*b*cs
gn(I*c*x^n)^3+1/2*I/x*Pi*b*n*csgn(I*d*(f*x+e)^m)^3-1/2*I/x*Pi*a*csgn(I*d)*c
sgn(I*d*(f*x+e)^m)^2-1/2*I/x*Pi*a*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1
/2*I*Pi*csgn(I*d*(f*x+e)^m)^3*b/x*ln(x^n)+1/2*I/x*Pi*ln(d)*b*csgn(I*x^n)*cs
gn(I*c*x^n)*csgn(I*c)+1/2*I/x*Pi*b*n*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(
f*x+e)^m)+1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*c*x^n
)^2*csgn(I*c)+1/4*Pi^2*csgn(I*d*(f*x+e)^m)^3/x*b*csgn(I*x^n)*csgn(I*c*x^n)*
csgn(I*c)+1/4*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x*b*csgn
(I*c*x^n)^3+1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*x^n)*csgn(I
*c*x^n)^2-1/2*I/x*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I/e*f*m*ln(f*x
+e)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/2*I/e*f*m*ln(x)*b*Pi*csgn(I*x^n)*csgn(
I*c*x^n)^2-1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b/x*ln(x^n)-1/2*I*Pi*cs
```

gn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b/x*ln(x^n)-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/2*I/x*ln(c)*Pi*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)

maxima [A] time = 2.24, size = 199, normalized size = 1.21

$$-\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)bfmn}{e}-\frac{\left(afm+\left(fmn+fm\log(c)\right)b\right)\log\left(fx+e\right)}{e}+\frac{2bfmnx\log\left(fx+e\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*f*m*n/e - (a*f*m + (f*m*n + f*m*
log(c))*b)*log(f*x + e)/e + 1/2*(2*b*f*m*n*x*log(f*x + e)*log(x) - b*f*m*n*
x*log(x)^2 - 2*a*e*log(d) + 2*(a*f*m + (f*m*n + f*m*log(c))*b)*x*log(x) - 2
*(e*n*log(d) + e*log(c)*log(d))*b - 2*(b*e*log(x^n) + (e*n + e*log(c))*b +
a*e)*log((f*x + e)^m) - 2*(b*f*m*x*log(f*x + e) - b*f*m*x*log(x) + b*e*log(
d))*log(x^n))/(e*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x**2,x)
```

```
[Out] Timed out
```

3.76 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx)^m\right)}{x^3}\,dx$

Optimal. Leaf size=234

$$-\frac{(a+b\log(cx^n))\log\left(d(e+fx)^m\right)}{2x^2}-\frac{f^2m\log(x)\left(a+b\log(cx^n)\right)}{2e^2}+\frac{f^2m\log(e+fx)\left(a+b\log(cx^n)\right)}{2e^2}-\frac{fm\left(a\right.}{}$$

```
[Out] -3/4*b*f*m*n/e/x-1/4*b*f^2*m*n*ln(x)/e^2+1/4*b*f^2*m*n*ln(x)^2/e^2-1/2*f*m*
(a+b*ln(c*x^n))/e/x-1/2*f^2*m*ln(x)*(a+b*ln(c*x^n))/e^2+1/4*b*f^2*m*n*ln(f*
x+e)/e^2-1/2*b*f^2*m*n*ln(-f*x/e)*ln(f*x+e)/e^2+1/2*f^2*m*(a+b*ln(c*x^n))*l
n(f*x+e)/e^2-1/4*b*n*ln(d*(f*x+e)^m)/x^2-1/2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m
)/x^2-1/2*b*f^2*m*n*polylog(2,1+f*x/e)/e^2
```

Rubi [A] time = 0.16, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2395, 44, 2376, 2301, 2394, 2315}

$$-\frac{bf^2mn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{2e^2}-\frac{(a+b\log(cx^n))\log\left(d(e+fx)^m\right)}{2x^2}-\frac{f^2m\log(x)\left(a+b\log(cx^n)\right)}{2e^2}+\frac{f^2m\log(e+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] (-3*b*f*m*n)/(4*e*x) - (b*f^2*m*n*Log[x])/(4*e^2) + (b*f^2*m*n*Log[x]^2)/(4
*e^2) - (f*m*(a + b*Log[c*x^n]))/(2*e*x) - (f^2*m*Log[x]*(a + b*Log[c*x^n])
)/(2*e^2) + (b*f^2*m*n*Log[e + f*x])/(4*e^2) - (b*f^2*m*n*Log[-((f*x)/e)]*L
og[e + f*x])/(2*e^2) + (f^2*m*(a + b*Log[c*x^n])*Log[e + f*x])/(2*e^2) - (b
*n*Log[d*(e + f*x)^m])/(4*x^2) - ((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/(2
*x^2) - (b*f^2*m*n*PolyLog[2, 1 + (f*x)/e])/(2*e^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x^3}\,dx = -\frac{fm\left(a+b\log\left(cx^n\right)\right)}{2ex}-\frac{f^2m\log(x)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{f^2m\left(a+b\log\right.}{}$$

$$= -\frac{bfmn}{2ex}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{2ex}-\frac{f^2m\log(x)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{f^2m}{}$$

$$= -\frac{bfmn}{2ex}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{2ex}-\frac{f^2m\log(x)\left(a+b\right.}{2e^2}$$

$$= -\frac{bfmn}{2ex}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{2ex}-\frac{f^2m\log(x)\left(a+b\right.}{2e^2}$$

$$= -\frac{3bfmn}{4ex}-\frac{bf^2mn\log(x)}{4e^2}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{2ex}-$$

Mathematica [A] time = 0.15, size = 232, normalized size = 0.99

$$-\frac{f^2mx^2\log(x)\left(2a+2b\log\left(cx^n\right)+2bn\log(e+fx)-2bn\log\left(\frac{fx}{e}+1\right)+bn\right)+2ae^2\log\left(d(e+fx)^m\right)-2af^2mx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] -1/4*(2*a*e*f*m*x + 3*b*e*f*m*n*x - b*f^2*m*n*x^2*Log[x]^2 + 2*b*e*f*m*x*Lo
g[c*x^n] - 2*a*f^2*m*x^2*Log[e + f*x] - b*f^2*m*n*x^2*Log[e + f*x] - 2*b*f^
2*m*x^2*Log[c*x^n]*Log[e + f*x] + 2*a*e^2*Log[d*(e + f*x)^m] + b*e^2*n*Log[
d*(e + f*x)^m] + 2*b*e^2*Log[c*x^n]*Log[d*(e + f*x)^m] + f^2*m*x^2*Log[x]*(
2*a + b*n + 2*b*Log[c*x^n] + 2*b*n*Log[e + f*x] - 2*b*n*Log[1 + (f*x)/e]) -
 2*b*f^2*m*n*x^2*PolyLog[2, -((f*x)/e)])/(e^2*x^2)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx+e\right)^m d\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(fx+e\right)^{m}d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^3, x)
```

maple [C] time = 0.69, size = 2100, normalized size = 8.97

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x+e)^m)/x^3,x)
```

```
[Out] -1/2*m*f*b*ln(x^n)/e/x+1/2*m*f^2*b*ln(x^n)/e^2*ln(f*x+e)-1/2*m*f^2*b*ln(x^n
)/e^2*ln(x)+1/4*I/e^2*f^2*m*ln(f*x+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*
I/e^2*f^2*m*ln(f*x+e)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/4*I/e*f*m/x*b*Pi*csg
n(I*c*x^n)^2*csgn(I*c)-1/2/x^2*ln(c)*ln(d)*b-1/4/x^2*ln(d)*b*n+(-1/2*b/x^2*
ln(x^n)-1/4*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*c)*csgn(I*x^n
)*csgn(I*c*x^n)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+2*b
*ln(c)+b*n+2*a)/x^2)*ln((f*x+e)^m)+1/2/e^2*f^2*m*ln(f*x+e)*a-1/2/e^2*f^2*m*
ln(x)*a-1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/
8*Pi^2*csgn(I*d*(f*x+e)^m)^3/x^2*b*csgn(I*c*x^n)^2*csgn(I*c)-1/8*Pi^2*csgn(
I*d)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*c*x^n)^3+1/4*I*Pi*csgn(I*d*(f*x+e)^
m)^3*b/x^2*ln(x^n)+1/4*I/x^2*ln(c)*Pi*b*csgn(I*d*(f*x+e)^m)^3+1/4*I/x^2*Pi*
ln(d)*b*csgn(I*c*x^n)^3-1/2/x^2*ln(d)*a-1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d
*(f*x+e)^m)^2/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*ln(d)*b/x^2*ln(
x^n)-1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*c*x^n)^3
-3/4*b*f*m*n/e/x+1/8*I/x^2*Pi*b*n*csgn(I*d*(f*x+e)^m)^3-1/4*I/x^2*Pi*a*csgn
(I*d)*csgn(I*d*(f*x+e)^m)^2-1/4*I/x^2*Pi*a*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+
e)^m)^2+1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3/x^2*b*csgn(I*c*x^n)^3+1/4*I/x^2*Pi*a
*csgn(I*d*(f*x+e)^m)^3-1/8*I/x^2*Pi*b*n*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/8
*I/x^2*Pi*b*n*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/4*I/x^2*Pi*a*csgn(I
*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/4*I/e^2*f^2*m*ln(f*x+e)*b*Pi*cs
gn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*b*f^2*m*n*ln(x)/e^2+1/4*b*f^2*m*n*ln(
x)^2/e^2-1/4*I/e^2*f^2*m*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I/e^2*f
^2*m*ln(x)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/4*I/e*f*m/x*b*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)^2-1/2*b*f^2*m*n*ln(-1/e*f*x)*ln(f*x+e)/e^2+1/4*I/x^2*ln(c)*Pi*b
*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/4*I/x^2*Pi*ln(d)*b*csgn(
I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*I/x^2*Pi*b*n*csgn(I*d)*csgn(I*(f*x+e)^m)
*csgn(I*d*(f*x+e)^m)+1/4*I/e*f*m/x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)
-1/2/e*f*m/x*a-1/2*n*f^2*b*m/e^2*dilog(-1/e*f*x)-1/4*I/x^2*ln(c)*Pi*b*csgn(
I*d)*csgn(I*d*(f*x+e)^m)^2-1/2/e*f*m/x*b*ln(c)+1/2/e^2*f^2*m*ln(f*x+e)*b*ln
(c)-1/2/e^2*f^2*m*ln(x)*b*ln(c)+1/4*I/e^2*f^2*m*ln(x)*b*Pi*csgn(I*c*x^n)^3+
1/4*I/e*f*m/x*b*Pi*csgn(I*c*x^n)^3+1/4*I/e^2*f^2*m*ln(x)*b*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)*csgn(I*c)-1/4*I/e^2*f^2*m*ln(f*x+e)*b*Pi*csgn(I*c*x^n)^3+1/4*I
*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b/x^2*ln(x^n)-1/8*Pi^2*
csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^2*b*csgn(I*x^n)*csgn(I*c*
x^n)^2-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^2*b*csgn(
I*c*x^n)^2*csgn(I*c)-1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*
x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)
^2/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f
*x+e)^m)^2/x^2*b*csgn(I*c*x^n)^2*csgn(I*c)+1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3/x
^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^
```

```
m)*csgn(I*d*(f*x+e)^m)/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*Pi^2*c
sgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^2*b*csgn(I*c*x^n)^3+1/8*Pi
^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi
^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*c*x^n)^2*csgn(I*c)-1/4*I*Pi
*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b/x^2*ln(x^n)-1/4*I*Pi*csgn(I*(f*x+e)^m)*c
sgn(I*d*(f*x+e)^m)^2*b/x^2*ln(x^n)-1/4*I/x^2*ln(c)*Pi*b*csgn(I*(f*x+e)^m)*c
sgn(I*d*(f*x+e)^m)^2-1/4*I/x^2*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I
/x^2*Pi*ln(d)*b*csgn(I*c*x^n)^2*csgn(I*c)+1/4*b*f^2*m*n*ln(f*x+e)/e^2
```

maxima [A] time = 2.37, size = 285, normalized size = 1.22

$$\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\text{Li}_2\left(-\frac{fx}{e}\right)\right)bf^2mn}{2\,e^2}+\frac{\left(2\,af^2m+\left(f^2mn+2\,f^2m\log(c)\right)b\right)\log\left(fx+e\right)}{4\,e^2}-\frac{2\,bf^2mnx^2\log\left(fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*f^2*m*n/e^2 + 1/4*(2*a*f^2*m
+ (f^2*m*n + 2*f^2*m*log(c))*b)*log(f*x + e)/e^2 - 1/4*(2*b*f^2*m*n*x^2*log
(f*x + e)*log(x) - b*f^2*m*n*x^2*log(x)^2 + 2*a*e^2*log(d) + (2*a*f^2*m + (
f^2*m*n + 2*f^2*m*log(c))*b)*x^2*log(x) + (e^2*n*log(d) + 2*e^2*log(c)*log(
d))*b + (2*a*e*f*m + (3*e*f*m*n + 2*e*f*m*log(c))*b)*x + (2*b*e^2*log(x^n)
+ 2*a*e^2 + (e^2*n + 2*e^2*log(c))*b)*log((f*x + e)^m) - 2*(b*f^2*m*x^2*log
(f*x + e) - b*f^2*m*x^2*log(x) - b*e*f*m*x - b*e^2*log(d))*log(x^n))/(e^2*x
^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x**3,x)
```

```
[Out] Timed out
```

3.77 $\int \frac{\left(a+b\log(cx^n)\right)\log\left(d(e+fx)^m\right)}{x^4}\,dx$

Optimal. Leaf size=274

$$-\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{3x^3}+\frac{f^3m\log(x)\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{f^3m\log(e+fx)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+\frac{f^2m\left(a\right.}{}$$

```
[Out] -5/36*b*f*m*n/e/x^2+4/9*b*f^2*m*n/e^2/x+1/9*b*f^3*m*n*ln(x)/e^3-1/6*b*f^3*m
*n*ln(x)^2/e^3-1/6*f*m*(a+b*ln(c*x^n))/e/x^2+1/3*f^2*m*(a+b*ln(c*x^n))/e^2/
x+1/3*f^3*m*ln(x)*(a+b*ln(c*x^n))/e^3-1/9*b*f^3*m*n*ln(f*x+e)/e^3+1/3*b*f^3
*m*n*ln(-f*x/e)*ln(f*x+e)/e^3-1/3*f^3*m*(a+b*ln(c*x^n))*ln(f*x+e)/e^3-1/9*b
*n*ln(d*(f*x+e)^m)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^3+1/3*b*f^3*m*
n*polylog(2,1+f*x/e)/e^3
```

Rubi [A] time = 0.18, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2395, 44, 2376, 2301, 2394, 2315}

$$\frac{bf^3mn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{3e^3}-\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{3x^3}+\frac{f^3m\log(x)\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{f^3m\log(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^4,x]
```

```
[Out] (-5*b*f*m*n)/(36*e*x^2) + (4*b*f^2*m*n)/(9*e^2*x) + (b*f^3*m*n*Log[x])/(9*e
^3) - (b*f^3*m*n*Log[x]^2)/(6*e^3) - (f*m*(a + b*Log[c*x^n]))/(6*e*x^2) + (
f^2*m*(a + b*Log[c*x^n]))/(3*e^2*x) + (f^3*m*Log[x]*(a + b*Log[c*x^n]))/(3*
e^3) - (b*f^3*m*n*Log[e + f*x])/(9*e^3) + (b*f^3*m*n*Log[-((f*x)/e)]*Log[e
+ f*x])/(3*e^3) - (f^3*m*(a + b*Log[c*x^n])*Log[e + f*x])/(3*e^3) - (b*n*Lo
g[d*(e + f*x)^m])/(9*x^3) - ((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/(3*x^3)
 + (b*f^3*m*n*PolyLog[2, 1 + (f*x)/e])/(3*e^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x^4}\,dx = -\frac{fm\left(a+b\log\left(cx^n\right)\right)}{6ex^2}+\frac{f^2m\left(a+b\log\left(cx^n\right)\right)}{3e^2x}+\frac{f^3m\log(x)\left(a+b\log\right.}{3e^3}$$

$$= -\frac{bfmn}{12ex^2}+\frac{bf^2mn}{3e^2x}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{6ex^2}+\frac{f^2m\left(a+b\log\left(cx^n\right)\right)}{3e^2x}+$$

$$= -\frac{bfmn}{12ex^2}+\frac{bf^2mn}{3e^2x}-\frac{bf^3mn\log^2(x)}{6e^3}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{6ex^2}+\frac{f^2m\left(a\right.}{}$$

$$= -\frac{bfmn}{12ex^2}+\frac{bf^2mn}{3e^2x}-\frac{bf^3mn\log^2(x)}{6e^3}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{6ex^2}+\frac{f^2m\left(a\right.}{}$$

$$= -\frac{5bfmn}{36ex^2}+\frac{4bf^2mn}{9e^2x}+\frac{bf^3mn\log(x)}{9e^3}-\frac{bf^3mn\log^2(x)}{6e^3}-\frac{fm\left(a+b\log\right.}{6ex^2}$$

Mathematica [A] time = 0.17, size = 280, normalized size = 1.02

$$-\frac{-4f^3mx^3\log(x)\left(3a+3b\log\left(cx^n\right)+3bn\log(e+fx)-3bn\log\left(\frac{fx}{e}+1\right)+bn\right)+12ae^3\log\left(d(e+fx)^m\right)+6ae^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^4,x]
```

```
[Out] -1/36*(6*a*e^2*f*m*x + 5*b*e^2*f*m*n*x - 12*a*e*f^2*m*x^2 - 16*b*e*f^2*m*n*
x^2 + 6*b*f^3*m*n*x^3*Log[x]^2 + 6*b*e^2*f*m*x*Log[c*x^n] - 12*b*e*f^2*m*x^
2*Log[c*x^n] + 12*a*f^3*m*x^3*Log[e + f*x] + 4*b*f^3*m*n*x^3*Log[e + f*x] +
 12*b*f^3*m*x^3*Log[c*x^n]*Log[e + f*x] + 12*a*e^3*Log[d*(e + f*x)^m] + 4*b
*e^3*n*Log[d*(e + f*x)^m] + 12*b*e^3*Log[c*x^n]*Log[d*(e + f*x)^m] - 4*f^3*
m*x^3*Log[x]*(3*a + b*n + 3*b*Log[c*x^n] + 3*b*n*Log[e + f*x] - 3*b*n*Log[1
 + (f*x)/e]) + 12*b*f^3*m*n*x^3*PolyLog[2, -((f*x)/e)])/(e^3*x^3)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx+e\right)^m d\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^4,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(cx^{n}\right)+a\right) \log \left(\left(fx+e\right)^{m} d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^4, x)
```

maple [C] time = 0.80, size = 2282, normalized size = 8.33

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x+e)^m)/x^4,x)
[Out] -1/6*I/e^3*f^3*m*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/6/e*f*m/x
^2*a+1/3/e^3*f^3*m*ln(x)*a-1/3*ln(d)*b/x^3*ln(x^n)-1/3/x^3*ln(d)*a+1/12*Pi^
2*csgn(I*d*(f*x+e)^m)^3/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/12*Pi^2
*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^3*b*csgn(I*x^n)*csgn(I*c
*x^n)^2-1/12*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^3*b*csg
n(I*c*x^n)^2*csgn(I*c)-1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^3*b*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/12*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e
)^m)^2/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/18*I/x^3*Pi*b*n*csgn(I*d
)*csgn(I*d*(f*x+e)^m)^2-1/18*I/x^3*Pi*b*n*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e
)^m)^2+1/6*I/x^3*Pi*a*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/6*I
/x^3*ln(c)*Pi*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+(-1/3*b/x^3*ln(x^n)-1/18*(3
*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)-3*I*Pi*b*csgn(I*c*x^n)^3+3*I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+6*b*ln(c)
+2*b*n+6*a)/x^3)*ln((f*x+e)^m)-1/6*m*f*b*ln(x^n)/e/x^2+1/3*m*f^2*b*ln(x^n)/
e^2/x-1/3*m*f^3*b*ln(x^n)/e^3*ln(f*x+e)+1/3*m*f^3*b*ln(x^n)/e^3*ln(x)-5/36*
b*f*m*n/e/x^2+4/9*b*f^2*m*n/e^2/x-1/3/x^3*ln(c)*ln(d)*b-1/9/x^3*ln(d)*b*n-1
/6*I/e^3*f^3*m*ln(f*x+e)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/12*I/e*f*m/x^2*b*
Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/6*I/e^2*f^2*m/x*b*Pi*csgn(I*c*x^n)^2*csgn(
I*c)-1/6/e*f*m/x^2*b*ln(c)+1/3/e^2*f^2*m/x*b*ln(c)-1/3/e^3*f^3*m*ln(f*x+e)*
b*ln(c)+1/3/e^3*f^3*m*ln(x)*b*ln(c)+1/6*I/e^3*f^3*m*ln(x)*b*Pi*csgn(I*c*x^n
)^2*csgn(I*c)+1/3/e^2*f^2*m/x*a+1/6*I/x^3*Pi*a*csgn(I*d*(f*x+e)^m)^3+1/12*P
i^2*csgn(I*d*(f*x+e)^m)^3/x^3*b*csgn(I*c*x^n)^3-1/3/e^3*f^3*m*ln(f*x+e)*a+1
/6*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b/x^3*ln(x^n)-1/6*I
/e^2*f^2*m/x*b*Pi*csgn(I*c*x^n)^3+1/6*I/e^3*f^3*m*ln(f*x+e)*b*Pi*csgn(I*c*x
^n)^3+1/6*I/x^3*ln(c)*Pi*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-
1/12*Pi^2*csgn(I*d*(f*x+e)^m)^3/x^3*b*csgn(I*c*x^n)^2*csgn(I*c)-1/12*Pi^2*c
sgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^3*b*csgn(I*c*x^n)^3-1/12*Pi^2*csgn(I*(f*x+
e)^m)*csgn(I*d*(f*x+e)^m)^2/x^3*b*csgn(I*c*x^n)^3-1/12*Pi^2*csgn(I*d*(f*x+e
)^m)^3/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/6*I*Pi*csgn(I*d*(f*x+e)^m)^3*b/x
^3*ln(x^n)+1/12*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^3*b*
csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/12*I/e*f*m/x^2*b*Pi*csgn(I*c*x^n)^2*c
sgn(I*c)+1/3*b*f^3*m*n*ln(-1/e*f*x)*ln(f*x+e)/e^3+1/6*I/x^3*Pi*ln(d)*b*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/18*I/x^3*Pi*b*n*csgn(I*d)*csgn(I*(f*x+e)^
m)*csgn(I*d*(f*x+e)^m)+1/12*I/e*f*m/x^2*b*Pi*csgn(I*c*x^n)^3-1/6*I/e^3*f^3*
m*ln(x)*b*Pi*csgn(I*c*x^n)^3-1/6*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b/x^3
*ln(x^n)-1/6*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b/x^3*ln(x^n)-1/6
*I/x^3*Pi*ln(d)*b*csgn(I*c*x^n)^2*csgn(I*c)+1/12*Pi^2*csgn(I*d)*csgn(I*d*(f
*x+e)^m)^2/x^3*b*csgn(I*c*x^n)^2*csgn(I*c)+1/12*Pi^2*csgn(I*(f*x+e)^m)*csgn
```

```
(I*d*(f*x+e)^m)^2/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/12*Pi^2*csgn(I*(f*x+e
)^m)*csgn(I*d*(f*x+e)^m)^2/x^3*b*csgn(I*c*x^n)^2*csgn(I*c)+1/6*I/e^3*f^3*m*
ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/18*I/x^3*Pi*b*n*csgn(I*d*(f*x+e)^m
)^3-1/6*I/x^3*Pi*a*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/6*I/x^3*Pi*a*csgn(I*(f
*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/6*I/x^3*ln(c)*Pi*b*csgn(I*d*(f*x+e)^m)^3+1
/6*I/x^3*Pi*ln(d)*b*csgn(I*c*x^n)^3+1/6*I/e^3*f^3*m*ln(f*x+e)*b*Pi*csgn(I*x
^n)*csgn(I*c*x^n)*csgn(I*c)+1/12*I/e*f*m/x^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)
*csgn(I*c)-1/6*I/e^2*f^2*m/x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/3*n
*b*f^3*m/e^3*dilog(-1/e*f*x)-1/6*I/e^3*f^3*m*ln(f*x+e)*b*Pi*csgn(I*x^n)*csg
n(I*c*x^n)^2+1/6*I/e^2*f^2*m/x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/9*b*f^3*m
*n*ln(x)/e^3-1/6*b*f^3*m*n*ln(x)^2/e^3-1/9*b*f^3*m*n*ln(f*x+e)/e^3-1/6*I/x^
3*ln(c)*Pi*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/6*I/x^3*Pi*ln(d)*b*c
sgn(I*x^n)*csgn(I*c*x^n)^2+1/12*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(
f*x+e)^m)/x^3*b*csgn(I*c*x^n)^3+1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x
^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2
```

maxima [A] time = 2.34, size = 342, normalized size = 1.25

$$-\frac{\left(\log\left(\frac{fx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{fx}{e}\right)\right)bf^3mn}{3e^3}-\frac{\left(3af^3m+\left(f^3mn+3f^3m\log(c)\right)b\right)\log\left(fx+e\right)}{9e^3}+\frac{12bf^3mnx^3\log(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(log(f*x/e + 1)*log(x) + dilog(-f*x/e))*b*f^3*m*n/e^3 - 1/9*(3*a*f^3*m
 + (f^3*m*n + 3*f^3*m*log(c))*b)*log(f*x + e)/e^3 + 1/36*(12*b*f^3*m*n*x^3*
log(f*x + e)*log(x) - 6*b*f^3*m*n*x^3*log(x)^2 - 12*a*e^3*log(d) + 4*(3*a*f
^3*m + (f^3*m*n + 3*f^3*m*log(c))*b)*x^3*log(x) + 4*(3*a*e*f^2*m + (4*e*f^2
*m*n + 3*e*f^2*m*log(c))*b)*x^2 - 4*(e^3*n*log(d) + 3*e^3*log(c)*log(d))*b
- (6*a*e^2*f*m + (5*e^2*f*m*n + 6*e^2*f*m*log(c))*b)*x - 4*(3*b*e^3*log(x^n
) + 3*a*e^3 + (e^3*n + 3*e^3*log(c))*b)*log((f*x + e)^m) - 6*(2*b*f^3*m*x^3
*log(f*x + e) - 2*b*f^3*m*x^3*log(x) - 2*b*e*f^2*m*x^2 + b*e^2*f*m*x + 2*b*
e^3*log(d))*log(x^n))/(e^3*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x**4,x)
```

```
[Out] Timed out
```

3.78 $\int x^2 \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d(e+fx)^m\right)\,dx$

Optimal. Leaf size=452

$$\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)+\frac{2be^3mn\text{Li}_2\left(-\frac{fx}{e}\right)\left(a+b\log\right.}{3f^3}$$

```
[Out] 8/9*a*b*e^2*m*n*x/f^2-26/27*b^2*e^2*m*n^2*x/f^2+19/108*b^2*e*m*n^2*x^2/f-2/
27*b^2*m*n^2*x^3+8/9*b^2*e^2*m*n*x*ln(c*x^n)/f^2-5/18*b*e*m*n*x^2*(a+b*ln(c
*x^n))/f+4/27*b*m*n*x^3*(a+b*ln(c*x^n))-1/3*e^2*m*x*(a+b*ln(c*x^n))^2/f^2+1
/6*e*m*x^2*(a+b*ln(c*x^n))^2/f-1/9*m*x^3*(a+b*ln(c*x^n))^2+2/27*b^2*e^3*m*n
^2*ln(f*x+e)/f^3+2/27*b^2*n^2*x^3*ln(d*(f*x+e)^m)-2/9*b*n*x^3*(a+b*ln(c*x^n
))*ln(d*(f*x+e)^m)+1/3*x^3*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)-2/9*b*e^3*m*n*
(a+b*ln(c*x^n))*ln(1+f*x/e)/f^3+1/3*e^3*m*(a+b*ln(c*x^n))^2*ln(1+f*x/e)/f^3
-2/9*b^2*e^3*m*n^2*polylog(2,-f*x/e)/f^3+2/3*b*e^3*m*n*(a+b*ln(c*x^n))*poly
log(2,-f*x/e)/f^3-2/3*b^2*e^3*m*n^2*polylog(3,-f*x/e)/f^3
```

Rubi [A] time = 0.68, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2305, 2304, 2378, 43, 2351, 2295, 2317, 2391, 2353, 2296, 2374, 6589}

$$\frac{2be^3mn\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{3f^3}-\frac{2b^2e^3mn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{9f^3}-\frac{2b^2e^3mn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{3f^3}+\frac{1}{3}x^3\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] (8*a*b*e^2*m*n*x)/(9*f^2) - (26*b^2*e^2*m*n^2*x)/(27*f^2) + (19*b^2*e*m*n^2
*x^2)/(108*f) - (2*b^2*m*n^2*x^3)/27 + (8*b^2*e^2*m*n*x*Log[c*x^n])/(9*f^2)
 - (5*b*e*m*n*x^2*(a + b*Log[c*x^n]))/(18*f) + (4*b*m*n*x^3*(a + b*Log[c*x^
n]))/27 - (e^2*m*x*(a + b*Log[c*x^n])^2)/(3*f^2) + (e*m*x^2*(a + b*Log[c*x^
n])^2)/(6*f) - (m*x^3*(a + b*Log[c*x^n])^2)/9 + (2*b^2*e^3*m*n^2*Log[e + f*
x])/(27*f^3) + (2*b^2*n^2*x^3*Log[d*(e + f*x)^m])/27 - (2*b*n*x^3*(a + b*Lo
g[c*x^n])*Log[d*(e + f*x)^m])/9 + (x^3*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)
^m])/3 - (2*b*e^3*m*n*(a + b*Log[c*x^n])*Log[1 + (f*x)/e])/(9*f^3) + (e^3*m
*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/(3*f^3) - (2*b^2*e^3*m*n^2*PolyLog[
2, -((f*x)/e)])/(9*f^3) + (2*b*e^3*m*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*x
)/e)])/(3*f^3) - (2*b^2*e^3*m*n^2*PolyLog[3, -((f*x)/e)])/(3*f^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d(e+fx)^m\right) dx = \frac{2}{27}b^2n^2x^3\log\left(d(e+fx)^m\right) - \frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+\right.$$

$$= \frac{2}{27}b^2n^2x^3\log\left(d(e+fx)^m\right) - \frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+\right.$$

$$= \frac{2}{27}b^2n^2x^3\log\left(d(e+fx)^m\right) - \frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+\right.$$

$$= -\frac{2b^2e^2mn^2x}{27f^2} + \frac{b^2emn^2x^2}{27f} - \frac{2}{81}b^2mn^2x^3 + \frac{2b^2e^3mn^2\log(e+f}{27f^3}$$

$$= \frac{2abe^2mnx}{9f^2} - \frac{2b^2e^2mn^2x}{27f^2} + \frac{5b^2emn^2x^2}{54f} - \frac{4}{81}b^2mn^2x^3 - \frac{bemnx^2}{}$$

$$= \frac{8abe^2mnx}{9f^2} - \frac{8b^2e^2mn^2x}{27f^2} + \frac{19b^2emn^2x^2}{108f} - \frac{2}{27}b^2mn^2x^3 + \frac{2b^2e^2}{}$$

$$= \frac{8abe^2mnx}{9f^2} - \frac{26b^2e^2mn^2x}{27f^2} + \frac{19b^2emn^2x^2}{108f} - \frac{2}{27}b^2mn^2x^3 + \frac{8b^2e}{}$$

Mathematica [A] time = 0.35, size = 788, normalized size = 1.74

$$\frac{36a^2f^3x^3\log\left(d(e+fx)^m\right) + 36a^2e^3m\log(e+fx) - 36a^2e^2fmx + 18a^2ef^2mx^2 - 12a^2f^3mx^3 + 72abf^3x^3\log(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] (-36*a^2*e^2*f*m*x + 96*a*b*e^2*f*m*n*x - 104*b^2*e^2*f*m*n^2*x + 18*a^2*e*
f^2*m*x^2 - 30*a*b*e*f^2*m*n*x^2 + 19*b^2*e*f^2*m*n^2*x^2 - 12*a^2*f^3*m*x^
3 + 16*a*b*f^3*m*n*x^3 - 8*b^2*f^3*m*n^2*x^3 - 72*a*b*e^2*f*m*x*Log[c*x^n]
+ 96*b^2*e^2*f*m*n*x*Log[c*x^n] + 36*a*b*e*f^2*m*x^2*Log[c*x^n] - 30*b^2*e*
f^2*m*n*x^2*Log[c*x^n] - 24*a*b*f^3*m*x^3*Log[c*x^n] + 16*b^2*f^3*m*n*x^3*L
og[c*x^n] - 36*b^2*e^2*f*m*x*Log[c*x^n]^2 + 18*b^2*e*f^2*m*x^2*Log[c*x^n]^2
 - 12*b^2*f^3*m*x^3*Log[c*x^n]^2 + 36*a^2*e^3*m*Log[e + f*x] - 24*a*b*e^3*m
*n*Log[e + f*x] + 8*b^2*e^3*m*n^2*Log[e + f*x] - 72*a*b*e^3*m*n*Log[x]*Log[
e + f*x] + 24*b^2*e^3*m*n^2*Log[x]*Log[e + f*x] + 36*b^2*e^3*m*n^2*Log[x]^2
*Log[e + f*x] + 72*a*b*e^3*m*Log[c*x^n]*Log[e + f*x] - 24*b^2*e^3*m*n*Log[c
*x^n]*Log[e + f*x] - 72*b^2*e^3*m*n*Log[x]*Log[c*x^n]*Log[e + f*x] + 36*b^2
*e^3*m*Log[c*x^n]^2*Log[e + f*x] + 36*a^2*f^3*x^3*Log[d*(e + f*x)^m] - 24*a
*b*f^3*n*x^3*Log[d*(e + f*x)^m] + 8*b^2*f^3*n^2*x^3*Log[d*(e + f*x)^m] + 72
*a*b*f^3*x^3*Log[c*x^n]*Log[d*(e + f*x)^m] - 24*b^2*f^3*n*x^3*Log[c*x^n]*Lo
g[d*(e + f*x)^m] + 36*b^2*f^3*x^3*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 72*a*b*
e^3*m*n*Log[x]*Log[1 + (f*x)/e] - 24*b^2*e^3*m*n^2*Log[x]*Log[1 + (f*x)/e]
- 36*b^2*e^3*m*n^2*Log[x]^2*Log[1 + (f*x)/e] + 72*b^2*e^3*m*n*Log[x]*Log[c*
```

```
x^n]*Log[1 + (f*x)/e] + 24*b*e^3*m*n*(3*a - b*n + 3*b*Log[c*x^n])*PolyLog[2
, -((f*x)/e)] - 72*b^2*e^3*m*n^2*PolyLog[3, -((f*x)/e)])/(108*f^3)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2x^2\log\left(cx^n\right)^2+2\,abx^2\log\left(cx^n\right)+a^2x^2\right)\log\left(\left(fx+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log((f*x +
 e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^2x^2\log\left(\left(fx+e\right)^m d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*x + e)^m*d), x)
```

maple [F] time = 4.81, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^2x^2\ln\left(d\left(fx+e\right)^m\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(3\,b^2ef^2mx^2-6\,b^2e^2fmx+6\,b^2e^3m\log\left(fx+e\right)-2\left(f^3m-3\,f^3\log(d)\right)b^2x^3\right)\log\left(x^n\right)^2+2\left(9\,b^2f^3x^3\log\left(x^n\right)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] 1/54*(3*(3*b^2*e*f^2*m*x^2 - 6*b^2*e^2*f*m*x + 6*b^2*e^3*m*log(f*x + e) - 2
*(f^3*m - 3*f^3*log(d))*b^2*x^3)*log(x^n)^2 + 2*(9*b^2*f^3*x^3*log(x^n)^2 +
 6*(3*a*b*f^3 - (f^3*n - 3*f^3*log(c))*b^2)*x^3*log(x^n) + (9*a^2*f^3 - 6*(
f^3*n - 3*f^3*log(c))*a*b + (2*f^3*n^2 - 6*f^3*n*log(c) + 9*f^3*log(c)^2)*b
^2)*x^3)*log((f*x + e)^m))/f^3 - integrate(1/27*((9*(f^4*m - 3*f^4*log(d))*
a^2 - 6*(f^4*m*n - 3*(f^4*m - 3*f^4*log(d))*log(c))*a*b + (2*f^4*m*n^2 - 6*
f^4*m*n*log(c) + 9*(f^4*m - 3*f^4*log(d))*log(c)^2)*b^2)*x^4 - 27*(b^2*e*f^
3*log(c)^2*log(d) + 2*a*b*e*f^3*log(c)*log(d) + a^2*e*f^3*log(d))*x^3 - 3*(
3*b^2*e^2*f^2*m*n*x^2 + 6*b^2*e^3*f*m*n*x - 2*(3*(f^4*m - 3*f^4*log(d))*a*b
 - (2*f^4*m*n - 3*f^4*n*log(d) - 3*(f^4*m - 3*f^4*log(d))*log(c))*b^2)*x^4
+ (18*a*b*e*f^3*log(d) - (e*f^3*m*n + 6*e*f^3*n*log(d) - 18*e*f^3*log(c)*lo
g(d))*b^2)*x^3 - 6*(b^2*e^3*f*m*n*x + b^2*e^4*m*n)*log(f*x + e))*log(x^n))/
(f^4*x^2 + e*f^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2,x)
[Out] int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m),x)
[Out] Timed out
```

3.79 $\int x \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d(e+fx)^m\right)\,dx$

Optimal. Leaf size=373

$$-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)+\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)-\frac{be^2mn\text{Li}_2\left(-\frac{fx}{e}\right)\left(a+b\log(c\right.}{f^2}$$

```
[Out] -3/2*a*b*e*m*n*x/f+7/4*b^2*e*m*n^2*x/f-3/8*b^2*m*n^2*x^2-3/2*b^2*e*m*n*x*ln
(c*x^n)/f+1/2*b*m*n*x^2*(a+b*ln(c*x^n))+1/2*e*m*x*(a+b*ln(c*x^n))^2/f-1/4*m
*x^2*(a+b*ln(c*x^n))^2-1/4*b^2*e^2*m*n^2*ln(f*x+e)/f^2+1/4*b^2*n^2*x^2*ln(d
*(f*x+e)^m)-1/2*b*n*x^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)+1/2*x^2*(a+b*ln(c*x
^n))^2*ln(d*(f*x+e)^m)+1/2*b*e^2*m*n*(a+b*ln(c*x^n))*ln(1+f*x/e)/f^2-1/2*e^
2*m*(a+b*ln(c*x^n))^2*ln(1+f*x/e)/f^2+1/2*b^2*e^2*m*n^2*polylog(2,-f*x/e)/f
^2-b*e^2*m*n*(a+b*ln(c*x^n))*polylog(2,-f*x/e)/f^2+b^2*e^2*m*n^2*polylog(3,
-f*x/e)/f^2
```

Rubi [A] time = 0.53, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2305, 2304, 2378, 43, 2351, 2295, 2317, 2391, 2353, 2296, 2374, 6589}

$$-\frac{be^2mn\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}+\frac{b^2e^2mn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{2f^2}+\frac{b^2e^2mn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{f^2}-\frac{1}{2}bnx^2\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] (-3*a*b*e*m*n*x)/(2*f) + (7*b^2*e*m*n^2*x)/(4*f) - (3*b^2*m*n^2*x^2)/8 - (3
*b^2*e*m*n*x*Log[c*x^n])/(2*f) + (b*m*n*x^2*(a + b*Log[c*x^n]))/2 + (e*m*x*
(a + b*Log[c*x^n])^2)/(2*f) - (m*x^2*(a + b*Log[c*x^n])^2)/4 - (b^2*e^2*m*n
^2*Log[e + f*x])/(4*f^2) + (b^2*n^2*x^2*Log[d*(e + f*x)^m])/4 - (b*n*x^2*(a
 + b*Log[c*x^n])*Log[d*(e + f*x)^m])/2 + (x^2*(a + b*Log[c*x^n])^2*Log[d*(e
 + f*x)^m])/2 + (b*e^2*m*n*(a + b*Log[c*x^n])*Log[1 + (f*x)/e])/(2*f^2) - (
e^2*m*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/(2*f^2) + (b^2*e^2*m*n^2*PolyL
og[2, -((f*x)/e)])/(2*f^2) - (b*e^2*m*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*
x)/e)])/f^2 + (b^2*e^2*m*n^2*PolyLog[3, -((f*x)/e)])/f^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)\,dx=\frac{1}{4}b^2n^2x^2\log\left(d(e+fx)^m\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^n\right.$$

$$=\frac{1}{4}b^2n^2x^2\log\left(d(e+fx)^m\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^n\right.$$

$$=\frac{1}{4}b^2n^2x^2\log\left(d(e+fx)^m\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^n\right.$$

$$=\frac{b^2emn^2x}{4f}-\frac{1}{8}b^2mn^2x^2-\frac{b^2e^2mn^2\log(e+fx)}{4f^2}+\frac{1}{4}b^2n^2x^2\log\left(d(e+\right.$$

$$=-\frac{abemnx}{2f}+\frac{b^2emn^2x}{4f}-\frac{1}{4}b^2mn^2x^2+\frac{1}{4}bmnx^2\left(a+b\log\left(cx^n\right)\right)+\frac{e}{-}$$

$$=-\frac{3abemnx}{2f}+\frac{3b^2emn^2x}{4f}-\frac{3}{8}b^2mn^2x^2-\frac{b^2emnx\log\left(cx^n\right)}{2f}+\frac{1}{2}bmnx$$

$$=-\frac{3abemnx}{2f}+\frac{7b^2emn^2x}{4f}-\frac{3}{8}b^2mn^2x^2-\frac{3b^2emnx\log\left(cx^n\right)}{2f}+\frac{1}{2}bmn$$

Mathematica [A] time = 0.29, size = 674, normalized size = 1.81

$$\frac{4a^2f^2x^2\log\left(d(e+fx)^m\right)-4a^2e^2m\log(e+fx)+4a^2efmx-2a^2f^2mx^2+8abf^2x^2\log\left(cx^n\right)\log\left(d(e+fx)^m\right)+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] (4*a^2*e*f*m*x - 12*a*b*e*f*m*n*x + 14*b^2*e*f*m*n^2*x - 2*a^2*f^2*m*x^2 +
4*a*b*f^2*m*n*x^2 - 3*b^2*f^2*m*n^2*x^2 + 8*a*b*e*f*m*x*Log[c*x^n] - 12*b^2
*e*f*m*n*x*Log[c*x^n] - 4*a*b*f^2*m*x^2*Log[c*x^n] + 4*b^2*f^2*m*n*x^2*Log[
c*x^n] + 4*b^2*e*f*m*x*Log[c*x^n]^2 - 2*b^2*f^2*m*x^2*Log[c*x^n]^2 - 4*a^2*
e^2*m*Log[e + f*x] + 4*a*b*e^2*m*n*Log[e + f*x] - 2*b^2*e^2*m*n^2*Log[e + f
*x] + 8*a*b*e^2*m*n*Log[x]*Log[e + f*x] - 4*b^2*e^2*m*n^2*Log[x]*Log[e + f*
x] - 4*b^2*e^2*m*n^2*Log[x]^2*Log[e + f*x] - 8*a*b*e^2*m*Log[c*x^n]*Log[e +
 f*x] + 4*b^2*e^2*m*n*Log[c*x^n]*Log[e + f*x] + 8*b^2*e^2*m*n*Log[x]*Log[c*
x^n]*Log[e + f*x] - 4*b^2*e^2*m*Log[c*x^n]^2*Log[e + f*x] + 4*a^2*f^2*x^2*L
og[d*(e + f*x)^m] - 4*a*b*f^2*n*x^2*Log[d*(e + f*x)^m] + 2*b^2*f^2*n^2*x^2*
Log[d*(e + f*x)^m] + 8*a*b*f^2*x^2*Log[c*x^n]*Log[d*(e + f*x)^m] - 4*b^2*f^
2*n*x^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 4*b^2*f^2*x^2*Log[c*x^n]^2*Log[d*(e
 + f*x)^m] - 8*a*b*e^2*m*n*Log[x]*Log[1 + (f*x)/e] + 4*b^2*e^2*m*n^2*Log[x]
*Log[1 + (f*x)/e] + 4*b^2*e^2*m*n^2*Log[x]^2*Log[1 + (f*x)/e] - 8*b^2*e^2*m
*n*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 4*b*e^2*m*n*(-2*a + b*n - 2*b*Log[c
*x^n])*PolyLog[2, -((f*x)/e)] + 8*b^2*e^2*m*n^2*PolyLog[3, -((f*x)/e)])/(8*
f^2)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x\log\left(cx^n\right)^2+2\,abx\log\left(cx^n\right)+a^2x\right)\log\left(\left(fx+e\right)^md\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log((f*x + e)^m*
d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^{n}\right) + a\right)^{2} x \log\left(\left(fx + e\right)^{m} d\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*x + e)^m*d), x)
```

maple [F] time = 6.90, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\,x^{n}\right) + a\right)^{2} x \ln\left(d \left(fx + e\right)^{m}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(2\,b^{2}efmx - 2\,b^{2}e^{2}m \log\left(fx + e\right) - \left(f^{2}m - 2\,f^{2}\log(d)\right)b^{2}x^{2}\right) \log\left(x^{n}\right)^{2} + \left(2\,b^{2}f^{2}x^{2}\log\left(x^{n}\right)^{2} + 2\left(2\,abf^{2} - \left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] 1/4*((2*b^2*e*f*m*x - 2*b^2*e^2*m*log(f*x + e) - (f^2*m - 2*f^2*log(d))*b^2
*x^2)*log(x^n)^2 + (2*b^2*f^2*x^2*log(x^n)^2 + 2*(2*a*b*f^2 - (f^2*n - 2*f^
2*log(c))*b^2)*x^2*log(x^n) + (2*a^2*f^2 - 2*(f^2*n - 2*f^2*log(c))*a*b + (
f^2*n^2 - 2*f^2*n*log(c) + 2*f^2*log(c)^2)*b^2)*x^2)*log((f*x + e)^m))/f^2
+ integrate(-1/4*((2*(f^3*m - 2*f^3*log(d))*a^2 - 2*(f^3*m*n - 2*(f^3*m - 2
*f^3*log(d))*log(c))*a*b + (f^3*m*n^2 - 2*f^3*m*n*log(c) + 2*(f^3*m - 2*f^3
*log(d))*log(c)^2)*b^2)*x^3 - 4*(b^2*e*f^2*log(c)^2*log(d) + 2*a*b*e*f^2*lo
g(c)*log(d) + a^2*e*f^2*log(d))*x^2 + 2*(2*b^2*e^2*f*m*n*x + 2*((f^3*m - 2*
f^3*log(d))*a*b - (f^3*m*n - f^3*n*log(d) - (f^3*m - 2*f^3*log(d))*log(c))*
b^2)*x^3 - (4*a*b*e*f^2*log(d) - (e*f^2*m*n + 2*e*f^2*n*log(d) - 4*e*f^2*lo
g(c)*log(d))*b^2)*x^2 - 2*(b^2*e^2*f*m*n*x + b^2*e^3*m*n)*log(f*x + e))*log
(x^n))/(f^3*x^2 + e*f^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\left(e + f\,x\right)^{m}\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

3.80 $\int \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d(e+fx)^m\right)\,dx$

Optimal. Leaf size=288

$$x\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)+\frac{2bemn\text{Li}_2\left(-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f}+\frac{em\log\left(\frac{fx}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{f}-r$$

```
[Out] 2*a*b*m*n*x-4*b^2*m*n^2*x+2*b*m*n*(-b*n+a)*x+4*b^2*m*n*x*ln(c*x^n)-m*x*(a+b
*ln(c*x^n))^2-2*b*e*m*n*(-b*n+a)*ln(f*x+e)/f-2*a*b*n*x*ln(d*(f*x+e)^m)+2*b^
2*n^2*x*ln(d*(f*x+e)^m)-2*b^2*n*x*ln(c*x^n)*ln(d*(f*x+e)^m)+x*(a+b*ln(c*x^n
))^2*ln(d*(f*x+e)^m)-2*b^2*e*m*n*ln(c*x^n)*ln(1+f*x/e)/f+e*m*(a+b*ln(c*x^n)
)^2*ln(1+f*x/e)/f-2*b^2*e*m*n^2*polylog(2,-f*x/e)/f+2*b*e*m*n*(a+b*ln(c*x^n
))*polylog(2,-f*x/e)/f-2*b^2*e*m*n^2*polylog(3,-f*x/e)/f
```

Rubi [A] time = 0.35, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2296, 2295, 2371, 6, 43, 2351, 2317, 2391, 2353, 2374, 6589}

$$\frac{2bemn\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f}-\frac{2b^2emn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{f}-\frac{2b^2emn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{f}+x\left(a+b\,l\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] 2*a*b*m*n*x - 4*b^2*m*n^2*x + 2*b*m*n*(a - b*n)*x + 4*b^2*m*n*x*Log[c*x^n]
- m*x*(a + b*Log[c*x^n])^2 - (2*b*e*m*n*(a - b*n)*Log[e + f*x])/f - 2*a*b*n
*x*Log[d*(e + f*x)^m] + 2*b^2*n^2*x*Log[d*(e + f*x)^m] - 2*b^2*n*x*Log[c*x^
n]*Log[d*(e + f*x)^m] + x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m] - (2*b^2*
e*m*n*Log[c*x^n]*Log[1 + (f*x)/e])/f + (e*m*(a + b*Log[c*x^n])^2*Log[1 + (f
*x)/e])/f - (2*b^2*e*m*n^2*PolyLog[2, -((f*x)/e)])/f + (2*b*e*m*n*(a + b*Lo
g[c*x^n])*PolyLog[2, -((f*x)/e)])/f - (2*b^2*e*m*n^2*PolyLog[3, -((f*x)/e)]
)/f
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2371

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)\,dx = -2abnx\log\left(d(e+fx)^m\right)+2b^2n^2x\log\left(d(e+fx)^m\right)-2b^2nx\log\left(\right.$$

$$= -2abnx\log\left(d(e+fx)^m\right)+2b^2n^2x\log\left(d(e+fx)^m\right)-2b^2nx\log\left(\right.$$

$$= -2abnx\log\left(d(e+fx)^m\right)+2b^2n^2x\log\left(d(e+fx)^m\right)-2b^2nx\log\left(\right.$$

$$= -2abnx\log\left(d(e+fx)^m\right)+2b^2n^2x\log\left(d(e+fx)^m\right)-2b^2nx\log\left(\right.$$

$$= 2bmn(a-bn)x-\frac{2bemn(a-bn)\log(e+fx)}{f}-2abnx\log\left(d(e+f\right.$$

$$= -2b^2mn^2x+2bmn(a-bn)x+2b^2mnx\log\left(cx^n\right)-mx\left(a+b\log\left(\right.\right.$$

$$= 2abmnx-2b^2mn^2x+2bmn(a-bn)x+2b^2mnx\log\left(cx^n\right)-mx\left(a\right.$$

$$= 2abmnx-4b^2mn^2x+2bmn(a-bn)x+4b^2mnx\log\left(cx^n\right)-mx\left(a\right.$$

Mathematica [A] time = 0.23, size = 507, normalized size = 1.76

$$\frac{a^2fx\log\left(d(e+fx)^m\right)+a^2em\log(e+fx)+a^2(-f)mx+2abfx\log\left(cx^n\right)\log\left(d(e+fx)^m\right)+2bemn\mathrm{Li}_2\left(-\frac{fx}{e}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] (-(a^2*f*m*x) + 4*a*b*f*m*n*x - 6*b^2*f*m*n^2*x - 2*a*b*f*m*x*Log[c*x^n] +
4*b^2*f*m*n*x*Log[c*x^n] - b^2*f*m*x*Log[c*x^n]^2 + a^2*e*m*Log[e + f*x] -
2*a*b*e*m*n*Log[e + f*x] + 2*b^2*e*m*n^2*Log[e + f*x] - 2*a*b*e*m*n*Log[x]*
Log[e + f*x] + 2*b^2*e*m*n^2*Log[x]*Log[e + f*x] + b^2*e*m*n^2*Log[x]^2*Log
[e + f*x] + 2*a*b*e*m*Log[c*x^n]*Log[e + f*x] - 2*b^2*e*m*n*Log[c*x^n]*Log[
e + f*x] - 2*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[e + f*x] + b^2*e*m*Log[c*x^n]^
2*Log[e + f*x] + a^2*f*x*Log[d*(e + f*x)^m] - 2*a*b*f*n*x*Log[d*(e + f*x)^m
] + 2*b^2*f*n^2*x*Log[d*(e + f*x)^m] + 2*a*b*f*x*Log[c*x^n]*Log[d*(e + f*x)
^m] - 2*b^2*f*n*x*Log[c*x^n]*Log[d*(e + f*x)^m] + b^2*f*x*Log[c*x^n]^2*Log[
d*(e + f*x)^m] + 2*a*b*e*m*n*Log[x]*Log[1 + (f*x)/e] - 2*b^2*e*m*n^2*Log[x]
*Log[1 + (f*x)/e] - b^2*e*m*n^2*Log[x]^2*Log[1 + (f*x)/e] + 2*b^2*e*m*n*Log
[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 2*b*e*m*n*(a - b*n + b*Log[c*x^n])*PolyLo
g[2, -((f*x)/e)] - 2*b^2*e*m*n^2*PolyLog[3, -((f*x)/e)])/f
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\log\left(cx^n\right)^2+2\,ab\log\left(cx^n\right)+a^2\right)\log\left(\left(fx+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="fricas")
```

[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^{n}\right)+a\right)^{2}\log\left(\left(fx+e\right)^{m}d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="giac")

[Out] integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d), x)

maple [F] time = 5.15, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^{n}\right)+a\right)^{2}\ln\left(d\left(fx+e\right)^{m}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m),x)

[Out] int((b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^{2}em\log\left(fx+e\right)-\left(fm-f\log(d)\right)b^{2}x\right)\log\left(x^{n}\right)^{2}+\left(b^{2}fx\log\left(x^{n}\right)^{2}-2\left(\left(fn-f\log(c)\right)b^{2}-abf\right)x\log\left(x^{n}\right)-\left(\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="maxima")

[Out] ((b^2*e*m*log(f*x + e) - (f*m - f*log(d))*b^2*x)*log(x^n)^2 + (b^2*f*x*log(x^n)^2 - 2*((f*n - f*log(c))*b^2 - a*b*f)*x*log(x^n) - (2*(f*n - f*log(c))*a*b - (2*f*n^2 - 2*f*n*log(c) + f*log(c)^2)*b^2 - a^2*f)*x)*log((f*x + e)^m))/f - integrate((((f^2*m - f^2*log(d))*a^2 - 2*(f^2*m*n - (f^2*m - f^2*log(d))*log(c))*a*b + (2*f^2*m*n^2 - 2*f^2*m*n*log(c) + (f^2*m - f^2*log(d))*log(c)^2)*b^2)*x^2 - (b^2*e*f*log(c)^2*log(d) + 2*a*b*e*f*log(c)*log(d) + a^2*e*f*log(d))*x + 2*(((f^2*m - f^2*log(d))*a*b - (2*f^2*m*n - f^2*n*log(d) - (f^2*m - f^2*log(d))*log(c))*b^2)*x^2 - (a*b*e*f*log(d) + (e*f*m*n - e*f*n*log(d) + e*f*log(c)*log(d))*b^2)*x + (b^2*e*f*m*n*x + b^2*e^2*m*n)*log(f*x + e))*log(x^n))/(f^2*x^2 + e*f*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(e+f\,x\right)^{m}\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2,x)

[Out] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m),x)

[Out] Timed out

3.81 $\int \frac{(a+b\log(cx^n))^2 \log(d(e+fx)^m)}{x} \, dx$

Optimal. Leaf size=131

$$\frac{(a+b\log(cx^n))^3 \log(d(e+fx)^m)}{3bn} - m\text{Li}_2\left(-\frac{fx}{e}\right)(a+b\log(cx^n))^2 + 2bmn\text{Li}_3\left(-\frac{fx}{e}\right)(a+b\log(cx^n)) - \frac{m\log}{}$$

```
[Out] 1/3*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/b/n-1/3*m*(a+b*ln(c*x^n))^3*ln(1+f*x/
e)/b/n-m*(a+b*ln(c*x^n))^2*polylog(2,-f*x/e)+2*b*m*n*(a+b*ln(c*x^n))*polylo
g(3,-f*x/e)-2*b^2*m*n^2*polylog(4,-f*x/e)
```

Rubi [A] time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2375, 2317, 2374, 2383, 6589}

$$-m\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))^2 + 2bmn\text{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n)) - 2b^2mn^2\text{PolyLog}\left(4,-\frac{fx}{e}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/(3*b*n) - (m*(a + b*Log[c*x^n])^3
*Log[1 + (f*x)/e])/(3*b*n) - m*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x)/e)]
+ 2*b*m*n*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)] - 2*b^2*m*n^2*PolyLog[4
, -((f*x)/e)]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}dx = \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{(fm)\int\frac{(a+b\log(cx^n))^3}{e+fx}dx}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{m(a+b\log(cx^n))^3\log\left(1+\frac{fx}{e}\right)}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{m(a+b\log(cx^n))^3\log\left(1+\frac{fx}{e}\right)}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{m(a+b\log(cx^n))^3\log\left(1+\frac{fx}{e}\right)}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{m(a+b\log(cx^n))^3\log\left(1+\frac{fx}{e}\right)}{3bn}$$

Mathematica [B] time = 0.18, size = 329, normalized size = 2.51

$$a^2\log(x)\log\left(d(e+fx)^m\right)-a^2m\log(x)\log\left(\frac{fx}{e}+1\right)+2ab\log(x)\log(cx^n)\log\left(d(e+fx)^m\right)-m\text{Li}_2\left(-\frac{fx}{e}\right)(a+b\log$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] a^2*Log[x]*Log[d*(e + f*x)^m] - a*b*n*Log[x]^2*Log[d*(e + f*x)^m] + (b^2*n^
2*Log[x]^3*Log[d*(e + f*x)^m])/3 + 2*a*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x)^
m] - b^2*n*Log[x]^2*Log[c*x^n]*Log[d*(e + f*x)^m] + b^2*Log[x]*Log[c*x^n]^2
*Log[d*(e + f*x)^m] - a^2*m*Log[x]*Log[1 + (f*x)/e] + a*b*m*n*Log[x]^2*Log[
1 + (f*x)/e] - (b^2*m*n^2*Log[x]^3*Log[1 + (f*x)/e])/3 - 2*a*b*m*Log[x]*Log
[c*x^n]*Log[1 + (f*x)/e] + b^2*m*n*Log[x]^2*Log[c*x^n]*Log[1 + (f*x)/e] - b
^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] - m*(a + b*Log[c*x^n])^2*PolyLog[
2, -((f*x)/e)] + 2*b*m*n*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)] - 2*b^2*
m*n^2*PolyLog[4, -((f*x)/e)]
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\log(cx^n)^2+2\,ab\log(cx^n)+a^2\right)\log\left(\left(fx+e\right)^m d\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log(cx^n)+a\right)^2\log\left(\left(fx+e\right)^m d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x, x)
```

maple [C] time = 1.31, size = 21792, normalized size = 166.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m)/x,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(b^2n^2\log(x)^3+3\,b^2\log(x)\log\left(x^n\right)^2-3\left(b^2n\log(c)+abn\right)\log(x)^2-3\left(b^2n\log(x)^2-2\left(b^2\log(c)+ab\right)\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)*
log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*(b
^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log((f*x + e)^m) - integrate(1/3*
(b^2*f*m*n^2*x*log(x)^3 - 3*b^2*e*log(c)^2*log(d) - 6*a*b*e*log(c)*log(d) -
 3*a^2*e*log(d) - 3*(b^2*f*m*n*log(c) + a*b*f*m*n)*x*log(x)^2 + 3*(b^2*f*m*
log(c)^2 + 2*a*b*f*m*log(c) + a^2*f*m)*x*log(x) + 3*(b^2*f*m*x*log(x) - b^2
*f*x*log(d) - b^2*e*log(d))*log(x^n)^2 - 3*(b^2*f*log(c)^2*log(d) + 2*a*b*f
*log(c)*log(d) + a^2*f*log(d))*x - 3*(b^2*f*m*n*x*log(x)^2 + 2*b^2*e*log(c)
*log(d) + 2*a*b*e*log(d) - 2*(b^2*f*m*log(c) + a*b*f*m)*x*log(x) + 2*(b^2*f
*log(c)*log(d) + a*b*f*log(d))*x)*log(x^n))/(f*x^2 + e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x,x)
```

```
[Out] Timed out
```

3.82 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx$

Optimal. Leaf size=248

$$-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x}-\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{x}+\frac{2bfmn\text{Li}_2\left(-\frac{e}{fx}\right)\left(a+b\log\left(cx^n\right)\right)}{e}$$

```
[Out] 2*b^2*f*m*n^2*ln(x)/e-2*b*f*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))/e-f*m*ln(1+e/f/
x)*(a+b*ln(c*x^n))^2/e-2*b^2*f*m*n^2*ln(f*x+e)/e-2*b^2*n^2*ln(d*(f*x+e)^m)/
x-2*b*n*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x-(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)
/x+2*b^2*f*m*n^2*polylog(2,-e/f/x)/e+2*b*f*m*n*(a+b*ln(c*x^n))*polylog(2,-e
/f/x)/e+2*b^2*f*m*n^2*polylog(3,-e/f/x)/e
```

Rubi [A] time = 0.40, antiderivative size = 283, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2305, 2304, 2378, 36, 29, 31, 2344, 2301, 2317, 2391, 2302, 30, 2374, 6589}

$$-\frac{2bfmn\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{2b^2fmn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{e}+\frac{2b^2fmn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{e}-\left(a+b\log\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] (2*b^2*f*m*n^2*Log[x])/e + (f*m*(a + b*Log[c*x^n])^2)/e + (f*m*(a + b*Log[c
*x^n])^3)/(3*b*e*n) - (2*b^2*f*m*n^2*Log[e + f*x])/e - (2*b^2*n^2*Log[d*(e
+ f*x)^m])/x - (2*b*n*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x - ((a + b*Lo
g[c*x^n])^2*Log[d*(e + f*x)^m])/x - (2*b*f*m*n*(a + b*Log[c*x^n])*Log[1 + (
f*x)/e])/e - (f*m*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/e - (2*b^2*f*m*n^2
*PolyLog[2, -((f*x)/e)])/e - (2*b*f*m*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*
x)/e)])/e + (2*b^2*f*m*n^2*PolyLog[3, -((f*x)/e)])/e
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{x^2}\,dx = -\frac{2b^2n^2\log\left(d(e+fx)^m\right)}{x} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x} -$$

$$= -\frac{2b^2n^2\log\left(d(e+fx)^m\right)}{x} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x} -$$

$$= -\frac{2b^2n^2\log\left(d(e+fx)^m\right)}{x} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x} -$$

$$= \frac{2b^2fmn^2\log(x)}{e} + \frac{fm\left(a+b\log\left(cx^n\right)\right)^2}{e} - \frac{2b^2fmn^2\log(e+fx)}{e} -$$

$$= \frac{2b^2fmn^2\log(x)}{e} + \frac{fm\left(a+b\log\left(cx^n\right)\right)^2}{e} + \frac{fm\left(a+b\log\left(cx^n\right)\right)^3}{3ben} -$$

$$= \frac{2b^2fmn^2\log(x)}{e} + \frac{fm\left(a+b\log\left(cx^n\right)\right)^2}{e} + \frac{fm\left(a+b\log\left(cx^n\right)\right)^3}{3ben} -$$

Mathematica [B] time = 0.35, size = 600, normalized size = 2.42

$$-\frac{3a^2e\log\left(d(e+fx)^m\right)+3a^2fmx\log(e+fx)-3a^2fmx\log(x)+6abe\log\left(cx^n\right)\log\left(d(e+fx)^m\right)+6bfmnx\text{Li}_2\Big(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] -1/3*(-3*a^2*f*m*x*Log[x] - 6*a*b*f*m*n*x*Log[x] - 6*b^2*f*m*n^2*x*Log[x] +
 3*a*b*f*m*n*x*Log[x]^2 + 3*b^2*f*m*n^2*x*Log[x]^2 - b^2*f*m*n^2*x*Log[x]^3
 - 6*a*b*f*m*x*Log[x]*Log[c*x^n] - 6*b^2*f*m*n*x*Log[x]*Log[c*x^n] + 3*b^2*
f*m*n*x*Log[x]^2*Log[c*x^n] - 3*b^2*f*m*x*Log[x]*Log[c*x^n]^2 + 3*a^2*f*m*x
*Log[e + f*x] + 6*a*b*f*m*n*x*Log[e + f*x] + 6*b^2*f*m*n^2*x*Log[e + f*x] -
 6*a*b*f*m*n*x*Log[x]*Log[e + f*x] - 6*b^2*f*m*n^2*x*Log[x]*Log[e + f*x] +
3*b^2*f*m*n^2*x*Log[x]^2*Log[e + f*x] + 6*a*b*f*m*x*Log[c*x^n]*Log[e + f*x]
 + 6*b^2*f*m*n*x*Log[c*x^n]*Log[e + f*x] - 6*b^2*f*m*n*x*Log[x]*Log[c*x^n]*
Log[e + f*x] + 3*b^2*f*m*x*Log[c*x^n]^2*Log[e + f*x] + 3*a^2*e*Log[d*(e + f
*x)^m] + 6*a*b*e*n*Log[d*(e + f*x)^m] + 6*b^2*e*n^2*Log[d*(e + f*x)^m] + 6*
a*b*e*Log[c*x^n]*Log[d*(e + f*x)^m] + 6*b^2*e*n*Log[c*x^n]*Log[d*(e + f*x)^
m] + 3*b^2*e*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 6*a*b*f*m*n*x*Log[x]*Log[1 +
 (f*x)/e] + 6*b^2*f*m*n^2*x*Log[x]*Log[1 + (f*x)/e] - 3*b^2*f*m*n^2*x*Log[x
]^2*Log[1 + (f*x)/e] + 6*b^2*f*m*n*x*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 6
*b*f*m*n*x*(a + b*n + b*Log[c*x^n])*PolyLog[2, -((f*x)/e)] - 6*b^2*f*m*n^2*
x*PolyLog[3, -((f*x)/e)])/(e*x)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\log\left(cx^n\right)^2+2\,ab\log\left(cx^n\right)+a^2\right)\log\left(\left(fx+e\right)^m d\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x^2
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{2}\log\left(\left(fx+e\right)^{m}d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x^2, x)
```

maple [C] time = 1.27, size = 10991, normalized size = 44.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m)/x^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^{2}fmx\log\left(fx+e\right)-b^{2}fmx\log(x)+b^{2}e\log(d)\right)\log\left(x^{n}\right)^{2}+\left(b^{2}e\log\left(x^{n}\right)^{2}+2\left(en+e\log(c)\right)ab+\left(2\,en^{2}+\right.\right.}{ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -((b^2*f*m*x*log(f*x + e) - b^2*f*m*x*log(x) + b^2*e*log(d))*log(x^n)^2 + (
b^2*e*log(x^n)^2 + 2*(e*n + e*log(c))*a*b + (2*e*n^2 + 2*e*n*log(c) + e*log
(c)^2)*b^2 + a^2*e + 2*((e*n + e*log(c))*b^2 + a*b*e)*log(x^n))*log((f*x +
e)^m))/(e*x) + integrate((b^2*e^2*log(c)^2*log(d) + 2*a*b*e^2*log(c)*log(d)
 + a^2*e^2*log(d) + ((e*f*m + e*f*log(d))*a^2 + 2*(e*f*m*n + (e*f*m + e*f*l
og(d))*log(c))*a*b + (2*e*f*m*n^2 + 2*e*f*m*n*log(c) + (e*f*m + e*f*log(d))
*log(c)^2)*b^2)*x + 2*(a*b*e^2*log(d) + (e^2*n*log(d) + e^2*log(c)*log(d))*
b^2 + ((e*f*m + e*f*log(d))*a*b + (e*f*m*n + e*f*n*log(d) + (e*f*m + e*f*lo
g(d))*log(c))*b^2)*x + (b^2*f^2*m*n*x^2 + b^2*e*f*m*n*x)*log(f*x + e) - (b^
2*f^2*m*n*x^2 + b^2*e*f*m*n*x)*log(x))*log(x^n))/(e*f*x^3 + e^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^{m}\right)\,{\left(a+b\,\ln\left(c\,x^{n}\right)\right)}^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^2,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x**2,x)
```

```
[Out] Timed out
```

3.83 $\int \frac{\left(a+b\log(cx^n)\right)^2 \log\left(d(e+fx)^m\right)}{x^3} dx$

Optimal. Leaf size=344

$$-\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{2x^2}-\frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{2x^2}-\frac{bf^2mn\text{Li}_2\left(-\frac{e}{fx}\right)\left(a+b\log\left(cx^n\right)\right)}{e^2}+$$

```
[Out] -7/4*b^2*f*m*n^2/e/x-1/4*b^2*f^2*m*n^2*ln(x)/e^2-3/2*b*f*m*n*(a+b*ln(c*x^n)
)/e/x+1/2*b*f^2*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))/e^2-1/2*f*m*(a+b*ln(c*x^n))
^2/e/x+1/2*f^2*m*ln(1+e/f/x)*(a+b*ln(c*x^n))^2/e^2+1/4*b^2*f^2*m*n^2*ln(f*x
+e)/e^2-1/4*b^2*n^2*ln(d*(f*x+e)^m)/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*(f*x+e
)^m)/x^2-1/2*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^2-1/2*b^2*f^2*m*n^2*polylo
g(2,-e/f/x)/e^2-b*f^2*m*n*(a+b*ln(c*x^n))*polylog(2,-e/f/x)/e^2-b^2*f^2*m*n
^2*polylog(3,-e/f/x)/e^2
```

Rubi [A] time = 0.59, antiderivative size = 385, normalized size of antiderivative = 1.12, number of steps used = 19, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2305, 2304, 2378, 44, 2351, 2301, 2317, 2391, 2353, 2302, 30, 2374, 6589}

$$\frac{bf^2mn\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e^2}+\frac{b^2f^2mn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{2e^2}-\frac{b^2f^2mn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{e^2}-\frac{\left(a+b\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] (-7*b^2*f*m*n^2)/(4*e*x) - (b^2*f^2*m*n^2*Log[x])/(4*e^2) - (3*b*f*m*n*(a +
 b*Log[c*x^n]))/(2*e*x) - (f^2*m*(a + b*Log[c*x^n])^2)/(4*e^2) - (f*m*(a +
b*Log[c*x^n])^2)/(2*e*x) - (f^2*m*(a + b*Log[c*x^n])^3)/(6*b*e^2*n) + (b^2*
f^2*m*n^2*Log[e + f*x])/(4*e^2) - (b^2*n^2*Log[d*(e + f*x)^m])/(4*x^2) - (b
*n*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/(2*x^2) - ((a + b*Log[c*x^n])^2*L
og[d*(e + f*x)^m])/(2*x^2) + (b*f^2*m*n*(a + b*Log[c*x^n])*Log[1 + (f*x)/e]
)/(2*e^2) + (f^2*m*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/(2*e^2) + (b^2*f^
2*m*n^2*PolyLog[2, -((f*x)/e)])/(2*e^2) + (b*f^2*m*n*(a + b*Log[c*x^n])*Pol
yLog[2, -((f*x)/e)])/e^2 - (b^2*f^2*m*n^2*PolyLog[3, -((f*x)/e)])/e^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{x^3}\,dx = -\frac{b^2n^2\log\left(d(e+fx)^m\right)}{4x^2} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{2x^2} - \frac{(a}{}$$

$$= -\frac{b^2n^2\log\left(d(e+fx)^m\right)}{4x^2} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{2x^2} - \frac{(a}{}$$

$$= -\frac{b^2n^2\log\left(d(e+fx)^m\right)}{4x^2} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{2x^2} - \frac{(a}{}$$

$$= -\frac{b^2fmn^2}{4ex} - \frac{b^2f^2mn^2\log(x)}{4e^2} + \frac{b^2f^2mn^2\log(e+fx)}{4e^2} - \frac{b^2n^2\log\left(d(e\right.}{4x^2}$$

$$= -\frac{3b^2fmn^2}{4ex} - \frac{b^2f^2mn^2\log(x)}{4e^2} - \frac{bfmn\left(a+b\log\left(cx^n\right)\right)}{2ex} - \frac{f^2m\left(a+\right.}{4}$$

$$= -\frac{7b^2fmn^2}{4ex} - \frac{b^2f^2mn^2\log(x)}{4e^2} - \frac{3bfmn\left(a+b\log\left(cx^n\right)\right)}{2ex} - \frac{f^2m\left(a+\right.}{}$$

$$= -\frac{7b^2fmn^2}{4ex} - \frac{b^2f^2mn^2\log(x)}{4e^2} - \frac{3bfmn\left(a+b\log\left(cx^n\right)\right)}{2ex} - \frac{f^2m\left(a+\right.}{}$$

Mathematica [B] time = 0.39, size = 796, normalized size = 2.31

$$-\frac{2b^2f^2mn^2x^2\log^3(x) - 3b^2f^2mn^2x^2\log^2(x) - 6abf^2mnx^2\log^2(x) - 6b^2f^2mnx^2\log\left(cx^n\right)\log^2(x) - 6b^2f^2mn^2x^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] -1/12*(6*a^2*e*f*m*x + 18*a*b*e*f*m*n*x + 21*b^2*e*f*m*n^2*x + 6*a^2*f^2*m*
x^2*Log[x] + 6*a*b*f^2*m*n*x^2*Log[x] + 3*b^2*f^2*m*n^2*x^2*Log[x] - 6*a*b*
f^2*m*n*x^2*Log[x]^2 - 3*b^2*f^2*m*n^2*x^2*Log[x]^2 + 2*b^2*f^2*m*n^2*x^2*L
og[x]^3 + 12*a*b*e*f*m*x*Log[c*x^n] + 18*b^2*e*f*m*n*x*Log[c*x^n] + 12*a*b*
f^2*m*x^2*Log[x]*Log[c*x^n] + 6*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n] - 6*b^2*f
^2*m*n*x^2*Log[x]^2*Log[c*x^n] + 6*b^2*e*f*m*x*Log[c*x^n]^2 + 6*b^2*f^2*m*x
^2*Log[x]*Log[c*x^n]^2 - 6*a^2*f^2*m*x^2*Log[e + f*x] - 6*a*b*f^2*m*n*x^2*L
og[e + f*x] - 3*b^2*f^2*m*n^2*x^2*Log[e + f*x] + 12*a*b*f^2*m*n*x^2*Log[x]*
Log[e + f*x] + 6*b^2*f^2*m*n^2*x^2*Log[x]*Log[e + f*x] - 6*b^2*f^2*m*n^2*x^
2*Log[x]^2*Log[e + f*x] - 12*a*b*f^2*m*x^2*Log[c*x^n]*Log[e + f*x] - 6*b^2*
f^2*m*n*x^2*Log[c*x^n]*Log[e + f*x] + 12*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n]*
Log[e + f*x] - 6*b^2*f^2*m*x^2*Log[c*x^n]^2*Log[e + f*x] + 6*a^2*e^2*Log[d*
(e + f*x)^m] + 6*a*b*e^2*n*Log[d*(e + f*x)^m] + 3*b^2*e^2*n^2*Log[d*(e + f*
x)^m] + 12*a*b*e^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 6*b^2*e^2*n*Log[c*x^n]*L
og[d*(e + f*x)^m] + 6*b^2*e^2*Log[c*x^n]^2*Log[d*(e + f*x)^m] - 12*a*b*f^2*
m*n*x^2*Log[x]*Log[1 + (f*x)/e] - 6*b^2*f^2*m*n^2*x^2*Log[x]*Log[1 + (f*x)/
e] + 6*b^2*f^2*m*n^2*x^2*Log[x]^2*Log[1 + (f*x)/e] - 12*b^2*f^2*m*n*x^2*Log
```

```
[x]*Log[c*x^n]*Log[1 + (f*x)/e] - 6*b*f^2*m*n*x^2*(2*a + b*n + 2*b*Log[c*x^
n])*PolyLog[2, -((f*x)/e)] + 12*b^2*f^2*m*n^2*x^2*PolyLog[3, -((f*x)/e)])/(
e^2*x^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^{2}\log\left(cx^{n}\right)^{2}+2\,ab\log\left(cx^{n}\right)+a^{2}\right)\log\left(\left(fx+e\right)^{m}d\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x^3
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^{n}\right)+a\right)^{2}\log\left(\left(fx+e\right)^{m}d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x^3, x)
```

maple [F] time = 5.04, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^{n}\right)+a\right)^{2}\ln\left(d\left(fx+e\right)^{m}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(b^{2}f^{2}mx^{2}\log\left(fx+e\right)-b^{2}f^{2}mx^{2}\log(x)-b^{2}efmx-b^{2}e^{2}\log(d)\right)\log\left(x^{n}\right)^{2}-\left(2\,b^{2}e^{2}\log\left(x^{n}\right)^{2}+2\,a^{2}e^{2}+2\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(b^2*f^2*m*x^2*log(f*x + e) - b^2*f^2*m*x^2*log(x) - b^2*e*f*m*x - b
^2*e^2*log(d))*log(x^n)^2 - (2*b^2*e^2*log(x^n)^2 + 2*a^2*e^2 + 2*(e^2*n +
2*e^2*log(c))*a*b + (e^2*n^2 + 2*e^2*n*log(c) + 2*e^2*log(c)^2)*b^2 + 2*(2*
a*b*e^2 + (e^2*n + 2*e^2*log(c))*b^2)*log(x^n))*log((f*x + e)^m))/(e^2*x^2)
 - integrate(-1/4*(4*b^2*e^3*log(c)^2*log(d) + 8*a*b*e^3*log(c)*log(d) + 4*
a^2*e^3*log(d) + (2*(e^2*f*m + 2*e^2*f*log(d))*a^2 + 2*(e^2*f*m*n + 2*(e^2*
f*m + 2*e^2*f*log(d))*log(c))*a*b + (e^2*f*m*n^2 + 2*e^2*f*m*n*log(c) + 2*(
e^2*f*m + 2*e^2*f*log(d))*log(c)^2)*b^2)*x + 2*(2*b^2*e*f^2*m*n*x^2 + 4*a*b
*e^3*log(d) + 2*(e^3*n*log(d) + 2*e^3*log(c)*log(d))*b^2 + (2*(e^2*f*m + 2*
e^2*f*log(d))*a*b + (3*e^2*f*m*n + 2*e^2*f*n*log(d) + 2*(e^2*f*m + 2*e^2*f*
log(d))*log(c))*b^2)*x - 2*(b^2*f^3*m*n*x^3 + b^2*e*f^2*m*n*x^2)*log(f*x +
```

```
e) + 2*(b^2*f^3*m*n*x^3 + b^2*e*f^2*m*n*x^2)*log(x))*log(x^n))/(e^2*f*x^4 +
 e^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+fx\right)^{m}\right)(a+b\,\ln\left(c\,x^{n}\right))^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x**3,x)
```

```
[Out] Timed out
```

3.84 $\int \frac{\left(a+b\log(cx^n)\right)^2\log\left(d(e+fx)^m\right)}{x^4}\,dx$

Optimal. Leaf size=420

$$-\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{3x^3}-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{9x^3}+\frac{2bf^3mn\text{Li}_2\left(-\frac{e}{fx}\right)\left(a+b\log\left(cx\right.\right.}{3e^3}$$

```
[Out] -19/108*b^2*f*m*n^2/e/x^2+26/27*b^2*f^2*m*n^2/e^2/x+2/27*b^2*f^3*m*n^2*ln(x
)/e^3-5/18*b*f*m*n*(a+b*ln(c*x^n))/e/x^2+8/9*b*f^2*m*n*(a+b*ln(c*x^n))/e^2/
x-2/9*b*f^3*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))/e^3-1/6*f*m*(a+b*ln(c*x^n))^2/e
/x^2+1/3*f^2*m*(a+b*ln(c*x^n))^2/e^2/x-1/3*f^3*m*ln(1+e/f/x)*(a+b*ln(c*x^n)
)^2/e^3-2/27*b^2*f^3*m*n^2*ln(f*x+e)/e^3-2/27*b^2*n^2*ln(d*(f*x+e)^m)/x^3-2
/9*b*n*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^3-1/3*(a+b*ln(c*x^n))^2*ln(d*(f*x+
e)^m)/x^3+2/9*b^2*f^3*m*n^2*polylog(2,-e/f/x)/e^3+2/3*b*f^3*m*n*(a+b*ln(c*x
^n))*polylog(2,-e/f/x)/e^3+2/3*b^2*f^3*m*n^2*polylog(3,-e/f/x)/e^3
```

Rubi [A] time = 0.72, antiderivative size = 462, normalized size of antiderivative = 1.10, number of steps used = 22, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2305, 2304, 2378, 44, 2351, 2301, 2317, 2391, 2353, 2302, 30, 2374, 6589}

$$-\frac{2bf^3mn\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{2b^2f^3mn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{9e^3}+\frac{2b^2f^3mn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{3e^3}-\frac{(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^4,x]
```

```
[Out] (-19*b^2*f*m*n^2)/(108*e*x^2) + (26*b^2*f^2*m*n^2)/(27*e^2*x) + (2*b^2*f^3*
m*n^2*Log[x])/(27*e^3) - (5*b*f*m*n*(a + b*Log[c*x^n]))/(18*e*x^2) + (8*b*f
^2*m*n*(a + b*Log[c*x^n]))/(9*e^2*x) + (f^3*m*(a + b*Log[c*x^n])^2)/(9*e^3)
 - (f*m*(a + b*Log[c*x^n])^2)/(6*e*x^2) + (f^2*m*(a + b*Log[c*x^n])^2)/(3*e
^2*x) + (f^3*m*(a + b*Log[c*x^n])^3)/(9*b*e^3*n) - (2*b^2*f^3*m*n^2*Log[e +
 f*x])/(27*e^3) - (2*b^2*n^2*Log[d*(e + f*x)^m])/(27*x^3) - (2*b*n*(a + b*L
og[c*x^n])*Log[d*(e + f*x)^m])/(9*x^3) - ((a + b*Log[c*x^n])^2*Log[d*(e + f
*x)^m])/(3*x^3) - (2*b*f^3*m*n*(a + b*Log[c*x^n])*Log[1 + (f*x)/e])/(9*e^3)
 - (f^3*m*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/(3*e^3) - (2*b^2*f^3*m*n^2
*PolyLog[2, -((f*x)/e)])/(9*e^3) - (2*b*f^3*m*n*(a + b*Log[c*x^n])*PolyLog[
2, -((f*x)/e)])/(3*e^3) + (2*b^2*f^3*m*n^2*PolyLog[3, -((f*x)/e)])/(3*e^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d(e+fx)^m\right)}{x^4}\,dx = -\frac{2b^2n^2\log\left(d(e+fx)^m\right)}{27x^3} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{9x^3}$$

$$= -\frac{2b^2n^2\log\left(d(e+fx)^m\right)}{27x^3} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{9x^3}$$

$$= -\frac{2b^2n^2\log\left(d(e+fx)^m\right)}{27x^3} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{9x^3}$$

$$= -\frac{b^2fmn^2}{27ex^2} + \frac{2b^2f^2mn^2}{27e^2x} + \frac{2b^2f^3mn^2\log(x)}{27e^3} - \frac{2b^2f^3mn^2\log(e+}{27e^3}$$

$$= -\frac{5b^2fmn^2}{54ex^2} + \frac{8b^2f^2mn^2}{27e^2x} + \frac{2b^2f^3mn^2\log(x)}{27e^3} - \frac{bfmn\left(a+b\log(\right.}{9ex^2}$$

$$= -\frac{19b^2fmn^2}{108ex^2} + \frac{26b^2f^2mn^2}{27e^2x} + \frac{2b^2f^3mn^2\log(x)}{27e^3} - \frac{5bfmn\left(a+b\,\mathrm{lo}\right.}{18ex^2}$$

$$= -\frac{19b^2fmn^2}{108ex^2} + \frac{26b^2f^2mn^2}{27e^2x} + \frac{2b^2f^3mn^2\log(x)}{27e^3} - \frac{5bfmn\left(a+b\,\mathrm{lo}\right.}{18ex^2}$$

Mathematica [B] time = 0.45, size = 909, normalized size = 2.16

$$-\frac{36a^2\log\left(d(e+fx)^m\right)e^3 + 8b^2n^2\log\left(d(e+fx)^m\right)e^3 + 36b^2\log^2\left(cx^n\right)\log\left(d(e+fx)^m\right)e^3 + 24abn\log\left(d(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^4,x]
```

```
[Out] -1/108*(18*a^2*e^2*f*m*x + 30*a*b*e^2*f*m*n*x + 19*b^2*e^2*f*m*n^2*x - 36*a
^2*e*f^2*m*x^2 - 96*a*b*e*f^2*m*n*x^2 - 104*b^2*e*f^2*m*n^2*x^2 - 36*a^2*f^
3*m*x^3*Log[x] - 24*a*b*f^3*m*n*x^3*Log[x] - 8*b^2*f^3*m*n^2*x^3*Log[x] + 3
6*a*b*f^3*m*n*x^3*Log[x]^2 + 12*b^2*f^3*m*n^2*x^3*Log[x]^2 - 12*b^2*f^3*m*n
^2*x^3*Log[x]^3 + 36*a*b*e^2*f*m*x*Log[c*x^n] + 30*b^2*e^2*f*m*n*x*Log[c*x^
n] - 72*a*b*e*f^2*m*x^2*Log[c*x^n] - 96*b^2*e*f^2*m*n*x^2*Log[c*x^n] - 72*a
*b*f^3*m*x^3*Log[x]*Log[c*x^n] - 24*b^2*f^3*m*n*x^3*Log[x]*Log[c*x^n] + 36*
b^2*f^3*m*n*x^3*Log[x]^2*Log[c*x^n] + 18*b^2*e^2*f*m*x*Log[c*x^n]^2 - 36*b^
2*e*f^2*m*x^2*Log[c*x^n]^2 - 36*b^2*f^3*m*x^3*Log[x]*Log[c*x^n]^2 + 36*a^2*
f^3*m*x^3*Log[e + f*x] + 24*a*b*f^3*m*n*x^3*Log[e + f*x] + 8*b^2*f^3*m*n^2*
x^3*Log[e + f*x] - 72*a*b*f^3*m*n*x^3*Log[x]*Log[e + f*x] - 24*b^2*f^3*m*n^
2*x^3*Log[x]*Log[e + f*x] + 36*b^2*f^3*m*n^2*x^3*Log[x]^2*Log[e + f*x] + 72
*a*b*f^3*m*x^3*Log[c*x^n]*Log[e + f*x] + 24*b^2*f^3*m*n*x^3*Log[c*x^n]*Log[
e + f*x] - 72*b^2*f^3*m*n*x^3*Log[x]*Log[c*x^n]*Log[e + f*x] + 36*b^2*f^3*m
*x^3*Log[c*x^n]^2*Log[e + f*x] + 36*a^2*e^3*Log[d*(e + f*x)^m] + 24*a*b*e^3
```

```
*n*Log[d*(e + f*x)^m] + 8*b^2*e^3*n^2*Log[d*(e + f*x)^m] + 72*a*b*e^3*Log[c
*x^n]*Log[d*(e + f*x)^m] + 24*b^2*e^3*n*Log[c*x^n]*Log[d*(e + f*x)^m] + 36*
b^2*e^3*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 72*a*b*f^3*m*n*x^3*Log[x]*Log[1 +
 (f*x)/e] + 24*b^2*f^3*m*n^2*x^3*Log[x]*Log[1 + (f*x)/e] - 36*b^2*f^3*m*n^2
*x^3*Log[x]^2*Log[1 + (f*x)/e] + 72*b^2*f^3*m*n*x^3*Log[x]*Log[c*x^n]*Log[1
 + (f*x)/e] + 24*b*f^3*m*n*x^3*(3*a + b*n + 3*b*Log[c*x^n])*PolyLog[2, -((f
*x)/e)] - 72*b^2*f^3*m*n^2*x^3*PolyLog[3, -((f*x)/e)])/(e^3*x^3)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^{2} \log \left(c x^{n}\right)^{2} + 2 \, a b \log \left(c x^{n}\right) + a^{2}\right) \log \left({\left(f x + e\right)}^{m} d\right)}{x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x^4
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(b \log \left(c x^{n}\right) + a\right)}^{2} \log \left({\left(f x + e\right)}^{m} d\right)}{x^{4}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x^4, x)
```

maple [F] time = 5.43, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c \,x^{n}\right)+a \right)^{2} \ln \left(d \left(f x +e \right)^{m}\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m)/x^4,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(d*(f*x+e)^m)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{9 \, {\left(2 \, b^{2} f^{3} m x^{3} \log \left(f x + e\right) - 2 \, b^{2} f^{3} m x^{3} \log\left(x\right) - 2 \, b^{2} e f^{2} m x^{2} + b^{2} e^{2} f m x + 2 \, b^{2} e^{3} \log\left(d\right)\right)} \log \left(x^{n}\right)^{2} + 2 \, {\left(9 \, b^{2} e^{3} \log \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^4,x, algorithm="maxima")
```

```
[Out] -1/54*(9*(2*b^2*f^3*m*x^3*log(f*x + e) - 2*b^2*f^3*m*x^3*log(x) - 2*b^2*e*f
^2*m*x^2 + b^2*e^2*f*m*x + 2*b^2*e^3*log(d))*log(x^n)^2 + 2*(9*b^2*e^3*log(
x^n)^2 + 9*a^2*e^3 + 6*(e^3*n + 3*e^3*log(c))*a*b + (2*e^3*n^2 + 6*e^3*n*lo
g(c) + 9*e^3*log(c)^2)*b^2 + 6*(3*a*b*e^3 + (e^3*n + 3*e^3*log(c))*b^2)*log
(x^n))*log((f*x + e)^m))/(e^3*x^3) + integrate(1/27*(27*b^2*e^4*log(c)^2*lo
g(d) + 54*a*b*e^4*log(c)*log(d) + 27*a^2*e^4*log(d) + (9*(e^3*f*m + 3*e^3*f
*log(d))*a^2 + 6*(e^3*f*m*n + 3*(e^3*f*m + 3*e^3*f*log(d))*log(c))*a*b + (2
*e^3*f*m*n^2 + 6*e^3*f*m*n*log(c) + 9*(e^3*f*m + 3*e^3*f*log(d))*log(c)^2)*
```

```
b^2)*x - 3*(6*b^2*e*f^3*m*n*x^3 + 3*b^2*e^2*f^2*m*n*x^2 - 18*a*b*e^4*log(d)
 - 6*(e^4*n*log(d) + 3*e^4*log(c)*log(d))*b^2 - (6*(e^3*f*m + 3*e^3*f*log(d
))*a*b + (5*e^3*f*m*n + 6*e^3*f*n*log(d) + 6*(e^3*f*m + 3*e^3*f*log(d))*log
(c))*b^2)*x - 6*(b^2*f^4*m*n*x^4 + b^2*e*f^3*m*n*x^3)*log(f*x + e) + 6*(b^2
*f^4*m*n*x^4 + b^2*e*f^3*m*n*x^3)*log(x))*log(x^n))/(e^3*f*x^5 + e^4*x^4),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^4,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x**4,x)
```

```
[Out] Timed out
```

3.85 $\int x\left(a+b\log\left(cx^n\right)\right)^3\log\left(d(e+fx)^m\right)\,dx$

Optimal. Leaf size=603

$$\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)+\frac{3b^2e^2mn^2\text{Li}_2\left(-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{2f^2}+\frac{3b^2e^2mn^2\text{Li}_3\left(-\frac{fx}{e}\right)\left(a+b\log\right.}{f^2}$$

```
[Out] 21/4*a*b^2*e*m*n^2*x/f-45/8*b^3*e*m*n^3*x/f+3/4*b^3*m*n^3*x^2+21/4*b^3*e*m*
n^2*x*ln(c*x^n)/f-9/8*b^2*m*n^2*x^2*(a+b*ln(c*x^n))-9/4*b*e*m*n*x*(a+b*ln(c
*x^n))^2/f+3/4*b*m*n*x^2*(a+b*ln(c*x^n))^2+1/2*e*m*x*(a+b*ln(c*x^n))^3/f-1/
4*m*x^2*(a+b*ln(c*x^n))^3+3/8*b^3*e^2*m*n^3*ln(f*x+e)/f^2-3/8*b^3*n^3*x^2*l
n(d*(f*x+e)^m)+3/4*b^2*n^2*x^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)-3/4*b*n*x^2*
(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)+1/2*x^2*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)
-3/4*b^2*e^2*m*n^2*(a+b*ln(c*x^n))*ln(1+f*x/e)/f^2+3/4*b*e^2*m*n*(a+b*ln(c*
x^n))^2*ln(1+f*x/e)/f^2-1/2*e^2*m*(a+b*ln(c*x^n))^3*ln(1+f*x/e)/f^2-3/4*b^3
*e^2*m*n^3*polylog(2,-f*x/e)/f^2+3/2*b^2*e^2*m*n^2*(a+b*ln(c*x^n))*polylog(
2,-f*x/e)/f^2-3/2*b*e^2*m*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x/e)/f^2-3/2*b^3
*e^2*m*n^3*polylog(3,-f*x/e)/f^2+3*b^2*e^2*m*n^2*(a+b*ln(c*x^n))*polylog(3,
-f*x/e)/f^2-3*b^3*e^2*m*n^3*polylog(4,-f*x/e)/f^2
```

Rubi [A] time = 0.97, antiderivative size = 603, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {2305, 2304, 2378, 43, 2351, 2295, 2317, 2391, 2353, 2296, 2374, 6589, 2383}

$$\frac{3b^2e^2mn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{2f^2}+\frac{3b^2e^2mn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}-\frac{3be^2mn\text{PolyLog}\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

```
[Out] (21*a*b^2*e*m*n^2*x)/(4*f) - (45*b^3*e*m*n^3*x)/(8*f) + (3*b^3*m*n^3*x^2)/4
 + (21*b^3*e*m*n^2*x*Log[c*x^n])/(4*f) - (9*b^2*m*n^2*x^2*(a + b*Log[c*x^n]
))/8 - (9*b*e*m*n*x*(a + b*Log[c*x^n])^2)/(4*f) + (3*b*m*n*x^2*(a + b*Log[c
*x^n])^2)/4 + (e*m*x*(a + b*Log[c*x^n])^3)/(2*f) - (m*x^2*(a + b*Log[c*x^n]
)^3)/4 + (3*b^3*e^2*m*n^3*Log[e + f*x])/(8*f^2) - (3*b^3*n^3*x^2*Log[d*(e +
 f*x)^m])/8 + (3*b^2*n^2*x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/4 - (3*
b*n*x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/4 + (x^2*(a + b*Log[c*x^n]
)^3*Log[d*(e + f*x)^m])/2 - (3*b^2*e^2*m*n^2*(a + b*Log[c*x^n])*Log[1 + (f*
x)/e])/(4*f^2) + (3*b*e^2*m*n*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/(4*f^2
) - (e^2*m*(a + b*Log[c*x^n])^3*Log[1 + (f*x)/e])/(2*f^2) - (3*b^3*e^2*m*n^
3*PolyLog[2, -((f*x)/e)])/(4*f^2) + (3*b^2*e^2*m*n^2*(a + b*Log[c*x^n])*Pol
yLog[2, -((f*x)/e)])/(2*f^2) - (3*b*e^2*m*n*(a + b*Log[c*x^n])^2*PolyLog[2,
 -((f*x)/e)])/(2*f^2) - (3*b^3*e^2*m*n^3*PolyLog[3, -((f*x)/e)])/(2*f^2) +
(3*b^2*e^2*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)])/f^2 - (3*b^3*e^
2*m*n^3*PolyLog[4, -((f*x)/e)])/f^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)^3\log\left(d(e+fx)^m\right)\,dx=-\frac{3}{8}b^3n^3x^2\log\left(d(e+fx)^m\right)+\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+$$

$$=-\frac{3}{8}b^3n^3x^2\log\left(d(e+fx)^m\right)+\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+$$

$$=-\frac{3}{8}b^3n^3x^2\log\left(d(e+fx)^m\right)+\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+$$

$$=-\frac{3b^3emn^3x}{8f}+\frac{3}{16}b^3mn^3x^2+\frac{3b^3e^2mn^3\log(e+fx)}{8f^2}-\frac{3}{8}b^3n^3x^2\log($$

$$=\frac{3ab^2emn^2x}{4f}-\frac{3b^3emn^3x}{8f}+\frac{3}{8}b^3mn^3x^2-\frac{3}{8}b^2mn^2x^2\left(a+b\log\left(cx^n\right)\right)$$

$$=\frac{9ab^2emn^2x}{4f}-\frac{9b^3emn^3x}{8f}+\frac{9}{16}b^3mn^3x^2+\frac{3b^3emn^2x\log\left(cx^n\right)}{4f}-\frac{3}{4}b$$

$$=\frac{21ab^2emn^2x}{4f}-\frac{21b^3emn^3x}{8f}+\frac{3}{4}b^3mn^3x^2+\frac{9b^3emn^2x\log\left(cx^n\right)}{4f}-\frac{9}{8}$$

$$=\frac{21ab^2emn^2x}{4f}-\frac{45b^3emn^3x}{8f}+\frac{3}{4}b^3mn^3x^2+\frac{21b^3emn^2x\log\left(cx^n\right)}{4f}-$$

Mathematica [B] time = 0.60, size = 1431, normalized size = 2.37

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

```
[Out] (4*a^3*e*f*m*x - 18*a^2*b*e*f*m*n*x + 42*a*b^2*e*f*m*n^2*x - 45*b^3*e*f*m*n
^3*x - 2*a^3*f^2*m*x^2 + 6*a^2*b*f^2*m*n*x^2 - 9*a*b^2*f^2*m*n^2*x^2 + 6*b^
3*f^2*m*n^3*x^2 + 12*a^2*b*e*f*m*x*Log[c*x^n] - 36*a*b^2*e*f*m*n*x*Log[c*x^
n] + 42*b^3*e*f*m*n^2*x*Log[c*x^n] - 6*a^2*b*f^2*m*x^2*Log[c*x^n] + 12*a*b^
```

```
2*f^2*m*n*x^2*Log[c*x^n] - 9*b^3*f^2*m*n^2*x^2*Log[c*x^n] + 12*a*b^2*e*f*m*
x*Log[c*x^n]^2 - 18*b^3*e*f*m*n*x*Log[c*x^n]^2 - 6*a*b^2*f^2*m*x^2*Log[c*x^
n]^2 + 6*b^3*f^2*m*n*x^2*Log[c*x^n]^2 + 4*b^3*e*f*m*x*Log[c*x^n]^3 - 2*b^3*
f^2*m*x^2*Log[c*x^n]^3 - 4*a^3*e^2*m*Log[e + f*x] + 6*a^2*b*e^2*m*n*Log[e +
 f*x] - 6*a*b^2*e^2*m*n^2*Log[e + f*x] + 3*b^3*e^2*m*n^3*Log[e + f*x] + 12*
a^2*b*e^2*m*n*Log[x]*Log[e + f*x] - 12*a*b^2*e^2*m*n^2*Log[x]*Log[e + f*x]
+ 6*b^3*e^2*m*n^3*Log[x]*Log[e + f*x] - 12*a*b^2*e^2*m*n^2*Log[x]^2*Log[e +
 f*x] + 6*b^3*e^2*m*n^3*Log[x]^2*Log[e + f*x] + 4*b^3*e^2*m*n^3*Log[x]^3*Lo
g[e + f*x] - 12*a^2*b*e^2*m*Log[c*x^n]*Log[e + f*x] + 12*a*b^2*e^2*m*n*Log[
c*x^n]*Log[e + f*x] - 6*b^3*e^2*m*n^2*Log[c*x^n]*Log[e + f*x] + 24*a*b^2*e^
2*m*n*Log[x]*Log[c*x^n]*Log[e + f*x] - 12*b^3*e^2*m*n^2*Log[x]*Log[c*x^n]*L
og[e + f*x] - 12*b^3*e^2*m*n^2*Log[x]^2*Log[c*x^n]*Log[e + f*x] - 12*a*b^2*
e^2*m*Log[c*x^n]^2*Log[e + f*x] + 6*b^3*e^2*m*n*Log[c*x^n]^2*Log[e + f*x] +
 12*b^3*e^2*m*n*Log[x]*Log[c*x^n]^2*Log[e + f*x] - 4*b^3*e^2*m*Log[c*x^n]^3
*Log[e + f*x] + 4*a^3*f^2*x^2*Log[d*(e + f*x)^m] - 6*a^2*b*f^2*n*x^2*Log[d*
(e + f*x)^m] + 6*a*b^2*f^2*n^2*x^2*Log[d*(e + f*x)^m] - 3*b^3*f^2*n^3*x^2*L
og[d*(e + f*x)^m] + 12*a^2*b*f^2*x^2*Log[c*x^n]*Log[d*(e + f*x)^m] - 12*a*b
^2*f^2*n*x^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 6*b^3*f^2*n^2*x^2*Log[c*x^n]*L
og[d*(e + f*x)^m] + 12*a*b^2*f^2*x^2*Log[c*x^n]^2*Log[d*(e + f*x)^m] - 6*b^
3*f^2*n*x^2*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 4*b^3*f^2*x^2*Log[c*x^n]^3*Lo
g[d*(e + f*x)^m] - 12*a^2*b*e^2*m*n*Log[x]*Log[1 + (f*x)/e] + 12*a*b^2*e^2*
m*n^2*Log[x]*Log[1 + (f*x)/e] - 6*b^3*e^2*m*n^3*Log[x]*Log[1 + (f*x)/e] + 1
2*a*b^2*e^2*m*n^2*Log[x]^2*Log[1 + (f*x)/e] - 6*b^3*e^2*m*n^3*Log[x]^2*Log[
1 + (f*x)/e] - 4*b^3*e^2*m*n^3*Log[x]^3*Log[1 + (f*x)/e] - 24*a*b^2*e^2*m*n
*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 12*b^3*e^2*m*n^2*Log[x]*Log[c*x^n]*Lo
g[1 + (f*x)/e] + 12*b^3*e^2*m*n^2*Log[x]^2*Log[c*x^n]*Log[1 + (f*x)/e] - 12
*b^3*e^2*m*n*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] - 6*b*e^2*m*n*(2*a^2 - 2*
a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog
[2, -((f*x)/e)] + 12*b^2*e^2*m*n^2*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[3,
-((f*x)/e)] - 24*b^3*e^2*m*n^3*PolyLog[4, -((f*x)/e)])/(8*f^2)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 x \log(cx^n)^3 + 3\,ab^2 x \log(cx^n)^2 + 3\,a^2 bx \log(cx^n) + a^3 x\right)\log\left(\left(fx+e\right)^m d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^n
) + a^3*x)*log((f*x + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log(cx^n) + a\right)^3 x \log\left(\left(fx+e\right)^m d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*x + e)^m*d), x)
```

maple [F] time = 87.77, size = 0, normalized size = 0.00

$$\int \left(b \ln(c\,x^n) + a\right)^3 x \ln\left(d\left(fx+e\right)^m\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^3*ln(d*(f*x+e)^m),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^3*ln(d*(f*x+e)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*b^3*e*f*m*x - 2*b^3*e^2*m*log(f*x + e) - (f^2*m - 2*f^2*log(d))*b
^3*x^2)*log(x^n)^3 + (4*b^3*f^2*x^2*log(x^n)^3 + 6*(2*a*b^2*f^2 - (f^2*n -
2*f^2*log(c))*b^3)*x^2*log(x^n)^2 + 6*(2*a^2*b*f^2 - 2*(f^2*n - 2*f^2*log(c
))*a*b^2 + (f^2*n^2 - 2*f^2*n*log(c) + 2*f^2*log(c)^2)*b^3)*x^2*log(x^n) +
(4*a^3*f^2 - 6*(f^2*n - 2*f^2*log(c))*a^2*b + 6*(f^2*n^2 - 2*f^2*n*log(c) +
 2*f^2*log(c)^2)*a*b^2 - (3*f^2*n^3 - 6*f^2*n^2*log(c) + 6*f^2*n*log(c)^2 -
 4*f^2*log(c)^3)*b^3)*x^2)*log((f*x + e)^m))/f^2 + integrate(-1/8*((4*(f^3*
m - 2*f^3*log(d))*a^3 - 6*(f^3*m*n - 2*(f^3*m - 2*f^3*log(d))*log(c))*a^2*b
 + 6*(f^3*m*n^2 - 2*f^3*m*n*log(c) + 2*(f^3*m - 2*f^3*log(d))*log(c)^2)*a*b
^2 - (3*f^3*m*n^3 - 6*f^3*m*n^2*log(c) + 6*f^3*m*n*log(c)^2 - 4*(f^3*m - 2*
f^3*log(d))*log(c)^3)*b^3)*x^3 - 8*(b^3*e*f^2*log(c)^3*log(d) + 3*a*b^2*e*f
^2*log(c)^2*log(d) + 3*a^2*b*e*f^2*log(c)*log(d) + a^3*e*f^2*log(d))*x^2 +
6*(2*b^3*e^2*f*m*n*x + 2*((f^3*m - 2*f^3*log(d))*a*b^2 - (f^3*m*n - f^3*n*l
og(d) - (f^3*m - 2*f^3*log(d))*log(c))*b^3)*x^3 - (4*a*b^2*e*f^2*log(d) - (
e*f^2*m*n + 2*e*f^2*n*log(d) - 4*e*f^2*log(c)*log(d))*b^3)*x^2 - 2*(b^3*e^2
*f*m*n*x + b^3*e^3*m*n)*log(f*x + e))*log(x^n)^2 + 6*((2*(f^3*m - 2*f^3*log
(d))*a^2*b - 2*(f^3*m*n - 2*(f^3*m - 2*f^3*log(d))*log(c))*a*b^2 + (f^3*m*n
^2 - 2*f^3*m*n*log(c) + 2*(f^3*m - 2*f^3*log(d))*log(c)^2)*b^3)*x^3 - 4*(b^
3*e*f^2*log(c)^2*log(d) + 2*a*b^2*e*f^2*log(c)*log(d) + a^2*b*e*f^2*log(d))
*x^2)*log(x^n))/(f^3*x^2 + e*f^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

3.86 $\int \left(a+b\log\left(cx^n\right)\right)^3 \log\left(d(e+fx)^m\right)\,dx$

Optimal. Leaf size=473

$$-\frac{6b^2emn^2\text{Li}_2\left(-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f}-\frac{6b^2emn^2\text{Li}_3\left(-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f}+6ab^2n^2x\log\left(d(e+fx)^m\right)+\frac{6b^2em}{}$$

```
[Out] -12*a*b^2*m*n^2*x+18*b^3*m*n^3*x-6*b^2*m*n^2*(-b*n+a)*x-18*b^3*m*n^2*x*ln(c
*x^n)+6*b*m*n*x*(a+b*ln(c*x^n))^2-m*x*(a+b*ln(c*x^n))^3+6*b^2*e*m*n^2*(-b*n
+a)*ln(f*x+e)/f+6*a*b^2*n^2*x*ln(d*(f*x+e)^m)-6*b^3*n^3*x*ln(d*(f*x+e)^m)+6
*b^3*n^2*x*ln(c*x^n)*ln(d*(f*x+e)^m)-3*b*n*x*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)
^m)+x*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)+6*b^3*e*m*n^2*ln(c*x^n)*ln(1+f*x/e)
/f-3*b*e*m*n*(a+b*ln(c*x^n))^2*ln(1+f*x/e)/f+e*m*(a+b*ln(c*x^n))^3*ln(1+f*x
/e)/f+6*b^3*e*m*n^3*polylog(2,-f*x/e)/f-6*b^2*e*m*n^2*(a+b*ln(c*x^n))*polyl
og(2,-f*x/e)/f+3*b*e*m*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x/e)/f+6*b^3*e*m*n^
3*polylog(3,-f*x/e)/f-6*b^2*e*m*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x/e)/f+6*b
^3*e*m*n^3*polylog(4,-f*x/e)/f
```

Rubi [A] time = 0.65, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2296, 2295, 2371, 6, 43, 2351, 2317, 2391, 2353, 2374, 6589, 2383}

$$-\frac{6b^2emn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f}-\frac{6b^2emn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f}+\frac{3bemn\text{PolyLog}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

```
[Out] -12*a*b^2*m*n^2*x + 18*b^3*m*n^3*x - 6*b^2*m*n^2*(a - b*n)*x - 18*b^3*m*n^2
*x*Log[c*x^n] + 6*b*m*n*x*(a + b*Log[c*x^n])^2 - m*x*(a + b*Log[c*x^n])^3 +
 (6*b^2*e*m*n^2*(a - b*n)*Log[e + f*x])/f + 6*a*b^2*n^2*x*Log[d*(e + f*x)^m
] - 6*b^3*n^3*x*Log[d*(e + f*x)^m] + 6*b^3*n^2*x*Log[c*x^n]*Log[d*(e + f*x)
^m] - 3*b*n*x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m] + x*(a + b*Log[c*x^n]
)^3*Log[d*(e + f*x)^m] + (6*b^3*e*m*n^2*Log[c*x^n]*Log[1 + (f*x)/e])/f - (3
*b*e*m*n*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/f + (e*m*(a + b*Log[c*x^n])
^3*Log[1 + (f*x)/e])/f + (6*b^3*e*m*n^3*PolyLog[2, -((f*x)/e)])/f - (6*b^2*
e*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*x)/e)])/f + (3*b*e*m*n*(a + b*Lo
g[c*x^n])^2*PolyLog[2, -((f*x)/e)])/f + (6*b^3*e*m*n^3*PolyLog[3, -((f*x)/e
)])/f - (6*b^2*e*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)])/f + (6*b^
3*e*m*n^3*PolyLog[4, -((f*x)/e)])/f
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2371

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)^3 \log\left(d(e+fx)^m\right)\,dx = 6ab^2n^2x\log\left(d(e+fx)^m\right) - 6b^3n^3x\log\left(d(e+fx)^m\right) + 6b^3n^2x\log$$

$$= 6ab^2n^2x\log\left(d(e+fx)^m\right) - 6b^3n^3x\log\left(d(e+fx)^m\right) + 6b^3n^2x\log$$

$$= 6ab^2n^2x\log\left(d(e+fx)^m\right) - 6b^3n^3x\log\left(d(e+fx)^m\right) + 6b^3n^2x\log$$

$$= 6ab^2n^2x\log\left(d(e+fx)^m\right) - 6b^3n^3x\log\left(d(e+fx)^m\right) + 6b^3n^2x\log$$

$$= -6b^2mn^2(a-bn)x + \frac{6b^2emn^2(a-bn)\log(e+fx)}{f} + 6ab^2n^2x\log$$

$$= 6b^3mn^3x - 6b^2mn^2(a-bn)x - 6b^3mn^2x\log\left(cx^n\right) + 3bmnx\left(a+b\right.$$

$$= -6ab^2mn^2x + 6b^3mn^3x - 6b^2mn^2(a-bn)x - 6b^3mn^2x\log\left(cx^n\right) +$$

$$= -12ab^2mn^2x + 12b^3mn^3x - 6b^2mn^2(a-bn)x - 12b^3mn^2x\log(cx$$

$$= -12ab^2mn^2x + 18b^3mn^3x - 6b^2mn^2(a-bn)x - 18b^3mn^2x\log(cx$$

Mathematica [B] time = 0.44, size = 1122, normalized size = 2.37

$$\frac{-fmxa^3 + em\log(e+fx)a^3 + fx\log\left(d(e+fx)^m\right)a^3 + 6bfmnxa^2 - 3bfmx\log\left(cx^n\right)a^2 - 3bemn\log(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

```
[Out] (-(a^3*f*m*x) + 6*a^2*b*f*m*n*x - 18*a*b^2*f*m*n^2*x + 24*b^3*f*m*n^3*x - 3
*a^2*b*f*m*x*Log[c*x^n] + 12*a*b^2*f*m*n*x*Log[c*x^n] - 18*b^3*f*m*n^2*x*Lo
g[c*x^n] - 3*a*b^2*f*m*x*Log[c*x^n]^2 + 6*b^3*f*m*n*x*Log[c*x^n]^2 - b^3*f*
m*x*Log[c*x^n]^3 + a^3*e*m*Log[e + f*x] - 3*a^2*b*e*m*n*Log[e + f*x] + 6*a*
b^2*e*m*n^2*Log[e + f*x] - 6*b^3*e*m*n^3*Log[e + f*x] - 3*a^2*b*e*m*n*Log[x
]*Log[e + f*x] + 6*a*b^2*e*m*n^2*Log[x]*Log[e + f*x] - 6*b^3*e*m*n^3*Log[x]
*Log[e + f*x] + 3*a*b^2*e*m*n^2*Log[x]^2*Log[e + f*x] - 3*b^3*e*m*n^3*Log[x
]^2*Log[e + f*x] - b^3*e*m*n^3*Log[x]^3*Log[e + f*x] + 3*a^2*b*e*m*Log[c*x^
n]*Log[e + f*x] - 6*a*b^2*e*m*n*Log[c*x^n]*Log[e + f*x] + 6*b^3*e*m*n^2*Log
[c*x^n]*Log[e + f*x] - 6*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[e + f*x] + 6*b^3
*e*m*n^2*Log[x]*Log[c*x^n]*Log[e + f*x] + 3*b^3*e*m*n^2*Log[x]^2*Log[c*x^n]
```

```
*Log[e + f*x] + 3*a*b^2*e*m*Log[c*x^n]^2*Log[e + f*x] - 3*b^3*e*m*n*Log[c*x
^n]^2*Log[e + f*x] - 3*b^3*e*m*n*Log[x]*Log[c*x^n]^2*Log[e + f*x] + b^3*e*m
*Log[c*x^n]^3*Log[e + f*x] + a^3*f*x*Log[d*(e + f*x)^m] - 3*a^2*b*f*n*x*Log
[d*(e + f*x)^m] + 6*a*b^2*f*n^2*x*Log[d*(e + f*x)^m] - 6*b^3*f*n^3*x*Log[d*
(e + f*x)^m] + 3*a^2*b*f*x*Log[c*x^n]*Log[d*(e + f*x)^m] - 6*a*b^2*f*n*x*Lo
g[c*x^n]*Log[d*(e + f*x)^m] + 6*b^3*f*n^2*x*Log[c*x^n]*Log[d*(e + f*x)^m] +
 3*a*b^2*f*x*Log[c*x^n]^2*Log[d*(e + f*x)^m] - 3*b^3*f*n*x*Log[c*x^n]^2*Log
[d*(e + f*x)^m] + b^3*f*x*Log[c*x^n]^3*Log[d*(e + f*x)^m] + 3*a^2*b*e*m*n*L
og[x]*Log[1 + (f*x)/e] - 6*a*b^2*e*m*n^2*Log[x]*Log[1 + (f*x)/e] + 6*b^3*e*
m*n^3*Log[x]*Log[1 + (f*x)/e] - 3*a*b^2*e*m*n^2*Log[x]^2*Log[1 + (f*x)/e] +
 3*b^3*e*m*n^3*Log[x]^2*Log[1 + (f*x)/e] + b^3*e*m*n^3*Log[x]^3*Log[1 + (f*
x)/e] + 6*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] - 6*b^3*e*m*n^2*Lo
g[x]*Log[c*x^n]*Log[1 + (f*x)/e] - 3*b^3*e*m*n^2*Log[x]^2*Log[c*x^n]*Log[1
+ (f*x)/e] + 3*b^3*e*m*n*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] + 3*b*e*m*n*(
a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*Po
lyLog[2, -((f*x)/e)] - 6*b^2*e*m*n^2*(a - b*n + b*Log[c*x^n])*PolyLog[3, -(
(f*x)/e)] + 6*b^3*e*m*n^3*PolyLog[4, -((f*x)/e)])/f
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\log\left(cx^n\right)^3+3\,ab^2\log\left(cx^n\right)^2+3\,a^2b\log\left(cx^n\right)+a^3\right)\log\left(\left(fx+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(fx+e\right)^m d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d), x)
```

maple [F] time = 11.85, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^3\ln\left(d\left(fx+e\right)^m\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(f*x+e)^m),x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(d*(f*x+e)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3em\log\left(fx+e\right)-\left(fm-f\log(d)\right)b^3x\right)\log\left(x^n\right)^3+\left(b^3fx\log\left(x^n\right)^3-3\left(\left(fn-f\log(c)\right)b^3-ab^2f\right)x\log\left(x^n\right)^2-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] ((b^3*e*m*log(f*x + e) - (f*m - f*log(d))*b^3*x)*log(x^n)^3 + (b^3*f*x*log(
x^n)^3 - 3*((f*n - f*log(c))*b^3 - a*b^2*f)*x*log(x^n)^2 - 3*(2*(f*n - f*lo
```

```
g(c))*a*b^2 - (2*f*n^2 - 2*f*n*log(c) + f*log(c)^2)*b^3 - a^2*b*f)*x*log(x^
n) - (3*(f*n - f*log(c))*a^2*b - 3*(2*f*n^2 - 2*f*n*log(c) + f*log(c)^2)*a*
b^2 + (6*f*n^3 - 6*f*n^2*log(c) + 3*f*n*log(c)^2 - f*log(c)^3)*b^3 - a^3*f)
*x)*log((f*x + e)^m))/f - integrate((((f^2*m - f^2*log(d))*a^3 - 3*(f^2*m*n
 - (f^2*m - f^2*log(d))*log(c))*a^2*b + 3*(2*f^2*m*n^2 - 2*f^2*m*n*log(c) +
 (f^2*m - f^2*log(d))*log(c)^2)*a*b^2 - (6*f^2*m*n^3 - 6*f^2*m*n^2*log(c) +
 3*f^2*m*n*log(c)^2 - (f^2*m - f^2*log(d))*log(c)^3)*b^3)*x^2 + 3*(((f^2*m
- f^2*log(d))*a*b^2 - (2*f^2*m*n - f^2*n*log(d) - (f^2*m - f^2*log(d))*log(
c))*b^3)*x^2 - (a*b^2*e*f*log(d) + (e*f*m*n - e*f*n*log(d) + e*f*log(c)*log
(d))*b^3)*x + (b^3*e*f*m*n*x + b^3*e^2*m*n)*log(f*x + e))*log(x^n)^2 - (b^3
*e*f*log(c)^3*log(d) + 3*a*b^2*e*f*log(c)^2*log(d) + 3*a^2*b*e*f*log(c)*log
(d) + a^3*e*f*log(d))*x + 3*(((f^2*m - f^2*log(d))*a^2*b - 2*(f^2*m*n - (f^
2*m - f^2*log(d))*log(c))*a*b^2 + (2*f^2*m*n^2 - 2*f^2*m*n*log(c) + (f^2*m
- f^2*log(d))*log(c)^2)*b^3)*x^2 - (b^3*e*f*log(c)^2*log(d) + 2*a*b^2*e*f*l
og(c)*log(d) + a^2*b*e*f*log(d))*x)*log(x^n))/(f^2*x^2 + e*f*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

3.87 $\int \frac{\left(a+b\log(cx^n)\right)^3\log\left(d(e+fx)^m\right)}{x}\,dx$

Optimal. Leaf size=161

$$-6b^2mn^2\mathrm{Li}_4\left(-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d(e+fx)^m\right)}{4bn}-m\mathrm{Li}_2\left(-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)^3+3bmn$$

```
[Out] 1/4*(a+b*ln(c*x^n))^4*ln(d*(f*x+e)^m)/b/n-1/4*m*(a+b*ln(c*x^n))^4*ln(1+f*x/
e)/b/n-m*(a+b*ln(c*x^n))^3*polylog(2,-f*x/e)+3*b*m*n*(a+b*ln(c*x^n))^2*poly
log(3,-f*x/e)-6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(4,-f*x/e)+6*b^3*m*n^3*pol
ylog(5,-f*x/e)
```

Rubi [A] time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2375, 2317, 2374, 2383, 6589}

$$-6b^2mn^2\mathrm{PolyLog}\left(4,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)-m\mathrm{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)^3+3bmn\mathrm{PolyLog}\left(3,-\frac{fx}{e}\right)($$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^4*Log[d*(e + f*x)^m])/(4*b*n) - (m*(a + b*Log[c*x^n])^4
*Log[1 + (f*x)/e])/(4*b*n) - m*(a + b*Log[c*x^n])^3*PolyLog[2, -((f*x)/e)]
+ 3*b*m*n*(a + b*Log[c*x^n])^2*PolyLog[3, -((f*x)/e)] - 6*b^2*m*n^2*(a + b*
Log[c*x^n])*PolyLog[4, -((f*x)/e)] + 6*b^3*m*n^3*PolyLog[5, -((f*x)/e)]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d(e+fx)^m\right)}{x}dx = \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d(e+fx)^m\right)}{4bn} - \frac{(fm)\int\frac{\left(a+b\log\left(cx^n\right)\right)^4}{e+fx}dx}{4bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d(e+fx)^m\right)}{4bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^4\log\left(1+\right.}{4bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d(e+fx)^m\right)}{4bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^4\log\left(1+\right.}{4bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d(e+fx)^m\right)}{4bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^4\log\left(1+\right.}{4bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d(e+fx)^m\right)}{4bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^4\log\left(1+\right.}{4bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d(e+fx)^m\right)}{4bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^4\log\left(1+\right.}{4bn}$$

Mathematica [B] time = 0.26, size = 602, normalized size = 3.74

$$a^3\log(x)\log\left(d(e+fx)^m\right)-a^3m\log(x)\log\left(\frac{fx}{e}+1\right)+3a^2b\log(x)\log\left(cx^n\right)\log\left(d(e+fx)^m\right)-3a^2bm\log(x)\log$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] a^3*Log[x]*Log[d*(e + f*x)^m] - (3*a^2*b*n*Log[x]^2*Log[d*(e + f*x)^m])/2 +
 a*b^2*n^2*Log[x]^3*Log[d*(e + f*x)^m] - (b^3*n^3*Log[x]^4*Log[d*(e + f*x)^
m])/4 + 3*a^2*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x)^m] - 3*a*b^2*n*Log[x]^2*L
og[c*x^n]*Log[d*(e + f*x)^m] + b^3*n^2*Log[x]^3*Log[c*x^n]*Log[d*(e + f*x)^
m] + 3*a*b^2*Log[x]*Log[c*x^n]^2*Log[d*(e + f*x)^m] - (3*b^3*n*Log[x]^2*Log
[c*x^n]^2*Log[d*(e + f*x)^m])/2 + b^3*Log[x]*Log[c*x^n]^3*Log[d*(e + f*x)^m
] - a^3*m*Log[x]*Log[1 + (f*x)/e] + (3*a^2*b*m*n*Log[x]^2*Log[1 + (f*x)/e])
/2 - a*b^2*m*n^2*Log[x]^3*Log[1 + (f*x)/e] + (b^3*m*n^3*Log[x]^4*Log[1 + (f
*x)/e])/4 - 3*a^2*b*m*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 3*a*b^2*m*n*Log[
x]^2*Log[c*x^n]*Log[1 + (f*x)/e] - b^3*m*n^2*Log[x]^3*Log[c*x^n]*Log[1 + (f
*x)/e] - 3*a*b^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] + (3*b^3*m*n*Log[x]
^2*Log[c*x^n]^2*Log[1 + (f*x)/e])/2 - b^3*m*Log[x]*Log[c*x^n]^3*Log[1 + (f*
x)/e] - m*(a + b*Log[c*x^n])^3*PolyLog[2, -((f*x)/e)] + 3*b*m*n*(a + b*Log[
c*x^n])^2*PolyLog[3, -((f*x)/e)] - 6*a*b^2*m*n^2*PolyLog[4, -((f*x)/e)] - 6
*b^3*m*n^2*Log[c*x^n]*PolyLog[4, -((f*x)/e)] + 6*b^3*m*n^3*PolyLog[5, -((f*
x)/e)]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\log\left(cx^n\right)^3+3\,ab^2\log\left(cx^n\right)^2+3\,a^2b\log\left(cx^n\right)+a^3\right)\log\left(\left(fx+e\right)^m d\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x + e)^m*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{3}\log\left(\left(fx+e\right)^{m}d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d)/x, x)
```

maple [C] time = 2.96, size = 60520, normalized size = 375.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(f*x+e)^m)/x,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b^
2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^2
+ 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3*n^
2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*a*b^
2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3
*a^2*b*log(c) + a^3)*log(x))*log((f*x + e)^m) - integrate(-1/4*(b^3*f*m*n^3
*x*log(x)^4 + 4*b^3*e*log(c)^3*log(d) + 12*a*b^2*e*log(c)^2*log(d) + 12*a^2
*b*e*log(c)*log(d) + 4*a^3*e*log(d) - 4*(b^3*f*m*n^2*log(c) + a*b^2*f*m*n^2
)*x*log(x)^3 + 6*(b^3*f*m*n*log(c)^2 + 2*a*b^2*f*m*n*log(c) + a^2*b*f*m*n)*
x*log(x)^2 - 4*(b^3*f*m*x*log(x) - b^3*f*x*log(d) - b^3*e*log(d))*log(x^n)^
3 - 4*(b^3*f*m*log(c)^3 + 3*a*b^2*f*m*log(c)^2 + 3*a^2*b*f*m*log(c) + a^3*f
*m)*x*log(x) + 6*(b^3*f*m*n*x*log(x)^2 + 2*b^3*e*log(c)*log(d) + 2*a*b^2*e*
log(d) - 2*(b^3*f*m*log(c) + a*b^2*f*m)*x*log(x) + 2*(b^3*f*log(c)*log(d) +
 a*b^2*f*log(d))*x)*log(x^n)^2 + 4*(b^3*f*log(c)^3*log(d) + 3*a*b^2*f*log(c
)^2*log(d) + 3*a^2*b*f*log(c)*log(d) + a^3*f*log(d))*x - 4*(b^3*f*m*n^2*x*l
og(x)^3 - 3*b^3*e*log(c)^2*log(d) - 6*a*b^2*e*log(c)*log(d) - 3*a^2*b*e*log
(d) - 3*(b^3*f*m*n*log(c) + a*b^2*f*m*n)*x*log(x)^2 + 3*(b^3*f*m*log(c)^2 +
 2*a*b^2*f*m*log(c) + a^2*b*f*m)*x*log(x) - 3*(b^3*f*log(c)^2*log(d) + 2*a*
b^2*f*log(c)*log(d) + a^2*b*f*log(d))*x)*log(x^n))/(f*x^2 + e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\,x\right)^{m}\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m)/x,x)
```

```
[Out] Timed out
```

3.88 $\int \frac{\left(a+b\log(cx^n)\right)^3\log\left(d(e+fx)^m\right)}{x^2}\,dx$

Optimal. Leaf size=411

$$-\frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x}+\frac{6b^2fmn^2\mathrm{Li}_2\left(-\frac{e}{fx}\right)\left(a+b\log\left(cx^n\right)\right)}{e}+\frac{6b^2fmn^2\mathrm{Li}_3\left(-\frac{e}{fx}\right)\left(a+b\log\left(c\right.\right.}{e}$$

```
[Out] 6*b^3*f*m*n^3*ln(x)/e-6*b^2*f*m*n^2*ln(1+e/f/x)*(a+b*ln(c*x^n))/e-3*b*f*m*n
*ln(1+e/f/x)*(a+b*ln(c*x^n))^2/e-f*m*ln(1+e/f/x)*(a+b*ln(c*x^n))^3/e-6*b^3*
f*m*n^3*ln(f*x+e)/e-6*b^3*n^3*ln(d*(f*x+e)^m)/x-6*b^2*n^2*(a+b*ln(c*x^n))*l
n(d*(f*x+e)^m)/x-3*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x-(a+b*ln(c*x^n))^
3*ln(d*(f*x+e)^m)/x+6*b^3*f*m*n^3*polylog(2,-e/f/x)/e+6*b^2*f*m*n^2*(a+b*ln
(c*x^n))*polylog(2,-e/f/x)/e+3*b*f*m*n*(a+b*ln(c*x^n))^2*polylog(2,-e/f/x)/
e+6*b^3*f*m*n^3*polylog(3,-e/f/x)/e+6*b^2*f*m*n^2*(a+b*ln(c*x^n))*polylog(3
,-e/f/x)/e+6*b^3*f*m*n^3*polylog(4,-e/f/x)/e
```

Rubi [A] time = 0.70, antiderivative size = 459, normalized size of antiderivative = 1.12, number of steps used = 22, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {2305, 2304, 2378, 36, 29, 31, 2344, 2301, 2317, 2391, 2302, 30, 2374, 6589, 2383}

$$-\frac{6b^2fmn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e}+\frac{6b^2fmn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{3bfmn\text{PolyLog}\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] (6*b^3*f*m*n^3*Log[x])/e + (3*b*f*m*n*(a + b*Log[c*x^n])^2)/e + (f*m*(a + b
*Log[c*x^n])^3)/e + (f*m*(a + b*Log[c*x^n])^4)/(4*b*e*n) - (6*b^3*f*m*n^3*L
og[e + f*x])/e - (6*b^3*n^3*Log[d*(e + f*x)^m])/x - (6*b^2*n^2*(a + b*Log[c
*x^n])*Log[d*(e + f*x)^m])/x - (3*b*n*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^
m])/x - ((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x - (6*b^2*f*m*n^2*(a + b
*Log[c*x^n])*Log[1 + (f*x)/e])/e - (3*b*f*m*n*(a + b*Log[c*x^n])^2*Log[1 +
(f*x)/e])/e - (f*m*(a + b*Log[c*x^n])^3*Log[1 + (f*x)/e])/e - (6*b^3*f*m*n^
3*PolyLog[2, -((f*x)/e)])/e - (6*b^2*f*m*n^2*(a + b*Log[c*x^n])*PolyLog[2,
-((f*x)/e)])/e - (3*b*f*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x)/e)])/e
+ (6*b^3*f*m*n^3*PolyLog[3, -((f*x)/e)])/e + (6*b^2*f*m*n^2*(a + b*Log[c*x^
n])*PolyLog[3, -((f*x)/e)])/e - (6*b^3*f*m*n^3*PolyLog[4, -((f*x)/e)])/e
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d(e+fx)^m\right)}{x^2}\,dx = -\frac{6b^3n^3\log\left(d(e+fx)^m\right)}{x} - \frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x}$$

$$= -\frac{6b^3n^3\log\left(d(e+fx)^m\right)}{x} - \frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x}$$

$$= -\frac{6b^3n^3\log\left(d(e+fx)^m\right)}{x} - \frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x}$$

$$= \frac{6b^3fmn^3\log(x)}{e} + \frac{3bfmn\left(a+b\log\left(cx^n\right)\right)^2}{e} - \frac{6b^3fmn^3\log(e+fx)}{e}$$

$$= \frac{6b^3fmn^3\log(x)}{e} + \frac{3bfmn\left(a+b\log\left(cx^n\right)\right)^2}{e} + \frac{fm\left(a+b\log\left(cx^n\right)\right)^3}{e}$$

$$= \frac{6b^3fmn^3\log(x)}{e} + \frac{3bfmn\left(a+b\log\left(cx^n\right)\right)^2}{e} + \frac{fm\left(a+b\log\left(cx^n\right)\right)^3}{e}$$

$$= \frac{6b^3fmn^3\log(x)}{e} + \frac{3bfmn\left(a+b\log\left(cx^n\right)\right)^2}{e} + \frac{fm\left(a+b\log\left(cx^n\right)\right)^3}{e}$$

Mathematica [B] time = 0.69, size = 1347, normalized size = 3.28

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] -1/4*(-4*a^3*f*m*x*Log[x] - 12*a^2*b*f*m*n*x*Log[x] - 24*a*b^2*f*m*n^2*x*Lo
g[x] - 24*b^3*f*m*n^3*x*Log[x] + 6*a^2*b*f*m*n*x*Log[x]^2 + 12*a*b^2*f*m*n^
2*x*Log[x]^2 + 12*b^3*f*m*n^3*x*Log[x]^2 - 4*a*b^2*f*m*n^2*x*Log[x]^3 - 4*b
^3*f*m*n^3*x*Log[x]^3 + b^3*f*m*n^3*x*Log[x]^4 - 12*a^2*b*f*m*x*Log[x]*Log[
c*x^n] - 24*a*b^2*f*m*n*x*Log[x]*Log[c*x^n] - 24*b^3*f*m*n^2*x*Log[x]*Log[c
*x^n] + 12*a*b^2*f*m*n*x*Log[x]^2*Log[c*x^n] + 12*b^3*f*m*n^2*x*Log[x]^2*Lo
g[c*x^n] - 4*b^3*f*m*n^2*x*Log[x]^3*Log[c*x^n] - 12*a*b^2*f*m*x*Log[x]*Log[
c*x^n]^2 - 12*b^3*f*m*n*x*Log[x]*Log[c*x^n]^2 + 6*b^3*f*m*n*x*Log[x]^2*Log[
```

```
c*x^n]^2 - 4*b^3*f*m*x*Log[x]*Log[c*x^n]^3 + 4*a^3*f*m*x*Log[e + f*x] + 12*
a^2*b*f*m*n*x*Log[e + f*x] + 24*a*b^2*f*m*n^2*x*Log[e + f*x] + 24*b^3*f*m*n
^3*x*Log[e + f*x] - 12*a^2*b*f*m*n*x*Log[x]*Log[e + f*x] - 24*a*b^2*f*m*n^2
*x*Log[x]*Log[e + f*x] - 24*b^3*f*m*n^3*x*Log[x]*Log[e + f*x] + 12*a*b^2*f*
m*n^2*x*Log[x]^2*Log[e + f*x] + 12*b^3*f*m*n^3*x*Log[x]^2*Log[e + f*x] - 4*
b^3*f*m*n^3*x*Log[x]^3*Log[e + f*x] + 12*a^2*b*f*m*x*Log[c*x^n]*Log[e + f*x
] + 24*a*b^2*f*m*n*x*Log[c*x^n]*Log[e + f*x] + 24*b^3*f*m*n^2*x*Log[c*x^n]*
Log[e + f*x] - 24*a*b^2*f*m*n*x*Log[x]*Log[c*x^n]*Log[e + f*x] - 24*b^3*f*m
*n^2*x*Log[x]*Log[c*x^n]*Log[e + f*x] + 12*b^3*f*m*n^2*x*Log[x]^2*Log[c*x^n
]*Log[e + f*x] + 12*a*b^2*f*m*x*Log[c*x^n]^2*Log[e + f*x] + 12*b^3*f*m*n*x*
Log[c*x^n]^2*Log[e + f*x] - 12*b^3*f*m*n*x*Log[x]*Log[c*x^n]^2*Log[e + f*x]
 + 4*b^3*f*m*x*Log[c*x^n]^3*Log[e + f*x] + 4*a^3*e*Log[d*(e + f*x)^m] + 12*
a^2*b*e*n*Log[d*(e + f*x)^m] + 24*a*b^2*e*n^2*Log[d*(e + f*x)^m] + 24*b^3*e
*n^3*Log[d*(e + f*x)^m] + 12*a^2*b*e*Log[c*x^n]*Log[d*(e + f*x)^m] + 24*a*b
^2*e*n*Log[c*x^n]*Log[d*(e + f*x)^m] + 24*b^3*e*n^2*Log[c*x^n]*Log[d*(e + f
*x)^m] + 12*a*b^2*e*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 12*b^3*e*n*Log[c*x^n]
^2*Log[d*(e + f*x)^m] + 4*b^3*e*Log[c*x^n]^3*Log[d*(e + f*x)^m] + 12*a^2*b*
f*m*n*x*Log[x]*Log[1 + (f*x)/e] + 24*a*b^2*f*m*n^2*x*Log[x]*Log[1 + (f*x)/e
] + 24*b^3*f*m*n^3*x*Log[x]*Log[1 + (f*x)/e] - 12*a*b^2*f*m*n^2*x*Log[x]^2*
Log[1 + (f*x)/e] - 12*b^3*f*m*n^3*x*Log[x]^2*Log[1 + (f*x)/e] + 4*b^3*f*m*n
^3*x*Log[x]^3*Log[1 + (f*x)/e] + 24*a*b^2*f*m*n*x*Log[x]*Log[c*x^n]*Log[1 +
 (f*x)/e] + 24*b^3*f*m*n^2*x*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] - 12*b^3*f*
m*n^2*x*Log[x]^2*Log[c*x^n]*Log[1 + (f*x)/e] + 12*b^3*f*m*n*x*Log[x]*Log[c*
x^n]^2*Log[1 + (f*x)/e] + 12*b*f*m*n*x*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a
+ b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -((f*x)/e)] - 24*b^2*f*m*n
^2*x*(a + b*n + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)] + 24*b^3*f*m*n^3*x*Pol
yLog[4, -((f*x)/e)])/(e*x)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^3\log\left(cx^n\right)^3 + 3\,ab^2\log\left(cx^n\right)^2 + 3\,a^2b\log\left(cx^n\right) + a^3\right)\log\left(\left(fx+e\right)^m d\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x + e)^m*d)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right) + a\right)^3\log\left(\left(fx+e\right)^m d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d)/x^2, x)
```

maple [C] time = 2.66, size = 42181, normalized size = 102.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(f*x+e)^m)/x^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^3 f m x \log\left(f x + e\right) - b^3 f m x \log(x) + b^3 e \log(d)\right) \log\left(x^n\right)^3 + \left(b^3 e \log\left(x^n\right)^3 + 3\left(e n + e \log(c)\right) a^2 b + 3\left(2\, e n^2 + \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -((b^3*f*m*x*log(f*x + e) - b^3*f*m*x*log(x) + b^3*e*log(d))*log(x^n)^3 + (
b^3*e*log(x^n)^3 + 3*(e*n + e*log(c))*a^2*b + 3*(2*e*n^2 + 2*e*n*log(c) + e
*log(c)^2)*a*b^2 + (6*e*n^3 + 6*e*n^2*log(c) + 3*e*n*log(c)^2 + e*log(c)^3)
*b^3 + a^3*e + 3*((e*n + e*log(c))*b^3 + a*b^2*e)*log(x^n)^2 + 3*(2*(e*n +
e*log(c))*a*b^2 + (2*e*n^2 + 2*e*n*log(c) + e*log(c)^2)*b^3 + a^2*b*e)*log(
x^n))*log((f*x + e)^m))/(e*x) + integrate((b^3*e^2*log(c)^3*log(d) + 3*a*b^
2*e^2*log(c)^2*log(d) + 3*a^2*b*e^2*log(c)*log(d) + a^3*e^2*log(d) + 3*(a*b
^2*e^2*log(d) + (e^2*n*log(d) + e^2*log(c)*log(d))*b^3 + ((e*f*m + e*f*log(
d))*a*b^2 + (e*f*m*n + e*f*n*log(d) + (e*f*m + e*f*log(d))*log(c))*b^3)*x +
 (b^3*f^2*m*n*x^2 + b^3*e*f*m*n*x)*log(f*x + e) - (b^3*f^2*m*n*x^2 + b^3*e*
f*m*n*x)*log(x))*log(x^n)^2 + ((e*f*m + e*f*log(d))*a^3 + 3*(e*f*m*n + (e*f
*m + e*f*log(d))*log(c))*a^2*b + 3*(2*e*f*m*n^2 + 2*e*f*m*n*log(c) + (e*f*m
 + e*f*log(d))*log(c)^2)*a*b^2 + (6*e*f*m*n^3 + 6*e*f*m*n^2*log(c) + 3*e*f*
m*n*log(c)^2 + (e*f*m + e*f*log(d))*log(c)^3)*b^3)*x + 3*(b^3*e^2*log(c)^2*
log(d) + 2*a*b^2*e^2*log(c)*log(d) + a^2*b*e^2*log(d) + ((e*f*m + e*f*log(d
))*a^2*b + 2*(e*f*m*n + (e*f*m + e*f*log(d))*log(c))*a*b^2 + (2*e*f*m*n^2 +
 2*e*f*m*n*log(c) + (e*f*m + e*f*log(d))*log(c)^2)*b^3)*x)*log(x^n))/(e*f*x
^3 + e^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\,x\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^2,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m)/x**2,x)
```

```
[Out] Timed out
```

3.89 $\int \frac{\left(a+b\log(cx^n)\right)^3\log\left(d(e+fx)^m\right)}{x^3}\,dx$

Optimal. Leaf size=555

$$-\frac{3b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{4x^2}-\frac{3b^2f^2mn^2\text{Li}_2\left(-\frac{e}{fx}\right)\left(a+b\log\left(cx^n\right)\right)}{2e^2}-\frac{3b^2f^2mn^2\text{Li}_3\left(-\frac{e}{fx}\right)\left(a+b\log\right.}{e^2}$$

```
[Out] -45/8*b^3*f*m*n^3/e/x-3/8*b^3*f^2*m*n^3*ln(x)/e^2-21/4*b^2*f*m*n^2*(a+b*ln(
c*x^n))/e/x+3/4*b^2*f^2*m*n^2*ln(1+e/f/x)*(a+b*ln(c*x^n))/e^2-9/4*b*f*m*n*(
a+b*ln(c*x^n))^2/e/x+3/4*b*f^2*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))^2/e^2-1/2*f*
m*(a+b*ln(c*x^n))^3/e/x+1/2*f^2*m*ln(1+e/f/x)*(a+b*ln(c*x^n))^3/e^2+3/8*b^3
*f^2*m*n^3*ln(f*x+e)/e^2-3/8*b^3*n^3*ln(d*(f*x+e)^m)/x^2-3/4*b^2*n^2*(a+b*l
n(c*x^n))*ln(d*(f*x+e)^m)/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^2
-1/2*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/x^2-3/4*b^3*f^2*m*n^3*polylog(2,-e/f
/x)/e^2-3/2*b^2*f^2*m*n^2*(a+b*ln(c*x^n))*polylog(2,-e/f/x)/e^2-3/2*b*f^2*m
*n*(a+b*ln(c*x^n))^2*polylog(2,-e/f/x)/e^2-3/2*b^3*f^2*m*n^3*polylog(3,-e/f
/x)/e^2-3*b^2*f^2*m*n^2*(a+b*ln(c*x^n))*polylog(3,-e/f/x)/e^2-3*b^3*f^2*m*n
^3*polylog(4,-e/f/x)/e^2
```

Rubi [A] time = 1.01, antiderivative size = 614, normalized size of antiderivative = 1.11, number of steps used = 30, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2305, 2304, 2378, 44, 2351, 2301, 2317, 2391, 2353, 2302, 30, 2374, 6589, 2383}

$$\frac{3b^2f^2mn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{2e^2}-\frac{3b^2f^2mn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e^2}+\frac{3bf^2mn\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] (-45*b^3*f*m*n^3)/(8*e*x) - (3*b^3*f^2*m*n^3*Log[x])/(8*e^2) - (21*b^2*f*m*
n^2*(a + b*Log[c*x^n]))/(4*e*x) - (3*b*f^2*m*n*(a + b*Log[c*x^n])^2)/(8*e^2
) - (9*b*f*m*n*(a + b*Log[c*x^n])^2)/(4*e*x) - (f^2*m*(a + b*Log[c*x^n])^3)
/(4*e^2) - (f*m*(a + b*Log[c*x^n])^3)/(2*e*x) - (f^2*m*(a + b*Log[c*x^n])^4
)/(8*b*e^2*n) + (3*b^3*f^2*m*n^3*Log[e + f*x])/(8*e^2) - (3*b^3*n^3*Log[d*(
e + f*x)^m])/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/(4
*x^2) - (3*b*n*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/(4*x^2) - ((a + b*L
og[c*x^n])^3*Log[d*(e + f*x)^m])/(2*x^2) + (3*b^2*f^2*m*n^2*(a + b*Log[c*x^
n])*Log[1 + (f*x)/e])/(4*e^2) + (3*b*f^2*m*n*(a + b*Log[c*x^n])^2*Log[1 + (
f*x)/e])/(4*e^2) + (f^2*m*(a + b*Log[c*x^n])^3*Log[1 + (f*x)/e])/(2*e^2) +
(3*b^3*f^2*m*n^3*PolyLog[2, -((f*x)/e)])/(4*e^2) + (3*b^2*f^2*m*n^2*(a + b*
Log[c*x^n])*PolyLog[2, -((f*x)/e)])/(2*e^2) + (3*b*f^2*m*n*(a + b*Log[c*x^n
])^2*PolyLog[2, -((f*x)/e)])/(2*e^2) - (3*b^3*f^2*m*n^3*PolyLog[3, -((f*x)/
e)])/(2*e^2) - (3*b^2*f^2*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)])/
e^2 + (3*b^3*f^2*m*n^3*PolyLog[4, -((f*x)/e)])/e^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
```

```
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d(e+fx)^m\right)}{x^3}\,dx = -\frac{3b^3n^3\log\left(d(e+fx)^m\right)}{8x^2}-\frac{3b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^{m}\right)}{4x^2}$$

$$= -\frac{3b^3n^3\log\left(d(e+fx)^m\right)}{8x^2}-\frac{3b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^{m}\right)}{4x^2}$$

$$= -\frac{3b^3n^3\log\left(d(e+fx)^m\right)}{8x^2}-\frac{3b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^{m}\right)}{4x^2}$$

$$= -\frac{3b^3fmn^3}{8ex}-\frac{3b^3f^2mn^3\log(x)}{8e^2}+\frac{3b^3f^2mn^3\log(e+fx)}{8e^2}-\frac{3b^3n^3}{}$$

$$= -\frac{9b^3fmn^3}{8ex}-\frac{3b^3f^2mn^3\log(x)}{8e^2}-\frac{3b^2fmn^2\left(a+b\log\left(cx^n\right)\right)}{4ex}-\frac{3b}{}$$

$$= -\frac{21b^3fmn^3}{8ex}-\frac{3b^3f^2mn^3\log(x)}{8e^2}-\frac{9b^2fmn^2\left(a+b\log\left(cx^n\right)\right)}{4ex}-\frac{3}{}$$

$$= -\frac{45b^3fmn^3}{8ex}-\frac{3b^3f^2mn^3\log(x)}{8e^2}-\frac{21b^2fmn^2\left(a+b\log\left(cx^n\right)\right)}{4ex}-$$

$$= -\frac{45b^3fmn^3}{8ex}-\frac{3b^3f^2mn^3\log(x)}{8e^2}-\frac{21b^2fmn^2\left(a+b\log\left(cx^n\right)\right)}{4ex}-$$

Mathematica [B] time = 0.79, size = 1736, normalized size = 3.13

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] -1/8*(4*a^3*e*f*m*x + 18*a^2*b*e*f*m*n*x + 42*a*b^2*e*f*m*n^2*x + 45*b^3*e*
f*m*n^3*x + 4*a^3*f^2*m*x^2*Log[x] + 6*a^2*b*f^2*m*n*x^2*Log[x] + 6*a*b^2*f
^2*m*n^2*x^2*Log[x] + 3*b^3*f^2*m*n^3*x^2*Log[x] - 6*a^2*b*f^2*m*n*x^2*Log[
x]^2 - 6*a*b^2*f^2*m*n^2*x^2*Log[x]^2 - 3*b^3*f^2*m*n^3*x^2*Log[x]^2 + 4*a*
b^2*f^2*m*n^2*x^2*Log[x]^3 + 2*b^3*f^2*m*n^3*x^2*Log[x]^3 - b^3*f^2*m*n^3*x
^2*Log[x]^4 + 12*a^2*b*e*f*m*x*Log[c*x^n] + 36*a*b^2*e*f*m*n*x*Log[c*x^n] +
 42*b^3*e*f*m*n^2*x*Log[c*x^n] + 12*a^2*b*f^2*m*x^2*Log[x]*Log[c*x^n] + 12*
a*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n] + 6*b^3*f^2*m*n^2*x^2*Log[x]*Log[c*x^n]
 - 12*a*b^2*f^2*m*n*x^2*Log[x]^2*Log[c*x^n] - 6*b^3*f^2*m*n^2*x^2*Log[x]^2*
Log[c*x^n] + 4*b^3*f^2*m*n^2*x^2*Log[x]^3*Log[c*x^n] + 12*a*b^2*e*f*m*x*Log
[c*x^n]^2 + 18*b^3*e*f*m*n*x*Log[c*x^n]^2 + 12*a*b^2*f^2*m*x^2*Log[x]*Log[c
*x^n]^2 + 6*b^3*f^2*m*n*x^2*Log[x]*Log[c*x^n]^2 - 6*b^3*f^2*m*n*x^2*Log[x]^
2*Log[c*x^n]^2 + 4*b^3*e*f*m*x*Log[c*x^n]^3 + 4*b^3*f^2*m*x^2*Log[x]*Log[c*
x^n]^3 - 4*a^3*f^2*m*x^2*Log[e + f*x] - 6*a^2*b*f^2*m*n*x^2*Log[e + f*x] -
6*a*b^2*f^2*m*n^2*x^2*Log[e + f*x] - 3*b^3*f^2*m*n^3*x^2*Log[e + f*x] + 12*
a^2*b*f^2*m*n*x^2*Log[x]*Log[e + f*x] + 12*a*b^2*f^2*m*n^2*x^2*Log[x]*Log[e
 + f*x] + 6*b^3*f^2*m*n^3*x^2*Log[x]*Log[e + f*x] - 12*a*b^2*f^2*m*n^2*x^2*
Log[x]^2*Log[e + f*x] - 6*b^3*f^2*m*n^3*x^2*Log[x]^2*Log[e + f*x] + 4*b^3*f
^2*m*n^3*x^2*Log[x]^3*Log[e + f*x] - 12*a^2*b*f^2*m*x^2*Log[c*x^n]*Log[e +
f*x] - 12*a*b^2*f^2*m*n*x^2*Log[c*x^n]*Log[e + f*x] - 6*b^3*f^2*m*n^2*x^2*L
og[c*x^n]*Log[e + f*x] + 24*a*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n]*Log[e + f*x
] + 12*b^3*f^2*m*n^2*x^2*Log[x]*Log[c*x^n]*Log[e + f*x] - 12*b^3*f^2*m*n^2*
x^2*Log[x]^2*Log[c*x^n]*Log[e + f*x] - 12*a*b^2*f^2*m*x^2*Log[c*x^n]^2*Log[
e + f*x] - 6*b^3*f^2*m*n*x^2*Log[c*x^n]^2*Log[e + f*x] + 12*b^3*f^2*m*n*x^2
*Log[x]*Log[c*x^n]^2*Log[e + f*x] - 4*b^3*f^2*m*x^2*Log[c*x^n]^3*Log[e + f*
x] + 4*a^3*e^2*Log[d*(e + f*x)^m] + 6*a^2*b*e^2*n*Log[d*(e + f*x)^m] + 6*a*
b^2*e^2*n^2*Log[d*(e + f*x)^m] + 3*b^3*e^2*n^3*Log[d*(e + f*x)^m] + 12*a^2*
b*e^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 12*a*b^2*e^2*n*Log[c*x^n]*Log[d*(e +
f*x)^m] + 6*b^3*e^2*n^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 12*a*b^2*e^2*Log[c*
x^n]^2*Log[d*(e + f*x)^m] + 6*b^3*e^2*n*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 4
*b^3*e^2*Log[c*x^n]^3*Log[d*(e + f*x)^m] - 12*a^2*b*f^2*m*n*x^2*Log[x]*Log[
1 + (f*x)/e] - 12*a*b^2*f^2*m*n^2*x^2*Log[x]*Log[1 + (f*x)/e] - 6*b^3*f^2*m
*n^3*x^2*Log[x]*Log[1 + (f*x)/e] + 12*a*b^2*f^2*m*n^2*x^2*Log[x]^2*Log[1 +
(f*x)/e] + 6*b^3*f^2*m*n^3*x^2*Log[x]^2*Log[1 + (f*x)/e] - 4*b^3*f^2*m*n^3*
x^2*Log[x]^3*Log[1 + (f*x)/e] - 24*a*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n]*Log[
1 + (f*x)/e] - 12*b^3*f^2*m*n^2*x^2*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 12
*b^3*f^2*m*n^2*x^2*Log[x]^2*Log[c*x^n]*Log[1 + (f*x)/e] - 12*b^3*f^2*m*n*x^
2*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] - 6*b*f^2*m*n*x^2*(2*a^2 + 2*a*b*n +
 b^2*n^2 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, -((f
*x)/e)] + 12*b^2*f^2*m*n^2*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[3, -((f
*x)/e)] - 24*b^3*f^2*m*n^3*x^2*PolyLog[4, -((f*x)/e)])/(e^2*x^2)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3 \log\left(cx^n\right)^3 + 3\,ab^2 \log\left(cx^n\right)^2 + 3\,a^2 b \log\left(cx^n\right) + a^3\right) \log\left(\left(fx+e\right)^m d\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x + e)^m*d)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^n\right) + a\right)^3 \log\left(\left(fx+e\right)^m d\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d)/x^3, x)
```

maple [F] time = 11.45, size = 0, normalized size = 0.00

$$\int \frac{(b\ln(c\,x^n)+a)^3\ln\left(d\left(fx+e\right)^m\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(f*x+e)^m)/x^3,x)
[Out] int((b*ln(c*x^n)+a)^3*ln(d*(f*x+e)^m)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^3,x, algorithm="maxima")
[Out] 1/8*(4*(b^3*f^2*m*x^2*log(f*x + e) - b^3*f^2*m*x^2*log(x) - b^3*e*f*m*x - b
^3*e^2*log(d))*log(x^n)^3 - (4*b^3*e^2*log(x^n)^3 + 4*a^3*e^2 + 6*(e^2*n +
2*e^2*log(c))*a^2*b + 6*(e^2*n^2 + 2*e^2*n*log(c) + 2*e^2*log(c)^2)*a*b^2 +
 (3*e^2*n^3 + 6*e^2*n^2*log(c) + 6*e^2*n*log(c)^2 + 4*e^2*log(c)^3)*b^3 + 6
*(2*a*b^2*e^2 + (e^2*n + 2*e^2*log(c))*b^3)*log(x^n)^2 + 6*(2*a^2*b*e^2 + 2
*(e^2*n + 2*e^2*log(c))*a*b^2 + (e^2*n^2 + 2*e^2*n*log(c) + 2*e^2*log(c)^2)
*b^3)*log(x^n))*log((f*x + e)^m))/(e^2*x^2) - integrate(-1/8*(8*b^3*e^3*log
(c)^3*log(d) + 24*a*b^2*e^3*log(c)^2*log(d) + 24*a^2*b*e^3*log(c)*log(d) +
8*a^3*e^3*log(d) + 6*(2*b^3*e*f^2*m*n*x^2 + 4*a*b^2*e^3*log(d) + 2*(e^3*n*l
og(d) + 2*e^3*log(c)*log(d))*b^3 + (2*(e^2*f*m + 2*e^2*f*log(d))*a*b^2 + (3
*e^2*f*m*n + 2*e^2*f*n*log(d) + 2*(e^2*f*m + 2*e^2*f*log(d))*log(c))*b^3)*x
 - 2*(b^3*f^3*m*n*x^3 + b^3*e*f^2*m*n*x^2)*log(f*x + e) + 2*(b^3*f^3*m*n*x^
3 + b^3*e*f^2*m*n*x^2)*log(x))*log(x^n)^2 + (4*(e^2*f*m + 2*e^2*f*log(d))*a
^3 + 6*(e^2*f*m*n + 2*(e^2*f*m + 2*e^2*f*log(d))*log(c))*a^2*b + 6*(e^2*f*m
*n^2 + 2*e^2*f*m*n*log(c) + 2*(e^2*f*m + 2*e^2*f*log(d))*log(c)^2)*a*b^2 +
(3*e^2*f*m*n^3 + 6*e^2*f*m*n^2*log(c) + 6*e^2*f*m*n*log(c)^2 + 4*(e^2*f*m +
 2*e^2*f*log(d))*log(c)^3)*b^3)*x + 6*(4*b^3*e^3*log(c)^2*log(d) + 8*a*b^2*
e^3*log(c)*log(d) + 4*a^2*b*e^3*log(d) + (2*(e^2*f*m + 2*e^2*f*log(d))*a^2*
b + 2*(e^2*f*m*n + 2*(e^2*f*m + 2*e^2*f*log(d))*log(c))*a*b^2 + (e^2*f*m*n^
2 + 2*e^2*f*m*n*log(c) + 2*(e^2*f*m + 2*e^2*f*log(d))*log(c)^2)*b^3)*x)*log
(x^n))/(e^2*f*x^4 + e^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\,x\right)^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^3,x)
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m)/x**3,x)
[Out] Timed out
```

3.90 $\int x^3 (a + b\log(cx^n)) \log\left(d(e + fx^2)^m\right) dx$

Optimal. Leaf size=221

$$\frac{1}{4}x^4(a + b\log(cx^n))\log\left(d(e + fx^2)^m\right) - \frac{e^2 m\log(e + fx^2)(a + b\log(cx^n))}{4f^2} + \frac{emx^2(a + b\log(cx^n))}{4f} - \frac{1}{8}mx^4(a +$$

```
[Out] -3/16*b*e*m*n*x^2/f+1/16*b*m*n*x^4+1/4*e*m*x^2*(a+b*ln(c*x^n))/f-1/8*m*x^4*
(a+b*ln(c*x^n))+1/16*b*e^2*m*n*ln(f*x^2+e)/f^2+1/8*b*e^2*m*n*ln(-f*x^2/e)*l
n(f*x^2+e)/f^2-1/4*e^2*m*(a+b*ln(c*x^n))*ln(f*x^2+e)/f^2-1/16*b*n*x^4*ln(d*
(f*x^2+e)^m)+1/4*x^4*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)+1/8*b*e^2*m*n*polylo
g(2,1+f*x^2/e)/f^2
```

Rubi [A] time = 0.22, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2454, 2395, 43, 2376, 2394, 2315}

$$\frac{be^2 mn\text{PolyLog}\left(2, \frac{fx^2}{e} + 1\right)}{8f^2} + \frac{1}{4}x^4(a + b\log(cx^n))\log\left(d(e + fx^2)^m\right) - \frac{e^2 m\log(e + fx^2)(a + b\log(cx^n))}{4f^2} + \frac{emx^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-3*b*e*m*n*x^2)/(16*f) + (b*m*n*x^4)/16 + (e*m*x^2*(a + b*Log[c*x^n]))/(4*
f) - (m*x^4*(a + b*Log[c*x^n]))/8 + (b*e^2*m*n*Log[e + f*x^2])/(16*f^2) + (
b*e^2*m*n*Log[-((f*x^2)/e)]*Log[e + f*x^2])/(8*f^2) - (e^2*m*(a + b*Log[c*x
^n])*Log[e + f*x^2])/(4*f^2) - (b*n*x^4*Log[d*(e + f*x^2)^m])/16 + (x^4*(a
+ b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/4 + (b*e^2*m*n*PolyLog[2, 1 + (f*x^2)
/e])/(8*f^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)dx=\frac{emx^2\left(a+b\log\left(cx^n\right)\right)}{4f}-\frac{1}{8}mx^4\left(a+b\log\left(cx^n\right)\right)-\frac{e^2m\left(a+b\,l\right.}{}$$

$$=-\frac{bemnx^2}{8f}+\frac{1}{32}bmnx^4+\frac{emx^2\left(a+b\log\left(cx^n\right)\right)}{4f}-\frac{1}{8}mx^4\left(a+b\right.$$

$$=-\frac{bemnx^2}{8f}+\frac{1}{32}bmnx^4+\frac{emx^2\left(a+b\log\left(cx^n\right)\right)}{4f}-\frac{1}{8}mx^4\left(a+b\right.$$

$$=-\frac{bemnx^2}{8f}+\frac{1}{32}bmnx^4+\frac{emx^2\left(a+b\log\left(cx^n\right)\right)}{4f}-\frac{1}{8}mx^4\left(a+b\right.$$

$$=-\frac{bemnx^2}{8f}+\frac{1}{32}bmnx^4+\frac{emx^2\left(a+b\log\left(cx^n\right)\right)}{4f}-\frac{1}{8}mx^4\left(a+b\right.$$

$$=-\frac{3bemnx^2}{16f}+\frac{1}{16}bmnx^4+\frac{emx^2\left(a+b\log\left(cx^n\right)\right)}{4f}-\frac{1}{8}mx^4\left(a+\right.$$

Mathematica [C] time = 0.15, size = 324, normalized size = 1.47

$$-\frac{-4af^2x^4\log\left(d\left(e+fx^2\right)^m\right)+4ae^2m\log\left(e+fx^2\right)-4aefmx^2+2af^2mx^4-4bf^2x^4\log\left(cx^n\right)\log\left(d\left(e+fx^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] -1/16*(-4*a*e*f*m*x^2 + 3*b*e*f*m*n*x^2 + 2*a*f^2*m*x^4 - b*f^2*m*n*x^4 - 4
*b*e*f*m*x^2*Log[c*x^n] + 2*b*f^2*m*x^4*Log[c*x^n] + 4*b*e^2*m*n*Log[x]*Log
[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 4*b*e^2*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt
[e]] + 4*a*e^2*m*Log[e + f*x^2] - b*e^2*m*n*Log[e + f*x^2] - 4*b*e^2*m*n*Lo
g[x]*Log[e + f*x^2] + 4*b*e^2*m*Log[c*x^n]*Log[e + f*x^2] - 4*a*f^2*x^4*Log
[d*(e + f*x^2)^m] + b*f^2*n*x^4*Log[d*(e + f*x^2)^m] - 4*b*f^2*x^4*Log[c*x^
n]*Log[d*(e + f*x^2)^m] + 4*b*e^2*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]]
+ 4*b*e^2*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/f^2
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^3\log\left(cx^n\right)+ax^3\right)\log\left(\left(fx^2+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*log(c*x^n) + a*x^3)*log((f*x^2 + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x^3\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3*log((f*x^2 + e)^m*d), x)
```

maple [C] time = 0.91, size = 2259, normalized size = 10.22

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m),x)
```

```
[Out] 1/16*I*Pi*b*m*x^4*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/16*Pi^2*csgn(I*d)*c
sgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^4*b*csgn(I*c*x^n)^2*csgn(I*c)-1/
16*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1
/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^4*b*csgn(I*c
*x^n)^3-1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*x^n)*csgn(
I*c*x^n)^2-1/8*a*m*x^4+1/4*a*x^4*ln(d)-1/32*I*Pi*b*n*x^4*csgn(I*d)*csgn(I*d
*(f*x^2+e)^m)^2-1/32*I*Pi*b*n*x^4*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
^2+1/8*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b*x^4*ln(x^n)-1/8*b*m*x^4*ln(
x^n)+1/4*b*x^4*ln(d)*ln(x^n)+1/4*b*e/f*m*x^2*ln(c)-1/8*I/f^2*e^2*m*ln(f*x^2
+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I*Pi*b*x^4*csgn(I*c)*csgn(I*x^n)*c
sgn(I*c*x^n)*ln(d)+1/4*a*e/f*m*x^2+1/8*I/f^2*e^2*m*ln(f*x^2+e)*Pi*b*csgn(I*
x^n)*csgn(I*c*x^n)*csgn(I*c)-3/16*b*e/f*m*n*x^2+(1/4*b*x^4*ln(x^n)+1/16*(-2
*I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+2*I*Pi*b*csgn(I*c)*csgn(I*c*x^n
)^2+2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-2*I*Pi*b*csgn(I*c*x^n)^3-b*n+4*b*l
n(c)+4*a)*x^4)*ln((f*x^2+e)^m)-1/8*I*ln(c)*Pi*b*x^4*csgn(I*d)*csgn(I*(f*x^2
+e)^m)*csgn(I*d*(f*x^2+e)^m)-1/8*I/f*Pi*b*e*m*csgn(I*c*x^n)^3*x^2-1/8*I/f^2
*e^2*m*ln(f*x^2+e)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/16*Pi^2*csgn(I*d*(f*x^2
+e)^m)^3*x^4*b*csgn(I*c*x^n)^3-1/8*I*Pi*a*x^4*csgn(I*d*(f*x^2+e)^m)^3+1/4*b
*x^4*ln(c)*ln(d)-1/8*b*m*x^4*ln(c)-1/16*b*n*x^4*ln(d)-1/8*I/f*Pi*b*e*m*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*x^2-1/8*I*Pi*b*x^4*csgn(I*c*x^n)^3*ln(d)+1/
32*I*Pi*b*n*x^4*csgn(I*d*(f*x^2+e)^m)^3+1/8*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I
*d*(f*x^2+e)^m)^2*b*x^4*ln(x^n)+1/8*I*ln(c)*Pi*b*x^4*csgn(I*d)*csgn(I*d*(f*
x^2+e)^m)^2+1/8*I*ln(c)*Pi*b*x^4*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^
2+1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*c*x^n)^3+1/16*Pi
^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*c*x^n)^3+1/16*P
i^2*csgn(I*d*(f*x^2+e)^m)^3*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/16*Pi^2*csg
n(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/
16*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*x^n)*csgn(
I*c*x^n)*csgn(I*c)+1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+
e)^m)*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)
^m)*csgn(I*d*(f*x^2+e)^m)*b*x^4*ln(x^n)+1/32*I*Pi*b*n*x^4*csgn(I*d)*csgn(I*
(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/8*I*Pi*b*x^4*csgn(I*x^n)*csgn(I*c*x^n)
```

```
^2*ln(d)-1/4/f^2*e^2*m*ln(f*x^2+e)*a-1/4/f^2*e^2*m*ln(f*x^2+e)*b*ln(c)+1/16
*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^4*b*csgn(I*c*x^n)^2*csgn(I*c)-1/8*I*Pi*csgn
(I*d*(f*x^2+e)^m)^3*b*x^4*ln(x^n)+1/8*I*Pi*b*x^4*csgn(I*c)*csgn(I*c*x^n)^2*
ln(d)+1/16*b*m*n*x^4+1/8*I/f^2*e^2*m*ln(f*x^2+e)*Pi*b*csgn(I*c*x^n)^3+1/8*I
/f*Pi*b*e*m*csgn(I*x^n)*csgn(I*c*x^n)^2*x^2+1/8*I/f*Pi*b*e*m*csgn(I*c*x^n)^
2*csgn(I*c)*x^2-1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^
m)*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*b*e^2/f^2*m*n*ln(x)*ln((-f
*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/4*b*e^2/f^2*m*n*ln(x)*ln((f*x+(-e*f)^(1/2)
)/(-e*f)^(1/2))+1/4*b*e^2/f^2*m*n*ln(x)*ln(f*x^2+e)+1/16*I*Pi*b*m*x^4*csgn(
I*c*x^n)^3+1/8*I*Pi*a*x^4*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2+1/8*I*Pi*a*x^4*
csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/16*I*Pi*b*m*x^4*csgn(I*x^n)*c
sgn(I*c*x^n)^2-1/16*I*Pi*b*m*x^4*csgn(I*c*x^n)^2*csgn(I*c)-1/8*I*Pi*a*x^4*c
sgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)-1/16*Pi^2*csgn(I*d)*csgn
(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*c*x^n)^2*csgn(I*c)+1/4*b*e/f*m*x^2*ln(x^n)
-1/16*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*x^n)*cs
gn(I*c*x^n)^2-1/16*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*c
sgn(I*c*x^n)^2*csgn(I*c)-1/4*b*e^2/f^2*m*n*dilog((-f*x+(-e*f)^(1/2))/(-e*f)
^(1/2))-1/4*b*e^2/f^2*m*n*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/16*b*e^2
*m*n*ln(f*x^2+e)/f^2-1/8*I*ln(c)*Pi*b*x^4*csgn(I*d*(f*x^2+e)^m)^3-1/4*m/f^2
*b*ln(x^n)*e^2*ln(f*x^2+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\left(4\,bmx^4\log\left(x^n\right)-\left(\left(mn-4\,m\log(c)\right)b-4\,am\right)x^4\right)\log\left(fx^2+e\right)+\int-\frac{\left(4\left(fm-2\,f\log(d)\right)a-\left(fmn-4\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/16*(4*b*m*x^4*log(x^n) - ((m*n - 4*m*log(c))*b - 4*a*m)*x^4)*log(f*x^2 +
e) + integrate(-1/8*((4*(f*m - 2*f*log(d))*a - (f*m*n - 4*(f*m - 2*f*log(d)
)*log(c))*b)*x^5 - 8*(b*e*log(c)*log(d) + a*e*log(d))*x^3 + 4*((f*m - 2*f*l
og(d))*b*x^5 - 2*b*e*x^3*log(d))*log(x^n))/(f*x^2 + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^3*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

3.91 $\int x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)\,dx$

Optimal. Leaf size=148

$$\frac{\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{2f}-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)-\frac{bn\left(e+fx^2\right)\log\left(d\left(e+fx^2\right)^m\right)}{4f}-\frac{ben\log}{}$$

```
[Out] 1/2*b*m*n*x^2-1/2*m*x^2*(a+b*ln(c*x^n))-1/4*b*n*(f*x^2+e)*ln(d*(f*x^2+e)^m)
/f-1/4*b*e*n*ln(-f*x^2/e)*ln(d*(f*x^2+e)^m)/f+1/2*(f*x^2+e)*(a+b*ln(c*x^n))
*ln(d*(f*x^2+e)^m)/f-1/4*b*e*m*n*polylog(2,1+f*x^2/e)/f
```

Rubi [A] time = 0.22, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2454, 2389, 2295, 2376, 2475, 2411, 43, 2351, 2317, 2391}

$$-\frac{bemn\text{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{4f}+\frac{\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{2f}-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)-\frac{bn\left(e+fx^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (b*m*n*x^2)/2 - (m*x^2*(a + b*Log[c*x^n]))/2 - (b*n*(e + f*x^2)*Log[d*(e +
f*x^2)^m])/(4*f) - (b*e*n*Log[-((f*x^2)/e)]*Log[d*(e + f*x^2)^m])/(4*f) + (
(e + f*x^2)*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(2*f) - (b*e*m*n*PolyL
og[2, 1 + (f*x^2)/e])/(4*f)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
```

```
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)dx=-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\right.\right.}{2f}$$

$$=\frac{1}{4}bmnx^2-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)\mathrm{log}}{2f}$$

$$=\frac{1}{4}bmnx^2-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)\mathrm{log}}{2f}$$

$$=\frac{1}{4}bmnx^2-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)\mathrm{log}}{2f}$$

$$=\frac{1}{4}bmnx^2-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)\mathrm{log}}{2f}$$

$$=\frac{1}{4}bmnx^2-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)\mathrm{log}}{2f}$$

$$=\frac{1}{2}bmnx^2-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)-\frac{bn\left(e+fx^2\right)\log\left(d\left(e+fx^2\right)^m\right.}{4f}$$

$$=\frac{1}{2}bmnx^2-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)-\frac{bn\left(e+fx^2\right)\log\left(d\left(e+fx^2\right)^m\right.}{4f}$$

Mathematica [C] time = 0.09, size = 266, normalized size = 1.80

$$\frac{2afx^2\log\left(d\left(e+fx^2\right)^m\right)+2ae\log\left(d\left(e+fx^2\right)^m\right)-2afmx^2+2bfx^2\log\left(cx^n\right)\log\left(d\left(e+fx^2\right)^m\right)+2bem\log\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-2*a*f*m*x^2 + 2*b*f*m*n*x^2 - 2*b*f*m*x^2*Log[c*x^n] + 2*b*e*m*n*Log[x]*L
og[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 2*b*e*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt
[e]] - b*e*m*n*Log[e + f*x^2] - 2*b*e*m*n*Log[x]*Log[e + f*x^2] + 2*b*e*m*L
og[c*x^n]*Log[e + f*x^2] + 2*a*e*Log[d*(e + f*x^2)^m] + 2*a*f*x^2*Log[d*(e
+ f*x^2)^m] - b*f*n*x^2*Log[d*(e + f*x^2)^m] + 2*b*f*x^2*Log[c*x^n]*Log[d*(
e + f*x^2)^m] + 2*b*e*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*b*e*m*n*
PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(4*f)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx\log\left(cx^n\right)+ax\right)\log\left(\left(fx^2+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)*log((f*x^2 + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x\log\left(\left(fx^2+e\right)^m d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*log((f*x^2 + e)^m*d), x)
```

maple [C] time = 0.72, size = 2068, normalized size = 13.97

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m),x)
```

```
[Out] -1/4*I*e*m/f*ln(f*x^2+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*a*m*x
^2+(1/2*b*x^2*ln(x^n)+1/4*x^2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c)*c
sgn(I*c*x^n)^2+2*b*ln(c)-b*n+2*a))*ln((f*x^2+e)^m)+1/2*b*e*n*m/f*dilog((-f*
x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/2*b*e*n*m/f*dilog((f*x+(-e*f)^(1/2))/(-e*f)
^(1/2))-1/4*I*Pi*a*x^2*csgn(I*d*(f*x^2+e)^m)^3+1/2*a*x^2*ln(d)+1/2*m/f*b*ln
(x^n)*e*ln(f*x^2+e)-1/8*I*Pi*b*n*x^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)
^m)^2+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b*x^2*ln(x^n)+1/
4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b*x^2*ln(x^n)-1/4*I*x^2*Pi*ln(d)*b
*csgn(I*c*x^n)^3-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^
m)*b*x^2*ln(x^n)+1/4*I*Pi*b*m*x^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*I
*Pi*b*n*x^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)-1/2*b*m*x^2
*ln(x^n)+1/2*b*x^2*ln(d)*ln(x^n)-1/4*b*n*x^2*ln(d)+1/2*b*x^2*ln(c)*ln(d)-1/
2*b*m*x^2*ln(c)-1/8*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^2*b*csgn(I*c*x^n)^3+1/4*
I*x^2*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*d)*csgn(I*d*(f
*x^2+e)^m)^2*x^2*b*csgn(I*c*x^n)^2*csgn(I*c)-1/8*Pi^2*csgn(I*(f*x^2+e)^m)*c
sgn(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*(f
*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*c*x^n)^2*csgn(I*c)-1/8*Pi^2
*csgn(I*d*(f*x^2+e)^m)^3*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/8*Pi^2
*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^2*b*csgn(I*c*x^n)^3-
1/4*I*Pi*a*x^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*I*ln
(c)*Pi*b*x^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/8*I*Pi*b*n*x^2*c
sgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I*Pi*b*m*x^2*csgn(I*x^n)*csgn(I*c*x^n)
^2-1/4*I*Pi*b*m*x^2*csgn(I*c*x^n)^2*csgn(I*c)+1/4*I*ln(c)*Pi*b*x^2*csgn(I*d
)*csgn(I*d*(f*x^2+e)^m)^2-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(
f*x^2+e)^m)*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/4*I*e*m/f*ln(f*x^2+
e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I*e*m/f*ln(f*x^2+e)*Pi*b*csgn(I*c*x
^n)^2*csgn(I*c)-1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*x^n
)*csgn(I*c*x^n)^2+1/8*I*Pi*b*n*x^2*csgn(I*d*(f*x^2+e)^m)^3+1/4*I*Pi*a*x^2*c
sgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I*ln(c)*Pi*b*x^2*csgn(I*d*(f
*x^2+e)^m)^3+1/2*e*m/f*ln(f*x^2+e)*a-1/4*b*e/f*m*n*ln(f*x^2+e)+1/2*e*m/f*ln
(f*x^2+e)*b*ln(c)+1/4*I*Pi*b*m*x^2*csgn(I*c*x^n)^3+1/4*I*Pi*a*x^2*csgn(I*d)
*csgn(I*d*(f*x^2+e)^m)^2-1/4*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b*x^2*ln(x^n)+1/8
*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*b*m*n*x
^2+1/4*I*x^2*Pi*ln(d)*b*csgn(I*c*x^n)^2*csgn(I*c)-1/4*I*x^2*Pi*ln(d)*b*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn
(I*d*(f*x^2+e)^m)*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*(f*x^2+
e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*c*x^n)^3+1/8*Pi^2*csgn(I*d*(f*x^
2+e)^m)^3*x^2*b*csgn(I*c*x^n)^2*csgn(I*c)+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^
2+e)^m)^2*x^2*b*csgn(I*c*x^n)^3-1/2*b*e/f*m*n*ln(x)*ln(f*x^2+e)+1/2*b*e*n*m
/f*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/2*b*e*n*m/f*ln(x)*ln((f*x+(
-e*f)^(1/2))/(-e*f)^(1/2))+1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*
(f*x^2+e)^m)*x^2*b*csgn(I*c*x^n)^2*csgn(I*c)+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f
*x^2+e)^m)^2*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*Pi^2*csgn(I*(f*x
^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-
1/4*I*e*m/f*ln(f*x^2+e)*Pi*b*csgn(I*c*x^n)^3-1/4*I*ln(c)*Pi*b*x^2*csgn(I*d)
*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(2\,bmx^2\log\left(x^n\right) - \left(\left(mn - 2\,m\log(c)\right)b - 2\,am\right)x^2\right)\log\left(fx^2 + e\right) + \int -\frac{\left(2\left(fm - f\log(d)\right)a - \left(fmn - 2\left(fm - \right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*m*x^2*log(x^n) - ((m*n - 2*m*log(c))*b - 2*a*m)*x^2)*log(f*x^2 + e
) + integrate(-1/2*((2*(f*m - f*log(d))*a - (f*m*n - 2*(f*m - f*log(d))*log
(c))*b)*x^3 - 2*(b*e*log(c)*log(d) + a*e*log(d))*x + 2*((f*m - f*log(d))*b*
x^3 - b*e*x*log(d))*log(x^n))/(f*x^2 + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

3.92 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx$

Optimal. Leaf size=113

$$\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{2bn}-\frac{1}{2}m\text{Li}_2\left(-\frac{fx^2}{e}\right)(a+b\log(cx^n))-\frac{m\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^2}{2bn}+\frac{1}{4}$$

```
[Out] 1/2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/b/n-1/2*m*(a+b*ln(c*x^n))^2*ln(1+f*
x^2/e)/b/n-1/2*m*(a+b*ln(c*x^n))*polylog(2,-f*x^2/e)+1/4*b*m*n*polylog(3,-f
*x^2/e)
```

Rubi [A] time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2375, 2337, 2374, 6589}

$$-\frac{1}{2}m\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))+\frac{1}{4}bmn\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)+\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{2bn}-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/(2*b*n) - (m*(a + b*Log[c*x^n])
^2*Log[1 + (f*x^2)/e])/(2*b*n) - (m*(a + b*Log[c*x^n])*PolyLog[2, -((f*x^2)
/e)])/2 + (b*m*n*PolyLog[3, -((f*x^2)/e)])/4
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx = \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{2bn} - \frac{(fm)\int \frac{x(a+b\log(cx^n))^2}{e+fx^2}dx}{bn}$$
$$= \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{2bn} - \frac{m(a+b\log(cx^n))^2\log(1+}{2bn}$$
$$= \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{2bn} - \frac{m(a+b\log(cx^n))^2\log(1+}{2bn}$$
$$= \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{2bn} - \frac{m(a+b\log(cx^n))^2\log(1+}{2bn}$$

Mathematica [C] time = 0.09, size = 297, normalized size = 2.63

$$\frac{1}{2}\left(a\log\left(-\frac{fx^2}{e}\right)\log\left(d\left(e+fx^2\right)^m\right)+am\text{Li}_2\left(\frac{fx^2}{e}+1\right)+2b\log(x)\log(cx^n)\log\left(d\left(e+fx^2\right)^m\right)-2bm\log(cx^n)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] (b*m*n*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 2*b*m*Log[x]*Log[c*x^n]*Lo
g[1 - (I*Sqrt[f]*x)/Sqrt[e]] + b*m*n*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]
] - 2*b*m*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - b*n*Log[x]^2*L
og[d*(e + f*x^2)^m] + a*Log[-((f*x^2)/e)]*Log[d*(e + f*x^2)^m] + 2*b*Log[x]
*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 2*b*m*Log[c*x^n]*PolyLog[2, ((-I)*Sqrt[f
]*x)/Sqrt[e]] - 2*b*m*Log[c*x^n]*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + a*m*Po
lyLog[2, 1 + (f*x^2)/e] + 2*b*m*n*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*
b*m*n*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]])/2
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\log(cx^n)+a)\log\left(\left(fx^2+e\right)^m d\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\log(cx^n)+a)\log\left(\left(fx^2+e\right)^m d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x, x)
```

maple [F] time = 3.57, size = 0, normalized size = 0.00

$$\int \frac{(b\ln(c\,x^n)+a)\ln\left(d\left(f\,x^2+e\right)^m\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(bmn\log(x)^2-2\,bm\log(x)\log\left(x^n\right)-2\left(bm\log(c)+am\right)\log(x)\right)\log\left(fx^2+e\right)-\int-\frac{bfmnx^2\log(x)^2+be}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*m*n*log(x)^2 - 2*b*m*log(x)*log(x^n) - 2*(b*m*log(c) + a*m)*log(x))
*log(f*x^2 + e) - integrate(-(b*f*m*n*x^2*log(x)^2 + b*e*log(c)*log(d) - 2*
(b*f*m*log(c) + a*f*m)*x^2*log(x) + (b*f*log(c)*log(d) + a*f*log(d))*x^2 +
a*e*log(d) - (2*b*f*m*x^2*log(x) - b*f*x^2*log(d) - b*e*log(d))*log(x^n))/(
f*x^3 + e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x,x)
```

```
[Out] Timed out
```

3.93 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx$

Optimal. Leaf size=195

$$-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}+\frac{fm\log(x)\,(a+b\log(cx^n))}{e}-\frac{fm\log\left(e+fx^2\right)(a+b\log(cx^n))}{2e}-\frac{bn\log\left(\right.}{}$$

```
[Out] 1/2*b*f*m*n*ln(x)/e-1/2*b*f*m*n*ln(x)^2/e+f*m*ln(x)*(a+b*ln(c*x^n))/e-1/4*b
*f*m*n*ln(f*x^2+e)/e+1/4*b*f*m*n*ln(-f*x^2/e)*ln(f*x^2+e)/e-1/2*f*m*(a+b*ln
(c*x^n))*ln(f*x^2+e)/e-1/4*b*n*ln(d*(f*x^2+e)^m)/x^2-1/2*(a+b*ln(c*x^n))*ln
(d*(f*x^2+e)^m)/x^2+1/4*b*f*m*n*polylog(2,1+f*x^2/e)/e
```

Rubi [A] time = 0.18, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {2454, 2395, 36, 29, 31, 2376, 2301, 2394, 2315}

$$\frac{bfmn\text{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{4e}-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}+\frac{fm\log(x)\,(a+b\log(cx^n))}{e}-\frac{fm\log\left(e+fx^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] (b*f*m*n*Log[x])/(2*e) - (b*f*m*n*Log[x]^2)/(2*e) + (f*m*Log[x]*(a + b*Log[
c*x^n]))/e - (b*f*m*n*Log[e + f*x^2])/(4*e) + (b*f*m*n*Log[-((f*x^2)/e)]*Lo
g[e + f*x^2])/(4*e) - (f*m*(a + b*Log[c*x^n])*Log[e + f*x^2])/(2*e) - (b*n*
Log[d*(e + f*x^2)^m])/(4*x^2) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(
2*x^2) + (b*f*m*n*PolyLog[2, 1 + (f*x^2)/e])/(4*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
```

```
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx = \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{fm\left(a+b\log\left(cx^n\right)\right)\log\left(e+fx^2\right)}{2e}$$

$$= \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{fm\left(a+b\log\left(cx^n\right)\right)\log\left(e+fx^2\right)}{2e}$$

$$= -\frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{fm\left(a+b\log\left(c\right.\right.}{}$$

$$= -\frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} + \frac{bfmn\log\left(-\frac{fx^2}{e}\right.}{4}$$

$$= -\frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} + \frac{bfmn\log\left(-\frac{fx^2}{e}\right.}{4}$$

$$= \frac{bfmn\log(x)}{2e} - \frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{b}{}$$

Mathematica [C] time = 0.13, size = 298, normalized size = 1.53

$$-\frac{2ae\log\left(d\left(e+fx^2\right)^m\right)+2afmx^2\log\left(e+fx^2\right)-4afmx^2\log(x)+2be\log\left(cx^n\right)\log\left(d\left(e+fx^2\right)^m\right)+2bfm}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] -1/4*(-4*a*f*m*x^2*Log[x] - 2*b*f*m*n*x^2*Log[x] + 2*b*f*m*n*x^2*Log[x]^2 -
 4*b*f*m*x^2*Log[x]*Log[c*x^n] + 2*b*f*m*n*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)
/Sqrt[e]] + 2*b*f*m*n*x^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 2*a*f*m*x
^2*Log[e + f*x^2] + b*f*m*n*x^2*Log[e + f*x^2] - 2*b*f*m*n*x^2*Log[x]*Log[e
 + f*x^2] + 2*b*f*m*x^2*Log[c*x^n]*Log[e + f*x^2] + 2*a*e*Log[d*(e + f*x^2)
^m] + b*e*n*Log[d*(e + f*x^2)^m] + 2*b*e*Log[c*x^n]*Log[d*(e + f*x^2)^m] +
2*b*f*m*n*x^2*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*b*f*m*n*x^2*PolyLog[
2, (I*Sqrt[f]*x)/Sqrt[e]])/(e*x^2)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^3, x)
```

maple [C] time = 0.68, size = 2101, normalized size = 10.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m)/x^3,x)
```

```
[Out] -1/2*I/e*f*m*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*I/x^2*Pi*b*
n*csgn(I*d*(f*x^2+e)^m)^3-1/2*b/x^2*ln(c)*ln(d)-1/4*b*n/x^2*ln(d)+1/4*I*Pi*
b/x^2*csgn(I*c*x^n)^3*ln(d)-1/2*a/x^2*ln(d)-1/2*f*m/e*ln(f*x^2+e)*a-1/4*I*f
*m/e*ln(f*x^2+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I*f*m/e*ln(f*x^2+e)*b
*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/4*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b/x^2*ln(x^n
)-1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b/x^2*ln(x^n)-1/2*b/e*f*m*n*di
log((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+(-1/2*b/x^2*ln(x^n)-1/4*(-I*Pi*b*csgn(
I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+I*Pi*b*csgn
(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*c*x^n)^3+b*n+2*b*ln(c)+2*a)/x^2)*ln((
f*x^2+e)^m)-1/2*b/x^2*ln(d)*ln(x^n)-1/2*I/e*f*m*ln(x)*b*Pi*csgn(I*c*x^n)^3+
1/8*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^2*b*csgn(I*c*x^n)^3-1/4*I/x^2*Pi*a*csgn(
I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I/x^2*Pi*a*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f
*x^2+e)^m)^2+1/4*I/x^2*ln(c)*Pi*b*csgn(I*d*(f*x^2+e)^m)^3-1/8*I/x^2*Pi*b*n*
csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/8*I/x^2*Pi*b*n*csgn(I*(f*x^2+e)^m)*csgn
(I*d*(f*x^2+e)^m)^2-1/2*b/e*f*m*n*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2)
)-1/2*b/e*f*m*n*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/2*b/e*f*m*n*ln(
x)*ln(f*x^2+e)+1/2*I/e*f*m*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I/e*f
*m*ln(x)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+
e)^m)^2/x^2*b*csgn(I*c*x^n)^3-1/8*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+
```

```
e)^m)^2/x^2*b*csgn(I*c*x^n)^3+m*f*b*ln(x^n)*ln(x)/e-1/2*m*f*b*ln(x^n)/e*ln(
f*x^2+e)-1/4*I/x^2*ln(c)*Pi*b*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I/x^2*l
n(c)*Pi*b*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+1/4*I/x^2*Pi*a*csgn(I
*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/2*b/e*f*m*n*ln(x)-1/2*b/e*f
*m*n*ln(x)^2-1/2*b/e*f*m*n*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/8*Pi^2
*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^2*b*csgn(I*x^n)*csgn(I*c*x^n
)^2-1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b/x^2*ln(x^n)+1/8*
Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^2*b*csgn(I*x^n)*
csgn(I*c*x^n)*csgn(I*c)+a/e*f*m*ln(x)+1/8*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d
*(f*x^2+e)^m)^2/x^2*b*csgn(I*c*x^n)^2*csgn(I*c)+1/8*Pi^2*csgn(I*d*(f*x^2+e)
^m)^3/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+b/e*f*m*ln(c)*ln(x)+1/4*I*P
i*b/x^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(d)-1/2*f*m/e*ln(f*x^2+e)*b*l
n(c)-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^2*b*csg
n(I*x^n)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f
*x^2+e)^m)/x^2*b*csgn(I*c*x^n)^2*csgn(I*c)-1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x
^2+e)^m)^2/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/8*Pi^2*csgn(I*(f*x^2
+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/
4*I*Pi*b/x^2*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(d)-1/4*I*Pi*b/x^2*csgn(I*c)*csg
n(I*c*x^n)^2*ln(d)+1/4*I*f*m/e*ln(f*x^2+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*c
sgn(I*c)-1/8*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2
-1/8*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^2*b*csgn(I*c*x^n)^2*csgn(I*c)-1/4*b*f*m
*n*ln(f*x^2+e)/e+1/4*I/x^2*ln(c)*Pi*b*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*
d*(f*x^2+e)^m)+1/8*I/x^2*Pi*b*n*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x
^2+e)^m)+1/4*I*f*m/e*ln(f*x^2+e)*b*Pi*csgn(I*c*x^n)^3+1/4*I*Pi*csgn(I*d)*cs
gn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b/x^2*ln(x^n)+1/4*I/x^2*Pi*a*csgn(I
*d*(f*x^2+e)^m)^3+1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)
^m)/x^2*b*csgn(I*c*x^n)^3+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^2*b*
csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^2*
b*csgn(I*c*x^n)^2*csgn(I*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,bm\log\left(x^{n}\right)+\left(mn+2\,m\log(c)\right)b+2\,am\right)\log\left(fx^{2}+e\right)}{4\,x^{2}}+\int\frac{2\,be\log(c)\log(d)+\left(2\left(fm+f\log(d)\right)a+\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*m*log(x^n) + (m*n + 2*m*log(c))*b + 2*a*m)*log(f*x^2 + e)/x^2 + i
ntegrate(1/2*(2*b*e*log(c)*log(d) + (2*(f*m + f*log(d))*a + (f*m*n + 2*(f*m
 + f*log(d))*log(c))*b)*x^2 + 2*a*e*log(d) + 2*((f*m + f*log(d))*b*x^2 + b*
e*log(d))*log(x^n))/(f*x^5 + e*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**3,x)

[Out] Timed out
```

3.94 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^5}\,dx$

Optimal. Leaf size=248

$$-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{4x^4}-\frac{f^2m\log(x)\left(a+b\log(cx^n)\right)}{2e^2}+\frac{f^2m\log\left(e+fx^2\right)\left(a+b\log(cx^n)\right)}{4e^2}-\frac{fm}{}$$

```
[Out] -3/16*b*f*m*n/e/x^2-1/8*b*f^2*m*n*ln(x)/e^2+1/4*b*f^2*m*n*ln(x)^2/e^2-1/4*f
*m*(a+b*ln(c*x^n))/e/x^2-1/2*f^2*m*ln(x)*(a+b*ln(c*x^n))/e^2+1/16*b*f^2*m*n
*ln(f*x^2+e)/e^2-1/8*b*f^2*m*n*ln(-f*x^2/e)*ln(f*x^2+e)/e^2+1/4*f^2*m*(a+b*
ln(c*x^n))*ln(f*x^2+e)/e^2-1/16*b*n*ln(d*(f*x^2+e)^m)/x^4-1/4*(a+b*ln(c*x^n
))*ln(d*(f*x^2+e)^m)/x^4-1/8*b*f^2*m*n*polylog(2,1+f*x^2/e)/e^2
```

Rubi [A] time = 0.23, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2454, 2395, 44, 2376, 2301, 2394, 2315}

$$-\frac{bf^2mn\text{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{8e^2}-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{4x^4}-\frac{f^2m\log(x)\left(a+b\log(cx^n)\right)}{2e^2}+\frac{f^2m\log(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^5,x]
```

```
[Out] (-3*b*f*m*n)/(16*e*x^2) - (b*f^2*m*n*Log[x])/(8*e^2) + (b*f^2*m*n*Log[x]^2)
/(4*e^2) - (f*m*(a + b*Log[c*x^n]))/(4*e*x^2) - (f^2*m*Log[x]*(a + b*Log[c*
x^n]))/(2*e^2) + (b*f^2*m*n*Log[e + f*x^2])/(16*e^2) - (b*f^2*m*n*Log[-((f*
x^2)/e)]*Log[e + f*x^2])/(8*e^2) + (f^2*m*(a + b*Log[c*x^n])*Log[e + f*x^2]
)/(4*e^2) - (b*n*Log[d*(e + f*x^2)^m])/(16*x^4) - ((a + b*Log[c*x^n])*Log[d
*(e + f*x^2)^m])/(4*x^4) - (b*f^2*m*n*PolyLog[2, 1 + (f*x^2)/e])/(8*e^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{x^5}\,dx = -\frac{fm\left(a+b\log\left(cx^n\right)\right)}{4ex^2}-\frac{f^2m\log(x)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{f^2m\left(a+b\right.}{}$$

$$=-\frac{bfmn}{8ex^2}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{4ex^2}-\frac{f^2m\log(x)\left(a+b\log\left(cx^n\right)\right)}{2e^2}+\frac{f^2}{}$$

$$=-\frac{bfmn}{8ex^2}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{4ex^2}-\frac{f^2m\log(x)\left(a+\right.}{2e^2}$$

$$=-\frac{bfmn}{8ex^2}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{4ex^2}-\frac{f^2m\log(x)\left(a+\right.}{2e^2}$$

$$=-\frac{bfmn}{8ex^2}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{4ex^2}-\frac{f^2m\log(x)\left(a+\right.}{2e^2}$$

$$=-\frac{3bfmn}{16ex^2}-\frac{bf^2mn\log(x)}{8e^2}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{fm\left(a+b\log\left(cx^n\right)\right)}{4ex^2}$$

Mathematica [C] time = 0.14, size = 363, normalized size = 1.46

$$-\frac{4ae^2\log\left(d\left(e+fx^2\right)^m\right)-4af^2mx^4\log\left(e+fx^2\right)+4aefmx^2+8af^2mx^4\log(x)+4be^2\log\left(cx^n\right)\log\left(d\left(e+fx^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^5,x]
```

```
[Out] -1/16*(4*a*e*f*m*x^2 + 3*b*e*f*m*n*x^2 + 8*a*f^2*m*x^4*Log[x] + 2*b*f^2*m*n
*x^4*Log[x] - 4*b*f^2*m*n*x^4*Log[x]^2 + 4*b*e*f*m*x^2*Log[c*x^n] + 8*b*f^2
*m*x^4*Log[x]*Log[c*x^n] - 4*b*f^2*m*n*x^4*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqr
```

```
t[e]] - 4*b*f^2*m*n*x^4*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 4*a*f^2*m*x
^4*Log[e + f*x^2] - b*f^2*m*n*x^4*Log[e + f*x^2] + 4*b*f^2*m*n*x^4*Log[x]*L
og[e + f*x^2] - 4*b*f^2*m*x^4*Log[c*x^n]*Log[e + f*x^2] + 4*a*e^2*Log[d*(e
+ f*x^2)^m] + b*e^2*n*Log[d*(e + f*x^2)^m] + 4*b*e^2*Log[c*x^n]*Log[d*(e +
f*x^2)^m] - 4*b*f^2*m*n*x^4*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 4*b*f^2*
m*n*x^4*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(e^2*x^4)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^5, x)
```

maple [C] time = 0.73, size = 2313, normalized size = 9.33

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m)/x^5,x)
```

```
[Out] 1/8*I/e*f*m/x^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*b/e^2*f^2*m*ln
(x)*ln(x^n)-1/16*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csg
n(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*cs
gn(I*d*(f*x^2+e)^m)/x^4*b*csgn(I*c*x^n)^2*csgn(I*c)-1/16*Pi^2*csgn(I*d)*csg
n(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/8*I/x^4*Pi
*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*ln(d)*b/x^4*ln(x^n)-1/2*a/
e^2*f^2*m*ln(x)-1/4*a/e*f*m/x^2-1/4/x^4*ln(d)*a-1/4/x^4*ln(c)*ln(d)*b-1/16/
x^4*ln(d)*b*n+1/32*I/x^4*Pi*b*n*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x
^2+e)^m)-1/4*b/e*f*m/x^2*ln(x^n)+1/16*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*
x^2+e)^m)^2/x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/16*Pi^2*csgn(I*(f*x^2+e)^m)
*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*c*x^n)^2*csgn(I*c)+1/16*Pi^2*csgn(I*d
*(f*x^2+e)^m)^3/x^4*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3/16*b/e*f*m*n/x^
2+(-1/4*b/x^4*ln(x^n)-1/16*(2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-2*I*Pi*b*c
sgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-2*I*Pi*b*csgn(I*c*x^n)^3+2*I*Pi*b*csgn(I
*c)*csgn(I*c*x^n)^2+4*b*ln(c)+b*n+4*a)/x^4)*ln((f*x^2+e)^m)+1/4*b/e^2*f^2*m
*n*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/4*b/e^2*f^2*m*n*dilog((f*x+(-e
*f)^(1/2))/(-e*f)^(1/2))+1/8*I/e^2*f^2*m*ln(f*x^2+e)*b*Pi*csgn(I*c*x^n)^2*c
sgn(I*c)+1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^4*
b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*b/e*f*m/x^2*ln(c)-1/8*I/x^4*Pi*ln
(d)*b*csgn(I*c*x^n)^2*csgn(I*c)-1/8*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^
2+e)^m)^2*b/x^4*ln(x^n)-1/8*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b/x^4*ln
(x^n)-1/32*I/x^4*Pi*b*n*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/8*b/e^2*f^2*m*n
*ln(x)+1/4*b/e^2*f^2*m*n*ln(x)^2-1/4*I*Pi*b/e^2*f^2*m*csgn(I*x^n)*csgn(I*c*
```

```
x^n)^2*ln(x)+1/4*m*f^2*b*ln(x^n)/e^2*ln(f*x^2+e)+1/16*Pi^2*csgn(I*d)*csgn(I
*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/16*Pi^2*csgn(I*d)*csg
n(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*c*x^n)^2*csgn(I*c)+1/4/e^2*f^2*m*ln(f*x^2
+e)*a+1/8*I/x^4*Pi*a*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)-1/
8*I/x^4*ln(c)*Pi*b*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/8*I/x^4*ln(c)*Pi*b*c
sgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+1/16*Pi^2*csgn(I*d*(f*x^2+e)^m)^
3/x^4*b*csgn(I*c*x^n)^3+1/8*I/x^4*Pi*a*csgn(I*d*(f*x^2+e)^m)^3-1/4*I*Pi*b/e
^2*f^2*m*csgn(I*c)*csgn(I*c*x^n)^2*ln(x)-1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+
e)^m)*csgn(I*d*(f*x^2+e)^m)/x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*b/e^2*f^2
*m*n*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/4*b/e^2*f^2*m*n*ln(x)*ln(
(f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/4*b/e^2*f^2*m*n*ln(x)*ln(f*x^2+e)-1/2*b/
e^2*f^2*m*ln(c)*ln(x)+1/4*I*Pi*b/e^2*f^2*m*csgn(I*c*x^n)^3*ln(x)-1/8*I/e^2*
f^2*m*ln(f*x^2+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/4*I*Pi*b/e^2*f
^2*m*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(x)-1/32*I/x^4*Pi*b*n*csgn(I*(f*
x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*c
sgn(I*d*(f*x^2+e)^m)/x^4*b*csgn(I*c*x^n)^3+1/8*I/e*f*m/x^2*b*Pi*csgn(I*c*x^
n)^3-1/8*I/e^2*f^2*m*ln(f*x^2+e)*b*Pi*csgn(I*c*x^n)^3+1/4/e^2*f^2*m*ln(f*x^
2+e)*b*ln(c)-1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*c*x^n
)^3-1/16*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^4*b*csgn(I*c*x^n)^2*csgn(I*c)+1/8*I
/x^4*ln(c)*Pi*b*csgn(I*d*(f*x^2+e)^m)^3+1/8*I/x^4*Pi*ln(d)*b*csgn(I*c*x^n)^
3-1/8*I/e*f*m/x^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I/e*f*m/x^2*b*Pi*csg
n(I*c*x^n)^2*csgn(I*c)+1/8*I/e^2*f^2*m*ln(f*x^2+e)*b*Pi*csgn(I*x^n)*csgn(I*
c*x^n)^2+1/32*I/x^4*Pi*b*n*csgn(I*d*(f*x^2+e)^m)^3-1/8*I/x^4*Pi*a*csgn(I*d)
*csgn(I*d*(f*x^2+e)^m)^2-1/8*I/x^4*Pi*a*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2
+e)^m)^2+1/8*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b/x^4*ln(x^n)-1/16*Pi^2*csgn(I*(f
*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*c*x^n)^3-1/16*Pi^2*csgn(I*d
*(f*x^2+e)^m)^3/x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I/x^4*Pi*ln(d)*b*csgn
(I*x^n)*csgn(I*c*x^n)^2+1/8*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*
x^2+e)^m)*b/x^4*ln(x^n)+1/8*I/x^4*ln(c)*Pi*b*csgn(I*d)*csgn(I*(f*x^2+e)^m)*
csgn(I*d*(f*x^2+e)^m)+1/16*b*f^2*m*n*ln(f*x^2+e)/e^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(4\,bm\log\left(x^{n}\right)+\left(mn+4\,m\log(c)\right)b+4\,am\right)\log\left(fx^{2}+e\right)}{16\,x^{4}}+\int\frac{8\,be\log(c)\log(d)+\left(4\left(fm+2\,f\log(d)\right)a+\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*(4*b*m*log(x^n) + (m*n + 4*m*log(c))*b + 4*a*m)*log(f*x^2 + e)/x^4 +
integrate(1/8*(8*b*e*log(c)*log(d) + (4*(f*m + 2*f*log(d))*a + (f*m*n + 4*(
f*m + 2*f*log(d))*log(c))*b)*x^2 + 8*a*e*log(d) + 4*((f*m + 2*f*log(d))*b*x
^2 + 2*b*e*log(d))*log(x^n))/(f*x^7 + e*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^5,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**5,x)

[Out] Timed out
```

3.95 $\int x^2 \left(a + b\log\left(cx^n\right)\right)\log\left(d\left(e + fx^2\right)^m\right)\,dx$

Optimal. Leaf size=251

$$\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)-\frac{2e^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{3f^{3/2}}+\frac{2emx\left(a+b\log\left(cx^n\right)\right)}{3f}-\frac{2}{9}mx^3\left(a\right.$$

```
[Out] -8/9*b*e*m*n*x/f+4/27*b*m*n*x^3+2/9*b*e^(3/2)*m*n*arctan(x*f^(1/2)/e^(1/2))
/f^(3/2)+2/3*e*m*x*(a+b*ln(c*x^n))/f-2/9*m*x^3*(a+b*ln(c*x^n))-2/3*e^(3/2)*
m*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))/f^(3/2)-1/9*b*n*x^3*ln(d*(f*x^2
+e)^m)+1/3*x^3*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)+1/3*I*b*e^(3/2)*m*n*polylo
g(2,-I*x*f^(1/2)/e^(1/2))/f^(3/2)-1/3*I*b*e^(3/2)*m*n*polylog(2,I*x*f^(1/2)
/e^(1/2))/f^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2455, 302, 205, 2376, 4848, 2391}

$$\frac{ibe^{3/2}mn\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3f^{3/2}}-\frac{ibe^{3/2}mn\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3f^{3/2}}+\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)-\frac{2e^{3/2}m\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-8*b*e*m*n*x)/(9*f) + (4*b*m*n*x^3)/27 + (2*b*e^(3/2)*m*n*ArcTan[(Sqrt[f]*
x)/Sqrt[e]])/(9*f^(3/2)) + (2*e*m*x*(a + b*Log[c*x^n]))/(3*f) - (2*m*x^3*(a
 + b*Log[c*x^n]))/9 - (2*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c
*x^n]))/(3*f^(3/2)) - (b*n*x^3*Log[d*(e + f*x^2)^m])/9 + (x^3*(a + b*Log[c*
x^n])*Log[d*(e + f*x^2)^m])/3 + ((I/3)*b*e^(3/2)*m*n*PolyLog[2, ((-I)*Sqrt[
f]*x)/Sqrt[e]])/f^(3/2) - ((I/3)*b*e^(3/2)*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqr
t[e]])/f^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)dx = \frac{2emx\left(a+b\log\left(cx^n\right)\right)}{3f} - \frac{2}{9}mx^3\left(a+b\log\left(cx^n\right)\right) - \frac{2e^{3/2}m\tan^{-1}}{}$$

$$= -\frac{2bemnx}{3f} + \frac{2}{27}bmnx^3 + \frac{2emx\left(a+b\log\left(cx^n\right)\right)}{3f} - \frac{2}{9}mx^3\left(a+b\right.$$

$$= -\frac{2bemnx}{3f} + \frac{2}{27}bmnx^3 + \frac{2emx\left(a+b\log\left(cx^n\right)\right)}{3f} - \frac{2}{9}mx^3\left(a+b\right.$$

$$= -\frac{2bemnx}{3f} + \frac{2}{27}bmnx^3 + \frac{2emx\left(a+b\log\left(cx^n\right)\right)}{3f} - \frac{2}{9}mx^3\left(a+b\right.$$

$$= -\frac{8bemnx}{9f} + \frac{4}{27}bmnx^3 + \frac{2emx\left(a+b\log\left(cx^n\right)\right)}{3f} - \frac{2}{9}mx^3\left(a+b\right.$$

$$= -\frac{8bemnx}{9f} + \frac{4}{27}bmnx^3 + \frac{2be^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{9f^{3/2}} + \frac{2emx\left(a+b\right.}{3}$$

Mathematica [A] time = 0.14, size = 389, normalized size = 1.55

$$\frac{9af^{3/2}x^3\log\left(d\left(e+fx^2\right)^m\right) - 18ae^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right) + 18ae\sqrt{f}mx - 6af^{3/2}mx^3 + 9bf^{3/2}x^3\log\left(cx^n\right)\log\left(d\left(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (18*a*e*Sqrt[f]*m*x - 24*b*e*Sqrt[f]*m*n*x - 6*a*f^(3/2)*m*x^3 + 4*b*f^(3/2
)*m*n*x^3 - 18*a*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 6*b*e^(3/2)*m*n*Ar
cTan[(Sqrt[f]*x)/Sqrt[e]] + 18*b*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Lo
g[x] + 18*b*e*Sqrt[f]*m*x*Log[c*x^n] - 6*b*f^(3/2)*m*x^3*Log[c*x^n] - 18*b*
e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - (9*I)*b*e^(3/2)*m*n*Log[
x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (9*I)*b*e^(3/2)*m*n*Log[x]*Log[1 + (I*S
qrt[f]*x)/Sqrt[e]] + 9*a*f^(3/2)*x^3*Log[d*(e + f*x^2)^m] - 3*b*f^(3/2)*n*x
^3*Log[d*(e + f*x^2)^m] + 9*b*f^(3/2)*x^3*Log[c*x^n]*Log[d*(e + f*x^2)^m] +
 (9*I)*b*e^(3/2)*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b*e^(3/2)
*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(27*f^(3/2))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2\log\left(cx^n\right)+ax^2\right)\log\left(\left(fx^2+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)*log((f*x^2 + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^{n}\right)+a\right)x^{2}\log\left(\left(fx^{2}+e\right)^{m}d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*x^2 + e)^m*d), x)
```

maple [C] time = 0.57, size = 2321, normalized size = 9.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m),x)
```

```
[Out] -1/3*I*m/f*x*e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/9*b*n*x^3*ln(d)+1
/3*b*x^3*ln(c)*ln(d)-2/9*b*m*x^3*ln(c)+(1/3*b*x^3*ln(x^n)+1/18*(-3*I*Pi*b*c
sgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+3*I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+3*I*P
i*b*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*Pi*b*csgn(I*c*x^n)^3-2*b*n+6*b*ln(c)+6*
a)*x^3)*ln((f*x^2+e)^m)-1/12*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
^2*x^3*b*csgn(I*c*x^n)^2*csgn(I*c)-1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^3*b*
csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(
f*x^2+e)^m)^2*b*x^3*ln(x^n)+1/3*a*x^3*ln(d)-2/9*b*m*x^3*ln(x^n)+1/3*b*x^3*l
n(d)*ln(x^n)-1/6*I*Pi*b*x^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(d)-2/9*a
*m*x^3-1/6*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b*x^3*l
n(x^n)+1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*a*x^3-1/6*I*Pi*
csgn(I*d*(f*x^2+e)^m)^3*x^3*b*ln(c)-1/6*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b*x^3*
ln(x^n)+1/3*I*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*Pi*b*csgn(I*x^n)*
csgn(I*c*x^n)*csgn(I*c)+1/9*I*m*x^3*Pi*b*csgn(I*c*x^n)^3+1/6*I*Pi*csgn(I*d)
*csgn(I*d*(f*x^2+e)^m)^2*a*x^3-2/3*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/
2))*b*ln(c)+2/9*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*n-1/3*m/f*b*n
*e^2/(-e*f)^(1/2)*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/3*m/f*b*n*e^2/(
-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/18*I*Pi*csgn(I*d*(f*x^
2+e)^m)^3*b*x^3*n+1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I*
c*x^n)^3+1/12*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I
*c*x^n)^3-8/9*b*e/f*m*n*x-1/18*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^
m)^2*b*x^3*n-1/6*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*a
*x^3+1/6*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*ln(c)+1/6*I*Pi*csgn(I
*d)*csgn(I*d*(f*x^2+e)^m)^2*b*x^3*ln(x^n)-1/18*I*Pi*csgn(I*d)*csgn(I*d*(f*x
^2+e)^m)^2*b*x^3*n-1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I
*x^n)*csgn(I*c*x^n)^2-1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csg
n(I*c*x^n)^2*csgn(I*c)-1/12*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^
2*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/3*I*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e
*f)^(1/2))*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)-1/3*I*m/f*e^2/(e*f)^(1/2)*arctan(
x*f/(e*f)^(1/2))*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-2/3*m/f*e^2/(e*f)^(1/2)*a
rctan(x*f/(e*f)^(1/2))*a-1/6*I*Pi*b*x^3*csgn(I*c*x^n)^3*ln(d)-1/6*I*Pi*csgn
(I*d*(f*x^2+e)^m)^3*a*x^3-1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^3*b*csgn(I*c*
x^n)^3+1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*ln(c)+1/1
2*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^3*b*csgn(I*x^n
)*csgn(I*c*x^n)^2+1/12*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e
)^m)*x^3*b*csgn(I*c*x^n)^2*csgn(I*c)+1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)
^m)^2*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/12*Pi^2*csgn(I*(f*x^2+e)^
```

```
m)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+2/3*a*
e/f*m*x+1/9*I*m*x^3*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/3*I*m/f*x*e*
Pi*b*csgn(I*c*x^n)^3+1/18*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^
2+e)^m)*b*x^3*n+2/3*m/f*b*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*n*ln(x)-1
/3*m/f*b*n*e^2/(-e*f)^(1/2)*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/3*
m/f*b*n*e^2/(-e*f)^(1/2)*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/6*I*Pi
*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^3*b*ln(c)-2/3*m/f*b*
e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*ln(x^n)+1/3*I*m/f*x*e*Pi*b*csgn(I*c
*x^n)^2*csgn(I*c)+1/3*I*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*Pi*b*cs
gn(I*c*x^n)^3+1/3*I*m/f*x*e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*Pi^2*csgn
(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^3*b*csgn(I*x^n)*csgn(I*c*
x^n)*csgn(I*c)+1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^3*b*csgn(I*x^n)*csgn(I*c
*x^n)^2+1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^3*b*csgn(I*c*x^n)^2*csgn(I*c)+4
/27*b*m*n*x^3-1/9*I*m*x^3*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/9*I*m*x^3*Pi*b
*csgn(I*c*x^n)^2*csgn(I*c)+2/3*b*e/f*m*x*ln(c)+1/6*I*Pi*b*x^3*csgn(I*c)*csg
n(I*c*x^n)^2*ln(d)-1/12*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+
e)^m)*x^3*b*csgn(I*c*x^n)^3+1/6*I*Pi*b*x^3*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(d
)+2/3*b*e/f*m*x*ln(x^n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{9}\left(3\,bmx^3\log\left(x^n\right)-\left(\left(mn-3\,m\log(c)\right)b-3\,am\right)x^3\right)\log\left(fx^2+e\right)+\int-\frac{\left(3\left(2\,fm-3\,f\log(d)\right)a-\left(2\,fmn-3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/9*(3*b*m*x^3*log(x^n) - ((m*n - 3*m*log(c))*b - 3*a*m)*x^3)*log(f*x^2 + e
) + integrate(-1/9*((3*(2*f*m - 3*f*log(d))*a - (2*f*m*n - 3*(2*f*m - 3*f*l
og(d))*log(c))*b)*x^4 - 9*(b*e*log(c)*log(d) + a*e*log(d))*x^2 + 3*((2*f*m
- 3*f*log(d))*b*x^4 - 3*b*e*x^2*log(d))*log(x^n))/(f*x^2 + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

3.96 $\int \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)\,dx$

Optimal. Leaf size=194

$$x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)+\frac{2\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{f}}-2mx\left(a+b\log\left(cx^n\right)\right)-bnx\log\left(d\left(e\right.\right.$$

```
[Out] 4*b*m*n*x-2*m*x*(a+b*ln(c*x^n))-b*n*x*ln(d*(f*x^2+e)^m)+x*(a+b*ln(c*x^n))*l
n(d*(f*x^2+e)^m)-2*b*m*n*arctan(x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)+2*m*arct
an(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))*e^(1/2)/f^(1/2)-I*b*m*n*polylog(2,-I*
x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)+I*b*m*n*polylog(2,I*x*f^(1/2)/e^(1/2))*e
^(1/2)/f^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2448, 321, 205, 2370, 4848, 2391}

$$-\frac{ib\sqrt{e}\,mn\text{PolyLog}\left(2,-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}+\frac{ib\sqrt{e}\,mn\text{PolyLog}\left(2,\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}+x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)+\frac{2\sqrt{e}\,m\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] 4*b*m*n*x - (2*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] - 2*m*x*(
a + b*Log[c*x^n]) + (2*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x
^n]))/Sqrt[f] - b*n*x*Log[d*(e + f*x^2)^m] + x*(a + b*Log[c*x^n])*Log[d*(e
+ f*x^2)^m] - (I*b*Sqrt[e]*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f
] + (I*b*Sqrt[e]*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)dx = -2mx\left(a+b\log\left(cx^n\right)\right)+\frac{2\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{f}}+$$

$$= 2bmnx-2mx\left(a+b\log\left(cx^n\right)\right)+\frac{2\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(\right.\right.}{\sqrt{f}}$$

$$= 2bmnx-2mx\left(a+b\log\left(cx^n\right)\right)+\frac{2\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(\right.\right.}{\sqrt{f}}$$

$$= 4bmnx-2mx\left(a+b\log\left(cx^n\right)\right)+\frac{2\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(\right.\right.}{\sqrt{f}}$$

$$= 4bmnx-\frac{2b\sqrt{e}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}-2mx\left(a+b\log\left(cx^n\right)\right)+\frac{2\sqrt{e}\,m}{}$$

Mathematica [A] time = 0.09, size = 332, normalized size = 1.71

$$\frac{a\sqrt{f}\,x\log\left(d\left(e+fx^2\right)^m\right)+2a\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)-2a\sqrt{f}\,mx+b\sqrt{f}\,x\log\left(cx^n\right)\log\left(d\left(e+fx^2\right)^m\right)+2b\sqrt{e}\,m\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-2*a*Sqrt[f]*m*x + 4*b*Sqrt[f]*m*n*x + 2*a*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sq
rt[e]] - 2*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 2*b*Sqrt[e]*m*n*ArcT
an[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 2*b*Sqrt[f]*m*x*Log[c*x^n] + 2*b*Sqrt[e]*m
*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + I*b*Sqrt[e]*m*n*Log[x]*Log[1 - (I
*Sqrt[f]*x)/Sqrt[e]] - I*b*Sqrt[e]*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]
] + a*Sqrt[f]*x*Log[d*(e + f*x^2)^m] - b*Sqrt[f]*n*x*Log[d*(e + f*x^2)^m] +
 b*Sqrt[f]*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] - I*b*Sqrt[e]*m*n*PolyLog[2, (
(-I)*Sqrt[f]*x)/Sqrt[e]] + I*b*Sqrt[e]*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]
])/Sqrt[f]
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^2+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^n\right) + a\right) \log\left(\left(fx^2 + e\right)^m d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d), x)
```

maple [C] time = 0.50, size = 2001, normalized size = 10.31

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m),x)
```

```
[Out] -2*m*b*ln(x^n)*x+ln(d)*ln(x^n)*x*b-I*m*e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f
*x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+(b*x*ln(x^n)+1/2*(I*Pi*b*csgn(
I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*Pi*b*cs
gn(I*c*x^n)^3+I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+2*b*ln(c)-2*b*n+2*a)*x)*ln((
f*x^2+e)^m)-2*b*m*x*ln(c)+b*x*ln(c)*ln(d)-b*n*x*ln(d)-1/4*Pi^2*x*b*csgn(I*d
)*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*d*(f*x^2+e)^m)^2-1/4*Pi^2*x*b*csgn(I*d
)*csgn(I*c*x^n)^2*csgn(I*d*(f*x^2+e)^m)^2*csgn(I*c)-1/4*Pi^2*x*b*csgn(I*x^n
)*csgn(I*c*x^n)^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+m*b*n*e/(-e*f
)^(1/2)*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-m*b*n*e/(-e*f)^(1/2)*ln(
x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+4*b*m*n*x-2*a*m*x+a*x*ln(d)+2*m*e/(e
*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*ln(c)+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*x
^n)*csgn(I*c*x^n)*csgn(I*d*(f*x^2+e)^m)^2*csgn(I*c)+1/4*Pi^2*x*b*csgn(I*x^n
)*csgn(I*c*x^n)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*csgn(I*c)+I*m*x
*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*ln(c)*Pi*x*b*csgn(I*d*(f*x^
2+e)^m)^3+1/2*I*Pi*x*b*n*csgn(I*d*(f*x^2+e)^m)^3-I*m*x*b*Pi*csgn(I*x^n)*csg
n(I*c*x^n)^2-I*m*x*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)-1/4*Pi^2*x*b*csgn(I*d)*cs
gn(I*c*x^n)^3*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+2*a*m*e/(e*f)^(1/2)
*arctan(1/(e*f)^(1/2)*f*x)-2*m*b*e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*n*
ln(x)-1/4*Pi^2*x*b*csgn(I*c*x^n)^3*csgn(I*d*(f*x^2+e)^m)^3-1/2*I*Pi*x*a*csg
n(I*d*(f*x^2+e)^m)^3-1/2*I*ln(x^n)*Pi*x*b*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csg
n(I*d*(f*x^2+e)^m)-1/2*I*ln(c)*Pi*x*b*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*
d*(f*x^2+e)^m)+1/2*I*Pi*x*b*n*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2
+e)^m)-I*m*e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I*c*x^n)^3+1/4
*Pi^2*x*b*csgn(I*d)*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*(f*x^2+e)^m)*csgn(I*
d*(f*x^2+e)^m)+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*c*x^n)^2*csgn(I*(f*x^2+e)^m)*c
sgn(I*d*(f*x^2+e)^m)*csgn(I*c)-1/2*I*Pi*ln(d)*b*x*csgn(I*x^n)*csgn(I*c*x^n)
*csgn(I*c)-1/4*Pi^2*x*b*csgn(I*d)*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*(f*x^2+e
)^m)*csgn(I*d*(f*x^2+e)^m)*csgn(I*c)+I*m*e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)
*f*x)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+I*m*e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)
*f*x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*Pi*ln(d)*b*x*csgn(I*x^n)*csgn(
I*c*x^n)^2+1/2*I*ln(c)*Pi*x*b*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+1
/2*I*ln(x^n)*Pi*x*b*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/2*I*Pi*ln
(d)*b*x*csgn(I*c*x^n)^3+1/4*Pi^2*x*b*csgn(I*c*x^n)^3*csgn(I*(f*x^2+e)^m)*cs
gn(I*d*(f*x^2+e)^m)^2+1/4*Pi^2*x*b*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*d*(f*
x^2+e)^m)^3+I*m*x*b*Pi*csgn(I*c*x^n)^3+1/2*I*Pi*x*a*csgn(I*d)*csgn(I*d*(f*x
^2+e)^m)^2-1/4*Pi^2*x*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*d*(f*x^2+e)^m)^3*c
sgn(I*c)-1/4*Pi^2*x*b*csgn(I*c*x^n)^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e
)^m)^2*csgn(I*c)-1/2*I*Pi*x*b*n*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/2*I*Pi*
x*b*n*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/2*I*Pi*x*a*csgn(I*d)*cs
```

```
gn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/2*I*ln(x^n)*Pi*x*b*csgn(I*d)*csgn
(I*d*(f*x^2+e)^m)^2-2*m*e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*n+m*b*n*e
/(-e*f)^(1/2)*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-m*b*n*e/(-e*f)^(1/2)*
dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+2*m*b*e/(e*f)^(1/2)*arctan(1/(e*f)^(
1/2)*f*x)*ln(x^n)+1/2*I*ln(c)*Pi*x*b*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2+1/2*
I*Pi*x*a*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+1/4*Pi^2*x*b*csgn(I*c*
x^n)^2*csgn(I*d*(f*x^2+e)^m)^3*csgn(I*c)+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*c*x^
n)^3*csgn(I*d*(f*x^2+e)^m)^2-1/2*I*ln(x^n)*Pi*x*b*csgn(I*d*(f*x^2+e)^m)^3+1
/2*I*Pi*ln(d)*b*x*csgn(I*c*x^n)^2*csgn(I*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(bmx\log\left(x^{n}\right)-\left(\left(mn-m\log(c)\right)b-am\right)x\right)\log\left(fx^{2}+e\right)+\int\frac{be\log(c)\log(d)-\left(\left(2fm-f\log(d)\right)a-\left(2fm\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] (b*m*x*log(x^n) - ((m*n - m*log(c))*b - a*m)*x)*log(f*x^2 + e) + integrate(
(b*e*log(c)*log(d) - ((2*f*m - f*log(d))*a - (2*f*m*n - (2*f*m - f*log(d))*
log(c))*b)*x^2 + a*e*log(d) - ((2*f*m - f*log(d))*b*x^2 - b*e*log(d))*log(x
^n))/(f*x^2 + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

3.97 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx$

Optimal. Leaf size=179

$$-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x}+\frac{2\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}}-\frac{bn\log\left(d\left(e+fx^2\right)^m\right)}{x}-\frac{ib\sqrt{f}\,mn\mathrm{Li}}{\sqrt{e}}$$

```
[Out] -b*n*ln(d*(f*x^2+e)^m)/x-(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x+2*b*m*n*arctan
(x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1/2)+2*m*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c
*x^n))*f^(1/2)/e^(1/2)-I*b*m*n*polylog(2,-I*x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1
/2)+I*b*m*n*polylog(2,I*x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2455, 205, 2376, 4848, 2391}

$$-\frac{ib\sqrt{f}\,mn\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{ib\sqrt{f}\,mn\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}}-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x}+2\sqrt{f}\,m\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (2*b*Sqrt[f]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + (2*Sqrt[f]*m*ArcTan
[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/Sqrt[e] - (b*n*Log[d*(e + f*x^2)^
m])/x - ((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x - (I*b*Sqrt[f]*m*n*Poly
Log[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + (I*b*Sqrt[f]*m*n*PolyLog[2, (I*
Sqrt[f]*x)/Sqrt[e]])/Sqrt[e]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx = \frac{2\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}} - \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e\right.\right.}{x}$$

$$= \frac{2\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}} - \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e\right.\right.}{x}$$

$$= \frac{2\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}} - \frac{bn\log\left(d\left(e+fx^2\right)^m\right)}{x} - $$

$$= \frac{2b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}} - $$

Mathematica [A] time = 0.09, size = 305, normalized size = 1.70

$$\frac{-a\sqrt{e}\,\log\left(d\left(e+fx^2\right)^m\right)+2a\sqrt{f}\,mx\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)-b\sqrt{e}\,\log\left(cx^n\right)\log\left(d\left(e+fx^2\right)^m\right)+2b\sqrt{f}\,mx\log\left(cx^n\right)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (2*a*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 2*b*Sqrt[f]*m*n*x*ArcTan[(Sq
rt[f]*x)/Sqrt[e]] - 2*b*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] +
2*b*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + I*b*Sqrt[f]*m*n*x*
Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - I*b*Sqrt[f]*m*n*x*Log[x]*Log[1 + (I
*Sqrt[f]*x)/Sqrt[e]] - a*Sqrt[e]*Log[d*(e + f*x^2)^m] - b*Sqrt[e]*n*Log[d*(
e + f*x^2)^m] - b*Sqrt[e]*Log[c*x^n]*Log[d*(e + f*x^2)^m] - I*b*Sqrt[f]*m*n
*x*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + I*b*Sqrt[f]*m*n*x*PolyLog[2, (I*S
qrt[f]*x)/Sqrt[e]])/(Sqrt[e]*x)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^2, x)
```

maple [C] time = 0.55, size = 1972, normalized size = 11.02

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m)/x^2,x)
```

```
[Out] -a/x*ln(d)-I*m*f/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I*x^n)*csg
n(I*c*x^n)*csgn(I*c)-ln(d)*b/x*ln(x^n)+(-b/x*ln(x^n)-1/2*(I*Pi*b*csgn(I*x^n
)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*Pi*b*csgn(I*
c*x^n)^3+I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+2*b*ln(c)+2*b*n+2*a)/x)*ln((f*x^2
+e)^m)-1/2*I/x*Pi*ln(d)*b*csgn(I*c*x^n)^2*csgn(I*c)+1/2*I*Pi*csgn(I*d)*csgn
(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*a/x-b/x*ln(c)*ln(d)-b*n/x*ln(d)+1/2*I
*Pi*csgn(I*d*(f*x^2+e)^m)^3*b/x*ln(x^n)-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+
e)^m)^2/x*b*csgn(I*c*x^n)^3-1/4*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)
^m)^2/x*b*csgn(I*c*x^n)^3+1/4*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m
)^2/x*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(
f*x^2+e)^m)^2/x*b*csgn(I*c*x^n)^2*csgn(I*c)-2*m*f*b/(e*f)^(1/2)*arctan(1/(e
*f)^(1/2)*f*x)*n*ln(x)+m*f*b*n/(-e*f)^(1/2)*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-
e*f)^(1/2))-m*f*b*n/(-e*f)^(1/2)*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-
1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x*b*ln(c)-1/2*I*Pi*csgn(I*d)*csg
n(I*d*(f*x^2+e)^m)^2*b/x*ln(x^n)+2*m*f/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x
)*a+1/4*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c
)+1/2*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b*n/x-1/4*Pi
^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x*b*csgn(I*x^n)*csgn
(I*c*x^n)^2+2*m*f*b/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*ln(x^n)+2*m*f/(e*
f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*n+m*f*b*n/(-e*f)^(1/2)*dilog((-f*x+(-e
*f)^(1/2))/(-e*f)^(1/2))-m*f*b*n/(-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-e*
f)^(1/2))+2*m*f/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*ln(c)-I*m*f/(e*f)^(
1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I*c*x^n)^3+1/2*I*Pi*csgn(I*d)*csgn
(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x*b*ln(c)-1/4*Pi^2*csgn(I*(f*x^2+e)^m
)*csgn(I*d*(f*x^2+e)^m)^2/x*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*Pi^2*
csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x*b*csgn(I*c*x^n)^2*csg
n(I*c)-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x*b*csgn(I*x^n)*csgn(I*c*
x^n)*csgn(I*c)+1/2*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
*b/x*ln(x^n)+1/2*I/x*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+I*m*f/(
e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+I*m*f
/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/2*I
*Pi*csgn(I*d*(f*x^2+e)^m)^3*a/x+1/4*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x*b*csgn(I
*c*x^n)^3+1/4*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x*b*
csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(
f*x^2+e)^m)^2*b/x*ln(x^n)-1/2*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m
)^2*b*n/x-1/4*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x*b*csgn(I*x^n)*csgn(I*c*x^n)^2-
1/4*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x*b*csgn(I*c*x^n)^2*csgn(I*c)+1/2*I*Pi*csg
n(I*d*(f*x^2+e)^m)^3*b*n/x+1/2*I/x*Pi*ln(d)*b*csgn(I*c*x^n)^3-1/2*I*Pi*csgn
(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b*n/x-1/2*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(
f*x^2+e)^m)^2/x*b*ln(c)-1/2*I/x*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*
Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x*b*csgn(I*c*x^n)^
3+1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x*b*csgn(I*x^n)*csgn(I*c*x^n)^
2+1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x*b*csgn(I*c*x^n)^2*csgn(I*c)-
1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*a/x-1/2*I*Pi*csgn(I*(f*x^2+e)^m)
*csgn(I*d*(f*x^2+e)^m)^2*a/x+1/2*I*Pi*csgn(I*d*(f*x^2+e)^m)^3/x*b*ln(c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(bm\log\left(x^{n}\right)+\left(mn+m\log(c)\right)b+am\right)\log\left(fx^{2}+e\right)}{x}+\int\frac{be\log(c)\log(d)+\left(\left(2\,fm+f\log(d)\right)a+\left(2\,fmn+\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -(b*m*log(x^n) + (m*n + m*log(c))*b + a*m)*log(f*x^2 + e)/x + integrate((b*
e*log(c)*log(d) + ((2*f*m + f*log(d))*a + (2*f*m*n + (2*f*m + f*log(d))*log
(c))*b)*x^2 + a*e*log(d) + ((2*f*m + f*log(d))*b*x^2 + b*e*log(d))*log(x^n)
)/(f*x^4 + e*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**2,x)
```

```
[Out] Timed out
```

3.98 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx$

Optimal. Leaf size=227

$$-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{3x^3}-\frac{2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{3e^{3/2}}-\frac{2fm(a+b\log(cx^n))}{3ex}-\frac{bn\log\left(d\left(e\right.\right.}{9x}$$

[Out] -8/9*b*f*m*n/e/x-2/9*b*f^(3/2)*m*n*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)-2/3*f*m*(a+b*ln(c*x^n))/e/x-2/3*f^(3/2)*m*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))/e^(3/2)-1/9*b*n*ln(d*(f*x^2+e)^m)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^3+1/3*I*b*f^(3/2)*m*n*polylog(2,-I*x*f^(1/2)/e^(1/2))/e^(3/2)-1/3*I*b*f^(3/2)*m*n*polylog(2,I*x*f^(1/2)/e^(1/2))/e^(3/2)

Rubi [A] time = 0.16, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2455, 325, 205, 2376, 4848, 2391}

$$\frac{ibf^{3/2}mn\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3e^{3/2}}-\frac{ibf^{3/2}mn\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{3e^{3/2}}-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{3x^3}-\frac{2f^{3/2}m\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^4,x]

[Out] (-8*b*f*m*n)/(9*e*x) - (2*b*f^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(9*e^(3/2)) - (2*f*m*(a + b*Log[c*x^n]))/(3*e*x) - (2*f^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(3*e^(3/2)) - (b*n*Log[d*(e + f*x^2)^m])/(9*x^3) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(3*x^3) + ((I/3)*b*f^(3/2)*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) - ((I/3)*b*f^(3/2)*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/e^(3/2)

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx = -\frac{2fm\left(a+b\log\left(cx^n\right)\right)}{3ex}-\frac{2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{3e^{3/2}}-$$

$$=-\frac{2bfmn}{3ex}-\frac{2fm\left(a+b\log\left(cx^n\right)\right)}{3ex}-\frac{2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\right.}{3e^{3/2}}$$

$$=-\frac{2bfmn}{3ex}-\frac{2fm\left(a+b\log\left(cx^n\right)\right)}{3ex}-\frac{2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\right.}{3e^{3/2}}$$

$$=-\frac{8bfmn}{9ex}-\frac{2fm\left(a+b\log\left(cx^n\right)\right)}{3ex}-\frac{2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\right.}{3e^{3/2}}$$

$$=-\frac{8bfmn}{9ex}-\frac{2bf^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{9e^{3/2}}-\frac{2fm\left(a+b\log\left(cx^n\right)\right)}{3ex}-\frac{2f}{}$$

Mathematica [C] time = 0.12, size = 362, normalized size = 1.59

$$\frac{-3ae^{3/2}\log\left(d\left(e+fx^2\right)^m\right)-6a\sqrt{e}fmx^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{fx^2}{e}\right)-3be^{3/2}\log\left(cx^n\right)\log\left(d\left(e+fx^2\right)^m\right)-6bf^{3/2}mx^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^4,x]
```

```
[Out] (-8*b*Sqrt[e]*f*m*n*x^2 - 2*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] -
 6*a*Sqrt[e]*f*m*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -((f*x^2)/e)] + 6*b*f^
(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 6*b*Sqrt[e]*f*m*x^2*Log[
c*x^n] - 6*b*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - (3*I)*b
*f^(3/2)*m*n*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b*f^(3/2)*m*
n*x^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 3*a*e^(3/2)*Log[d*(e + f*x^2)
^m] - b*e^(3/2)*n*Log[d*(e + f*x^2)^m] - 3*b*e^(3/2)*Log[c*x^n]*Log[d*(e +
f*x^2)^m] + (3*I)*b*f^(3/2)*m*n*x^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] -
(3*I)*b*f^(3/2)*m*n*x^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(9*e^(3/2)*x^3)
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^4,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right)+a\right) \log\left(\left(fx^{2}+e\right)^{m} d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^4, x)
```

maple [C] time = 0.59, size = 2204, normalized size = 9.71

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m)/x^4,x)
[Out] -1/3*I*m*f^2/e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I*c*x^n)^2*c
sgn(I*c)+(-1/3*b/x^3*ln(x^n)-1/18*(3*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I
*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-3*I*Pi*b*csgn(I*c*x^n)^3+3*I*Pi*b
*csgn(I*c)*csgn(I*c*x^n)^2+6*b*ln(c)+2*b*n+6*a)/x^3)*ln((f*x^2+e)^m)+1/3*I*
m*f/e/x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-2/3*b/e*f*m/x*ln(x^n)-1/3*
ln(d)*b/x^3*ln(x^n)-1/3*a/x^3*ln(d)+1/3*I*m*f^2/e/(e*f)^(1/2)*arctan(1/(e*f
)^(1/2)*f*x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/18*I*Pi*csgn(I*d)*c
sgn(I*d*(f*x^2+e)^m)^2*b*n/x^3-8/9*b/e*f*m*n/x-1/3*b/x^3*ln(c)*ln(d)-1/9*b*
n/x^3*ln(d)-1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*ln(c
)+1/6*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*a/x^3-1/6*I*
Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*ln(c)+1/12*Pi^2*csgn(I*d*(f*x^2+
e)^m)^3/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/6*I/x^3*Pi*ln(d)*b*csgn
(I*x^n)*csgn(I*c*x^n)^2-1/18*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
^2*b*n/x^3+1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*x^n)*cs
gn(I*c*x^n)^2+1/6*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b/x^3*ln(x^n)+1/18*I*Pi*csgn
(I*d*(f*x^2+e)^m)^3*b*n/x^3-1/6*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*a/x^
3-1/12*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*c*x^n)
^3-1/3*I*m*f^2/e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I*x^n)*csg
n(I*c*x^n)^2-2/9*m*f^2/e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*n-1/3*m*f^
2*b*n/e/(-e*f)^(1/2)*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/3*m*f^2*b*n/
e/(-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-2/3*m*f^2/e/(e*f)^(1/
2)*arctan(1/(e*f)^(1/2)*f*x)*b*ln(c)+1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)
^m)^2/x^3*b*csgn(I*c*x^n)^2*csgn(I*c)+1/12*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*
d*(f*x^2+e)^m)^2/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/12*Pi^2*csgn(I*(f*x^2+
e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*c*x^n)^2*csgn(I*c)+1/6*I/x^3*Pi*
ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/3*I*m*f/e/x*b*Pi*csgn(I*c*x^n
)^2*csgn(I*c)-1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b/x^3*ln
(x^n)+2/3*m*f^2*b/e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*n*ln(x)-1/3*m*f^2
*b*n/e/(-e*f)^(1/2)*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/3*m*f^2*b*
n/e/(-e*f)^(1/2)*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/6*I*Pi*csgn(I*
d*(f*x^2+e)^m)^3*a/x^3-1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*cs
gn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/12*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f
*x^2+e)^m)^2/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/3*I*m*f/e/x*b*Pi*c
sgn(I*c*x^n)^3+1/6*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
/x^3*b*ln(c)+1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^3*b*csgn(I*c*x^n)^3+1/18*I
*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b*n/x^3+1/6*I*Pi*cs
gn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b/x^3*ln(x^n)-1/12*Pi^2*c
sgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^3*b*csgn(I*x^n)*csgn(I
```

```
*c*x^n)^2-1/12*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^3
*b*csgn(I*c*x^n)^2*csgn(I*c)+1/12*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I
*d*(f*x^2+e)^m)/x^3*b*csgn(I*c*x^n)^3-1/6*I/x^3*Pi*ln(d)*b*csgn(I*c*x^n)^2*
csgn(I*c)-2/3*m*f^2*b/e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*ln(x^n)-2/3*a
/e*f*m/x-2/3*b/e*f*m/x*ln(c)-1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e
)^m)^2*a/x^3+1/6*I*Pi*csgn(I*d*(f*x^2+e)^m)^3/x^3*b*ln(c)-1/6*I*Pi*csgn(I*d
)*csgn(I*d*(f*x^2+e)^m)^2*b/x^3*ln(x^n)-1/3*I*m*f/e/x*b*Pi*csgn(I*x^n)*csgn
(I*c*x^n)^2+1/3*I*m*f^2/e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I
*c*x^n)^3+1/6*I/x^3*Pi*ln(d)*b*csgn(I*c*x^n)^3+1/12*Pi^2*csgn(I*d)*csgn(I*(
f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)
-2/3*m*f^2/e/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*a-1/12*Pi^2*csgn(I*d)*cs
gn(I*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*c*x^n)^3-1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)
^3/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^3*
b*csgn(I*c*x^n)^2*csgn(I*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3\,bm\log\left(x^{n}\right)+\left(mn+3\,m\log(c)\right)b+3\,am\right)\log\left(fx^{2}+e\right)}{9\,x^{3}}+\int\frac{9\,be\log(c)\log(d)+\left(3\left(2\,fm+3\,f\log(d)\right)a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*(3*b*m*log(x^n) + (m*n + 3*m*log(c))*b + 3*a*m)*log(f*x^2 + e)/x^3 + i
ntegrate(1/9*(9*b*e*log(c)*log(d) + (3*(2*f*m + 3*f*log(d))*a + (2*f*m*n +
3*(2*f*m + 3*f*log(d))*log(c))*b)*x^2 + 9*a*e*log(d) + 3*((2*f*m + 3*f*log(
d))*b*x^2 + 3*b*e*log(d))*log(x^n))/(f*x^6 + e*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**4,x)
```

```
[Out] Timed out
```

3.99 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^6}\,dx$

Optimal. Leaf size=267

$$-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{5x^5}+\frac{2f^{5/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(a+b\log(cx^n))}{5e^{5/2}}+\frac{2f^2m(a+b\log(cx^n))}{5e^2x}-\frac{2fm(a+\cdots}{15\cdots}$$

```
[Out] -16/225*b*f*m*n/e/x^3+12/25*b*f^2*m*n/e^2/x+2/25*b*f^(5/2)*m*n*arctan(x*f^(
1/2)/e^(1/2))/e^(5/2)-2/15*f*m*(a+b*ln(c*x^n))/e/x^3+2/5*f^2*m*(a+b*ln(c*x^
n))/e^2/x+2/5*f^(5/2)*m*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))/e^(5/2)-1
/25*b*n*ln(d*(f*x^2+e)^m)/x^5-1/5*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^5-1/5
*I*b*f^(5/2)*m*n*polylog(2,-I*x*f^(1/2)/e^(1/2))/e^(5/2)+1/5*I*b*f^(5/2)*m*
n*polylog(2,I*x*f^(1/2)/e^(1/2))/e^(5/2)
```

Rubi [A] time = 0.19, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2455, 325, 205, 2376, 4848, 2391}

$$-\frac{ibf^{5/2}mn\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{5e^{5/2}}+\frac{ibf^{5/2}mn\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{5e^{5/2}}-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{5x^5}+\frac{2f^2m(a+b\cdots}{5e^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^6,x]
```

```
[Out] (-16*b*f*m*n)/(225*e*x^3) + (12*b*f^2*m*n)/(25*e^2*x) + (2*b*f^(5/2)*m*n*Ar
cTan[(Sqrt[f]*x)/Sqrt[e]])/(25*e^(5/2)) - (2*f*m*(a + b*Log[c*x^n]))/(15*e*
x^3) + (2*f^2*m*(a + b*Log[c*x^n]))/(5*e^2*x) + (2*f^(5/2)*m*ArcTan[(Sqrt[f
]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(5*e^(5/2)) - (b*n*Log[d*(e + f*x^2)^m])/
(25*x^5) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(5*x^5) - ((I/5)*b*f^(
5/2)*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^(5/2) + ((I/5)*b*f^(5/2)*m
*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/e^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^6}\,dx = -\frac{2fm(a+b\log(cx^n))}{15ex^3}+\frac{2f^2m(a+b\log(cx^n))}{5e^2x}+\frac{2f^{5/2}m\tan^{-1}}{}$$

$$= -\frac{2bfmn}{45ex^3}+\frac{2bf^2mn}{5e^2x}-\frac{2fm(a+b\log(cx^n))}{15ex^3}+\frac{2f^2m(a+b\log}{5e^2x}$$

$$= -\frac{2bfmn}{45ex^3}+\frac{2bf^2mn}{5e^2x}-\frac{2fm(a+b\log(cx^n))}{15ex^3}+\frac{2f^2m(a+b\log}{5e^2x}$$

$$= -\frac{16bfmn}{225ex^3}+\frac{2bf^2mn}{5e^2x}-\frac{2fm(a+b\log(cx^n))}{15ex^3}+\frac{2f^2m(a+b\log}{5e^2x}$$

$$= -\frac{16bfmn}{225ex^3}+\frac{12bf^2mn}{25e^2x}-\frac{2fm(a+b\log(cx^n))}{15ex^3}+\frac{2f^2m(a+b\log}{5e^2x}$$

$$= -\frac{16bfmn}{225ex^3}+\frac{12bf^2mn}{25e^2x}+\frac{2bf^{5/2}mn\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{25e^{5/2}}-\frac{2fm(a+b\log}{15ex^3}$$

Mathematica [C] time = 0.18, size = 399, normalized size = 1.49

$$-\frac{45ae^{5/2}\log\left(d\left(e+fx^2\right)^m\right)+30ae^{3/2}fmx^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{fx^2}{e}\right)+45be^{5/2}\log(cx^n)\log\left(d\left(e+fx^2\right)^m\right)+30be}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^6,x]
```

```
[Out] -1/225*(16*b*e^(3/2)*f*m*n*x^2 - 108*b*Sqrt[e]*f^2*m*n*x^4 - 18*b*f^(5/2)*m
*n*x^5*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 30*a*e^(3/2)*f*m*x^2*Hypergeometric2F1
[-3/2, 1, -1/2, -((f*x^2)/e)] + 90*b*f^(5/2)*m*n*x^5*ArcTan[(Sqrt[f]*x)/Sqr
t[e]]*Log[x] + 30*b*e^(3/2)*f*m*x^2*Log[c*x^n] - 90*b*Sqrt[e]*f^2*m*x^4*Log
[c*x^n] - 90*b*f^(5/2)*m*x^5*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - (45*I
)*b*f^(5/2)*m*n*x^5*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (45*I)*b*f^(5/2
)*m*n*x^5*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 45*a*e^(5/2)*Log[d*(e + f
*x^2)^m] + 9*b*e^(5/2)*n*Log[d*(e + f*x^2)^m] + 45*b*e^(5/2)*Log[c*x^n]*Log
[d*(e + f*x^2)^m] + (45*I)*b*f^(5/2)*m*n*x^5*PolyLog[2, ((-I)*Sqrt[f]*x)/Sq
rt[e]] - (45*I)*b*f^(5/2)*m*n*x^5*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(e^(5/
2)*x^5)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(fx^{2}+e\right)^{m}d\right)}{x^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(fx^{2}+e\right)^{m}d\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^6, x)
```

maple [C] time = 0.59, size = 2385, normalized size = 8.93

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^2+e)^m)/x^6,x)
```

```
[Out] -1/5*ln(d)/x^5*a-1/5*ln(d)/x^5*b*ln(c)-1/25*ln(d)*b*n/x^5+(-1/5*b/x^5*ln(x^
n)-1/50*(5*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-5*I*b*Pi*csgn(I*x^n)*csgn(I*c
*x^n)*csgn(I*c)-5*I*b*Pi*csgn(I*c*x^n)^3+5*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)
+10*b*ln(c)+2*b*n+10*a)/x^5)*ln((f*x^2+e)^m)-1/5*ln(d)*b/x^5*ln(x^n)-1/5*I*
m*f^3/e^2/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I*x^n)*csgn(I*c*x
^n)*csgn(I*c)+1/15*I*m*f/e/x^3*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/5
*I*m*f^2/e^2/x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/15*I*m*f/e/x^3*b*
Pi*csgn(I*c*x^n)^3-1/5*I*m*f^2/e^2/x*b*Pi*csgn(I*c*x^n)^3+1/50*I*Pi*csgn(I*
d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b*n/x^5+1/20*Pi^2*csgn(I*d)*cs
gn(I*d*(f*x^2+e)^m)^2/x^5*b*csgn(I*x^n)*csgn(I*c*x^n)^2+12/25*b/e^2*f^2*m*n
/x+1/20*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b*csgn(I*c*x^n)^2*csgn(I
*c)+1/20*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^5*b*csg
n(I*c*x^n)^3-1/10*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b*ln
(c)-1/10*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b/x^5*ln(x^n)+2/5
*m*f^3/e^2/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*a+1/5*I*m*f^3/e^2/(e*f)^(1
/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/15*I*m*f/e
/x^3*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/20*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^
m)*csgn(I*d*(f*x^2+e)^m)/x^5*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+2/5*b/e^
2*f^2*m/x*ln(c)+1/10*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^
m)/x^5*b*ln(c)+1/10*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m
)*b/x^5*ln(x^n)+1/10*I*Pi*csgn(I*d*(f*x^2+e)^m)^3/x^5*b*ln(c)+1/10*I*Pi*csg
n(I*d*(f*x^2+e)^m)^3*b/x^5*ln(x^n)+1/50*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b*n/x^
5-1/20*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^5*b*csgn(I*x^n)*csgn(I*c*x^n)^2+2/5*m
*f^3*b/e^2/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*ln(x^n)+1/5*I*m*f^3/e^2/(e
*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/5*I*m*
f^2/e^2/x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/5*I*m*f^2/e^2/x*b*Pi*csgn(I*c*
x^n)^2*csgn(I*c)-1/5*I*m*f^3/e^2/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*Pi
*csgn(I*c*x^n)^3+2/5*a/e^2*f^2*m/x-1/20*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^5*b*
csgn(I*c*x^n)^2*csgn(I*c)-1/20*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b
```

```
*csgn(I*c*x^n)^3-1/20*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^5*
b*csgn(I*c*x^n)^3+1/10*I*ln(d)/x^5*b*Pi*csgn(I*c*x^n)^3+1/10*I*Pi*csgn(I*d)
*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^5*a-1/10*I*Pi*csgn(I*d)*csgn(I
*d*(f*x^2+e)^m)^2/x^5*b*ln(c)-1/20*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x
^5*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/20*Pi^2*csgn(I*d)*csgn(I*(f*x^2+
e)^m)*csgn(I*d*(f*x^2+e)^m)/x^5*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/20*Pi^2*csg
n(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^5*b*csgn(I*c*x^n)^2*csgn
(I*c)+1/10*I*ln(d)/x^5*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/50*I*Pi*c
sgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b*n/x^5+1/20*Pi^2*csgn(I*d*(f*x^
2+e)^m)^3/x^5*b*csgn(I*c*x^n)^3-2/15*m*f/e/x^3*a+1/5*m*f^3*b*n/e^2/(-e*f)^(
1/2)*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/5*m*f^3*b*n/e^2/(-e*f)^(1
/2)*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-2/5*m*f^3*b/e^2/(e*f)^(1/2)*a
rctan(1/(e*f)^(1/2)*f*x)*n*ln(x)+1/10*I*Pi*csgn(I*d*(f*x^2+e)^m)^3/x^5*a-1/
20*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b*csgn(I*x^n)*csgn(
I*c*x^n)*csgn(I*c)+2/5*m*f^3/e^2/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*ln
(c)-1/10*I*ln(d)/x^5*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/10*I*ln(d)/x^5*b*Pi
*csgn(I*c*x^n)^2*csgn(I*c)+1/20*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)
^m)^2/x^5*b*csgn(I*x^n)*csgn(I*c*x^n)^2-2/15*m*f*b/e/x^3*ln(x^n)+1/20*Pi^2*
csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b*csgn(I*c*x^n)^2*csgn(I*c)
+1/20*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^5*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c
)+2/5*m*f^2*b*ln(x^n)/e^2/x-1/10*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^5
*a-1/10*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^5*a-2/15*m*f/e/x
^3*b*ln(c)-1/10*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b/x^5*ln(x^n)-1/50*I
*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b*n/x^5-1/15*I*m*f/e/x^3*b*Pi*csgn(I*
c*x^n)^2*csgn(I*c)+2/25*m*f^3/e^2/(e*f)^(1/2)*arctan(1/(e*f)^(1/2)*f*x)*b*n
+1/5*m*f^3*b*n/e^2/(-e*f)^(1/2)*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/5
*m*f^3*b*n/e^2/(-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-16/225*b
*f*m*n/e/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(5\,bm\log\left(x^{n}\right)+\left(mn+5\,m\log(c)\right)b+5\,am\right)\log\left(fx^{2}+e\right)}{25\,x^{5}}+\int\frac{25\,be\log(c)\log(d)+\left(5\left(2\,fm+5\,f\log(d)\right)a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^6,x, algorithm="maxima")
```

```
[Out] -1/25*(5*b*m*log(x^n) + (m*n + 5*m*log(c))*b + 5*a*m)*log(f*x^2 + e)/x^5 +
integrate(1/25*(25*b*e*log(c)*log(d) + (5*(2*f*m + 5*f*log(d))*a + (2*f*m*n
 + 5*(2*f*m + 5*f*log(d))*log(c))*b)*x^2 + 25*a*e*log(d) + 5*((2*f*m + 5*f*
log(d))*b*x^2 + 5*b*e*log(d))*log(x^n))/(f*x^8 + e*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^6,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**6,x)
```

```
[Out] Timed out
```

3.100 $\int x\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)\,dx$

Optimal. Leaf size=310

$$-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)+\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)+\frac{bemn\text{Li}_2\left(-\frac{fx^2}{e}\right)\left(a+b\log\right.}{2f}$$

```
[Out] -3/4*b^2*m*n^2*x^2+b*m*n*x^2*(a+b*ln(c*x^n))-1/2*m*x^2*(a+b*ln(c*x^n))^2+1/
4*b^2*e*m*n^2*ln(f*x^2+e)/f+1/4*b^2*n^2*x^2*ln(d*(f*x^2+e)^m)-1/2*b*n*x^2*(
a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)+1/2*x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m
)-1/2*b*e*m*n*(a+b*ln(c*x^n))*ln(1+f*x^2/e)/f+1/2*e*m*(a+b*ln(c*x^n))^2*ln(
1+f*x^2/e)/f-1/4*b^2*e*m*n^2*polylog(2,-f*x^2/e)/f+1/2*b*e*m*n*(a+b*ln(c*x^
n))*polylog(2,-f*x^2/e)/f-1/4*b^2*e*m*n^2*polylog(3,-f*x^2/e)/f
```

Rubi [A] time = 0.54, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {2305, 2304, 2378, 266, 43, 2351, 2337, 2391, 2353, 2374, 6589}

$$\frac{bemn\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{2f}-\frac{b^2emn^2\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{4f}-\frac{b^2emn^2\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)}{4f}-\frac{1}{2}bnx^2\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-3*b^2*m*n^2*x^2)/4 + b*m*n*x^2*(a + b*Log[c*x^n]) - (m*x^2*(a + b*Log[c*x
^n])^2)/2 + (b^2*e*m*n^2*Log[e + f*x^2])/(4*f) + (b^2*n^2*x^2*Log[d*(e + f*
x^2)^m])/4 - (b*n*x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/2 + (x^2*(a
+ b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/2 - (b*e*m*n*(a + b*Log[c*x^n])*Log
[1 + (f*x^2)/e])/(2*f) + (e*m*(a + b*Log[c*x^n])^2*Log[1 + (f*x^2)/e])/(2*f
) - (b^2*e*m*n^2*PolyLog[2, -((f*x^2)/e)])/(4*f) + (b*e*m*n*(a + b*Log[c*x^
n])*PolyLog[2, -((f*x^2)/e)])/(2*f) - (b^2*e*m*n^2*PolyLog[3, -((f*x^2)/e)]
)/(4*f)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
```

```
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)dx = \frac{1}{4}b^2n^2x^2\log\left(d\left(e+fx^2\right)^m\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+\right.\right.$$

$$= \frac{1}{4}b^2n^2x^2\log\left(d\left(e+fx^2\right)^m\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+\right.\right.$$

$$= \frac{1}{4}b^2n^2x^2\log\left(d\left(e+fx^2\right)^m\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+\right.\right.$$

$$= \frac{1}{4}b^2n^2x^2\log\left(d\left(e+fx^2\right)^m\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+\right.\right.$$

$$= -\frac{1}{2}b^2mn^2x^2+\frac{1}{2}bmnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)$$

$$= -\frac{3}{4}b^2mn^2x^2+bmnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)^2$$

$$= -\frac{3}{4}b^2mn^2x^2+bmnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}mx^2\left(a+b\log\left(cx^n\right)\right)^2$$

Mathematica [C] time = 0.27, size = 814, normalized size = 2.63

$$\frac{-2fmx^2a^2+2em\log\left(fx^2+e\right)a^2+2fx^2\log\left(d\left(fx^2+e\right)^m\right)a^2+4bfmnx^2a-4bfmx^2\log\left(cx^n\right)a+4bemn\log(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-2*a^2*f*m*x^2 + 4*a*b*f*m*n*x^2 - 3*b^2*f*m*n^2*x^2 - 4*a*b*f*m*x^2*Log[c
*x^n] + 4*b^2*f*m*n*x^2*Log[c*x^n] - 2*b^2*f*m*x^2*Log[c*x^n]^2 + 4*a*b*e*m
*n*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]*Log[1 - (I*
Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]]
 + 4*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 4*a*b*e*m
*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]*Log[1 + (I*
Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]]
 + 4*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 2*a^2*e*m
*Log[e + f*x^2] - 2*a*b*e*m*n*Log[e + f*x^2] + b^2*e*m*n^2*Log[e + f*x^2] -
 4*a*b*e*m*n*Log[x]*Log[e + f*x^2] + 2*b^2*e*m*n^2*Log[x]*Log[e + f*x^2] +
2*b^2*e*m*n^2*Log[x]^2*Log[e + f*x^2] + 4*a*b*e*m*Log[c*x^n]*Log[e + f*x^2]
 - 2*b^2*e*m*n*Log[c*x^n]*Log[e + f*x^2] - 4*b^2*e*m*n*Log[x]*Log[c*x^n]*Lo
g[e + f*x^2] + 2*b^2*e*m*Log[c*x^n]^2*Log[e + f*x^2] + 2*a^2*f*x^2*Log[d*(e
 + f*x^2)^m] - 2*a*b*f*n*x^2*Log[d*(e + f*x^2)^m] + b^2*f*n^2*x^2*Log[d*(e
+ f*x^2)^m] + 4*a*b*f*x^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 2*b^2*f*n*x^2*L
og[c*x^n]*Log[d*(e + f*x^2)^m] + 2*b^2*f*x^2*Log[c*x^n]^2*Log[d*(e + f*x^2)
^m] + 2*b*e*m*n*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[2, ((-I)*Sqrt[f]*x)/Sq
rt[e]] + 2*b*e*m*n*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[2, (I*Sqrt[f]*x)/Sq
rt[e]] - 4*b^2*e*m*n^2*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 4*b^2*e*m*n^2
*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]])/(4*f)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b^2x\log\left(cx^n\right)^2+2\,abx\log\left(cx^n\right)+a^2x\right)\log\left(\left(fx^2+e\right)^md\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log((f*x^2 + e)^
m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)^2 x\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*x^2 + e)^m*d), x)
```

maple [F] time = 5.87, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)^2 x\ln\left(d\left(f\,x^2+e\right)^m\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(2\,b^2mx^2\log\left(x^n\right)^2-2\left(\left(mn-2\,m\log(c)\right)b^2-2\,abm\right)x^2\log\left(x^n\right)-\left(2\left(mn-2\,m\log(c)\right)ab-\left(mn^2-2\,mn\,\mathrm{lo}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*m*x^2*log(x^n)^2 - 2*((m*n - 2*m*log(c))*b^2 - 2*a*b*m)*x^2*log(
x^n) - (2*(m*n - 2*m*log(c))*a*b - (m*n^2 - 2*m*n*log(c) + 2*m*log(c)^2)*b^
2 - 2*a^2*m)*x^2)*log(f*x^2 + e) + integrate(-1/2*((2*(f*m - f*log(d))*a^2
- 2*(f*m*n - 2*(f*m - f*log(d))*log(c))*a*b + (f*m*n^2 - 2*f*m*n*log(c) + 2
*(f*m - f*log(d))*log(c)^2)*b^2)*x^3 + 2*((f*m - f*log(d))*b^2*x^3 - b^2*e*
x*log(d))*log(x^n)^2 - 2*(b^2*e*log(c)^2*log(d) + 2*a*b*e*log(c)*log(d) + a
^2*e*log(d))*x + 2*((2*(f*m - f*log(d))*a*b - (f*m*n - 2*(f*m - f*log(d))*l
og(c))*b^2)*x^3 - 2*(b^2*e*log(c)*log(d) + a*b*e*log(d))*x)*log(x^n))/(f*x^
2 + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\left(f\,x^2+e\right)^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

3.101 $\int \frac{\left(a+b\log(cx^n)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx$

Optimal. Leaf size=147

$$\frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn}-\frac{1}{2}m\text{Li}_2\left(-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2+\frac{1}{2}bmn\text{Li}_3\left(-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{m\,l}{}$$

```
[Out] 1/3*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/b/n-1/3*m*(a+b*ln(c*x^n))^3*ln(1+f*
x^2/e)/b/n-1/2*m*(a+b*ln(c*x^n))^2*polylog(2,-f*x^2/e)+1/2*b*m*n*(a+b*ln(c*
x^n))*polylog(3,-f*x^2/e)-1/4*b^2*m*n^2*polylog(4,-f*x^2/e)
```

Rubi [A] time = 0.18, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2375, 2337, 2374, 2383, 6589}

$$-\frac{1}{2}m\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2+\frac{1}{2}bmn\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}b^2mn^2\text{PolyLog}\left(4,-\frac{f}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/(3*b*n) - (m*(a + b*Log[c*x^n])
^3*Log[1 + (f*x^2)/e])/(3*b*n) - (m*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x^
2)/e)])/2 + (b*m*n*(a + b*Log[c*x^n])*PolyLog[3, -((f*x^2)/e)])/2 - (b^2*m*
n^2*PolyLog[4, -((f*x^2)/e)])/4
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{x}dx = \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn} - \frac{(2fm)\int\frac{x\left(a+b\log\left(cx^n\right)\right)^3}{e+fx^2}d}{3bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^3\log}{3bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^3\log}{3bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^3\log}{3bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)}{3bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^3\log}{3bn}$$

Mathematica [C] time = 0.24, size = 736, normalized size = 5.01

$$a^2\log(x)\log\left(d\left(e+fx^2\right)^m\right) - a^2m\log(x)\log\left(1-\frac{i\sqrt{f}x}{\sqrt{e}}\right) - a^2m\log(x)\log\left(1+\frac{i\sqrt{f}x}{\sqrt{e}}\right) + 2ab\log(x)\log\left(cx^n\right)\log$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] -(a^2*m*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]]) + a*b*m*n*Log[x]^2*Log[1 - (
I*Sqrt[f]*x)/Sqrt[e]] - (b^2*m*n^2*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]])
/3 - 2*a*b*m*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + b^2*m*n*Log
[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - b^2*m*Log[x]*Log[c*x^n]^2
*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - a^2*m*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e
]] + a*b*m*n*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (b^2*m*n^2*Log[x]^3*
Log[1 + (I*Sqrt[f]*x)/Sqrt[e]])/3 - 2*a*b*m*Log[x]*Log[c*x^n]*Log[1 + (I*Sq
rt[f]*x)/Sqrt[e]] + b^2*m*n*Log[x]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[
e]] - b^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + a^2*Log[x]
*Log[d*(e + f*x^2)^m] - a*b*n*Log[x]^2*Log[d*(e + f*x^2)^m] + (b^2*n^2*Log[
x]^3*Log[d*(e + f*x^2)^m])/3 + 2*a*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^2)^m]
 - b^2*n*Log[x]^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + b^2*Log[x]*Log[c*x^n]^2
*Log[d*(e + f*x^2)^m] - m*(a + b*Log[c*x^n])^2*PolyLog[2, ((-I)*Sqrt[f]*x)/
Sqrt[e]] - m*(a + b*Log[c*x^n])^2*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + 2*a*b
*m*n*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*b^2*m*n*Log[c*x^n]*PolyLog[3,
 ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*a*b*m*n*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] +
2*b^2*m*n*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*m*n^2*PolyLo
g[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*m*n^2*PolyLog[4, (I*Sqrt[f]*x)/Sqrt[
e]]
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^2\log\left(cx^n\right)^2+2\,ab\log\left(cx^n\right)+a^2\right)\log\left(\left(fx^2+e\right)^m d\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/x
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{2}\log\left(\left(fx^{2}+e\right)^{m}d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x, x)
```

maple [F] time = 5.16, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^{n}\right)+a\right)^{2}\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(b^{2}mn^{2}\log(x)^{3}+3\,b^{2}m\log(x)\log\left(x^{n}\right)^{2}-3\left(b^{2}mn\log(c)+abmn\right)\log(x)^{2}-3\left(b^{2}mn\log(x)^{2}-2\left(b^{2}m\log(c)+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*m*n^2*log(x)^3 + 3*b^2*m*log(x)*log(x^n)^2 - 3*(b^2*m*n*log(c) + a
*b*m*n)*log(x)^2 - 3*(b^2*m*n*log(x)^2 - 2*(b^2*m*log(c) + a*b*m)*log(x))*l
og(x^n) + 3*(b^2*m*log(c)^2 + 2*a*b*m*log(c) + a^2*m)*log(x))*log(f*x^2 + e
) - integrate(1/3*(2*b^2*f*m*n^2*x^2*log(x)^3 - 3*b^2*e*log(c)^2*log(d) - 6
*a*b*e*log(c)*log(d) - 6*(b^2*f*m*n*log(c) + a*b*f*m*n)*x^2*log(x)^2 - 3*a^
2*e*log(d) + 6*(b^2*f*m*log(c)^2 + 2*a*b*f*m*log(c) + a^2*f*m)*x^2*log(x) -
 3*(b^2*f*log(c)^2*log(d) + 2*a*b*f*log(c)*log(d) + a^2*f*log(d))*x^2 + 3*(
2*b^2*f*m*x^2*log(x) - b^2*f*x^2*log(d) - b^2*e*log(d))*log(x^n)^2 - 6*(b^2
*f*m*n*x^2*log(x)^2 + b^2*e*log(c)*log(d) + a*b*e*log(d) - 2*(b^2*f*m*log(c
) + a*b*f*m)*x^2*log(x) + (b^2*f*log(c)*log(d) + a*b*f*log(d))*x^2)*log(x^n
))/(f*x^3 + e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)\,{\left(a+b\,\ln\left(c\,x^{n}\right)\right)}^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x,x)
```

```
[Out] Timed out
```

3.102 $\int \frac{\left(a+b\log(cx^n)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx$

Optimal. Leaf size=276

$$-\frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}-\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}+\frac{bfmn\text{Li}_2\left(-\frac{e}{fx^2}\right)\left(a+b\log\left(cx^n\right)\right)}{2e}$$

```
[Out] 1/2*b^2*f*m*n^2*ln(x)/e-1/2*b*f*m*n*ln(1+e/f/x^2)*(a+b*ln(c*x^n))/e-1/2*f*m
*ln(1+e/f/x^2)*(a+b*ln(c*x^n))^2/e-1/4*b^2*f*m*n^2*ln(f*x^2+e)/e-1/4*b^2*n^
2*ln(d*(f*x^2+e)^m)/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^2-1/2*(
a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^2+1/4*b^2*f*m*n^2*polylog(2,-e/f/x^2)/
e+1/2*b*f*m*n*(a+b*ln(c*x^n))*polylog(2,-e/f/x^2)/e+1/4*b^2*f*m*n^2*polylog
(3,-e/f/x^2)/e
```

Rubi [A] time = 0.33, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2305, 2304, 2378, 266, 36, 29, 31, 2345, 2391, 2374, 6589}

$$\frac{bfmn\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)\left(a+b\log\left(cx^n\right)\right)}{2e}+\frac{b^2fmn^2\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{4e}+\frac{b^2fmn^2\text{PolyLog}\left(3,-\frac{e}{fx^2}\right)}{4e}-\frac{bn\left(a+b\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] (b^2*f*m*n^2*Log[x])/(2*e) - (b*f*m*n*Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])
)/(2*e) - (f*m*Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])^2)/(2*e) - (b^2*f*m*n^
2*Log[e + f*x^2])/(4*e) - (b^2*n^2*Log[d*(e + f*x^2)^m])/(4*x^2) - (b*n*(a
+ b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(2*x^2) - ((a + b*Log[c*x^n])^2*Log[d
*(e + f*x^2)^m])/(2*x^2) + (b^2*f*m*n^2*PolyLog[2, -(e/(f*x^2))])/(4*e) + (
b*f*m*n*(a + b*Log[c*x^n])*PolyLog[2, -(e/(f*x^2))])/(2*e) + (b^2*f*m*n^2*P
olyLog[3, -(e/(f*x^2))])/(4*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2345

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> -Simp[(Log[1 + d/(e*x^r)]*(a + b*Log[c*x^n])^p)/(d*r),
x] + Dist[(b*n*p)/(d*r), Int[(Log[1 + d/(e*x^r)]*(a + b*Log[c*x^n])^(p - 1)
)/x, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx = -\frac{b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{4x^2} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}$$

$$= -\frac{b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{4x^2} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}$$

$$= -\frac{bfmn\log\left(1+\frac{e}{fx^2}\right)\left(a+b\log\left(cx^n\right)\right)}{2e} - \frac{fm\log\left(1+\frac{e}{fx^2}\right)\left(a+b\log\right.}{2e}$$

$$= -\frac{bfmn\log\left(1+\frac{e}{fx^2}\right)\left(a+b\log\left(cx^n\right)\right)}{2e} - \frac{fm\log\left(1+\frac{e}{fx^2}\right)\left(a+b\log\right.}{2e}$$

$$= \frac{b^2fmn^2\log(x)}{2e} - \frac{bfmn\log\left(1+\frac{e}{fx^2}\right)\left(a+b\log\left(cx^n\right)\right)}{2e} - \frac{fm\log\left(1\right.}{}$$

Mathematica [C] time = 0.44, size = 946, normalized size = 3.43

$$-\frac{-4b^2fmn^2x^2\log^3(x)+6b^2fmn^2x^2\log^2(x)+12abfmnx^2\log^2(x)+12b^2fmnx^2\log\left(cx^n\right)\log^2(x)-6b^2fmn^2x^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] -1/12*(-12*a^2*f*m*x^2*Log[x] - 12*a*b*f*m*n*x^2*Log[x] - 6*b^2*f*m*n^2*x^2
*Log[x] + 12*a*b*f*m*n*x^2*Log[x]^2 + 6*b^2*f*m*n^2*x^2*Log[x]^2 - 4*b^2*f*
m*n^2*x^2*Log[x]^3 - 24*a*b*f*m*x^2*Log[x]*Log[c*x^n] - 12*b^2*f*m*n*x^2*Lo
g[x]*Log[c*x^n] + 12*b^2*f*m*n*x^2*Log[x]^2*Log[c*x^n] - 12*b^2*f*m*x^2*Log
[x]*Log[c*x^n]^2 + 12*a*b*f*m*n*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] +
 6*b^2*f*m*n^2*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 6*b^2*f*m*n^2*x^
2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^2*f*m*n*x^2*Log[x]*Log[c*x
^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*a*b*f*m*n*x^2*Log[x]*Log[1 + (I*Sqr
t[f]*x)/Sqrt[e]] + 6*b^2*f*m*n^2*x^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]]
- 6*b^2*f*m*n^2*x^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^2*f*m*n*
x^2*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 6*a^2*f*m*x^2*Log[e
+ f*x^2] + 6*a*b*f*m*n*x^2*Log[e + f*x^2] + 3*b^2*f*m*n^2*x^2*Log[e + f*x^2
] - 12*a*b*f*m*n*x^2*Log[x]*Log[e + f*x^2] - 6*b^2*f*m*n^2*x^2*Log[x]*Log[e
 + f*x^2] + 6*b^2*f*m*n^2*x^2*Log[x]^2*Log[e + f*x^2] + 12*a*b*f*m*x^2*Log[
c*x^n]*Log[e + f*x^2] + 6*b^2*f*m*n*x^2*Log[c*x^n]*Log[e + f*x^2] - 12*b^2*
f*m*n*x^2*Log[x]*Log[c*x^n]*Log[e + f*x^2] + 6*b^2*f*m*x^2*Log[c*x^n]^2*Log
[e + f*x^2] + 6*a^2*e*Log[d*(e + f*x^2)^m] + 6*a*b*e*n*Log[d*(e + f*x^2)^m]
 + 3*b^2*e*n^2*Log[d*(e + f*x^2)^m] + 12*a*b*e*Log[c*x^n]*Log[d*(e + f*x^2)
^m] + 6*b^2*e*n*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 6*b^2*e*Log[c*x^n]^2*Log[
d*(e + f*x^2)^m] + 6*b*f*m*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[2, ((
-I)*Sqrt[f]*x)/Sqrt[e]] + 6*b*f*m*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLo
g[2, (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^2*f*m*n^2*x^2*PolyLog[3, ((-I)*Sqrt[f]*x
)/Sqrt[e]] - 12*b^2*f*m*n^2*x^2*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]])/(e*x^2)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\log\left(cx^n\right)^2+2\,ab\log\left(cx^n\right)+a^2\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/x
^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{2}\log\left(\left(fx^{2}+e\right)^{m}d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^3, x)
```

maple [F] time = 5.35, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^{n}\right)+a\right)^{2}\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,b^{2}m\log\left(x^{n}\right)^{2}+2\left(mn+2\,m\log(c)\right)ab+\left(mn^{2}+2\,mn\log(c)+2\,m\log(c)^{2}\right)b^{2}+2\,a^{2}m+2\left(\left(mn+2\,m\log\right.\right.}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b^2*m*log(x^n)^2 + 2*(m*n + 2*m*log(c))*a*b + (m*n^2 + 2*m*n*log(c)
 + 2*m*log(c)^2)*b^2 + 2*a^2*m + 2*((m*n + 2*m*log(c))*b^2 + 2*a*b*m)*log(x
^n))*log(f*x^2 + e)/x^2 + integrate(1/2*(2*b^2*e*log(c)^2*log(d) + 4*a*b*e*
log(c)*log(d) + 2*a^2*e*log(d) + (2*(f*m + f*log(d))*a^2 + 2*(f*m*n + 2*(f*
m + f*log(d))*log(c))*a*b + (f*m*n^2 + 2*f*m*n*log(c) + 2*(f*m + f*log(d))*
log(c)^2)*b^2)*x^2 + 2*((f*m + f*log(d))*b^2*x^2 + b^2*e*log(d))*log(x^n)^2
 + 2*(2*b^2*e*log(c)*log(d) + 2*a*b*e*log(d) + (2*(f*m + f*log(d))*a*b + (f
*m*n + 2*(f*m + f*log(d))*log(c))*b^2)*x^2)*log(x^n))/(f*x^5 + e*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**3,x)
```

```
[Out] Timed out
```

3.103 $\int \frac{\left(a+b\log(cx^n)\right)^2 \log\left(d\left(e+fx^2\right)^m\right)}{x^5}\,dx$

Optimal. Leaf size=356

$$-\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{4x^4}-\frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{8x^4}-\frac{bf^2mn\mathrm{Li}_2\left(-\frac{e}{fx^2}\right)\left(a+b\log(\right.}{4e^2}$$

```
[Out] -7/32*b^2*f*m*n^2/e/x^2-1/16*b^2*f^2*m*n^2*ln(x)/e^2-3/8*b*f*m*n*(a+b*ln(c*
x^n))/e/x^2+1/8*b*f^2*m*n*ln(1+e/f/x^2)*(a+b*ln(c*x^n))/e^2-1/4*f*m*(a+b*ln
(c*x^n))^2/e/x^2+1/4*f^2*m*ln(1+e/f/x^2)*(a+b*ln(c*x^n))^2/e^2+1/32*b^2*f^2
*m*n^2*ln(f*x^2+e)/e^2-1/32*b^2*n^2*ln(d*(f*x^2+e)^m)/x^4-1/8*b*n*(a+b*ln(c
*x^n))*ln(d*(f*x^2+e)^m)/x^4-1/4*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^4-1/
16*b^2*f^2*m*n^2*polylog(2,-e/f/x^2)/e^2-1/4*b*f^2*m*n*(a+b*ln(c*x^n))*poly
log(2,-e/f/x^2)/e^2-1/8*b^2*f^2*m*n^2*polylog(3,-e/f/x^2)/e^2
```

Rubi [A] time = 0.67, antiderivative size = 408, normalized size of antiderivative = 1.15, number of steps used = 20, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2305, 2304, 2378, 266, 44, 2351, 2301, 2337, 2391, 2353, 2302, 30, 2374, 6589}

$$\frac{bf^2mn\mathrm{PolyLog}\left(2,-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{4e^2}+\frac{b^2f^2mn^2\mathrm{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{16e^2}-\frac{b^2f^2mn^2\mathrm{PolyLog}\left(3,-\frac{fx^2}{e}\right)}{8e^2}-\frac{(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^5,x]
```

```
[Out] (-7*b^2*f*m*n^2)/(32*e*x^2) - (b^2*f^2*m*n^2*Log[x])/(16*e^2) - (3*b*f*m*n*
(a + b*Log[c*x^n]))/(8*e*x^2) - (f^2*m*(a + b*Log[c*x^n])^2)/(8*e^2) - (f*m
*(a + b*Log[c*x^n])^2)/(4*e*x^2) - (f^2*m*(a + b*Log[c*x^n])^3)/(6*b*e^2*n)
 + (b^2*f^2*m*n^2*Log[e + f*x^2])/(32*e^2) - (b^2*n^2*Log[d*(e + f*x^2)^m])
/(32*x^4) - (b*n*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(8*x^4) - ((a + b
*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/(4*x^4) + (b*f^2*m*n*(a + b*Log[c*x^n]
)*Log[1 + (f*x^2)/e])/(8*e^2) + (f^2*m*(a + b*Log[c*x^n])^2*Log[1 + (f*x^2)
/e])/(4*e^2) + (b^2*f^2*m*n^2*PolyLog[2, -((f*x^2)/e)])/(16*e^2) + (b*f^2*m
*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*x^2)/e)])/(4*e^2) - (b^2*f^2*m*n^2*Po
lyLog[3, -((f*x^2)/e)])/(8*e^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
```

```
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{x^5}\,dx = -\frac{b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{32x^4} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right.\right.}{8x^4}$$

$$= -\frac{b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{32x^4} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right.\right.}{8x^4}$$

$$= -\frac{b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{32x^4} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right.\right.}{8x^4}$$

$$= -\frac{b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{32x^4} - \frac{bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right.\right.}{8x^4}$$

$$= -\frac{3b^2fmn^2}{32ex^2} - \frac{b^2f^2mn^2\log(x)}{16e^2} - \frac{bfmn\left(a+b\log\left(cx^n\right)\right)}{8ex^2} - \frac{f^2m}{}$$

$$= -\frac{7b^2fmn^2}{32ex^2} - \frac{b^2f^2mn^2\log(x)}{16e^2} - \frac{3bfmn\left(a+b\log\left(cx^n\right)\right)}{8ex^2} - \frac{f^2m}{}$$

$$= -\frac{7b^2fmn^2}{32ex^2} - \frac{b^2f^2mn^2\log(x)}{16e^2} - \frac{3bfmn\left(a+b\log\left(cx^n\right)\right)}{8ex^2} - \frac{f^2m}{}$$

Mathematica [C] time = 0.48, size = 1111, normalized size = 3.12

$$-\frac{16b^2f^2mn^2\log^3(x)x^4 - 12b^2f^2mn^2\log^2(x)x^4 - 48abf^2mn\log^2(x)x^4 + 48b^2f^2m\log(x)\log^2\left(cx^n\right)x^4 + 6b^2f}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^5,x]
```

```
[Out] -1/96*(24*a^2*e*f*m*x^2 + 36*a*b*e*f*m*n*x^2 + 21*b^2*e*f*m*n^2*x^2 + 48*a^
2*f^2*m*x^4*Log[x] + 24*a*b*f^2*m*n*x^4*Log[x] + 6*b^2*f^2*m*n^2*x^4*Log[x]
 - 48*a*b*f^2*m*n*x^4*Log[x]^2 - 12*b^2*f^2*m*n^2*x^4*Log[x]^2 + 16*b^2*f^2
*m*n^2*x^4*Log[x]^3 + 48*a*b*e*f*m*x^2*Log[c*x^n] + 36*b^2*e*f*m*n*x^2*Log[
c*x^n] + 96*a*b*f^2*m*x^4*Log[x]*Log[c*x^n] + 24*b^2*f^2*m*n*x^4*Log[x]*Log
[c*x^n] - 48*b^2*f^2*m*n*x^4*Log[x]^2*Log[c*x^n] + 24*b^2*e*f*m*x^2*Log[c*x
^n]^2 + 48*b^2*f^2*m*x^4*Log[x]*Log[c*x^n]^2 - 48*a*b*f^2*m*n*x^4*Log[x]*Lo
```

```
g[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^2*f^2*m*n^2*x^4*Log[x]*Log[1 - (I*Sqrt[
f]*x)/Sqrt[e]] + 24*b^2*f^2*m*n^2*x^4*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e
]] - 48*b^2*f^2*m*n*x^4*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] -
48*a*b*f^2*m*n*x^4*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^2*f^2*m*n^2
*x^4*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 24*b^2*f^2*m*n^2*x^4*Log[x]^2*
Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 48*b^2*f^2*m*n*x^4*Log[x]*Log[c*x^n]*Log[1
 + (I*Sqrt[f]*x)/Sqrt[e]] - 24*a^2*f^2*m*x^4*Log[e + f*x^2] - 12*a*b*f^2*m*
n*x^4*Log[e + f*x^2] - 3*b^2*f^2*m*n^2*x^4*Log[e + f*x^2] + 48*a*b*f^2*m*n*
x^4*Log[x]*Log[e + f*x^2] + 12*b^2*f^2*m*n^2*x^4*Log[x]*Log[e + f*x^2] - 24
*b^2*f^2*m*n^2*x^4*Log[x]^2*Log[e + f*x^2] - 48*a*b*f^2*m*x^4*Log[c*x^n]*Lo
g[e + f*x^2] - 12*b^2*f^2*m*n*x^4*Log[c*x^n]*Log[e + f*x^2] + 48*b^2*f^2*m*
n*x^4*Log[x]*Log[c*x^n]*Log[e + f*x^2] - 24*b^2*f^2*m*x^4*Log[c*x^n]^2*Log[
e + f*x^2] + 24*a^2*e^2*Log[d*(e + f*x^2)^m] + 12*a*b*e^2*n*Log[d*(e + f*x^
2)^m] + 3*b^2*e^2*n^2*Log[d*(e + f*x^2)^m] + 48*a*b*e^2*Log[c*x^n]*Log[d*(e
 + f*x^2)^m] + 12*b^2*e^2*n*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 24*b^2*e^2*Lo
g[c*x^n]^2*Log[d*(e + f*x^2)^m] - 12*b*f^2*m*n*x^4*(4*a + b*n + 4*b*Log[c*x
^n])*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 12*b*f^2*m*n*x^4*(4*a + b*n + 4
*b*Log[c*x^n])*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + 48*b^2*f^2*m*n^2*x^4*Pol
yLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 48*b^2*f^2*m*n^2*x^4*PolyLog[3, (I*Sqrt
[f]*x)/Sqrt[e]])/(e^2*x^4)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^2\log\left(cx^n\right)^2+2\,ab\log\left(cx^n\right)+a^2\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/x
^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(fx^2+e\right)^m d\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^5, x)
```

maple [F] time = 5.54, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^n\right)+a\right)^2\ln\left(d\left(f\,x^2+e\right)^m\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m)/x^5,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(8\,b^2m\log\left(x^n\right)^2+4\left(mn+4\,m\log(c)\right)ab+\left(mn^2+4\,mn\log(c)+8\,m\log(c)^2\right)b^2+8\,a^2m+4\left(\left(mn+4\,m\log(c\right.\right.}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^5,x, algorithm="maxima")
```

```
[Out] -1/32*(8*b^2*m*log(x^n)^2 + 4*(m*n + 4*m*log(c))*a*b + (m*n^2 + 4*m*n*log(c
) + 8*m*log(c)^2)*b^2 + 8*a^2*m + 4*((m*n + 4*m*log(c))*b^2 + 4*a*b*m)*log(
x^n))*log(f*x^2 + e)/x^4 + integrate(1/16*(16*b^2*e*log(c)^2*log(d) + 32*a*
b*e*log(c)*log(d) + 16*a^2*e*log(d) + (8*(f*m + 2*f*log(d))*a^2 + 4*(f*m*n
+ 4*(f*m + 2*f*log(d))*log(c))*a*b + (f*m*n^2 + 4*f*m*n*log(c) + 8*(f*m + 2
*f*log(d))*log(c)^2)*b^2)*x^2 + 8*((f*m + 2*f*log(d))*b^2*x^2 + 2*b^2*e*log
(d))*log(x^n)^2 + 4*(8*b^2*e*log(c)*log(d) + 8*a*b*e*log(d) + (4*(f*m + 2*f
*log(d))*a*b + (f*m*n + 4*(f*m + 2*f*log(d))*log(c))*b^2)*x^2)*log(x^n))/(f
*x^7 + e*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^5,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**5,x)
```

```
[Out] Timed out
```

3.104 $\int x^2 \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d\left(e+fx^2\right)^m\right)\,dx$

Optimal. Leaf size=630

$$\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)+\frac{4be^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+\right.}{9f^{3/2}}$$

```
[Out] -16/9*a*b*e*m*n*x/f+52/27*b^2*e*m*n^2*x/f-4/27*b^2*m*n^2*x^3-4/27*b^2*e^(3/
2)*m*n^2*arctan(x*f^(1/2)/e^(1/2))/f^(3/2)-16/9*b^2*e*m*n*x*ln(c*x^n)/f+8/2
7*b*m*n*x^3*(a+b*ln(c*x^n))+4/9*b*e^(3/2)*m*n*arctan(x*f^(1/2)/e^(1/2))*(a+
b*ln(c*x^n))/f^(3/2)+2/3*e*m*x*(a+b*ln(c*x^n))^2/f-2/9*m*x^3*(a+b*ln(c*x^n)
)^2+2/27*b^2*n^2*x^3*ln(d*(f*x^2+e)^m)-2/9*b*n*x^3*(a+b*ln(c*x^n))*ln(d*(f*
x^2+e)^m)+1/3*x^3*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)-1/3*(-e)^(3/2)*m*(a+b
*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))/f^(3/2)+1/3*(-e)^(3/2)*m*(a+b*ln(c
*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))/f^(3/2)+2/3*b*(-e)^(3/2)*m*n*(a+b*ln(c*
x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2/3*b*(-e)^(3/2)*m*n*(a+b*ln
(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2/9*I*b^2*e^(3/2)*m*n^2*po
lylog(2,-I*x*f^(1/2)/e^(1/2))/f^(3/2)+2/9*I*b^2*e^(3/2)*m*n^2*polylog(2,I*x
*f^(1/2)/e^(1/2))/f^(3/2)-2/3*b^2*(-e)^(3/2)*m*n^2*polylog(3,-x*f^(1/2)/(-e
)^(1/2))/f^(3/2)+2/3*b^2*(-e)^(3/2)*m*n^2*polylog(3,x*f^(1/2)/(-e)^(1/2))/f
^(3/2)
```

Rubi [A] time = 1.07, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.607, Rules used = {2305, 2304, 2378, 302, 205, 2351, 2295, 2324, 12, 4848, 2391, 2353, 2296, 2330, 2317, 2374, 6589}

$$\frac{2b(-e)^{3/2}mn\text{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\left(cx^n\right)\right)}{3f^{3/2}}-\frac{2b(-e)^{3/2}mn\text{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\left(cx^n\right)\right)}{3f^{3/2}}-\frac{2ib^2e^{3/2}mn^2\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-16*a*b*e*m*n*x)/(9*f) + (52*b^2*e*m*n^2*x)/(27*f) - (4*b^2*m*n^2*x^3)/27
 - (4*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(27*f^(3/2)) - (16*b^2*
e*m*n*x*Log[c*x^n])/(9*f) + (8*b*m*n*x^3*(a + b*Log[c*x^n]))/27 + (4*b*e^(3
/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*f^(3/2)) + (2*e*
m*x*(a + b*Log[c*x^n])^2)/(3*f) - (2*m*x^3*(a + b*Log[c*x^n])^2)/9 - ((-e)^
(3/2)*m*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(3*f^(3/2)) + (
(-e)^(3/2)*m*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/(3*f^(3/2)
) + (2*b^2*n^2*x^3*Log[d*(e + f*x^2)^m])/27 - (2*b*n*x^3*(a + b*Log[c*x^n])
*Log[d*(e + f*x^2)^m])/9 + (x^3*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/
3 + (2*b*(-e)^(3/2)*m*n*(a + b*Log[c*x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e
])])/(3*f^(3/2)) - (2*b*(-e)^(3/2)*m*n*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[
f]*x)/Sqrt[-e]])/(3*f^(3/2)) - (((2*I)/9)*b^2*e^(3/2)*m*n^2*PolyLog[2, ((-I
)*Sqrt[f]*x)/Sqrt[e]])/f^(3/2) + (((2*I)/9)*b^2*e^(3/2)*m*n^2*PolyLog[2, (I
*Sqrt[f]*x)/Sqrt[e]])/f^(3/2) - (2*b^2*(-e)^(3/2)*m*n^2*PolyLog[3, -((Sqrt[
f]*x)/Sqrt[-e])])/(3*f^(3/2)) + (2*b^2*(-e)^(3/2)*m*n^2*PolyLog[3, (Sqrt[f]
*x)/Sqrt[-e]])/(3*f^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
```

```
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)dx=\frac{2}{27}b^2n^2x^3\log\left(d\left(e+fx^2\right)^m\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(d\right.$$

$$=\frac{2}{27}b^2n^2x^3\log\left(d\left(e+fx^2\right)^m\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(d\right.$$

$$=\frac{2}{27}b^2n^2x^3\log\left(d\left(e+fx^2\right)^m\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\log\left(d\right.$$

$$=\frac{4b^2emn^2x}{27f}-\frac{4}{81}b^2mn^2x^3+\frac{2}{27}b^2n^2x^3\log\left(d\left(e+fx^2\right)^m\right)-\frac{2}{9}b$$

$$=-\frac{4abemnx}{9f}+\frac{4b^2emn^2x}{27f}-\frac{8}{81}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan^{-1}\left(\right.}{27f^{3/2}}$$

$$=-\frac{16abemnx}{9f}+\frac{16b^2emn^2x}{27f}-\frac{4}{27}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan^{-1}}{27f^{3/2}}$$

$$=-\frac{16abemnx}{9f}+\frac{52b^2emn^2x}{27f}-\frac{4}{27}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan^{-1}}{27f^{3/2}}$$

$$=-\frac{16abemnx}{9f}+\frac{52b^2emn^2x}{27f}-\frac{4}{27}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan^{-1}}{27f^{3/2}}$$

$$=-\frac{16abemnx}{9f}+\frac{52b^2emn^2x}{27f}-\frac{4}{27}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan^{-1}}{27f^{3/2}}$$

Mathematica [A] time = 0.45, size = 1128, normalized size = 1.79

$$\frac{-4b^2f^{3/2}mn^2x^3-6b^2f^{3/2}m\log^2\left(cx^n\right)x^3-6a^2f^{3/2}mx^3+8abf^{3/2}mnx^3-12abf^{3/2}m\log\left(cx^n\right)x^3+8b^2f^{3/2}mn\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (18*a^2*e*Sqrt[f]*m*x - 48*a*b*e*Sqrt[f]*m*n*x + 52*b^2*e*Sqrt[f]*m*n^2*x -
 6*a^2*f^(3/2)*m*x^3 + 8*a*b*f^(3/2)*m*n*x^3 - 4*b^2*f^(3/2)*m*n^2*x^3 - 18
*a^2*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 12*a*b*e^(3/2)*m*n*ArcTan[(Sqr
t[f]*x)/Sqrt[e]] - 4*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 36*a*b
*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*b^2*e^(3/2)*m*n^2*ArcT
an[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 18*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sq
rt[e]]*Log[x]^2 + 36*a*b*e*Sqrt[f]*m*x*Log[c*x^n] - 48*b^2*e*Sqrt[f]*m*n*x*
Log[c*x^n] - 12*a*b*f^(3/2)*m*x^3*Log[c*x^n] + 8*b^2*f^(3/2)*m*n*x^3*Log[c*
x^n] - 36*a*b*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 12*b^2*e^(
3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 36*b^2*e^(3/2)*m*n*ArcTan
[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 18*b^2*e*Sqrt[f]*m*x*Log[c*x^n]^2
 - 6*b^2*f^(3/2)*m*x^3*Log[c*x^n]^2 - 18*b^2*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/S
qrt[e]]*Log[c*x^n]^2 - (18*I)*a*b*e^(3/2)*m*n*Log[x]*Log[1 - (I*Sqrt[f]*x)/
Sqrt[e]] + (6*I)*b^2*e^(3/2)*m*n^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] +
(9*I)*b^2*e^(3/2)*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^
```

```
2*e^(3/2)*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*a*b
*e^(3/2)*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^2*e^(3/2)*m*n^
2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b^2*e^(3/2)*m*n^2*Log[x]^2*
Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^2*e^(3/2)*m*n*Log[x]*Log[c*x^n]*L
og[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 9*a^2*f^(3/2)*x^3*Log[d*(e + f*x^2)^m] - 6*
a*b*f^(3/2)*n*x^3*Log[d*(e + f*x^2)^m] + 2*b^2*f^(3/2)*n^2*x^3*Log[d*(e + f
*x^2)^m] + 18*a*b*f^(3/2)*x^3*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 6*b^2*f^(3/
2)*n*x^3*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 9*b^2*f^(3/2)*x^3*Log[c*x^n]^2*L
og[d*(e + f*x^2)^m] + (6*I)*b*e^(3/2)*m*n*(3*a - b*n + 3*b*Log[c*x^n])*Poly
Log[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b*e^(3/2)*m*n*(-3*a + b*n - 3*b*Lo
g[c*x^n])*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^2*e^(3/2)*m*n^2*Poly
Log[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^2*e^(3/2)*m*n^2*PolyLog[3, (I*S
qrt[f]*x)/Sqrt[e]])/(27*f^(3/2))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^2\log\left(cx^n\right)^2+2\,abx^2\log\left(cx^n\right)+a^2x^2\right)\log\left(\left(fx^2+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log((f*x^2
 + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^2x^2\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*x^2 + e)^m*d), x)
```

maple [F] time = 111.55, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^2x^2\ln\left(d\left(f\,x^2+e\right)^m\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{27}\left(9\,b^2mx^3\log\left(x^n\right)^2-6\left(\left(mn-3\,m\log(c)\right)b^2-3\,abm\right)x^3\log\left(x^n\right)-\left(6\left(mn-3\,m\log(c)\right)ab-\left(2\,mn^2-6\,mn\log\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/27*(9*b^2*m*x^3*log(x^n)^2 - 6*((m*n - 3*m*log(c))*b^2 - 3*a*b*m)*x^3*log
(x^n) - (6*(m*n - 3*m*log(c))*a*b - (2*m*n^2 - 6*m*n*log(c) + 9*m*log(c)^2)
*b^2 - 9*a^2*m)*x^3)*log(f*x^2 + e) + integrate(-1/27*((9*(2*f*m - 3*f*log(
d))*a^2 - 6*(2*f*m*n - 3*(2*f*m - 3*f*log(d))*log(c))*a*b + (4*f*m*n^2 - 12
*f*m*n*log(c) + 9*(2*f*m - 3*f*log(d))*log(c)^2)*b^2)*x^4 - 27*(b^2*e*log(c
)^2*log(d) + 2*a*b*e*log(c)*log(d) + a^2*e*log(d))*x^2 + 9*((2*f*m - 3*f*lo
```

```
g(d))*b^2*x^4 - 3*b^2*e*x^2*log(d))*log(x^n)^2 + 6*((3*(2*f*m - 3*f*log(d))
*a*b - (2*f*m*n - 3*(2*f*m - 3*f*log(d))*log(c))*b^2)*x^4 - 9*(b^2*e*log(c)
*log(d) + a*b*e*log(d))*x^2)*log(x^n))/(f*x^2 + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln\left(d\left(f x^2+e\right)^m\right) (a+b \ln(c x^n))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

3.105 $\int \left(a+b\log\left(cx^n\right)\right)^2 \log\left(d\left(e+fx^2\right)^m\right)\,dx$

Optimal. Leaf size=546

$$x\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)+\frac{2b\sqrt{-e}\,mn\text{Li}_2\left(-\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{f}}-\frac{2b\sqrt{-e}\,mn\text{Li}_2\left(\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\left(\right.\right.}{\sqrt{f}}$$

```
[Out] 4*a*b*m*n*x-8*b^2*m*n^2*x+4*b*m*n*(-b*n+a)*x+8*b^2*m*n*x*ln(c*x^n)-2*m*x*(a
+b*ln(c*x^n))^2-2*a*b*n*x*ln(d*(f*x^2+e)^m)+2*b^2*n^2*x*ln(d*(f*x^2+e)^m)-2
*b^2*n*x*ln(c*x^n)*ln(d*(f*x^2+e)^m)+x*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)-
m*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+m*(a+b*ln
(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+2*b*m*n*(a+b*ln(c*
x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-2*b*m*n*(a+b*ln(c
*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-2*b^2*m*n^2*polyl
og(3,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+2*b^2*m*n^2*polylog(3,x*f^(1
/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-4*b*m*n*(-b*n+a)*arctan(x*f^(1/2)/e^(1/2
))*e^(1/2)/f^(1/2)-4*b^2*m*n*arctan(x*f^(1/2)/e^(1/2))*ln(c*x^n)*e^(1/2)/f^
(1/2)+2*I*b^2*m*n^2*polylog(2,-I*x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)-2*I*b^2
*m*n^2*polylog(2,I*x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {2296, 2295, 2371, 6, 321, 205, 2351, 2324, 12, 4848, 2391, 2353, 2330, 2317, 2374, 6589}

$$\frac{2b\sqrt{-e}\,mn\text{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{f}}-\frac{2b\sqrt{-e}\,mn\text{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{f}}+\frac{2ib^2\sqrt{e}\,mn^2\text{Poly}}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] 4*a*b*m*n*x - 8*b^2*m*n^2*x + 4*b*m*n*(a - b*n)*x - (4*b*Sqrt[e]*m*n*(a - b
*n)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] + 8*b^2*m*n*x*Log[c*x^n] - (4*b^2*
Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n])/Sqrt[f] - 2*m*x*(a + b*
Log[c*x^n])^2 - (Sqrt[-e]*m*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-
e]])/Sqrt[f] + (Sqrt[-e]*m*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e
]])/Sqrt[f] - 2*a*b*n*x*Log[d*(e + f*x^2)^m] + 2*b^2*n^2*x*Log[d*(e + f*x^2
)^m] - 2*b^2*n*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] + x*(a + b*Log[c*x^n])^2*L
og[d*(e + f*x^2)^m] + (2*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])*PolyLog[2, -((Sq
rt[f]*x)/Sqrt[-e])])/Sqrt[f] - (2*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])*PolyLog
[2, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[f] + ((2*I)*b^2*Sqrt[e]*m*n^2*PolyLog[2, ((
-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] - ((2*I)*b^2*Sqrt[e]*m*n^2*PolyLog[2, (I*S
qrt[f]*x)/Sqrt[e]])/Sqrt[f] - (2*b^2*Sqrt[-e]*m*n^2*PolyLog[3, -((Sqrt[f]*x
)/Sqrt[-e])])/Sqrt[f] + (2*b^2*Sqrt[-e]*m*n^2*PolyLog[3, (Sqrt[f]*x)/Sqrt[-
e]])/Sqrt[f]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2371

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)\,dx = -2abnx\log\left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-2b^2nx$$

$$= -2abnx\log\left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-2b^2nx$$

$$= -2abnx\log\left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-2b^2nx$$

$$= 4bmn(a-bn)x-2abnx\log\left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log\left(d\left(e+\right.\right.$$

$$= 4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}-2abnx\log\left(d\right.$$

$$= -4b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}+$$

$$= 4abmnx-4b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan^{-1}}{\sqrt{f}}$$

$$= 4abmnx-8b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan^{-1}}{\sqrt{f}}$$

$$= 4abmnx-8b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan^{-1}}{\sqrt{f}}$$

$$= 4abmnx-8b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan^{-1}}{\sqrt{f}}$$

Mathematica [A] time = 0.34, size = 993, normalized size = 1.82

$$\frac{-2\sqrt{f}\,mxa^2+2\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)a^2+\sqrt{f}\,x\log\left(d\left(fx^2+e\right)^m\right)a^2+8b\sqrt{f}\,mnxa-4b\sqrt{e}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)a-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-2*a^2*Sqrt[f]*m*x + 8*a*b*Sqrt[f]*m*n*x - 12*b^2*Sqrt[f]*m*n^2*x + 2*a^2*
Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*a*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x
)/Sqrt[e]] + 4*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*a*b*Sqrt[e
]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] + 4*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt
[f]*x)/Sqrt[e]]*Log[x] + 2*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Lo
g[x]^2 - 4*a*b*Sqrt[f]*m*x*Log[c*x^n] + 8*b^2*Sqrt[f]*m*n*x*Log[c*x^n] + 4*
a*b*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 4*b^2*Sqrt[e]*m*n*Ar
cTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 4*b^2*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)
/Sqrt[e]]*Log[x]*Log[c*x^n] - 2*b^2*Sqrt[f]*m*x*Log[c*x^n]^2 + 2*b^2*Sqrt[e
]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + (2*I)*a*b*Sqrt[e]*m*n*Log[x]
*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqrt[e]*m*n^2*Log[x]*Log[1 - (I
```

```
*Sqrt[f]*x)/Sqrt[e]] - I*b^2*Sqrt[e]*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/S
qrt[e]] + (2*I)*b^2*Sqrt[e]*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqr
t[e]] - (2*I)*a*b*Sqrt[e]*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (2*I)
*b^2*Sqrt[e]*m*n^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + I*b^2*Sqrt[e]*m*
n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqrt[e]*m*n*Log[x]*
Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + a^2*Sqrt[f]*x*Log[d*(e + f*x^2)
^m] - 2*a*b*Sqrt[f]*n*x*Log[d*(e + f*x^2)^m] + 2*b^2*Sqrt[f]*n^2*x*Log[d*(e
 + f*x^2)^m] + 2*a*b*Sqrt[f]*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 2*b^2*Sqrt
[f]*n*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] + b^2*Sqrt[f]*x*Log[c*x^n]^2*Log[d*
(e + f*x^2)^m] - (2*I)*b*Sqrt[e]*m*n*(a - b*n + b*Log[c*x^n])*PolyLog[2, ((
-I)*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b*Sqrt[e]*m*n*(a - b*n + b*Log[c*x^n])*Poly
Log[2, (I*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[e]*m*n^2*PolyLog[3, ((-I)*Sq
rt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqrt[e]*m*n^2*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e
]])/Sqrt[f]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\log\left(cx^n\right)^2+2\,ab\log\left(cx^n\right)+a^2\right)\log\left(\left(fx^2+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d), x)
```

maple [F] time = 130.68, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^2\ln\left(d\left(f\,x^2+e\right)^m\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m),x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(b^2mx\log\left(x^n\right)^2-2\left(\left(mn-m\log(c)\right)b^2-abm\right)x\log\left(x^n\right)-\left(2\left(mn-m\log(c)\right)ab-\left(2\,mn^2-2\,mn\log(c)+m\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] (b^2*m*x*log(x^n)^2 - 2*((m*n - m*log(c))*b^2 - a*b*m)*x*log(x^n) - (2*(m*n
 - m*log(c))*a*b - (2*m*n^2 - 2*m*n*log(c) + m*log(c)^2)*b^2 - a^2*m)*x)*lo
g(f*x^2 + e) + integrate((b^2*e*log(c)^2*log(d) + 2*a*b*e*log(c)*log(d) + a
^2*e*log(d) - ((2*f*m - f*log(d))*a^2 - 2*(2*f*m*n - (2*f*m - f*log(d))*log
(c))*a*b + (4*f*m*n^2 - 4*f*m*n*log(c) + (2*f*m - f*log(d))*log(c)^2)*b^2)*
x^2 - ((2*f*m - f*log(d))*b^2*x^2 - b^2*e*log(d))*log(x^n)^2 + 2*(b^2*e*log
```

```
(c)*log(d) + a*b*e*log(d) - ((2*f*m - f*log(d))*a*b - (2*f*m*n - (2*f*m - f
*log(d))*log(c))*b^2)*x^2)*log(x^n))/(f*x^2 + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

3.106 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx$

Optimal. Leaf size=478

$$-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{x}-\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{x}-\frac{2b\sqrt{f}\,mn\text{Li}_2\left(-\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\right.}{\sqrt{-e}}$$

```
[Out] -2*b^2*n^2*ln(d*(f*x^2+e)^m)/x-2*b*n*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x-(a
+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x+m*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)
^(1/2))*f^(1/2)/(-e)^(1/2)-m*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))*f
^(1/2)/(-e)^(1/2)-2*b*m*n*(a+b*ln(c*x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))*
f^(1/2)/(-e)^(1/2)+2*b*m*n*(a+b*ln(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))*
f^(1/2)/(-e)^(1/2)+2*b^2*m*n^2*polylog(3,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e
)^(1/2)-2*b^2*m*n^2*polylog(3,x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+4*b^
2*m*n^2*arctan(x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1/2)+4*b*m*n*arctan(x*f^(1/2)/
e^(1/2))*(a+b*ln(c*x^n))*f^(1/2)/e^(1/2)-2*I*b^2*m*n^2*polylog(2,-I*x*f^(1/
2)/e^(1/2))*f^(1/2)/e^(1/2)+2*I*b^2*m*n^2*polylog(2,I*x*f^(1/2)/e^(1/2))*f^
(1/2)/e^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2305, 2304, 2378, 205, 2324, 12, 4848, 2391, 2330, 2317, 2374, 6589}

$$-\frac{2b\sqrt{f}\,mn\text{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{-e}}+\frac{2b\sqrt{f}\,mn\text{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{-e}}-\frac{2ib^2\sqrt{f}\,mn^2\text{Poly}}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (4*b^2*Sqrt[f]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + (4*b*Sqrt[f]*m*
n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/Sqrt[e] + (Sqrt[f]*m*(a +
 b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] - (Sqrt[f]*m*(a +
b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] - (2*b^2*n^2*Log[d*
(e + f*x^2)^m])/x - (2*b*n*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x - ((a
 + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x - (2*b*Sqrt[f]*m*n*(a + b*Log[c*
x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[-e] + (2*b*Sqrt[f]*m*n*(a +
 b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] - ((2*I)*b^2*Sqrt
[f]*m*n^2*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + ((2*I)*b^2*Sqrt[f
]*m*n^2*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + (2*b^2*Sqrt[f]*m*n^2*P
olyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[-e] - (2*b^2*Sqrt[f]*m*n^2*PolyLog
[3, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx = -\frac{2b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{x} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)\right.}{x}$$

$$= -\frac{2b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{x} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)\right.}{x}$$

$$= \frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}}$$

$$= \frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}}$$

$$= \frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}}$$

$$= \frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}}$$

$$= \frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}}$$

Mathematica [A] time = 0.33, size = 917, normalized size = 1.92

$$\frac{2\sqrt{f}\,mx\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)a^2 - \sqrt{e}\log\left(d\left(fx^2+e\right)^m\right)a^2 + 4b\sqrt{f}\,mnx\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)a - 4b\sqrt{f}\,mnx\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(x)a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (2*a^2*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 4*a*b*Sqrt[f]*m*n*x*ArcTan
[(Sqrt[f]*x)/Sqrt[e]] + 4*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] -
 4*a*b*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 4*b^2*Sqrt[f]*m*n
^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] + 2*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqr
t[f]*x)/Sqrt[e]]*Log[x]^2 + 4*a*b*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*L
og[c*x^n] + 4*b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 4*
b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 2*b^2*Sqr
t[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + (2*I)*a*b*Sqrt[f]*m*n*x
*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[f]*m*n^2*x*Log[x]*L
og[1 - (I*Sqrt[f]*x)/Sqrt[e]] - I*b^2*Sqrt[f]*m*n^2*x*Log[x]^2*Log[1 - (I*S
qrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]*Log[1 - (I*S
qrt[f]*x)/Sqrt[e]] - (2*I)*a*b*Sqrt[f]*m*n*x*Log[x]*Log[1 + (I*Sqrt[f]*x)/S
qrt[e]] - (2*I)*b^2*Sqrt[f]*m*n^2*x*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] +
 I*b^2*Sqrt[f]*m*n^2*x*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*
Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - a^2*Sqrt[e
```

```
]*Log[d*(e + f*x^2)^m] - 2*a*b*Sqrt[e]*n*Log[d*(e + f*x^2)^m] - 2*b^2*Sqrt[
e]*n^2*Log[d*(e + f*x^2)^m] - 2*a*b*Sqrt[e]*Log[c*x^n]*Log[d*(e + f*x^2)^m]
 - 2*b^2*Sqrt[e]*n*Log[c*x^n]*Log[d*(e + f*x^2)^m] - b^2*Sqrt[e]*Log[c*x^n]
^2*Log[d*(e + f*x^2)^m] - (2*I)*b*Sqrt[f]*m*n*x*(a + b*n + b*Log[c*x^n])*Po
lyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b*Sqrt[f]*m*n*x*(a + b*n + b*Log
[c*x^n])*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[f]*m*n^2*x*Poly
Log[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqrt[f]*m*n^2*x*PolyLog[3, (I*
Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*x)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\log\left(cx^n\right)^2+2\,ab\log\left(cx^n\right)+a^2\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/x
^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(fx^2+e\right)^m d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^2, x)
```

maple [F] time = 69.22, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right)+a\right)^2\ln\left(d\left(f\,x^2+e\right)^m\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^2m\log\left(x^n\right)^2+2\left(mn+m\log(c)\right)ab+\left(2\,mn^2+2\,mn\log(c)+m\log(c)^2\right)b^2+a^2m+2\left(\left(mn+m\log(c)\right)b^2\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -(b^2*m*log(x^n)^2 + 2*(m*n + m*log(c))*a*b + (2*m*n^2 + 2*m*n*log(c) + m*l
og(c)^2)*b^2 + a^2*m + 2*((m*n + m*log(c))*b^2 + a*b*m)*log(x^n))*log(f*x^2
 + e)/x + integrate((b^2*e*log(c)^2*log(d) + 2*a*b*e*log(c)*log(d) + a^2*e*
log(d) + ((2*f*m + f*log(d))*a^2 + 2*(2*f*m*n + (2*f*m + f*log(d))*log(c))*
a*b + (4*f*m*n^2 + 4*f*m*n*log(c) + (2*f*m + f*log(d))*log(c)^2)*b^2)*x^2 +
 ((2*f*m + f*log(d))*b^2*x^2 + b^2*e*log(d))*log(x^n)^2 + 2*(b^2*e*log(c)*l
og(d) + a*b*e*log(d) + ((2*f*m + f*log(d))*a*b + (2*f*m*n + (2*f*m + f*log(
d))*log(c))*b^2)*x^2)*log(x^n))/(f*x^4 + e*x^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,(a+b\,\ln(c\,x^n))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^2,x)
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**2,x)
[Out] Timed out
```

3.107 $\int \frac{\left(a+b\log(cx^n)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx$

Optimal. Leaf size=571

$$-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{9x^3}-\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{3x^3}-\frac{4bf^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+\right.}{9e^{3/2}}$$

```
[Out] -52/27*b^2*f*m*n^2/e/x-4/27*b^2*f^(3/2)*m*n^2*arctan(x*f^(1/2)/e^(1/2))/e^(
3/2)-16/9*b*f*m*n*(a+b*ln(c*x^n))/e/x-4/9*b*f^(3/2)*m*n*arctan(x*f^(1/2)/e^
(1/2))*(a+b*ln(c*x^n))/e^(3/2)-2/3*f*m*(a+b*ln(c*x^n))^2/e/x-2/27*b^2*n^2*l
n(d*(f*x^2+e)^m)/x^3-2/9*b*n*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^3-1/3*(a+b
*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^3+1/3*f^(3/2)*m*(a+b*ln(c*x^n))^2*ln(1-x*
f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-1/3*f^(3/2)*m*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/
2)/(-e)^(1/2))/(-e)^(3/2)-2/3*b*f^(3/2)*m*n*(a+b*ln(c*x^n))*polylog(2,-x*f^
(1/2)/(-e)^(1/2))/(-e)^(3/2)+2/3*b*f^(3/2)*m*n*(a+b*ln(c*x^n))*polylog(2,x*
f^(1/2)/(-e)^(1/2))/(-e)^(3/2)+2/9*I*b^2*f^(3/2)*m*n^2*polylog(2,-I*x*f^(1/
2)/e^(1/2))/e^(3/2)-2/9*I*b^2*f^(3/2)*m*n^2*polylog(2,I*x*f^(1/2)/e^(1/2))/
e^(3/2)+2/3*b^2*f^(3/2)*m*n^2*polylog(3,-x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-2
/3*b^2*f^(3/2)*m*n^2*polylog(3,x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)
```

Rubi [A] time = 0.93, antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {2305, 2304, 2378, 325, 205, 2351, 2324, 12, 4848, 2391, 2353, 2330, 2317, 2374, 6589}

$$-\frac{2bf^{3/2}mn\text{PolyLog}\left(2,-\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\left(cx^n\right)\right)}{3(-e)^{3/2}}+\frac{2bf^{3/2}mn\text{PolyLog}\left(2,\frac{\sqrt{f}x}{\sqrt{-e}}\right)\left(a+b\log\left(cx^n\right)\right)}{3(-e)^{3/2}}+\frac{2ib^2f^{3/2}mn^2\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^4,x]
```

```
[Out] (-52*b^2*f*m*n^2)/(27*e*x) - (4*b^2*f^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e
]])/(27*e^(3/2)) - (16*b*f*m*n*(a + b*Log[c*x^n]))/(9*e*x) - (4*b*f^(3/2)*m
*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*e^(3/2)) - (2*f*m*(a
+ b*Log[c*x^n])^2)/(3*e*x) + (f^(3/2)*m*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[
f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) - (f^(3/2)*m*(a + b*Log[c*x^n])^2*Log[1 + (
Sqrt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) - (2*b^2*n^2*Log[d*(e + f*x^2)^m])/(27
*x^3) - (2*b*n*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(9*x^3) - ((a + b*L
og[c*x^n])^2*Log[d*(e + f*x^2)^m])/(3*x^3) - (2*b*f^(3/2)*m*n*(a + b*Log[c*
x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3/2)) + (2*b*f^(3/2)*m*
n*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) + (((
2*I)/9)*b^2*f^(3/2)*m*n^2*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) - (
((2*I)/9)*b^2*f^(3/2)*m*n^2*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) + (2
*b^2*f^(3/2)*m*n^2*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3/2)) - (2
*b^2*f^(3/2)*m*n^2*PolyLog[3, (Sqrt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx = -\frac{2b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{27x^3} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)\right.}{9x^3}$$

$$= -\frac{2b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{27x^3} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)\right.}{9x^3}$$

$$= -\frac{4b^2fmn^2}{27ex} - \frac{2b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{27x^3} - \frac{2bn\left(a+b\log\left(cx^n\right)\right)\log}{9x^3}$$

$$= -\frac{4b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{2b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{27x^3}$$

$$= -\frac{16b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{4bfmn\left(a+b\log\left(cx^n\right)\right)}{9ex}$$

$$= -\frac{52b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{16bfmn\left(a+b\log\left(cx^n\right)\right.}{9ex}$$

$$= -\frac{52b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{16bfmn\left(a+b\log\left(cx^n\right)\right.}{9ex}$$

$$= -\frac{52b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{16bfmn\left(a+b\log\left(cx^n\right)\right.}{9ex}$$

$$= -\frac{52b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{16bfmn\left(a+b\log\left(cx^n\right)\right.}{9ex}$$

<u>**Mathematica**</u> **[A]** time = 0.43, size = 1083, normalized size = 1.90

$$\frac{-18b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\log^2(x)x^3 - 18b^2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\log^2\left(cx^n\right)x^3 - 4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)x^3 - 18a^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^4,x]
```

```
[Out] (-18*a^2*Sqrt[e]*f*m*x^2 - 48*a*b*Sqrt[e]*f*m*n*x^2 - 52*b^2*Sqrt[e]*f*m*n^
2*x^2 - 18*a^2*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 12*a*b*f^(3/2)*m
*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f
]*x)/Sqrt[e]] + 36*a*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] +
 12*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 18*b^2*f^(3/
2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 36*a*b*Sqrt[e]*f*m*x^2*
Log[c*x^n] - 48*b^2*Sqrt[e]*f*m*n*x^2*Log[c*x^n] - 36*a*b*f^(3/2)*m*x^3*Arc
Tan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 12*b^2*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f
]*x)/Sqrt[e]]*Log[c*x^n] + 36*b^2*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e
]]*Log[x]*Log[c*x^n] - 18*b^2*Sqrt[e]*f*m*x^2*Log[c*x^n]^2 - 18*b^2*f^(3/2)
*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - (18*I)*a*b*f^(3/2)*m*n*x^
3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^2*f^(3/2)*m*n^2*x^3*Log[x
]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (9*I)*b^2*f^(3/2)*m*n^2*x^3*Log[x]^2*Log
[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^2*f^(3/2)*m*n*x^3*Log[x]*Log[c*x^n]*
Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*a*b*f^(3/2)*m*n*x^3*Log[x]*Log[1 +
```

```
(I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^2*f^(3/2)*m*n^2*x^3*Log[x]*Log[1 + (I*Sqrt
[f]*x)/Sqrt[e]] - (9*I)*b^2*f^(3/2)*m*n^2*x^3*Log[x]^2*Log[1 + (I*Sqrt[f]*x
)/Sqrt[e]] + (18*I)*b^2*f^(3/2)*m*n*x^3*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f
]*x)/Sqrt[e]] - 9*a^2*e^(3/2)*Log[d*(e + f*x^2)^m] - 6*a*b*e^(3/2)*n*Log[d*
(e + f*x^2)^m] - 2*b^2*e^(3/2)*n^2*Log[d*(e + f*x^2)^m] - 18*a*b*e^(3/2)*Lo
g[c*x^n]*Log[d*(e + f*x^2)^m] - 6*b^2*e^(3/2)*n*Log[c*x^n]*Log[d*(e + f*x^2
)^m] - 9*b^2*e^(3/2)*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] + (6*I)*b*f^(3/2)*m*
n*x^3*(3*a + b*n + 3*b*Log[c*x^n])*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (
6*I)*b*f^(3/2)*m*n*x^3*(3*a + b*n + 3*b*Log[c*x^n])*PolyLog[2, (I*Sqrt[f]*x
)/Sqrt[e]] - (18*I)*b^2*f^(3/2)*m*n^2*x^3*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[
e]] + (18*I)*b^2*f^(3/2)*m*n^2*x^3*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]])/(27*e
^(3/2)*x^3)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\log\left(cx^n\right)^2+2\,ab\log\left(cx^n\right)+a^2\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/x
^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(fx^2+e\right)^m d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^4, x)
```

maple [F] time = 113.91, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^n\right)+a\right)^2\ln\left(d\left(f\,x^2+e\right)^m\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m)/x^4,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^2+e)^m)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(9\,b^2m\log\left(x^n\right)^2+6\left(mn+3\,m\log(c)\right)ab+\left(2\,mn^2+6\,mn\log(c)+9\,m\log(c)^2\right)b^2+9\,a^2m+6\left(\left(mn+3\,m\,\mathrm{l}\right.\right.}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(9*b^2*m*log(x^n)^2 + 6*(m*n + 3*m*log(c))*a*b + (2*m*n^2 + 6*m*n*log
(c) + 9*m*log(c)^2)*b^2 + 9*a^2*m + 6*((m*n + 3*m*log(c))*b^2 + 3*a*b*m)*lo
g(x^n))*log(f*x^2 + e)/x^3 + integrate(1/27*(27*b^2*e*log(c)^2*log(d) + 54*
a*b*e*log(c)*log(d) + 27*a^2*e*log(d) + (9*(2*f*m + 3*f*log(d))*a^2 + 6*(2*
```

```
f*m*n + 3*(2*f*m + 3*f*log(d))*log(c))*a*b + (4*f*m*n^2 + 12*f*m*n*log(c) +
 9*(2*f*m + 3*f*log(d))*log(c)^2)*b^2)*x^2 + 9*((2*f*m + 3*f*log(d))*b^2*x^
2 + 3*b^2*e*log(d))*log(x^n)^2 + 6*(9*b^2*e*log(c)*log(d) + 9*a*b*e*log(d)
+ (3*(2*f*m + 3*f*log(d))*a*b + (2*f*m*n + 3*(2*f*m + 3*f*log(d))*log(c))*b
^2)*x^2)*log(x^n))/(f*x^6 + e*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^4,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**4,x)
```

```
[Out] Timed out
```

3.108 $\int x\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)dx$

Optimal. Leaf size=514

$$\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)-\frac{3b^2emn^2\text{Li}_2\left(-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{4f}-\frac{3b^2emn^2\text{Li}_3\left(-\frac{fx^2}{e}\right)\left(a+b\right.}{4f}$$

```
[Out] 3/2*b^3*m*n^3*x^2-9/4*b^2*m*n^2*x^2*(a+b*ln(c*x^n))+3/2*b*m*n*x^2*(a+b*ln(c
*x^n))^2-1/2*m*x^2*(a+b*ln(c*x^n))^3-3/8*b^3*e*m*n^3*ln(f*x^2+e)/f-3/8*b^3*
n^3*x^2*ln(d*(f*x^2+e)^m)+3/4*b^2*n^2*x^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)
-3/4*b*n*x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)+1/2*x^2*(a+b*ln(c*x^n))^3*
ln(d*(f*x^2+e)^m)+3/4*b^2*e*m*n^2*(a+b*ln(c*x^n))*ln(1+f*x^2/e)/f-3/4*b*e*m
*n*(a+b*ln(c*x^n))^2*ln(1+f*x^2/e)/f+1/2*e*m*(a+b*ln(c*x^n))^3*ln(1+f*x^2/e
)/f+3/8*b^3*e*m*n^3*polylog(2,-f*x^2/e)/f-3/4*b^2*e*m*n^2*(a+b*ln(c*x^n))*p
olylog(2,-f*x^2/e)/f+3/4*b*e*m*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^2/e)/f+3/
8*b^3*e*m*n^3*polylog(3,-f*x^2/e)/f-3/4*b^2*e*m*n^2*(a+b*ln(c*x^n))*polylog
(3,-f*x^2/e)/f+3/8*b^3*e*m*n^3*polylog(4,-f*x^2/e)/f
```

Rubi [A] time = 0.92, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2305, 2304, 2378, 266, 43, 2351, 2337, 2391, 2353, 2374, 6589, 2383}

$$-\frac{3b^2emn^2\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{4f}-\frac{3b^2emn^2\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{4f}+\frac{3bemn\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (3*b^3*m*n^3*x^2)/2 - (9*b^2*m*n^2*x^2*(a + b*Log[c*x^n]))/4 + (3*b*m*n*x^2
*(a + b*Log[c*x^n])^2)/2 - (m*x^2*(a + b*Log[c*x^n])^3)/2 - (3*b^3*e*m*n^3*
Log[e + f*x^2])/(8*f) - (3*b^3*n^3*x^2*Log[d*(e + f*x^2)^m])/8 + (3*b^2*n^2
*x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/4 - (3*b*n*x^2*(a + b*Log[c*x
^n])^2*Log[d*(e + f*x^2)^m])/4 + (x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2
)^m])/2 + (3*b^2*e*m*n^2*(a + b*Log[c*x^n])*Log[1 + (f*x^2)/e])/(4*f) - (3*
b*e*m*n*(a + b*Log[c*x^n])^2*Log[1 + (f*x^2)/e])/(4*f) + (e*m*(a + b*Log[c*
x^n])^3*Log[1 + (f*x^2)/e])/(2*f) + (3*b^3*e*m*n^3*PolyLog[2, -((f*x^2)/e)]
)/(8*f) - (3*b^2*e*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*x^2)/e)])/(4*f)
 + (3*b*e*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x^2)/e)])/(4*f) + (3*b^3
*e*m*n^3*PolyLog[3, -((f*x^2)/e)])/(8*f) - (3*b^2*e*m*n^2*(a + b*Log[c*x^n]
)*PolyLog[3, -((f*x^2)/e)])/(4*f) + (3*b^3*e*m*n^3*PolyLog[4, -((f*x^2)/e)]
)/(8*f)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)dx=-\frac{3}{8}b^3n^3x^2\log\left(d\left(e+fx^2\right)^m\right)+\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)\log\left(\right.$$

$$=-\frac{3}{8}b^3n^3x^2\log\left(d\left(e+fx^2\right)^m\right)+\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)\log\left(\right.$$

$$=-\frac{3}{8}b^3n^3x^2\log\left(d\left(e+fx^2\right)^m\right)+\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)\log\left(\right.$$

$$=-\frac{3}{8}b^3n^3x^2\log\left(d\left(e+fx^2\right)^m\right)+\frac{3}{4}b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)\log\left(\right.$$

$$=\frac{3}{4}b^3mn^3x^2-\frac{3}{4}b^2mn^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{4}bmnx^2\left(a+b\log\right.$$

$$=\frac{9}{8}b^3mn^3x^2-\frac{3}{2}b^2mn^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bmnx^2\left(a+b\log\right.$$

$$=\frac{3}{2}b^3mn^3x^2-\frac{9}{4}b^2mn^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bmnx^2\left(a+b\log\right.$$

$$=\frac{3}{2}b^3mn^3x^2-\frac{9}{4}b^2mn^2x^2\left(a+b\log\left(cx^n\right)\right)+\frac{3}{2}bmnx^2\left(a+b\log\right.$$

Mathematica [C] time = 0.58, size = 1911, normalized size = 3.72

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-4*a^3*f*m*x^2 + 12*a^2*b*f*m*n*x^2 - 18*a*b^2*f*m*n^2*x^2 + 12*b^3*f*m*n^
3*x^2 - 12*a^2*b*f*m*x^2*Log[c*x^n] + 24*a*b^2*f*m*n*x^2*Log[c*x^n] - 18*b^
3*f*m*n^2*x^2*Log[c*x^n] - 12*a*b^2*f*m*x^2*Log[c*x^n]^2 + 12*b^3*f*m*n*x^2
*Log[c*x^n]^2 - 4*b^3*f*m*x^2*Log[c*x^n]^3 + 12*a^2*b*e*m*n*Log[x]*Log[1 -
(I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*e*m*n^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt
[e]] + 6*b^3*e*m*n^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*e*m*n
^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*e*m*n^3*Log[x]^2*Log[1 -
 (I*Sqrt[f]*x)/Sqrt[e]] + 4*b^3*e*m*n^3*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt
[e]] + 24*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12
*b^3*e*m*n^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^3*e*m*
n^2*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*e*m*n*Log[x
```

```
]*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*a^2*b*e*m*n*Log[x]*Log[1
 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*e*m*n^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/S
qrt[e]] + 6*b^3*e*m*n^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*e*
m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*e*m*n^3*Log[x]^2*Log[
1 + (I*Sqrt[f]*x)/Sqrt[e]] + 4*b^3*e*m*n^3*Log[x]^3*Log[1 + (I*Sqrt[f]*x)/S
qrt[e]] + 24*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] -
 12*b^3*e*m*n^2*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^3*e
*m*n^2*Log[x]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*e*m*n*Lo
g[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 4*a^3*e*m*Log[e + f*x^2]
 - 6*a^2*b*e*m*n*Log[e + f*x^2] + 6*a*b^2*e*m*n^2*Log[e + f*x^2] - 3*b^3*e*
m*n^3*Log[e + f*x^2] - 12*a^2*b*e*m*n*Log[x]*Log[e + f*x^2] + 12*a*b^2*e*m*
n^2*Log[x]*Log[e + f*x^2] - 6*b^3*e*m*n^3*Log[x]*Log[e + f*x^2] + 12*a*b^2*
e*m*n^2*Log[x]^2*Log[e + f*x^2] - 6*b^3*e*m*n^3*Log[x]^2*Log[e + f*x^2] - 4
*b^3*e*m*n^3*Log[x]^3*Log[e + f*x^2] + 12*a^2*b*e*m*Log[c*x^n]*Log[e + f*x^
2] - 12*a*b^2*e*m*n*Log[c*x^n]*Log[e + f*x^2] + 6*b^3*e*m*n^2*Log[c*x^n]*Lo
g[e + f*x^2] - 24*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[e + f*x^2] + 12*b^3*e*m
*n^2*Log[x]*Log[c*x^n]*Log[e + f*x^2] + 12*b^3*e*m*n^2*Log[x]^2*Log[c*x^n]*
Log[e + f*x^2] + 12*a*b^2*e*m*Log[c*x^n]^2*Log[e + f*x^2] - 6*b^3*e*m*n*Log
[c*x^n]^2*Log[e + f*x^2] - 12*b^3*e*m*n*Log[x]*Log[c*x^n]^2*Log[e + f*x^2]
+ 4*b^3*e*m*Log[c*x^n]^3*Log[e + f*x^2] + 4*a^3*f*x^2*Log[d*(e + f*x^2)^m]
- 6*a^2*b*f*n*x^2*Log[d*(e + f*x^2)^m] + 6*a*b^2*f*n^2*x^2*Log[d*(e + f*x^2
)^m] - 3*b^3*f*n^3*x^2*Log[d*(e + f*x^2)^m] + 12*a^2*b*f*x^2*Log[c*x^n]*Log
[d*(e + f*x^2)^m] - 12*a*b^2*f*n*x^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 6*b^
3*f*n^2*x^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 12*a*b^2*f*x^2*Log[c*x^n]^2*L
og[d*(e + f*x^2)^m] - 6*b^3*f*n*x^2*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] + 4*b
^3*f*x^2*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] + 6*b*e*m*n*(2*a^2 - 2*a*b*n + b
^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, ((-I)
*Sqrt[f]*x)/Sqrt[e]] + 6*b*e*m*n*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b
*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] - 24
*a*b^2*e*m*n^2*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*e*m*n^3*PolyLo
g[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 24*b^3*e*m*n^2*Log[c*x^n]*PolyLog[3, ((-I)
*Sqrt[f]*x)/Sqrt[e]] - 24*a*b^2*e*m*n^2*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] +
 12*b^3*e*m*n^3*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - 24*b^3*e*m*n^2*Log[c*x^
n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + 24*b^3*e*m*n^3*PolyLog[4, ((-I)*Sqrt
[f]*x)/Sqrt[e]] + 24*b^3*e*m*n^3*PolyLog[4, (I*Sqrt[f]*x)/Sqrt[e]])/(8*f)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3x\log\left(cx^n\right)^3+3\,ab^2x\log\left(cx^n\right)^2+3\,a^2bx\log\left(cx^n\right)+a^3x\right)\log\left(\left(fx^2+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^n
) + a^3*x)*log((f*x^2 + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^3x\log\left(\left(fx^2+e\right)^m d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*x^2 + e)^m*d), x)
```

maple [F] time = 10.37, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^3x\ln\left(d\left(f\,x^2+e\right)^m\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(4\,b^{3}mx^{2}\log\left(x^{n}\right)^{3}-6\left(\left(mn-2\,m\log(c)\right)b^{3}-2\,ab^{2}m\right)x^{2}\log\left(x^{n}\right)^{2}-6\left(2\left(mn-2\,m\log(c)\right)ab^{2}-\left(mn^{2}-2\,n\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/8*(4*b^3*m*x^2*log(x^n)^3 - 6*((m*n - 2*m*log(c))*b^3 - 2*a*b^2*m)*x^2*lo
g(x^n)^2 - 6*(2*(m*n - 2*m*log(c))*a*b^2 - (m*n^2 - 2*m*n*log(c) + 2*m*log(
c)^2)*b^3 - 2*a^2*b*m)*x^2*log(x^n) - (6*(m*n - 2*m*log(c))*a^2*b - 6*(m*n^
2 - 2*m*n*log(c) + 2*m*log(c)^2)*a*b^2 + (3*m*n^3 - 6*m*n^2*log(c) + 6*m*n*
log(c)^2 - 4*m*log(c)^3)*b^3 - 4*a^3*m)*x^2)*log(f*x^2 + e) + integrate(-1/
4*((4*(f*m - f*log(d))*a^3 - 6*(f*m*n - 2*(f*m - f*log(d))*log(c))*a^2*b +
6*(f*m*n^2 - 2*f*m*n*log(c) + 2*(f*m - f*log(d))*log(c)^2)*a*b^2 - (3*f*m*n
^3 - 6*f*m*n^2*log(c) + 6*f*m*n*log(c)^2 - 4*(f*m - f*log(d))*log(c)^3)*b^3
)*x^3 + 4*((f*m - f*log(d))*b^3*x^3 - b^3*e*x*log(d))*log(x^n)^3 + 6*((2*(f
*m - f*log(d))*a*b^2 - (f*m*n - 2*(f*m - f*log(d))*log(c))*b^3)*x^3 - 2*(b^
3*e*log(c)*log(d) + a*b^2*e*log(d))*x)*log(x^n)^2 - 4*(b^3*e*log(c)^3*log(d
) + 3*a*b^2*e*log(c)^2*log(d) + 3*a^2*b*e*log(c)*log(d) + a^3*e*log(d))*x +
 6*((2*(f*m - f*log(d))*a^2*b - 2*(f*m*n - 2*(f*m - f*log(d))*log(c))*a*b^2
 + (f*m*n^2 - 2*f*m*n*log(c) + 2*(f*m - f*log(d))*log(c)^2)*b^3)*x^3 - 2*(b
^3*e*log(c)^2*log(d) + 2*a*b^2*e*log(c)*log(d) + a^2*b*e*log(d))*x)*log(x^n
))/(f*x^2 + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

3.109 $\int \frac{\left(a+b\log(cx^n)\right)^3 \log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx$

Optimal. Leaf size=181

$$-\frac{3}{4}b^2mn^2\text{Li}_4\left(-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn}-\frac{1}{2}m\text{Li}_2\left(-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)^3+$$

```
[Out] 1/4*(a+b*ln(c*x^n))^4*ln(d*(f*x^2+e)^m)/b/n-1/4*m*(a+b*ln(c*x^n))^4*ln(1+f*
x^2/e)/b/n-1/2*m*(a+b*ln(c*x^n))^3*polylog(2,-f*x^2/e)+3/4*b*m*n*(a+b*ln(c*
x^n))^2*polylog(3,-f*x^2/e)-3/4*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(4,-f*x^2/
e)+3/8*b^3*m*n^3*polylog(5,-f*x^2/e)
```

Rubi [A] time = 0.21, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2375, 2337, 2374, 2383, 6589}

$$-\frac{3}{4}b^2mn^2\text{PolyLog}\left(4,-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}m\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)\left(a+b\log\left(cx^n\right)\right)^3+\frac{3}{4}bmn\text{PolyLog}\left(3,-\frac{f}{\ }\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^4*Log[d*(e + f*x^2)^m])/(4*b*n) - (m*(a + b*Log[c*x^n])
^4*Log[1 + (f*x^2)/e])/(4*b*n) - (m*(a + b*Log[c*x^n])^3*PolyLog[2, -((f*x^
2)/e)])/2 + (3*b*m*n*(a + b*Log[c*x^n])^2*PolyLog[3, -((f*x^2)/e)])/4 - (3*
b^2*m*n^2*(a + b*Log[c*x^n])*PolyLog[4, -((f*x^2)/e)])/4 + (3*b^3*m*n^3*Pol
yLog[5, -((f*x^2)/e)])/8
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
```

```
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx = \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn} - \frac{(fm)\int\frac{x\left(a+b\log\left(cx^n\right)\right)^4}{e+fx^2}\,dx}{2bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^4\log}{4bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^4\log}{4bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^4\log}{4bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^4\log}{4bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d\left(e+fx^2\right)^m\right)}{4bn} - \frac{m\left(a+b\log\left(cx^n\right)\right)^4\log}{4bn}$$

Mathematica [C] time = 0.37, size = 1348, normalized size = 7.45

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] -(a^3*m*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]]) + (3*a^2*b*m*n*Log[x]^2*Log[
1 - (I*Sqrt[f]*x)/Sqrt[e]])/2 - a*b^2*m*n^2*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/
Sqrt[e]] + (b^3*m*n^3*Log[x]^4*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]])/4 - 3*a^2*b*
m*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 3*a*b^2*m*n*Log[x]^2*L
og[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - b^3*m*n^2*Log[x]^3*Log[c*x^n]*Lo
g[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 3*a*b^2*m*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqr
t[f]*x)/Sqrt[e]] + (3*b^3*m*n*Log[x]^2*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/S
qrt[e]])/2 - b^3*m*Log[x]*Log[c*x^n]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - a^3
*m*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*a^2*b*m*n*Log[x]^2*Log[1 + (I
*Sqrt[f]*x)/Sqrt[e]])/2 - a*b^2*m*n^2*Log[x]^3*Log[1 + (I*Sqrt[f]*x)/Sqrt[e
]] + (b^3*m*n^3*Log[x]^4*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]])/4 - 3*a^2*b*m*Log[
x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 3*a*b^2*m*n*Log[x]^2*Log[c*x
^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - b^3*m*n^2*Log[x]^3*Log[c*x^n]*Log[1 +
(I*Sqrt[f]*x)/Sqrt[e]] - 3*a*b^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x
)/Sqrt[e]] + (3*b^3*m*n*Log[x]^2*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]
])/2 - b^3*m*Log[x]*Log[c*x^n]^3*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + a^3*Log[x
]*Log[d*(e + f*x^2)^m] - (3*a^2*b*n*Log[x]^2*Log[d*(e + f*x^2)^m])/2 + a*b^
2*n^2*Log[x]^3*Log[d*(e + f*x^2)^m] - (b^3*n^3*Log[x]^4*Log[d*(e + f*x^2)^m
])/4 + 3*a^2*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 3*a*b^2*n*Log[x]^2*
Log[c*x^n]*Log[d*(e + f*x^2)^m] + b^3*n^2*Log[x]^3*Log[c*x^n]*Log[d*(e + f*
```

```
x^2)^m] + 3*a*b^2*Log[x]*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] - (3*b^3*n*Log[x
]^2*Log[c*x^n]^2*Log[d*(e + f*x^2)^m])/2 + b^3*Log[x]*Log[c*x^n]^3*Log[d*(e
 + f*x^2)^m] - m*(a + b*Log[c*x^n])^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]]
- m*(a + b*Log[c*x^n])^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + 3*a^2*b*m*n*Po
lyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 6*a*b^2*m*n*Log[c*x^n]*PolyLog[3, ((-I
)*Sqrt[f]*x)/Sqrt[e]] + 3*b^3*m*n*Log[c*x^n]^2*PolyLog[3, ((-I)*Sqrt[f]*x)/
Sqrt[e]] + 3*a^2*b*m*n*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + 6*a*b^2*m*n*Log[
c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + 3*b^3*m*n*Log[c*x^n]^2*PolyLog[3
, (I*Sqrt[f]*x)/Sqrt[e]] - 6*a*b^2*m*n^2*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e
]] - 6*b^3*m*n^2*Log[c*x^n]*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 6*a*b^2*
m*n^2*PolyLog[4, (I*Sqrt[f]*x)/Sqrt[e]] - 6*b^3*m*n^2*Log[c*x^n]*PolyLog[4,
 (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*m*n^3*PolyLog[5, ((-I)*Sqrt[f]*x)/Sqrt[e]]
+ 6*b^3*m*n^3*PolyLog[5, (I*Sqrt[f]*x)/Sqrt[e]]
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\log\left(cx^n\right)^3+3\,ab^2\log\left(cx^n\right)^2+3\,a^2b\log\left(cx^n\right)+a^3\right)\log\left(\left(fx^2+e\right)^m d\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x^2 + e)^m*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(fx^2+e\right)^m d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x, x)
```

maple [F] time = 7.04, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^n\right)+a\right)^3\ln\left(d\left(f\,x^2+e\right)^m\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*m*n^3*log(x)^4 - 4*b^3*m*log(x)*log(x^n)^3 - 4*(b^3*m*n^2*log(c)
+ a*b^2*m*n^2)*log(x)^3 + 6*(b^3*m*n*log(c)^2 + 2*a*b^2*m*n*log(c) + a^2*b*
m*n)*log(x)^2 + 6*(b^3*m*n*log(x)^2 - 2*(b^3*m*log(c) + a*b^2*m)*log(x))*lo
g(x^n)^2 - 4*(b^3*m*n^2*log(x)^3 - 3*(b^3*m*n*log(c) + a*b^2*m*n)*log(x)^2
```

```
+ 3*(b^3*m*log(c)^2 + 2*a*b^2*m*log(c) + a^2*b*m)*log(x))*log(x^n) - 4*(b^3
*m*log(c)^3 + 3*a*b^2*m*log(c)^2 + 3*a^2*b*m*log(c) + a^3*m)*log(x))*log(f*
x^2 + e) - integrate(-1/2*(b^3*f*m*n^3*x^2*log(x)^4 + 2*b^3*e*log(c)^3*log(
d) + 6*a*b^2*e*log(c)^2*log(d) + 6*a^2*b*e*log(c)*log(d) - 4*(b^3*f*m*n^2*l
og(c) + a*b^2*f*m*n^2)*x^2*log(x)^3 + 2*a^3*e*log(d) + 6*(b^3*f*m*n*log(c)^
2 + 2*a*b^2*f*m*n*log(c) + a^2*b*f*m*n)*x^2*log(x)^2 - 4*(b^3*f*m*log(c)^3
+ 3*a*b^2*f*m*log(c)^2 + 3*a^2*b*f*m*log(c) + a^3*f*m)*x^2*log(x) - 2*(2*b^
3*f*m*x^2*log(x) - b^3*f*x^2*log(d) - b^3*e*log(d))*log(x^n)^3 + 2*(b^3*f*l
og(c)^3*log(d) + 3*a*b^2*f*log(c)^2*log(d) + 3*a^2*b*f*log(c)*log(d) + a^3*
f*log(d))*x^2 + 6*(b^3*f*m*n*x^2*log(x)^2 + b^3*e*log(c)*log(d) + a*b^2*e*l
og(d) - 2*(b^3*f*m*log(c) + a*b^2*f*m)*x^2*log(x) + (b^3*f*log(c)*log(d) +
a*b^2*f*log(d))*x^2)*log(x^n)^2 - 2*(2*b^3*f*m*n^2*x^2*log(x)^3 - 3*b^3*e*l
og(c)^2*log(d) - 6*a*b^2*e*log(c)*log(d) - 3*a^2*b*e*log(d) - 6*(b^3*f*m*n*
log(c) + a*b^2*f*m*n)*x^2*log(x)^2 + 6*(b^3*f*m*log(c)^2 + 2*a*b^2*f*m*log(
c) + a^2*b*f*m)*x^2*log(x) - 3*(b^3*f*log(c)^2*log(d) + 2*a*b^2*f*log(c)*lo
g(d) + a^2*b*f*log(d))*x^2)*log(x^n))/(f*x^3 + e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x,x)
```

```
[Out] Timed out
```

3.110 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx$

Optimal. Leaf size=451

$$-\frac{3b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{4x^2}+\frac{3b^2fmn^2\text{Li}_2\left(-\frac{e}{fx^2}\right)\left(a+b\log\left(cx^n\right)\right)}{4e}+\frac{3b^2fmn^2\text{Li}_3\left(-\frac{e}{fx^2}\right)\left(a+b\log\left(cx^n\right)\right)}{4e}$$

```
[Out] 3/4*b^3*f*m*n^3*ln(x)/e-3/4*b^2*f*m*n^2*ln(1+e/f/x^2)*(a+b*ln(c*x^n))/e-3/4
*b*f*m*n*ln(1+e/f/x^2)*(a+b*ln(c*x^n))^2/e-1/2*f*m*ln(1+e/f/x^2)*(a+b*ln(c*
x^n))^3/e-3/8*b^3*f*m*n^3*ln(f*x^2+e)/e-3/8*b^3*n^3*ln(d*(f*x^2+e)^m)/x^2-3
/4*b^2*n^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*
ln(d*(f*x^2+e)^m)/x^2-1/2*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^2+3/8*b^3*f
*m*n^3*polylog(2,-e/f/x^2)/e+3/4*b^2*f*m*n^2*(a+b*ln(c*x^n))*polylog(2,-e/f
/x^2)/e+3/4*b*f*m*n*(a+b*ln(c*x^n))^2*polylog(2,-e/f/x^2)/e+3/8*b^3*f*m*n^3
*polylog(3,-e/f/x^2)/e+3/4*b^2*f*m*n^2*(a+b*ln(c*x^n))*polylog(3,-e/f/x^2)/
e+3/8*b^3*f*m*n^3*polylog(4,-e/f/x^2)/e
```

Rubi [A] time = 0.57, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2305, 2304, 2378, 266, 36, 29, 31, 2345, 2391, 2374, 6589, 2383}

$$\frac{3b^2fmn^2\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)\left(a+b\log\left(cx^n\right)\right)}{4e}+\frac{3b^2fmn^2\text{PolyLog}\left(3,-\frac{e}{fx^2}\right)\left(a+b\log\left(cx^n\right)\right)}{4e}+\frac{3bfmn\text{PolyLog}\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] (3*b^3*f*m*n^3*Log[x])/(4*e) - (3*b^2*f*m*n^2*Log[1 + e/(f*x^2)]*(a + b*Log
[c*x^n]))/(4*e) - (3*b*f*m*n*Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])^2)/(4*e)
 - (f*m*Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])^3)/(2*e) - (3*b^3*f*m*n^3*Log
[e + f*x^2])/(8*e) - (3*b^3*n^3*Log[d*(e + f*x^2)^m])/(8*x^2) - (3*b^2*n^2*
(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(4*x^2) - (3*b*n*(a + b*Log[c*x^n]
)^2*Log[d*(e + f*x^2)^m])/(4*x^2) - ((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)
^m])/(2*x^2) + (3*b^3*f*m*n^3*PolyLog[2, -(e/(f*x^2))])/(8*e) + (3*b^2*f*m*
n^2*(a + b*Log[c*x^n])*PolyLog[2, -(e/(f*x^2))])/(4*e) + (3*b*f*m*n*(a + b*
Log[c*x^n])^2*PolyLog[2, -(e/(f*x^2))])/(4*e) + (3*b^3*f*m*n^3*PolyLog[3, -
(e/(f*x^2))])/(8*e) + (3*b^2*f*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e/(f*x
^2))])/(4*e) + (3*b^3*f*m*n^3*PolyLog[4, -(e/(f*x^2))])/(8*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2345

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> -Simp[(Log[1 + d/(e*x^r)]*(a + b*Log[c*x^n])^p)/(d*r),
x] + Dist[(b*n*p)/(d*r), Int[(Log[1 + d/(e*x^r)]*(a + b*Log[c*x^n])^(p - 1)
)/x, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x^3}dx = -\frac{3b^3n^3\log\left(d\left(e+fx^2\right)^m\right)}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))\log\left(d\left(e+fx\right.\right.}{4x^2}$$

$$= -\frac{3b^3n^3\log\left(d\left(e+fx^2\right)^m\right)}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))\log\left(d\left(e+fx\right.\right.}{4x^2}$$

$$= -\frac{3b^2fmn^2\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e} - \frac{3bfmn\log\left(1+\frac{e}{fx^2}\right)(}{4e}$$

$$= -\frac{3b^2fmn^2\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e} - \frac{3bfmn\log\left(1+\frac{e}{fx^2}\right)(}{4e}$$

$$= \frac{3b^3fmn^3\log(x)}{4e} - \frac{3b^2fmn^2\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e} - \frac{3bfn}{}$$

$$= \frac{3b^3fmn^3\log(x)}{4e} - \frac{3b^2fmn^2\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e} - \frac{3bfn}{}$$

Mathematica [C] time = 0.91, size = 2248, normalized size = 4.98

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] -1/8*(-8*a^3*f*m*x^2*Log[x] - 12*a^2*b*f*m*n*x^2*Log[x] - 12*a*b^2*f*m*n^2*
x^2*Log[x] - 6*b^3*f*m*n^3*x^2*Log[x] + 12*a^2*b*f*m*n*x^2*Log[x]^2 + 12*a*
b^2*f*m*n^2*x^2*Log[x]^2 + 6*b^3*f*m*n^3*x^2*Log[x]^2 - 8*a*b^2*f*m*n^2*x^2
*Log[x]^3 - 4*b^3*f*m*n^3*x^2*Log[x]^3 + 2*b^3*f*m*n^3*x^2*Log[x]^4 - 24*a^
2*b*f*m*x^2*Log[x]*Log[c*x^n] - 24*a*b^2*f*m*n*x^2*Log[x]*Log[c*x^n] - 12*b
^3*f*m*n^2*x^2*Log[x]*Log[c*x^n] + 24*a*b^2*f*m*n*x^2*Log[x]^2*Log[c*x^n] +
 12*b^3*f*m*n^2*x^2*Log[x]^2*Log[c*x^n] - 8*b^3*f*m*n^2*x^2*Log[x]^3*Log[c*
x^n] - 24*a*b^2*f*m*x^2*Log[x]*Log[c*x^n]^2 - 12*b^3*f*m*n*x^2*Log[x]*Log[c
*x^n]^2 + 12*b^3*f*m*n*x^2*Log[x]^2*Log[c*x^n]^2 - 8*b^3*f*m*x^2*Log[x]*Log
[c*x^n]^3 + 12*a^2*b*f*m*n*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*a
*b^2*f*m*n^2*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*f*m*n^3*x^2*
Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*f*m*n^2*x^2*Log[x]^2*Log[1
 - (I*Sqrt[f]*x)/Sqrt[e]] - 6*b^3*f*m*n^3*x^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x
)/Sqrt[e]] + 4*b^3*f*m*n^3*x^2*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 24
*a*b^2*f*m*n*x^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*
f*m*n^2*x^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^3*f*m*n
^2*x^2*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*f*m*n*x^
2*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*a^2*b*f*m*n*x^2*L
og[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*a*b^2*f*m*n^2*x^2*Log[x]*Log[1 +
(I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*f*m*n^3*x^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqr
t[e]] - 12*a*b^2*f*m*n^2*x^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 6*b^
3*f*m*n^3*x^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 4*b^3*f*m*n^3*x^2*L
og[x]^3*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 24*a*b^2*f*m*n*x^2*Log[x]*Log[c*x^
n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*f*m*n^2*x^2*Log[x]*Log[c*x^n]*Lo
g[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^3*f*m*n^2*x^2*Log[x]^2*Log[c*x^n]*Log[1
 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*f*m*n*x^2*Log[x]*Log[c*x^n]^2*Log[1 + (I
*Sqrt[f]*x)/Sqrt[e]] + 4*a^3*f*m*x^2*Log[e + f*x^2] + 6*a^2*b*f*m*n*x^2*Log
[e + f*x^2] + 6*a*b^2*f*m*n^2*x^2*Log[e + f*x^2] + 3*b^3*f*m*n^3*x^2*Log[e
```

```
+ f*x^2] - 12*a^2*b*f*m*n*x^2*Log[x]*Log[e + f*x^2] - 12*a*b^2*f*m*n^2*x^2*
Log[x]*Log[e + f*x^2] - 6*b^3*f*m*n^3*x^2*Log[x]*Log[e + f*x^2] + 12*a*b^2*
f*m*n^2*x^2*Log[x]^2*Log[e + f*x^2] + 6*b^3*f*m*n^3*x^2*Log[x]^2*Log[e + f*
x^2] - 4*b^3*f*m*n^3*x^2*Log[x]^3*Log[e + f*x^2] + 12*a^2*b*f*m*x^2*Log[c*x
^n]*Log[e + f*x^2] + 12*a*b^2*f*m*n*x^2*Log[c*x^n]*Log[e + f*x^2] + 6*b^3*f
*m*n^2*x^2*Log[c*x^n]*Log[e + f*x^2] - 24*a*b^2*f*m*n*x^2*Log[x]*Log[c*x^n]
*Log[e + f*x^2] - 12*b^3*f*m*n^2*x^2*Log[x]*Log[c*x^n]*Log[e + f*x^2] + 12*
b^3*f*m*n^2*x^2*Log[x]^2*Log[c*x^n]*Log[e + f*x^2] + 12*a*b^2*f*m*x^2*Log[c
*x^n]^2*Log[e + f*x^2] + 6*b^3*f*m*n*x^2*Log[c*x^n]^2*Log[e + f*x^2] - 12*b
^3*f*m*n*x^2*Log[x]*Log[c*x^n]^2*Log[e + f*x^2] + 4*b^3*f*m*x^2*Log[c*x^n]^
3*Log[e + f*x^2] + 4*a^3*e*Log[d*(e + f*x^2)^m] + 6*a^2*b*e*n*Log[d*(e + f*
x^2)^m] + 6*a*b^2*e*n^2*Log[d*(e + f*x^2)^m] + 3*b^3*e*n^3*Log[d*(e + f*x^2
)^m] + 12*a^2*b*e*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 12*a*b^2*e*n*Log[c*x^n]
*Log[d*(e + f*x^2)^m] + 6*b^3*e*n^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 12*a*
b^2*e*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] + 6*b^3*e*n*Log[c*x^n]^2*Log[d*(e +
 f*x^2)^m] + 4*b^3*e*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] + 6*b*f*m*n*x^2*(2*a
^2 + 2*a*b*n + b^2*n^2 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*P
olyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 6*b*f*m*n*x^2*(2*a^2 + 2*a*b*n + b^2*
n^2 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, (I*Sqrt[f
]*x)/Sqrt[e]] - 24*a*b^2*f*m*n^2*x^2*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] -
 12*b^3*f*m*n^3*x^2*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 24*b^3*f*m*n^2*x
^2*Log[c*x^n]*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 24*a*b^2*f*m*n^2*x^2*P
olyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^3*f*m*n^3*x^2*PolyLog[3, (I*Sqrt[f]
*x)/Sqrt[e]] - 24*b^3*f*m*n^2*x^2*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[
e]] + 24*b^3*f*m*n^3*x^2*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 24*b^3*f*m*
n^3*x^2*PolyLog[4, (I*Sqrt[f]*x)/Sqrt[e]])/(e*x^2)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\log\left(cx^n\right)^3+3\,ab^2\log\left(cx^n\right)^2+3\,a^2b\log\left(cx^n\right)+a^3\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x^2 + e)^m*d)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^3\log\left(\left(fx^2+e\right)^m d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x^3, x)
```

maple [F] time = 11.55, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right)+a\right)^3\ln\left(d\left(f\,x^2+e\right)^m\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(4\,b^3m\log\left(x^n\right)^3+6\left(mn+2\,m\log(c)\right)a^2b+6\left(mn^2+2\,mn\log(c)+2\,m\log(c)^2\right)ab^2+\left(3\,mn^3+6\,mn^2\log(c)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*m*log(x^n)^3 + 6*(m*n + 2*m*log(c))*a^2*b + 6*(m*n^2 + 2*m*n*lo
g(c) + 2*m*log(c)^2)*a*b^2 + (3*m*n^3 + 6*m*n^2*log(c) + 6*m*n*log(c)^2 + 4
*m*log(c)^3)*b^3 + 4*a^3*m + 6*((m*n + 2*m*log(c))*b^3 + 2*a*b^2*m)*log(x^n
)^2 + 6*(2*(m*n + 2*m*log(c))*a*b^2 + (m*n^2 + 2*m*n*log(c) + 2*m*log(c)^2)
*b^3 + 2*a^2*b*m)*log(x^n))*log(f*x^2 + e)/x^2 + integrate(1/4*(4*b^3*e*log
(c)^3*log(d) + 12*a*b^2*e*log(c)^2*log(d) + 12*a^2*b*e*log(c)*log(d) + 4*a^
3*e*log(d) + 4*((f*m + f*log(d))*b^3*x^2 + b^3*e*log(d))*log(x^n)^3 + (4*(f
*m + f*log(d))*a^3 + 6*(f*m*n + 2*(f*m + f*log(d))*log(c))*a^2*b + 6*(f*m*n
^2 + 2*f*m*n*log(c) + 2*(f*m + f*log(d))*log(c)^2)*a*b^2 + (3*f*m*n^3 + 6*f
*m*n^2*log(c) + 6*f*m*n*log(c)^2 + 4*(f*m + f*log(d))*log(c)^3)*b^3)*x^2 +
6*(2*b^3*e*log(c)*log(d) + 2*a*b^2*e*log(d) + (2*(f*m + f*log(d))*a*b^2 + (
f*m*n + 2*(f*m + f*log(d))*log(c))*b^3)*x^2)*log(x^n)^2 + 6*(2*b^3*e*log(c)
^2*log(d) + 4*a*b^2*e*log(c)*log(d) + 2*a^2*b*e*log(d) + (2*(f*m + f*log(d)
)*a^2*b + 2*(f*m*n + 2*(f*m + f*log(d))*log(c))*a*b^2 + (f*m*n^2 + 2*f*m*n*
log(c) + 2*(f*m + f*log(d))*log(c)^2)*b^3)*x^2)*log(x^n))/(f*x^5 + e*x^3),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^3,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x**3,x)
```

```
[Out] Timed out
```

3.111 $\int x^2 \left(a+b\log\left(cx^n\right)\right)^3 \log\left(d\left(e+fx^2\right)^m\right)\,dx$

Optimal. Leaf size=1092

$$\frac{16}{81}mn^3x^3b^3-\frac{160emn^3xb^3}{27f}+\frac{4e^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)b^3}{27f^{3/2}}+\frac{52emn^2x\log\left(cx^n\right)b^3}{9f}-\frac{2}{27}n^3x^3\log\left(d\left(fx^2+e\right)^m\right)b^3+-$$

```
[Out] 52/9*b^3*e*m*n^2*x*ln(c*x^n)/f-8/3*b*e*m*n*x*(a+b*ln(c*x^n))^2/f+1/3*x^3*(a
+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)-2/9*m*x^3*(a+b*ln(c*x^n))^3+16/81*b^3*m*n
^3*x^3-160/27*b^3*e*m*n^3*x/f-2/27*b^3*n^3*x^3*ln(d*(f*x^2+e)^m)+b*(-e)^(3/
2)*m*n*(a+b*ln(c*x^n))^2*polylog(2,-x*f^(1/2)/(-e)^(1/2))/f^(3/2)-b*(-e)^(3
/2)*m*n*(a+b*ln(c*x^n))^2*polylog(2,x*f^(1/2)/(-e)^(1/2))/f^(3/2)+2/9*I*b^3
*e^(3/2)*m*n^3*polylog(2,-I*x*f^(1/2)/e^(1/2))/f^(3/2)-4/9*b^2*e^(3/2)*m*n^
2*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))/f^(3/2)+1/3*b*(-e)^(3/2)*m*n*(a
+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))/f^(3/2)-1/3*b*(-e)^(3/2)*m*n*(a+
b*ln(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2/3*b^2*(-e)^(3/2)*m*n^2*
(a+b*ln(c*x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))/f^(3/2)+2/3*b^2*(-e)^(3/2)
*m*n^2*(a+b*ln(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2*b^2*(-e)^(
3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,-x*f^(1/2)/(-e)^(1/2))/f^(3/2)+2*b^2*(
-e)^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2/9
*I*b^3*e^(3/2)*m*n^3*polylog(2,I*x*f^(1/2)/e^(1/2))/f^(3/2)+2/3*e*m*x*(a+b*
ln(c*x^n))^3/f-4/9*b^2*m*n^2*x^3*(a+b*ln(c*x^n))+4/9*b*m*n*x^3*(a+b*ln(c*x^
n))^2+2/9*b^2*n^2*x^3*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)-1/3*b*n*x^3*(a+b*ln
(c*x^n))^2*ln(d*(f*x^2+e)^m)-1/3*(-e)^(3/2)*m*(a+b*ln(c*x^n))^3*ln(1-x*f^(1
/2)/(-e)^(1/2))/f^(3/2)+1/3*(-e)^(3/2)*m*(a+b*ln(c*x^n))^3*ln(1+x*f^(1/2)/(
-e)^(1/2))/f^(3/2)+4/27*b^3*e^(3/2)*m*n^3*arctan(x*f^(1/2)/e^(1/2))/f^(3/2)
+2/3*b^3*(-e)^(3/2)*m*n^3*polylog(3,-x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2/3*b^3*
(-e)^(3/2)*m*n^3*polylog(3,x*f^(1/2)/(-e)^(1/2))/f^(3/2)+2*b^3*(-e)^(3/2)*m
*n^3*polylog(4,-x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2*b^3*(-e)^(3/2)*m*n^3*polylo
g(4,x*f^(1/2)/(-e)^(1/2))/f^(3/2)+52/9*a*b^2*e*m*n^2*x/f
```

Rubi [A] time = 1.81, antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 18, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {2305, 2304, 2378, 302, 205, 2351, 2295, 2324, 12, 4848, 2391, 2353, 2296, 2330, 2317, 2374, 6589, 2383}

$$\frac{16}{81}mn^3x^3b^3-\frac{160emn^3xb^3}{27f}+\frac{4e^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)b^3}{27f^{3/2}}+\frac{52emn^2x\log\left(cx^n\right)b^3}{9f}-\frac{2}{27}n^3x^3\log\left(d\left(fx^2+e\right)^m\right)b^3+-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (52*a*b^2*e*m*n^2*x)/(9*f) - (160*b^3*e*m*n^3*x)/(27*f) + (16*b^3*m*n^3*x^3
)/81 + (4*b^3*e^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(27*f^(3/2)) + (52
*b^3*e*m*n^2*x*Log[c*x^n])/(9*f) - (4*b^2*m*n^2*x^3*(a + b*Log[c*x^n]))/9 -
 (4*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*f^
(3/2)) - (8*b*e*m*n*x*(a + b*Log[c*x^n])^2)/(3*f) + (4*b*m*n*x^3*(a + b*Log
[c*x^n])^2)/9 + (2*e*m*x*(a + b*Log[c*x^n])^3)/(3*f) - (2*m*x^3*(a + b*Log[
c*x^n])^3)/9 + (b*(-e)^(3/2)*m*n*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/S
qrt[-e]])/(3*f^(3/2)) - ((-e)^(3/2)*m*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]
*x)/Sqrt[-e]])/(3*f^(3/2)) - (b*(-e)^(3/2)*m*n*(a + b*Log[c*x^n])^2*Log[1 +
 (Sqrt[f]*x)/Sqrt[-e]])/(3*f^(3/2)) + ((-e)^(3/2)*m*(a + b*Log[c*x^n])^3*Lo
g[1 + (Sqrt[f]*x)/Sqrt[-e]])/(3*f^(3/2)) - (2*b^3*n^3*x^3*Log[d*(e + f*x^2)
^m])/27 + (2*b^2*n^2*x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/9 - (b*n*
x^3*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/3 + (x^3*(a + b*Log[c*x^n])^
3*Log[d*(e + f*x^2)^m])/3 - (2*b^2*(-e)^(3/2)*m*n^2*(a + b*Log[c*x^n])*Poly
```

```
Log[2, -((Sqrt[f]*x)/Sqrt[-e])])/(3*f^(3/2)) + (b*(-e)^(3/2)*m*n*(a + b*Log
[c*x^n])^2*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/f^(3/2) + (2*b^2*(-e)^(3/2)
*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(3*f^(3/2)) - (
b*(-e)^(3/2)*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/f^(
3/2) + (((2*I)/9)*b^3*e^(3/2)*m*n^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/f
^(3/2) - (((2*I)/9)*b^3*e^(3/2)*m*n^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/f^
(3/2) + (2*b^3*(-e)^(3/2)*m*n^3*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/(3*f^(
3/2)) - (2*b^2*(-e)^(3/2)*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((Sqrt[f]*x)
/Sqrt[-e])])/f^(3/2) - (2*b^3*(-e)^(3/2)*m*n^3*PolyLog[3, (Sqrt[f]*x)/Sqrt[
-e]])/(3*f^(3/2)) + (2*b^2*(-e)^(3/2)*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, (
Sqrt[f]*x)/Sqrt[-e]])/f^(3/2) + (2*b^3*(-e)^(3/2)*m*n^3*PolyLog[4, -((Sqrt[
f]*x)/Sqrt[-e])])/f^(3/2) - (2*b^3*(-e)^(3/2)*m*n^3*PolyLog[4, (Sqrt[f]*x)/
Sqrt[-e]])/f^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
```

```
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)dx=-\frac{2}{27}b^3n^3x^3\log\left(d\left(e+fx^2\right)^m\right)+\frac{2}{9}b^2n^2x^3\left(a+b\log\left(cx^n\right)\right)\log\left(\right.$$

$$=-\frac{2}{27}b^3n^3x^3\log\left(d\left(e+fx^2\right)^m\right)+\frac{2}{9}b^2n^2x^3\left(a+b\log\left(cx^n\right)\right)\log\left(\right.$$

$$=-\frac{2}{27}b^3n^3x^3\log\left(d\left(e+fx^2\right)^m\right)+\frac{2}{9}b^2n^2x^3\left(a+b\log\left(cx^n\right)\right)\log\left(\right.$$

$$=-\frac{4b^3emn^3x}{27f}+\frac{4}{81}b^3mn^3x^3-\frac{2}{27}b^3n^3x^3\log\left(d\left(e+fx^2\right)^m\right)+\frac{2}{9}b^2$$

$$=\frac{4ab^2emn^2x}{9f}-\frac{4b^3emn^3x}{27f}+\frac{8}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}}{\sqrt{e}}\right.}{27f^{3/2}}$$

$$=\frac{16ab^2emn^2x}{9f}-\frac{16b^3emn^3x}{27f}+\frac{4}{27}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\tan^{-1}\left(\right.}{27f^{3/2}}$$

$$=\frac{52ab^2emn^2x}{9f}-\frac{52b^3emn^3x}{27f}+\frac{16}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\tan^{-1}\left(\right.}{27f^{3/2}}$$

$$=\frac{52ab^2emn^2x}{9f}-\frac{160b^3emn^3x}{27f}+\frac{16}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\tan^{-1}\left(\right.}{27f^{3/2}}$$

$$=\frac{52ab^2emn^2x}{9f}-\frac{160b^3emn^3x}{27f}+\frac{16}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\tan^{-1}\left(\right.}{27f^{3/2}}$$

$$=\frac{52ab^2emn^2x}{9f}-\frac{160b^3emn^3x}{27f}+\frac{16}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\tan^{-1}\left(\right.}{27f^{3/2}}$$

Mathematica [B] time = 1.01, size = 2544, normalized size = 2.33

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (54*a^3*e*Sqrt[f]*m*x - 216*a^2*b*e*Sqrt[f]*m*n*x + 468*a*b^2*e*Sqrt[f]*m*n
^2*x - 480*b^3*e*Sqrt[f]*m*n^3*x - 18*a^3*f^(3/2)*m*x^3 + 36*a^2*b*f^(3/2)*
m*n*x^3 - 36*a*b^2*f^(3/2)*m*n^2*x^3 + 16*b^3*f^(3/2)*m*n^3*x^3 - 54*a^3*e^
(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 54*a^2*b*e^(3/2)*m*n*ArcTan[(Sqrt[f]*
x)/Sqrt[e]] - 36*a*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 12*b^3*e
^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 162*a^2*b*e^(3/2)*m*n*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[x] - 108*a*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[
e]]*Log[x] + 36*b^3*e^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 162*
a*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 + 54*b^3*e^(3/2)*m
*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 + 54*b^3*e^(3/2)*m*n^3*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[x]^3 + 162*a^2*b*e*Sqrt[f]*m*x*Log[c*x^n] - 432*a*b^2
*e*Sqrt[f]*m*n*x*Log[c*x^n] + 468*b^3*e*Sqrt[f]*m*n^2*x*Log[c*x^n] - 54*a^2
*b*f^(3/2)*m*x^3*Log[c*x^n] + 72*a*b^2*f^(3/2)*m*n*x^3*Log[c*x^n] - 36*b^3*
f^(3/2)*m*n^2*x^3*Log[c*x^n] - 162*a^2*b*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[
e]]*Log[c*x^n] + 108*a*b^2*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^
n] - 36*b^3*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 324*a*b^
2*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] - 108*b^3*e^(3/
2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] - 162*b^3*e^(3/2)*m*
n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2*Log[c*x^n] + 162*a*b^2*e*Sqrt[f]*m
*x*Log[c*x^n]^2 - 216*b^3*e*Sqrt[f]*m*n*x*Log[c*x^n]^2 - 54*a*b^2*f^(3/2)*m
*x^3*Log[c*x^n]^2 + 36*b^3*f^(3/2)*m*n*x^3*Log[c*x^n]^2 - 162*a*b^2*e^(3/2)
*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + 54*b^3*e^(3/2)*m*n*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + 162*b^3*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqr
t[e]]*Log[x]*Log[c*x^n]^2 + 54*b^3*e*Sqrt[f]*m*x*Log[c*x^n]^3 - 18*b^3*f^(3
/2)*m*x^3*Log[c*x^n]^3 - 54*b^3*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c
*x^n]^3 - (81*I)*a^2*b*e^(3/2)*m*n*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] +
(54*I)*a*b^2*e^(3/2)*m*n^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b
^3*e^(3/2)*m*n^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (81*I)*a*b^2*e^(3/
2)*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (27*I)*b^3*e^(3/2)*m*n^3
*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (27*I)*b^3*e^(3/2)*m*n^3*Log[x]^
3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (162*I)*a*b^2*e^(3/2)*m*n*Log[x]*Log[c*x
^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (54*I)*b^3*e^(3/2)*m*n^2*Log[x]*Log[c*
x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (81*I)*b^3*e^(3/2)*m*n^2*Log[x]^2*Log
[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (81*I)*b^3*e^(3/2)*m*n*Log[x]*Log[
c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (81*I)*a^2*b*e^(3/2)*m*n*Log[x]*L
og[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (54*I)*a*b^2*e^(3/2)*m*n^2*Log[x]*Log[1 + (
I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^3*e^(3/2)*m*n^3*Log[x]*Log[1 + (I*Sqrt[f]*
x)/Sqrt[e]] - (81*I)*a*b^2*e^(3/2)*m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqr
t[e]] + (27*I)*b^3*e^(3/2)*m*n^3*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] +
(27*I)*b^3*e^(3/2)*m*n^3*Log[x]^3*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (162*I)*
a*b^2*e^(3/2)*m*n*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (54*I)
*b^3*e^(3/2)*m*n^2*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (81*I
)*b^3*e^(3/2)*m*n^2*Log[x]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (8
1*I)*b^3*e^(3/2)*m*n*Log[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 2
7*a^3*f^(3/2)*x^3*Log[d*(e + f*x^2)^m] - 27*a^2*b*f^(3/2)*n*x^3*Log[d*(e +
f*x^2)^m] + 18*a*b^2*f^(3/2)*n^2*x^3*Log[d*(e + f*x^2)^m] - 6*b^3*f^(3/2)*n
^3*x^3*Log[d*(e + f*x^2)^m] + 81*a^2*b*f^(3/2)*x^3*Log[c*x^n]*Log[d*(e + f*
x^2)^m] - 54*a*b^2*f^(3/2)*n*x^3*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 18*b^3*f
^(3/2)*n^2*x^3*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 81*a*b^2*f^(3/2)*x^3*Log[c
*x^n]^2*Log[d*(e + f*x^2)^m] - 27*b^3*f^(3/2)*n*x^3*Log[c*x^n]^2*Log[d*(e +
 f*x^2)^m] + 27*b^3*f^(3/2)*x^3*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] + (9*I)*b
*e^(3/2)*m*n*(9*a^2 - 6*a*b*n + 2*b^2*n^2 - 6*b*(-3*a + b*n)*Log[c*x^n] + 9
*b^2*Log[c*x^n]^2)*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b*e^(3/2)*m
*n*(9*a^2 - 6*a*b*n + 2*b^2*n^2 - 6*b*(-3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c
*x^n]^2)*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] - (162*I)*a*b^2*e^(3/2)*m*n^2*Po
lyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (54*I)*b^3*e^(3/2)*m*n^3*PolyLog[3, ((
-I)*Sqrt[f]*x)/Sqrt[e]] - (162*I)*b^3*e^(3/2)*m*n^2*Log[c*x^n]*PolyLog[3, (
(-I)*Sqrt[f]*x)/Sqrt[e]] + (162*I)*a*b^2*e^(3/2)*m*n^2*PolyLog[3, (I*Sqrt[f
]*x)/Sqrt[e]] - (54*I)*b^3*e^(3/2)*m*n^3*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]]
```

```
+ (162*I)*b^3*e^(3/2)*m*n^2*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] +
(162*I)*b^3*e^(3/2)*m*n^3*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (162*I)*b^
3*e^(3/2)*m*n^3*PolyLog[4, (I*Sqrt[f]*x)/Sqrt[e]])/(81*f^(3/2))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3x^2\log\left(cx^n\right)^3 + 3\,ab^2x^2\log\left(cx^n\right)^2 + 3\,a^2bx^2\log\left(cx^n\right) + a^3x^2\right)\log\left(\left(fx^2+e\right)^m d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^2*log(c*x^n)^3 + 3*a*b^2*x^2*log(c*x^n)^2 + 3*a^2*b*x^2*log
(c*x^n) + a^3*x^2)*log((f*x^2 + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right) + a\right)^3 x^2 \log\left(\left(fx^2+e\right)^m d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x^2*log((f*x^2 + e)^m*d), x)
```

maple [F] time = 185.48, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right) + a\right)^3 x^2 \ln\left(d\left(f\,x^2+e\right)^m\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{27}\left(9\,b^3mx^3\log\left(x^n\right)^3 - 9\left(\left(mn - 3\,m\log(c)\right)b^3 - 3\,ab^2m\right)x^3\log\left(x^n\right)^2 - 3\left(6\left(mn - 3\,m\log(c)\right)ab^2 - \left(2\,mn^2 - 6\,m\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/27*(9*b^3*m*x^3*log(x^n)^3 - 9*((m*n - 3*m*log(c))*b^3 - 3*a*b^2*m)*x^3*l
og(x^n)^2 - 3*(6*(m*n - 3*m*log(c))*a*b^2 - (2*m*n^2 - 6*m*n*log(c) + 9*m*l
og(c)^2)*b^3 - 9*a^2*b*m)*x^3*log(x^n) - (9*(m*n - 3*m*log(c))*a^2*b - 3*(2
*m*n^2 - 6*m*n*log(c) + 9*m*log(c)^2)*a*b^2 + (2*m*n^3 - 6*m*n^2*log(c) + 9
*m*n*log(c)^2 - 9*m*log(c)^3)*b^3 - 9*a^3*m)*x^3)*log(f*x^2 + e) + integrat
e(-1/27*((9*(2*f*m - 3*f*log(d))*a^3 - 9*(2*f*m*n - 3*(2*f*m - 3*f*log(d))*
log(c))*a^2*b + 3*(4*f*m*n^2 - 12*f*m*n*log(c) + 9*(2*f*m - 3*f*log(d))*log
(c)^2)*a*b^2 - (4*f*m*n^3 - 12*f*m*n^2*log(c) + 18*f*m*n*log(c)^2 - 9*(2*f*
m - 3*f*log(d))*log(c)^3)*b^3)*x^4 + 9*((2*f*m - 3*f*log(d))*b^3*x^4 - 3*b^
3*e*x^2*log(d))*log(x^n)^3 - 27*(b^3*e*log(c)^3*log(d) + 3*a*b^2*e*log(c)^2
*log(d) + 3*a^2*b*e*log(c)*log(d) + a^3*e*log(d))*x^2 + 9*((3*(2*f*m - 3*f*
log(d))*a*b^2 - (2*f*m*n - 3*(2*f*m - 3*f*log(d))*log(c))*b^3)*x^4 - 9*(b^3
*e*log(c)*log(d) + a*b^2*e*log(d))*x^2)*log(x^n)^2 + 3*((9*(2*f*m - 3*f*log
(d))*a^2*b - 6*(2*f*m*n - 3*(2*f*m - 3*f*log(d))*log(c))*a*b^2 + (4*f*m*n^2
 - 12*f*m*n*log(c) + 9*(2*f*m - 3*f*log(d))*log(c)^2)*b^3)*x^4 - 27*(b^3*e*
log(c)^2*log(d) + 2*a*b^2*e*log(c)*log(d) + a^2*b*e*log(d))*x^2)*log(x^n))/
(f*x^2 + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln\left(d\left(f x^2+e\right)^m\right) (a+b \ln(c x^n))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3,x)
[Out] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m),x)
[Out] Timed out
```

3.112 $\int \left(a+b\log \left(cx^n\right)\right)^3 \log \left(d\left(e+fx^2\right)^m\right)\, dx$

Optimal. Leaf size=977

$$36mn^3xb^3 - 36mn^2x\log\left(cx^n\right)b^3 + \frac{12\sqrt{e}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log\left(cx^n\right)b^3}{\sqrt{f}} - 6n^3x\log\left(d\left(fx^2+e\right)^m\right)b^3 + 6n^2x\log\left(cx^n\right)$$

```
[Out] 36*b^3*m*n^3*x+6*b^3*m*n^3*polylog(3,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1
/2)-6*b^3*m*n^3*polylog(3,x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+6*b^3*m*
n^3*polylog(4,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-6*b^3*m*n^3*polylog
(4,x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+x*(a+b*ln(c*x^n))^3*ln(d*(f*x^2
+e)^m)-2*m*x*(a+b*ln(c*x^n))^3-12*b^2*m*n^2*(-b*n+a)*x-24*a*b^2*m*n^2*x-6*b
^3*n^3*x*ln(d*(f*x^2+e)^m)+6*I*b^3*m*n^3*polylog(2,I*x*f^(1/2)/e^(1/2))*e^(
1/2)/f^(1/2)+3*b*m*n*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2
)/f^(1/2)-3*b*m*n*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f
^(1/2)-6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1
/2)/f^(1/2)+3*b*m*n*(a+b*ln(c*x^n))^2*polylog(2,-x*f^(1/2)/(-e)^(1/2))*(-e)
^(1/2)/f^(1/2)+6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))*
(-e)^(1/2)/f^(1/2)-3*b*m*n*(a+b*ln(c*x^n))^2*polylog(2,x*f^(1/2)/(-e)^(1/2)
)*(-e)^(1/2)/f^(1/2)-6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(3,-x*f^(1/2)/(-e)^
(1/2))*(-e)^(1/2)/f^(1/2)+6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(3,x*f^(1/2)/(
-e)^(1/2))*(-e)^(1/2)/f^(1/2)+12*b^2*m*n^2*(-b*n+a)*arctan(x*f^(1/2)/e^(1/2
))*e^(1/2)/f^(1/2)+12*b^3*m*n^2*arctan(x*f^(1/2)/e^(1/2))*ln(c*x^n)*e^(1/2)
/f^(1/2)-6*I*b^3*m*n^3*polylog(2,-I*x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)-36*b
^3*m*n^2*x*ln(c*x^n)+12*b*m*n*x*(a+b*ln(c*x^n))^2-m*(a+b*ln(c*x^n))^3*ln(1-
x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+m*(a+b*ln(c*x^n))^3*ln(1+x*f^(1/2)
/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+6*a*b^2*n^2*x*ln(d*(f*x^2+e)^m)+6*b^3*n^2*x
*ln(c*x^n)*ln(d*(f*x^2+e)^m)-3*b*n*x*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)
```

Rubi [A] time = 1.50, antiderivative size = 977, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {2296, 2295, 2371, 6, 321, 205, 2351, 2324, 12, 4848, 2391, 2353, 2330, 2317, 2374, 6589, 2383}

$$36mn^3xb^3 - 36mn^2x\log\left(cx^n\right)b^3 + \frac{12\sqrt{e}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log\left(cx^n\right)b^3}{\sqrt{f}} - 6n^3x\log\left(d\left(fx^2+e\right)^m\right)b^3 + 6n^2x\log\left(cx^n\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] -24*a*b^2*m*n^2*x + 36*b^3*m*n^3*x - 12*b^2*m*n^2*(a - b*n)*x + (12*b^2*Sqr
t[e]*m*n^2*(a - b*n)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] - 36*b^3*m*n^2*x*
Log[c*x^n] + (12*b^3*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n])/
Sqrt[f] + 12*b*m*n*x*(a + b*Log[c*x^n])^2 - 2*m*x*(a + b*Log[c*x^n])^3 + (3
*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[f]
 - (Sqrt[-e]*m*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[f]
- (3*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/Sqr
t[f] + (Sqrt[-e]*m*(a + b*Log[c*x^n])^3*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/Sqrt
[f] + 6*a*b^2*n^2*x*Log[d*(e + f*x^2)^m] - 6*b^3*n^3*x*Log[d*(e + f*x^2)^m]
 + 6*b^3*n^2*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 3*b*n*x*(a + b*Log[c*x^n])
^2*Log[d*(e + f*x^2)^m] + x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m] - (6*
b^2*Sqrt[-e]*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/
Sqrt[f] + (3*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((Sqrt[f]*x)/S
qrt[-e])])/Sqrt[f] + (6*b^2*Sqrt[-e]*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, (S
qrt[f]*x)/Sqrt[-e]])/Sqrt[f] - (3*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])^2*PolyL
og[2, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[f] - ((6*I)*b^3*Sqrt[e]*m*n^3*PolyLog[2,
```

```
((-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] + ((6*I)*b^3*Sqrt[e]*m*n^3*PolyLog[2, (I
*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] + (6*b^3*Sqrt[-e]*m*n^3*PolyLog[3, -((Sqrt[f]
*x)/Sqrt[-e])])/Sqrt[f] - (6*b^2*Sqrt[-e]*m*n^2*(a + b*Log[c*x^n])*PolyLog[
3, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[f] - (6*b^3*Sqrt[-e]*m*n^3*PolyLog[3, (Sq
rt[f]*x)/Sqrt[-e]])/Sqrt[f] + (6*b^2*Sqrt[-e]*m*n^2*(a + b*Log[c*x^n])*Poly
Log[3, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[f] + (6*b^3*Sqrt[-e]*m*n^3*PolyLog[4, -(
(Sqrt[f]*x)/Sqrt[-e])])/Sqrt[f] - (6*b^3*Sqrt[-e]*m*n^3*PolyLog[4, (Sqrt[f]
*x)/Sqrt[-e]])/Sqrt[f]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
```

```
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2371

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)dx = 6ab^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-6b^3n^3x\log\left(d\left(e+fx^2\right)^m\right)+6b^3n^2$$

$$= 6ab^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-6b^3n^3x\log\left(d\left(e+fx^2\right)^m\right)+6b^3n^2$$

$$= 6ab^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-6b^3n^3x\log\left(d\left(e+fx^2\right)^m\right)+6b^3n^2$$

$$= -12b^2mn^2(a-bn)x+6ab^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-6b^3n^3x\log$$

$$= -12b^2mn^2(a-bn)x+\frac{12b^2\sqrt{e}\,mn^2(a-bn)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}+6ab^2n$$

$$= 12b^3mn^3x-12b^2mn^2(a-bn)x+\frac{12b^2\sqrt{e}\,mn^2(a-bn)\tan^{-1}\left(\frac{\sqrt{f}}{\sqrt{}}\right.}{\sqrt{f}}$$

$$= -12ab^2mn^2x+12b^3mn^3x-12b^2mn^2(a-bn)x+\frac{12b^2\sqrt{e}\,mn^2(a}{}$$

$$= -24ab^2mn^2x+24b^3mn^3x-12b^2mn^2(a-bn)x+\frac{12b^2\sqrt{e}\,mn^2(a}{}$$

$$= -24ab^2mn^2x+36b^3mn^3x-12b^2mn^2(a-bn)x+\frac{12b^2\sqrt{e}\,mn^2(a}{}$$

$$= -24ab^2mn^2x+36b^3mn^3x-12b^2mn^2(a-bn)x+\frac{12b^2\sqrt{e}\,mn^2(a}{}$$

$$= -24ab^2mn^2x+36b^3mn^3x-12b^2mn^2(a-bn)x+\frac{12b^2\sqrt{e}\,mn^2(a}{}$$

Mathematica [B] time = 0.75, size = 2302, normalized size = 2.36

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-2*a^3*Sqrt[f]*m*x + 12*a^2*b*Sqrt[f]*m*n*x - 36*a*b^2*Sqrt[f]*m*n^2*x + 4
8*b^3*Sqrt[f]*m*n^3*x + 2*a^3*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 6*a^2
*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 12*a*b^2*Sqrt[e]*m*n^2*ArcTan[
(Sqrt[f]*x)/Sqrt[e]] - 12*b^3*Sqrt[e]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 6
*a^2*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] + 12*a*b^2*Sqrt[e]*m*
n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*b^3*Sqrt[e]*m*n^3*ArcTan[(Sqrt[
f]*x)/Sqrt[e]]*Log[x] + 6*a*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*L
og[x]^2 - 6*b^3*Sqrt[e]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 2*b^3*
```

```
Sqrt[e]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^3 - 6*a^2*b*Sqrt[f]*m*x*Lo
g[c*x^n] + 24*a*b^2*Sqrt[f]*m*n*x*Log[c*x^n] - 36*b^3*Sqrt[f]*m*n^2*x*Log[c
*x^n] + 6*a^2*b*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 12*a*b^2
*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 12*b^3*Sqrt[e]*m*n^2*
ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 12*a*b^2*Sqrt[e]*m*n*ArcTan[(Sqrt[
f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 12*b^3*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/
Sqrt[e]]*Log[x]*Log[c*x^n] + 6*b^3*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]
]*Log[x]^2*Log[c*x^n] - 6*a*b^2*Sqrt[f]*m*x*Log[c*x^n]^2 + 12*b^3*Sqrt[f]*m
*n*x*Log[c*x^n]^2 + 6*a*b^2*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n
]^2 - 6*b^3*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - 6*b^3*Sq
rt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n]^2 - 2*b^3*Sqrt[f]*m
*x*Log[c*x^n]^3 + 2*b^3*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^3
+ (3*I)*a^2*b*Sqrt[e]*m*n*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*a*b
^2*Sqrt[e]*m*n^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[e]*
m*n^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*a*b^2*Sqrt[e]*m*n^2*Log
[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b^3*Sqrt[e]*m*n^3*Log[x]^2*Log
[1 - (I*Sqrt[f]*x)/Sqrt[e]] + I*b^3*Sqrt[e]*m*n^3*Log[x]^3*Log[1 - (I*Sqrt[
f]*x)/Sqrt[e]] + (6*I)*a*b^2*Sqrt[e]*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[
f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[e]*m*n^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[
f]*x)/Sqrt[e]] - (3*I)*b^3*Sqrt[e]*m*n^2*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqr
t[f]*x)/Sqrt[e]] + (3*I)*b^3*Sqrt[e]*m*n*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqr
t[f]*x)/Sqrt[e]] - (3*I)*a^2*b*Sqrt[e]*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqr
t[e]] + (6*I)*a*b^2*Sqrt[e]*m*n^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (
6*I)*b^3*Sqrt[e]*m*n^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*a*b^2*
Sqrt[e]*m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*b^3*Sqrt[e]*m
*n^3*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - I*b^3*Sqrt[e]*m*n^3*Log[x]^3
*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*a*b^2*Sqrt[e]*m*n*Log[x]*Log[c*x^n]
*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[e]*m*n^2*Log[x]*Log[c*x^n]
*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b^3*Sqrt[e]*m*n^2*Log[x]^2*Log[c*x^
n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*b^3*Sqrt[e]*m*n*Log[x]*Log[c*x^n]
^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + a^3*Sqrt[f]*x*Log[d*(e + f*x^2)^m] - 3*
a^2*b*Sqrt[f]*n*x*Log[d*(e + f*x^2)^m] + 6*a*b^2*Sqrt[f]*n^2*x*Log[d*(e + f
*x^2)^m] - 6*b^3*Sqrt[f]*n^3*x*Log[d*(e + f*x^2)^m] + 3*a^2*b*Sqrt[f]*x*Log
[c*x^n]*Log[d*(e + f*x^2)^m] - 6*a*b^2*Sqrt[f]*n*x*Log[c*x^n]*Log[d*(e + f*
x^2)^m] + 6*b^3*Sqrt[f]*n^2*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 3*a*b^2*Sqr
t[f]*x*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] - 3*b^3*Sqrt[f]*n*x*Log[c*x^n]^2*L
og[d*(e + f*x^2)^m] + b^3*Sqrt[f]*x*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] - (3*
I)*b*Sqrt[e]*m*n*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^
2*Log[c*x^n]^2)*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b*Sqrt[e]*m*n*
(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*P
olyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*a*b^2*Sqrt[e]*m*n^2*PolyLog[3, ((-
I)*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[e]*m*n^3*PolyLog[3, ((-I)*Sqrt[f]*x
)/Sqrt[e]] + (6*I)*b^3*Sqrt[e]*m*n^2*Log[c*x^n]*PolyLog[3, ((-I)*Sqrt[f]*x)
/Sqrt[e]] - (6*I)*a*b^2*Sqrt[e]*m*n^2*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + (
6*I)*b^3*Sqrt[e]*m*n^3*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[e
]*m*n^2*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[e]*m*
n^3*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[e]*m*n^3*PolyLog[
4, (I*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f]
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\log\left(cx^n\right)^3+3\,ab^2\log\left(cx^n\right)^2+3\,a^2b\log\left(cx^n\right)+a^3\right)\log\left(\left(fx^2+e\right)^m d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x^2 + e)^m*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)^3 \log\left(\left(fx^2+e\right)^m d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d), x)
```

maple [F] time = 123.98, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)^3 \ln\left(d\left(f\,x^2+e\right)^m\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m),x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\big(b^3mx\log\left(x^n\right)^3-3\big(\big(mn-m\log(c)\big)b^3-ab^2m\big)x\log\left(x^n\right)^2-3\big(2\big(mn-m\log(c)\big)ab^2-\big(2\,mn^2-2\,mn\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] (b^3*m*x*log(x^n)^3 - 3*((m*n - m*log(c))*b^3 - a*b^2*m)*x*log(x^n)^2 - 3*(
2*(m*n - m*log(c))*a*b^2 - (2*m*n^2 - 2*m*n*log(c) + m*log(c)^2)*b^3 - a^2*
b*m)*x*log(x^n) - (3*(m*n - m*log(c))*a^2*b - 3*(2*m*n^2 - 2*m*n*log(c) + m
*log(c)^2)*a*b^2 + (6*m*n^3 - 6*m*n^2*log(c) + 3*m*n*log(c)^2 - m*log(c)^3)
*b^3 - a^3*m)*x)*log(f*x^2 + e) + integrate((b^3*e*log(c)^3*log(d) + 3*a*b^
2*e*log(c)^2*log(d) + 3*a^2*b*e*log(c)*log(d) + a^3*e*log(d) - ((2*f*m - f*
log(d))*b^3*x^2 - b^3*e*log(d))*log(x^n)^3 - ((2*f*m - f*log(d))*a^3 - 3*(2
*f*m*n - (2*f*m - f*log(d))*log(c))*a^2*b + 3*(4*f*m*n^2 - 4*f*m*n*log(c) +
 (2*f*m - f*log(d))*log(c)^2)*a*b^2 - (12*f*m*n^3 - 12*f*m*n^2*log(c) + 6*f
*m*n*log(c)^2 - (2*f*m - f*log(d))*log(c)^3)*b^3)*x^2 + 3*(b^3*e*log(c)*log
(d) + a*b^2*e*log(d) - ((2*f*m - f*log(d))*a*b^2 - (2*f*m*n - (2*f*m - f*lo
g(d))*log(c))*b^3)*x^2)*log(x^n)^2 + 3*(b^3*e*log(c)^2*log(d) + 2*a*b^2*e*l
og(c)*log(d) + a^2*b*e*log(d) - ((2*f*m - f*log(d))*a^2*b - 2*(2*f*m*n - (2
*f*m - f*log(d))*log(c))*a*b^2 + (4*f*m*n^2 - 4*f*m*n*log(c) + (2*f*m - f*l
og(d))*log(c)^2)*b^3)*x^2)*log(x^n))/(f*x^2 + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(f\,x^2+e\right)^m\right)\left(a+b\,\ln\left(c\,x^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

3.113 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx$

Optimal. Leaf size=879

$$\frac{12b^3\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)n^3}{\sqrt{e}}-\frac{6b^3\log\left(d\left(fx^2+e\right)^m\right)n^3}{x}-\frac{6ib^3\sqrt{f}\,m\text{Li}_2\left(-\frac{i\sqrt{f}x}{\sqrt{e}}\right)n^3}{\sqrt{e}}+\frac{6ib^3\sqrt{f}\,m\text{Li}_2\left(\frac{i\sqrt{f}x}{\sqrt{e}}\right)n^3}{\sqrt{e}}+\frac{6b^3\sqrt{}}{}$$

```
[Out] -6*b^3*n^3*ln(d*(f*x^2+e)^m)/x-6*b^2*n^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/
x-3*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x-(a+b*ln(c*x^n))^3*ln(d*(f*x^2
+e)^m)/x+3*b*m*n*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^
(1/2)+m*(a+b*ln(c*x^n))^3*ln(1-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-3*b
*m*n*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-m*(a+b
*ln(c*x^n))^3*ln(1+x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-6*b^2*m*n^2*(a+
b*ln(c*x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-3*b*m*n*(a
+b*ln(c*x^n))^2*polylog(2,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+6*b^2*m
*n^2*(a+b*ln(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+3*b
*m*n*(a+b*ln(c*x^n))^2*polylog(2,x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+6
*b^3*m*n^3*polylog(3,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+6*b^2*m*n^2*
(a+b*ln(c*x^n))*polylog(3,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-6*b^3*m
*n^3*polylog(3,x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-6*b^2*m*n^2*(a+b*ln
(c*x^n))*polylog(3,x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-6*b^3*m*n^3*pol
ylog(4,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+6*b^3*m*n^3*polylog(4,x*f^
(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+12*b^3*m*n^3*arctan(x*f^(1/2)/e^(1/2))
*f^(1/2)/e^(1/2)+12*b^2*m*n^2*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))*f^(
1/2)/e^(1/2)-6*I*b^3*m*n^3*polylog(2,-I*x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1/2)+
6*I*b^3*m*n^3*polylog(2,I*x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 879, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {2305, 2304, 2378, 205, 2324, 12, 4848, 2391, 2330, 2317, 2374, 6589, 2383}

$$\frac{12b^3\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)n^3}{\sqrt{e}}-\frac{6b^3\log\left(d\left(fx^2+e\right)^m\right)n^3}{x}-\frac{6ib^3\sqrt{f}\,m\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)n^3}{\sqrt{e}}+\frac{6ib^3\sqrt{f}\,m\text{PolyLog}\left(2,\right.}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (12*b^3*Sqrt[f]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + (12*b^2*Sqrt[f
]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/Sqrt[e] + (3*b*Sqrt
[f]*m*n*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] + (Sqr
t[f]*m*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] - (3*b*
Sqrt[f]*m*n*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] -
(Sqrt[f]*m*(a + b*Log[c*x^n])^3*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] - (
6*b^3*n^3*Log[d*(e + f*x^2)^m])/x - (6*b^2*n^2*(a + b*Log[c*x^n])*Log[d*(e
+ f*x^2)^m])/x - (3*b*n*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x - ((a
+ b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x - (6*b^2*Sqrt[f]*m*n^2*(a + b*Log
[c*x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[-e] - (3*b*Sqrt[f]*m*n*(
a + b*Log[c*x^n])^2*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[-e] + (6*b^2*
Sqrt[f]*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e]
 + (3*b*Sqrt[f]*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/
Sqrt[-e] - ((6*I)*b^3*Sqrt[f]*m*n^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/S
qrt[e] + ((6*I)*b^3*Sqrt[f]*m*n^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/Sqrt[e
] + (6*b^3*Sqrt[f]*m*n^3*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[-e] + (6
*b^2*Sqrt[f]*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/
```

```
Sqrt[-e] - (6*b^3*Sqrt[f]*m*n^3*PolyLog[3, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e]
- (6*b^2*Sqrt[f]*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, (Sqrt[f]*x)/Sqrt[-e]])
/Sqrt[-e] - (6*b^3*Sqrt[f]*m*n^3*PolyLog[4, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[
-e] + (6*b^3*Sqrt[f]*m*n^3*PolyLog[4, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
```

```
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx = -\frac{6b^3n^3\log\left(d\left(e+fx^2\right)^m\right)}{x} - \frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+\right.\right.}{x}$$

$$= -\frac{6b^3n^3\log\left(d\left(e+fx^2\right)^m\right)}{x} - \frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+\right.\right.}{x}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\right.}{\sqrt{e}}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\right.}{\sqrt{e}}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\right.}{\sqrt{e}}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\right.}{\sqrt{e}}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\right.}{\sqrt{e}}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\right.}{\sqrt{e}}$$

Mathematica [B] time = 0.71, size = 2166, normalized size = 2.46

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (2*a^3*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 6*a^2*b*Sqrt[f]*m*n*x*ArcT
an[(Sqrt[f]*x)/Sqrt[e]] + 12*a*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[
e]] + 12*b^3*Sqrt[f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 6*a^2*b*Sqrt[f]*
m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*a*b^2*Sqrt[f]*m*n^2*x*ArcTan[
(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*b^3*Sqrt[f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqr
t[e]]*Log[x] + 6*a*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2
 + 6*b^3*Sqrt[f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 2*b^3*Sqrt[
f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^3 + 6*a^2*b*Sqrt[f]*m*x*ArcTa
n[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 12*a*b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*
x)/Sqrt[e]]*Log[c*x^n] + 12*b^3*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
*Log[c*x^n] - 12*a*b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log
[c*x^n] - 12*b^3*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x
^n] + 6*b^3*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2*Log[c*x^n]
 + 6*a*b^2*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + 6*b^3*Sqr
t[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - 6*b^3*Sqrt[f]*m*n*x*A
rcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n]^2 + 2*b^3*Sqrt[f]*m*x*ArcTan[(
Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^3 + (3*I)*a^2*b*Sqrt[f]*m*n*x*Log[x]*Log[1 -
 (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*a*b^2*Sqrt[f]*m*n^2*x*Log[x]*Log[1 - (I*Sqr
t[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[f]*m*n^3*x*Log[x]*Log[1 - (I*Sqrt[f]*x)/S
```

```
qrt[e]] - (3*I)*a*b^2*Sqrt[f]*m*n^2*x*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e
]] - (3*I)*b^3*Sqrt[f]*m*n^3*x*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + I*
b^3*Sqrt[f]*m*n^3*x*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*a*b^2*S
qrt[f]*m*n*x*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*S
qrt[f]*m*n^2*x*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*b^3
*Sqrt[f]*m*n^2*x*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)
*b^3*Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (3*
I)*a^2*b*Sqrt[f]*m*n*x*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*a*b^2*
Sqrt[f]*m*n^2*x*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[f]*m
*n^3*x*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*a*b^2*Sqrt[f]*m*n^2*x*
Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b^3*Sqrt[f]*m*n^3*x*Log[x]^
2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - I*b^3*Sqrt[f]*m*n^3*x*Log[x]^3*Log[1 + (
I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*a*b^2*Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]*Log[1 +
 (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[f]*m*n^2*x*Log[x]*Log[c*x^n]*Log[1
 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b^3*Sqrt[f]*m*n^2*x*Log[x]^2*Log[c*x^n]*L
og[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*b^3*Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]^2
*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - a^3*Sqrt[e]*Log[d*(e + f*x^2)^m] - 3*a^2*
b*Sqrt[e]*n*Log[d*(e + f*x^2)^m] - 6*a*b^2*Sqrt[e]*n^2*Log[d*(e + f*x^2)^m]
 - 6*b^3*Sqrt[e]*n^3*Log[d*(e + f*x^2)^m] - 3*a^2*b*Sqrt[e]*Log[c*x^n]*Log[
d*(e + f*x^2)^m] - 6*a*b^2*Sqrt[e]*n*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 6*b^
3*Sqrt[e]*n^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 3*a*b^2*Sqrt[e]*Log[c*x^n]^
2*Log[d*(e + f*x^2)^m] - 3*b^3*Sqrt[e]*n*Log[c*x^n]^2*Log[d*(e + f*x^2)^m]
- b^3*Sqrt[e]*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] - (3*I)*b*Sqrt[f]*m*n*x*(a^
2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*Poly
Log[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b*Sqrt[f]*m*n*x*(a^2 + 2*a*b*n + 2
*b^2*n^2 + 2*b*(a + b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, (I*Sqrt[
f]*x)/Sqrt[e]] + (6*I)*a*b^2*Sqrt[f]*m*n^2*x*PolyLog[3, ((-I)*Sqrt[f]*x)/Sq
rt[e]] + (6*I)*b^3*Sqrt[f]*m*n^3*x*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (
6*I)*b^3*Sqrt[f]*m*n^2*x*Log[c*x^n]*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] -
(6*I)*a*b^2*Sqrt[f]*m*n^2*x*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*S
qrt[f]*m*n^3*x*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[f]*m*n^2*
x*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[f]*m*n^3*x*
PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[f]*m*n^3*x*PolyLog[4,
 (I*Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*x)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3 \log\left(cx^n\right)^3 + 3\,ab^2 \log\left(cx^n\right)^2 + 3\,a^2 b \log\left(cx^n\right) + a^3\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x^2 + e)^m*d)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^3 \log\left(\left(fx^2+e\right)^m d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x^2, x)
```

maple [F] time = 112.50, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c\,x^{n}\right)+a\right)^{3} \ln \left(d \left(f\,x^{2}+e\right)^{m}\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^{3} m \log\left(x^{n}\right)^{3} + 3\left(mn + m\log(c)\right)a^{2}b + 3\left(2\,mn^{2} + 2\,mn\log(c) + m\log(c)^{2}\right)ab^{2} + \left(6\,mn^{3} + 6\,mn^{2}\log(c) + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -(b^3*m*log(x^n)^3 + 3*(m*n + m*log(c))*a^2*b + 3*(2*m*n^2 + 2*m*n*log(c) +
 m*log(c)^2)*a*b^2 + (6*m*n^3 + 6*m*n^2*log(c) + 3*m*n*log(c)^2 + m*log(c)^
3)*b^3 + a^3*m + 3*((m*n + m*log(c))*b^3 + a*b^2*m)*log(x^n)^2 + 3*(2*(m*n
+ m*log(c))*a*b^2 + (2*m*n^2 + 2*m*n*log(c) + m*log(c)^2)*b^3 + a^2*b*m)*lo
g(x^n))*log(f*x^2 + e)/x + integrate((b^3*e*log(c)^3*log(d) + 3*a*b^2*e*log
(c)^2*log(d) + 3*a^2*b*e*log(c)*log(d) + a^3*e*log(d) + ((2*f*m + f*log(d))
*b^3*x^2 + b^3*e*log(d))*log(x^n)^3 + ((2*f*m + f*log(d))*a^3 + 3*(2*f*m*n
+ (2*f*m + f*log(d))*log(c))*a^2*b + 3*(4*f*m*n^2 + 4*f*m*n*log(c) + (2*f*m
 + f*log(d))*log(c)^2)*a*b^2 + (12*f*m*n^3 + 12*f*m*n^2*log(c) + 6*f*m*n*lo
g(c)^2 + (2*f*m + f*log(d))*log(c)^3)*b^3)*x^2 + 3*(b^3*e*log(c)*log(d) + a
*b^2*e*log(d) + ((2*f*m + f*log(d))*a*b^2 + (2*f*m*n + (2*f*m + f*log(d))*l
og(c))*b^3)*x^2)*log(x^n)^2 + 3*(b^3*e*log(c)^2*log(d) + 2*a*b^2*e*log(c)*l
og(d) + a^2*b*e*log(d) + ((2*f*m + f*log(d))*a^2*b + 2*(2*f*m*n + (2*f*m +
f*log(d))*log(c))*a*b^2 + (4*f*m*n^2 + 4*f*m*n*log(c) + (2*f*m + f*log(d))*
log(c)^2)*b^3)*x^2)*log(x^n))/(f*x^4 + e*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^2,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x**2,x)
```

```
[Out] Timed out
```

3.114 $\int \frac{\left(a+b\log(cx^n)\right)^3\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx$

Optimal. Leaf size=1007

$$-\frac{4b^3f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)n^3}{27e^{3/2}}-\frac{2b^3\log\left(d\left(fx^2+e\right)^m\right)n^3}{27x^3}+\frac{2ib^3f^{3/2}m\text{Li}_2\left(-\frac{i\sqrt{f}x}{\sqrt{e}}\right)n^3}{9e^{3/2}}-\frac{2ib^3f^{3/2}m\text{Li}_2\left(\frac{i\sqrt{f}x}{\sqrt{e}}\right)n^3}{9e^{3/2}}+\frac{2b^3f}{}$$

```
[Out] -4/9*b^2*f^(3/2)*m*n^2*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))/e^(3/2)+1/
3*b*f^(3/2)*m*n*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-1/3
*b*f^(3/2)*m*n*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-2/3*
b^2*f^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))/(-e)^(3/
2)+2/3*b^2*f^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))/(-
e)^(3/2)+2*b^2*f^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,-x*f^(1/2)/(-e)^(1/2
))/(-e)^(3/2)-2*b^2*f^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,x*f^(1/2)/(-e)^
(1/2))/(-e)^(3/2)-2/9*I*b^3*f^(3/2)*m*n^3*polylog(2,I*x*f^(1/2)/e^(1/2))/e^
(3/2)-b*f^(3/2)*m*n*(a+b*ln(c*x^n))^2*polylog(2,-x*f^(1/2)/(-e)^(1/2))/(-e)
^(3/2)+b*f^(3/2)*m*n*(a+b*ln(c*x^n))^2*polylog(2,x*f^(1/2)/(-e)^(1/2))/(-e)
^(3/2)+2/9*I*b^3*f^(3/2)*m*n^3*polylog(2,-I*x*f^(1/2)/e^(1/2))/e^(3/2)-4/27
*b^3*f^(3/2)*m*n^3*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)+2/3*b^3*f^(3/2)*m*n^3*
polylog(3,-x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-2/3*b^3*f^(3/2)*m*n^3*polylog(3
,x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-2*b^3*f^(3/2)*m*n^3*polylog(4,-x*f^(1/2)/
(-e)^(1/2))/(-e)^(3/2)+2*b^3*f^(3/2)*m*n^3*polylog(4,x*f^(1/2)/(-e)^(1/2))/
(-e)^(3/2)-1/3*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^3-52/9*b^2*f*m*n^2*(a+
b*ln(c*x^n))/e/x-8/3*b*f*m*n*(a+b*ln(c*x^n))^2/e/x-2/27*b^3*n^3*ln(d*(f*x^2
+e)^m)/x^3-2/3*f*m*(a+b*ln(c*x^n))^3/e/x-2/9*b^2*n^2*(a+b*ln(c*x^n))*ln(d*(
f*x^2+e)^m)/x^3-1/3*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^3+1/3*f^(3/2)
*m*(a+b*ln(c*x^n))^3*ln(1-x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-1/3*f^(3/2)*m*(a
+b*ln(c*x^n))^3*ln(1+x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-160/27*b^3*f*m*n^3/e/
x
```

Rubi [A] time = 1.70, antiderivative size = 1007, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 16, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2305, 2304, 2378, 325, 205, 2351, 2324, 12, 4848, 2391, 2353, 2330, 2317, 2374, 6589, 2383}

$$-\frac{4b^3f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)n^3}{27e^{3/2}}-\frac{2b^3\log\left(d\left(fx^2+e\right)^m\right)n^3}{27x^3}+\frac{2ib^3f^{3/2}m\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)n^3}{9e^{3/2}}-\frac{2ib^3f^{3/2}m\text{PolyLog}\left(2\right.}{9e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^4,x]
```

```
[Out] (-160*b^3*f*m*n^3)/(27*e*x) - (4*b^3*f^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[
e]])/(27*e^(3/2)) - (52*b^2*f*m*n^2*(a + b*Log[c*x^n]))/(9*e*x) - (4*b^2*f^
(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*e^(3/2)) - (
8*b*f*m*n*(a + b*Log[c*x^n])^2)/(3*e*x) - (2*f*m*(a + b*Log[c*x^n])^3)/(3*e
*x) + (b*f^(3/2)*m*n*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(3
*(-e)^(3/2)) + (f^(3/2)*m*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]*x)/Sqrt[-e]
])/(3*(-e)^(3/2)) - (b*f^(3/2)*m*n*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)
/Sqrt[-e]])/(3*(-e)^(3/2)) - (f^(3/2)*m*(a + b*Log[c*x^n])^3*Log[1 + (Sqrt[
f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) - (2*b^3*n^3*Log[d*(e + f*x^2)^m])/(27*x^3)
 - (2*b^2*n^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(9*x^3) - (b*n*(a +
b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/(3*x^3) - ((a + b*Log[c*x^n])^3*Log[d
*(e + f*x^2)^m])/(3*x^3) - (2*b^2*f^(3/2)*m*n^2*(a + b*Log[c*x^n])*PolyLog[
2, -((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3/2)) - (b*f^(3/2)*m*n*(a + b*Log[c*x
^n])^2*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/(-e)^(3/2) + (2*b^2*f^(3/2)*m*n
^2*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) + (b
```

```
*f^(3/2)*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(-e)^(3
/2) + (((2*I)/9)*b^3*f^(3/2)*m*n^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^
(3/2) - (((2*I)/9)*b^3*f^(3/2)*m*n^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/e^(
3/2) + (2*b^3*f^(3/2)*m*n^3*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3
/2)) + (2*b^2*f^(3/2)*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((Sqrt[f]*x)/Sqr
t[-e])])/(-e)^(3/2) - (2*b^3*f^(3/2)*m*n^3*PolyLog[3, (Sqrt[f]*x)/Sqrt[-e]]
)/(3*(-e)^(3/2)) - (2*b^2*f^(3/2)*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, (Sqrt
[f]*x)/Sqrt[-e]])/(-e)^(3/2) - (2*b^3*f^(3/2)*m*n^3*PolyLog[4, -((Sqrt[f]*x
)/Sqrt[-e])])/(-e)^(3/2) + (2*b^3*f^(3/2)*m*n^3*PolyLog[4, (Sqrt[f]*x)/Sqrt
[-e]])/(-e)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

```
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2378

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^2\right)^m\right)}{x^4}dx = -\frac{2b^3n^3\log\left(d\left(e+fx^2\right)^m\right)}{27x^3} - \frac{2b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+\right.\right.}{9x^3}$$

$$= -\frac{2b^3n^3\log\left(d\left(e+fx^2\right)^m\right)}{27x^3} - \frac{2b^2n^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+\right.\right.}{9x^3}$$

$$= -\frac{4b^3fmn^3}{27ex} - \frac{2b^3n^3\log\left(d\left(e+fx^2\right)^m\right)}{27x^3} - \frac{2b^2n^2\left(a+b\log\left(cx^n\right)\right.}{9x^3}$$

$$= -\frac{4b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{2b^3n^3\log\left(d\left(e+fx^2\right)\right.}{27x^3}$$

$$= -\frac{16b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{4b^2fmn^2\left(a+b\log\left(\right.\right.}{9ex}$$

$$= -\frac{52b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{16b^2fmn^2\left(a+b\log\right.}{9ex}$$

$$= -\frac{160b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{52b^2fmn^2\left(a+b\log\right.}{9ex}$$

$$= -\frac{160b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{52b^2fmn^2\left(a+b\log\right.}{9ex}$$

$$= -\frac{160b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{52b^2fmn^2\left(a+b\log\right.}{9ex}$$

$$= -\frac{160b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{52b^2fmn^2\left(a+b\log\right.}{9ex}$$

Mathematica [B] time = 0.88, size = 2488, normalized size = 2.47

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^4,x]
```

```
[Out] (-18*a^3*Sqrt[e]*f*m*x^2 - 72*a^2*b*Sqrt[e]*f*m*n*x^2 - 156*a*b^2*Sqrt[e]*f
*m*n^2*x^2 - 160*b^3*Sqrt[e]*f*m*n^3*x^2 - 18*a^3*f^(3/2)*m*x^3*ArcTan[(Sqr
t[f]*x)/Sqrt[e]] - 18*a^2*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 1
2*a*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*b^3*f^(3/2)*m*n^3
*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 54*a^2*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]
*x)/Sqrt[e]]*Log[x] + 36*a*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]
]*Log[x] + 12*b^3*f^(3/2)*m*n^3*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 54
*a*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 18*b^3*f^(3
/2)*m*n^3*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 + 18*b^3*f^(3/2)*m*n^3*x
^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^3 - 54*a^2*b*Sqrt[e]*f*m*x^2*Log[c*x^
n] - 144*a*b^2*Sqrt[e]*f*m*n*x^2*Log[c*x^n] - 156*b^3*Sqrt[e]*f*m*n^2*x^2*L
og[c*x^n] - 54*a^2*b*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] -
 36*a*b^2*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 12*b^3*f
```

```
^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 108*a*b^2*f^(3/2)
*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 36*b^3*f^(3/2)*m*n
^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] - 54*b^3*f^(3/2)*m*n^2
*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2*Log[c*x^n] - 54*a*b^2*Sqrt[e]*f*m
*x^2*Log[c*x^n]^2 - 72*b^3*Sqrt[e]*f*m*n*x^2*Log[c*x^n]^2 - 54*a*b^2*f^(3/2
)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - 18*b^3*f^(3/2)*m*n*x^3*A
rcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + 54*b^3*f^(3/2)*m*n*x^3*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n]^2 - 18*b^3*Sqrt[e]*f*m*x^2*Log[c*x^n]^3
 - 18*b^3*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^3 - (27*I)*a
^2*b*f^(3/2)*m*n*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*a*b^2*f
^(3/2)*m*n^2*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*f^(3/2)*
m*n^3*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (27*I)*a*b^2*f^(3/2)*m*n^
2*x^3*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (9*I)*b^3*f^(3/2)*m*n^3*x^3
*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b^3*f^(3/2)*m*n^3*x^3*Log[
x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (54*I)*a*b^2*f^(3/2)*m*n*x^3*Log[x]*L
og[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^3*f^(3/2)*m*n^2*x^3*Log
[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (27*I)*b^3*f^(3/2)*m*n^2*x^
3*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (27*I)*b^3*f^(3/2)*m
*n*x^3*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (27*I)*a^2*b*f^
(3/2)*m*n*x^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*a*b^2*f^(3/2)*
m*n^2*x^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*f^(3/2)*m*n^3*x
^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (27*I)*a*b^2*f^(3/2)*m*n^2*x^3*L
og[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b^3*f^(3/2)*m*n^3*x^3*Log[x]
^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (9*I)*b^3*f^(3/2)*m*n^3*x^3*Log[x]^3*Lo
g[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (54*I)*a*b^2*f^(3/2)*m*n*x^3*Log[x]*Log[c*x^
n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^3*f^(3/2)*m*n^2*x^3*Log[x]*Log
[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (27*I)*b^3*f^(3/2)*m*n^2*x^3*Log[x
]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (27*I)*b^3*f^(3/2)*m*n*x^3*
Log[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 9*a^3*e^(3/2)*Log[d*(e
 + f*x^2)^m] - 9*a^2*b*e^(3/2)*n*Log[d*(e + f*x^2)^m] - 6*a*b^2*e^(3/2)*n^2
*Log[d*(e + f*x^2)^m] - 2*b^3*e^(3/2)*n^3*Log[d*(e + f*x^2)^m] - 27*a^2*b*e
^(3/2)*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 18*a*b^2*e^(3/2)*n*Log[c*x^n]*Log[
d*(e + f*x^2)^m] - 6*b^3*e^(3/2)*n^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 27*a
*b^2*e^(3/2)*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] - 9*b^3*e^(3/2)*n*Log[c*x^n]
^2*Log[d*(e + f*x^2)^m] - 9*b^3*e^(3/2)*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] +
 (3*I)*b*f^(3/2)*m*n*x^3*(9*a^2 + 6*a*b*n + 2*b^2*n^2 + 6*b*(3*a + b*n)*Log
[c*x^n] + 9*b^2*Log[c*x^n]^2)*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (3*I)*
b*f^(3/2)*m*n*x^3*(9*a^2 + 6*a*b*n + 2*b^2*n^2 + 6*b*(3*a + b*n)*Log[c*x^n]
 + 9*b^2*Log[c*x^n]^2)*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] - (54*I)*a*b^2*f^(
3/2)*m*n^2*x^3*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^3*f^(3/2)*m*
n^3*x^3*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (54*I)*b^3*f^(3/2)*m*n^2*x^3
*Log[c*x^n]*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (54*I)*a*b^2*f^(3/2)*m*n
^2*x^3*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^3*f^(3/2)*m*n^3*x^3*Pol
yLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + (54*I)*b^3*f^(3/2)*m*n^2*x^3*Log[c*x^n]*Po
lyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + (54*I)*b^3*f^(3/2)*m*n^3*x^3*PolyLog[4, (
(-I)*Sqrt[f]*x)/Sqrt[e]] - (54*I)*b^3*f^(3/2)*m*n^3*x^3*PolyLog[4, (I*Sqrt[
f]*x)/Sqrt[e]])/(27*e^(3/2)*x^3)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\log\left(cx^n\right)^3+3\,ab^2\log\left(cx^n\right)^2+3\,a^2b\log\left(cx^n\right)+a^3\right)\log\left(\left(fx^2+e\right)^m d\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x^2 + e)^m*d)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^n\right) + a\right)^3 \log\left(\left(fx^2 + e\right)^m d\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x^4, x)
```

maple [F] time = 146.86, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^n\right) + a\right)^3 \ln\left(d\left(f\,x^2 + e\right)^m\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m)/x^4,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^2+e)^m)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(9\, b^3 m \log\left(x^n\right)^3 + 9\left(mn + 3\, m \log(c)\right)a^2 b + 3\left(2\, mn^2 + 6\, mn \log(c) + 9\, m \log(c)^2\right)ab^2 + \left(2\, mn^3 + 6\, mn^2 \log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(9*b^3*m*log(x^n)^3 + 9*(m*n + 3*m*log(c))*a^2*b + 3*(2*m*n^2 + 6*m*n
*log(c) + 9*m*log(c)^2)*a*b^2 + (2*m*n^3 + 6*m*n^2*log(c) + 9*m*n*log(c)^2
+ 9*m*log(c)^3)*b^3 + 9*a^3*m + 9*((m*n + 3*m*log(c))*b^3 + 3*a*b^2*m)*log(
x^n)^2 + 3*(6*(m*n + 3*m*log(c))*a*b^2 + (2*m*n^2 + 6*m*n*log(c) + 9*m*log(
c)^2)*b^3 + 9*a^2*b*m)*log(x^n))*log(f*x^2 + e)/x^3 + integrate(1/27*(27*b^
3*e*log(c)^3*log(d) + 81*a*b^2*e*log(c)^2*log(d) + 81*a^2*b*e*log(c)*log(d)
 + 27*a^3*e*log(d) + 9*((2*f*m + 3*f*log(d))*b^3*x^2 + 3*b^3*e*log(d))*log(
x^n)^3 + (9*(2*f*m + 3*f*log(d))*a^3 + 9*(2*f*m*n + 3*(2*f*m + 3*f*log(d))*
log(c))*a^2*b + 3*(4*f*m*n^2 + 12*f*m*n*log(c) + 9*(2*f*m + 3*f*log(d))*log
(c)^2)*a*b^2 + (4*f*m*n^3 + 12*f*m*n^2*log(c) + 18*f*m*n*log(c)^2 + 9*(2*f*
m + 3*f*log(d))*log(c)^3)*b^3)*x^2 + 9*(9*b^3*e*log(c)*log(d) + 9*a*b^2*e*l
og(d) + (3*(2*f*m + 3*f*log(d))*a*b^2 + (2*f*m*n + 3*(2*f*m + 3*f*log(d))*l
og(c))*b^3)*x^2)*log(x^n)^2 + 3*(27*b^3*e*log(c)^2*log(d) + 54*a*b^2*e*log(
c)*log(d) + 27*a^2*b*e*log(d) + (9*(2*f*m + 3*f*log(d))*a^2*b + 6*(2*f*m*n
+ 3*(2*f*m + 3*f*log(d))*log(c))*a*b^2 + (4*f*m*n^2 + 12*f*m*n*log(c) + 9*(
2*f*m + 3*f*log(d))*log(c)^2)*b^3)*x^2)*log(x^n))/(f*x^6 + e*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2 + e\right)^m\right)\,\left(a + b\,\ln\left(c\,x^n\right)\right)^3}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^4,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x**4,x)
```

```
[Out] Timed out
```

3.115 $\int x^2 \log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=403

$$\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^6k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{3f^6}+\frac{e^5k\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{3f^5}-\frac{e^4kx}{}$$

```
[Out] 2/9*b*e^4*k*n*x/f^4-1/9*b*e^3*k*n*x^(3/2)/f^3+5/72*b*e^2*k*n*x^2/f^2-11/225
*b*e*k*n*x^(5/2)/f+1/27*b*k*n*x^3-1/6*e^4*k*x*(a+b*ln(c*x^n))/f^4+1/9*e^3*k
*x^(3/2)*(a+b*ln(c*x^n))/f^3-1/12*e^2*k*x^2*(a+b*ln(c*x^n))/f^2+1/15*e*k*x^
(5/2)*(a+b*ln(c*x^n))/f-1/18*k*x^3*(a+b*ln(c*x^n))+1/9*b*e^6*k*n*ln(e+f*x^(
1/2))/f^6-1/3*e^6*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/f^6+2/3*b*e^6*k*n*ln(-f
*x^(1/2)/e)*ln(e+f*x^(1/2))/f^6-1/9*b*n*x^3*ln(d*(e+f*x^(1/2))^k)+1/3*x^3*(
a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)+2/3*b*e^6*k*n*polylog(2,1+f*x^(1/2)/e)
/f^6-7/9*b*e^5*k*n*x^(1/2)/f^5+1/3*e^5*k*(a+b*ln(c*x^n))*x^(1/2)/f^5
```

Rubi [A] time = 0.34, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2395, 43, 2376, 2394, 2315}

$$\frac{2be^6kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{3f^6}+\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^6k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{3f^6}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-7*b*e^5*k*n*Sqrt[x])/(9*f^5) + (2*b*e^4*k*n*x)/(9*f^4) - (b*e^3*k*n*x^(3/
2))/(9*f^3) + (5*b*e^2*k*n*x^2)/(72*f^2) - (11*b*e*k*n*x^(5/2))/(225*f) + (
b*k*n*x^3)/27 + (b*e^6*k*n*Log[e + f*Sqrt[x]])/(9*f^6) - (b*n*x^3*Log[d*(e
+ f*Sqrt[x])^k])/9 + (2*b*e^6*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])
/(3*f^6) + (e^5*k*Sqrt[x]*(a + b*Log[c*x^n]))/(3*f^5) - (e^4*k*x*(a + b*Log
[c*x^n]))/(6*f^4) + (e^3*k*x^(3/2)*(a + b*Log[c*x^n]))/(9*f^3) - (e^2*k*x^2
*(a + b*Log[c*x^n]))/(12*f^2) + (e*k*x^(5/2)*(a + b*Log[c*x^n]))/(15*f) - (
k*x^3*(a + b*Log[c*x^n]))/18 - (e^6*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])
)/(3*f^6) + (x^3*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/3 + (2*b*e^6*
k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(3*f^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)dx = \frac{e^5k\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{3f^5} - \frac{e^4kx\left(a+b\log\left(cx^n\right)\right)}{6f^4} + \frac{e^3kx^{3/2}\left(a+b\right.}{9f^3}$$

$$= -\frac{2be^5kn\sqrt{x}}{3f^5} + \frac{be^4knx}{6f^4} - \frac{2be^3knx^{3/2}}{27f^3} + \frac{be^2knx^2}{24f^2} - \frac{2beknx^{5/2}}{75f} + \frac{1}{5}$$

$$= -\frac{2be^5kn\sqrt{x}}{3f^5} + \frac{be^4knx}{6f^4} - \frac{2be^3knx^{3/2}}{27f^3} + \frac{be^2knx^2}{24f^2} - \frac{2beknx^{5/2}}{75f} + \frac{1}{5}$$

$$= -\frac{2be^5kn\sqrt{x}}{3f^5} + \frac{be^4knx}{6f^4} - \frac{2be^3knx^{3/2}}{27f^3} + \frac{be^2knx^2}{24f^2} - \frac{2beknx^{5/2}}{75f} + \frac{1}{5}$$

$$= -\frac{2be^5kn\sqrt{x}}{3f^5} + \frac{be^4knx}{6f^4} - \frac{2be^3knx^{3/2}}{27f^3} + \frac{be^2knx^2}{24f^2} - \frac{2beknx^{5/2}}{75f} + \frac{1}{5}$$

$$= -\frac{7be^5kn\sqrt{x}}{9f^5} + \frac{2be^4knx}{9f^4} - \frac{be^3knx^{3/2}}{9f^3} + \frac{5be^2knx^2}{72f^2} - \frac{11beknx^{5/2}}{225f} +$$

Mathematica [A] time = 0.47, size = 434, normalized size = 1.08

$$-\frac{600e^6k\log\left(e+f\sqrt{x}\right)\left(3a+3b\log\left(cx^n\right)-3bn\log(x)-bn\right)-1800af^6x^3\log\left(d\left(e+f\sqrt{x}\right)^k\right)-1800ae^5fk\sqrt{x}+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/5400*(-1800*a*e^5*f*k*Sqrt[x] + 4200*b*e^5*f*k*n*Sqrt[x] + 900*a*e^4*f^2
*k*x - 1200*b*e^4*f^2*k*n*x - 600*a*e^3*f^3*k*x^(3/2) + 600*b*e^3*f^3*k*n*x
^(3/2) + 450*a*e^2*f^4*k*x^2 - 375*b*e^2*f^4*k*n*x^2 - 360*a*e*f^5*k*x^(5/2
```

```
) + 264*b*e*f^5*k*n*x^(5/2) + 300*a*f^6*k*x^3 - 200*b*f^6*k*n*x^3 - 1800*a*
f^6*x^3*Log[d*(e + f*Sqrt[x])^k] + 600*b*f^6*n*x^3*Log[d*(e + f*Sqrt[x])^k]
 + 1800*b*e^6*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x] - 1800*b*e^5*f*k*Sqrt[x]*Lo
g[c*x^n] + 900*b*e^4*f^2*k*x*Log[c*x^n] - 600*b*e^3*f^3*k*x^(3/2)*Log[c*x^n
] + 450*b*e^2*f^4*k*x^2*Log[c*x^n] - 360*b*e*f^5*k*x^(5/2)*Log[c*x^n] + 300
*b*f^6*k*x^3*Log[c*x^n] - 1800*b*f^6*x^3*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n
] + 600*e^6*k*Log[e + f*Sqrt[x]]*(3*a - b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n]
) + 3600*b*e^6*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/f^6
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2\log\left(cx^n\right)+ax^2\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)*log((f*sqrt(x) + e)^k*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x^2\log\left(\left(f\sqrt{x}+e\right)^k d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*sqrt(x) + e)^k*d), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)x^2\ln\left(d\left(f\sqrt{x}+e\right)^k\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)*ln(d*(e+f*x^(1/2))^k),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)*ln(d*(e+f*x^(1/2))^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{147\,bex^3\log(d)\log\left(x^n\right)+49\left(3\,ae\log(d)-\left(en\log(d)-3\,e\log(c)\log(d)\right)b\right)x^3+49\left(3\,bex^3\log\left(x^n\right)-\left(\left(en-3\right.\right.\right.}{441\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="maxima"
)
```

```
[Out] 1/441*(147*b*e*x^3*log(d)*log(x^n) + 49*(3*a*e*log(d) - (e*n*log(d) - 3*e*l
og(c)*log(d))*b)*x^3 + 49*(3*b*e*x^3*log(x^n) - ((e*n - 3*e*log(c))*b - 3*a
*e)*x^3)*k*log(f*sqrt(x) + e) - (21*b*f*k*x^4*log(x^n) + (21*a*f*k - (13*f*
k*n - 21*f*k*log(c))*b)*x^4)/sqrt(x))/e + integrate(1/18*(3*b*f^2*k*x^3*log
(x^n) + (3*a*f^2*k - (f^2*k*n - 3*f^2*k*log(c))*b)*x^3)/(e*f*sqrt(x) + e^2)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,{\left(e+f\,\sqrt{x}\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
[Out] int(x^2*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k),x)
[Out] Timed out
```

3.116 $\int x \log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)dx$

Optimal. Leaf size=313

$$\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^4k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{2f^4}+\frac{e^3k\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{2f^3}-\frac{e^2kx}{}$$

```
[Out] 3/8*b*e^2*k*n*x/f^2-7/36*b*e*k*n*x^(3/2)/f+1/8*b*k*n*x^2-1/4*e^2*k*x*(a+b*l
n(c*x^n))/f^2+1/6*e*k*x^(3/2)*(a+b*ln(c*x^n))/f-1/8*k*x^2*(a+b*ln(c*x^n))+1
/4*b*e^4*k*n*ln(e+f*x^(1/2))/f^4-1/2*e^4*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/
f^4+b*e^4*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^4-1/4*b*n*x^2*ln(d*(e+f*x^
(1/2))^k)+1/2*x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)+b*e^4*k*n*polylog(2
,1+f*x^(1/2)/e)/f^4-5/4*b*e^3*k*n*x^(1/2)/f^3+1/2*e^3*k*(a+b*ln(c*x^n))*x^(
1/2)/f^3
```

Rubi [A] time = 0.24, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2454, 2395, 43, 2376, 2394, 2315}

$$\frac{be^4kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{f^4}+\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^4k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{2f^4}+$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-5*b*e^3*k*n*Sqrt[x])/(4*f^3) + (3*b*e^2*k*n*x)/(8*f^2) - (7*b*e*k*n*x^(3/
2))/(36*f) + (b*k*n*x^2)/8 + (b*e^4*k*n*Log[e + f*Sqrt[x]])/(4*f^4) - (b*n*
x^2*Log[d*(e + f*Sqrt[x])^k])/4 + (b*e^4*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sq
rt[x])/e)])/f^4 + (e^3*k*Sqrt[x]*(a + b*Log[c*x^n]))/(2*f^3) - (e^2*k*x*(a
+ b*Log[c*x^n]))/(4*f^2) + (e*k*x^(3/2)*(a + b*Log[c*x^n]))/(6*f) - (k*x^2*
(a + b*Log[c*x^n]))/8 - (e^4*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*f^
4) + (x^2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/2 + (b*e^4*k*n*PolyL
og[2, 1 + (f*Sqrt[x])/e])/f^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x \log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)dx = \frac{e^3k\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{2f^3} - \frac{e^2kx\left(a+b\log\left(cx^n\right)\right)}{4f^2} + \frac{ekx^{3/2}\left(a+b\log\right.}{6f}$$

$$= -\frac{be^3kn\sqrt{x}}{f^3} + \frac{be^2knx}{4f^2} - \frac{beknx^{3/2}}{9f} + \frac{1}{16}bknx^2 + \frac{e^3k\sqrt{x}\left(a+b\log\right.}{2f^3}$$

$$= -\frac{be^3kn\sqrt{x}}{f^3} + \frac{be^2knx}{4f^2} - \frac{beknx^{3/2}}{9f} + \frac{1}{16}bknx^2 + \frac{e^3k\sqrt{x}\left(a+b\log\right.}{2f^3}$$

$$= -\frac{be^3kn\sqrt{x}}{f^3} + \frac{be^2knx}{4f^2} - \frac{beknx^{3/2}}{9f} + \frac{1}{16}bknx^2 - \frac{1}{4}bnx^2\log\left(d\left(e+\right.\right.$$

$$= -\frac{be^3kn\sqrt{x}}{f^3} + \frac{be^2knx}{4f^2} - \frac{beknx^{3/2}}{9f} + \frac{1}{16}bknx^2 - \frac{1}{4}bnx^2\log\left(d\left(e+\right.\right.$$

$$= -\frac{5be^3kn\sqrt{x}}{4f^3} + \frac{3be^2knx}{8f^2} - \frac{7beknx^{3/2}}{36f} + \frac{1}{8}bknx^2 + \frac{be^4kn\log\left(e+\right.}{4f^4}$$

Mathematica [A] time = 0.36, size = 336, normalized size = 1.07

$$-\frac{18e^4k\log\left(e+f\sqrt{x}\right)\left(2a+2b\log\left(cx^n\right)-2bn\log(x)-bn\right)-36af^4x^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)-36ae^3fk\sqrt{x}+18ae^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/72*(-36*a*e^3*f*k*Sqrt[x] + 90*b*e^3*f*k*n*Sqrt[x] + 18*a*e^2*f^2*k*x -
27*b*e^2*f^2*k*n*x - 12*a*e*f^3*k*x^(3/2) + 14*b*e*f^3*k*n*x^(3/2) + 9*a*f^
4*k*x^2 - 9*b*f^4*k*n*x^2 - 36*a*f^4*x^2*Log[d*(e + f*Sqrt[x])^k] + 18*b*f^
4*n*x^2*Log[d*(e + f*Sqrt[x])^k] + 36*b*e^4*k*n*Log[1 + (f*Sqrt[x])/e]*Log[
x] - 36*b*e^3*f*k*Sqrt[x]*Log[c*x^n] + 18*b*e^2*f^2*k*x*Log[c*x^n] - 12*b*e
*f^3*k*x^(3/2)*Log[c*x^n] + 9*b*f^4*k*x^2*Log[c*x^n] - 36*b*f^4*x^2*Log[d*(
```

```
e + f*Sqrt[x])^k]*Log[c*x^n] + 18*e^4*k*Log[e + f*Sqrt[x]]*(2*a - b*n - 2*b
*n*Log[x] + 2*b*Log[c*x^n]) + 72*b*e^4*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/f^
4
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx\log\left(cx^{n}\right)+ax\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="fricas")
[Out] integral((b*x*log(c*x^n) + a*x)*log((f*sqrt(x) + e)^k*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^{n}\right)+a\right)x\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x*log((f*sqrt(x) + e)^k*d), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^{n}\right)+a\right)x\ln\left(d\left(f\sqrt{x}+e\right)^{k}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k),x)
[Out] int(x*(b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{50\,bex^{2}\log(d)\log\left(x^{n}\right)+25\left(2\,ae\log(d)-\left(en\log(d)-2\,e\log(c)\log(d)\right)b\right)x^{2}+25\left(2\,bex^{2}\log\left(x^{n}\right)-\left(\left(en-2\,e\right.\right.\right.}{100\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="maxima")
[Out] 1/100*(50*b*e*x^2*log(d)*log(x^n) + 25*(2*a*e*log(d) - (e*n*log(d) - 2*e*lo
g(c)*log(d))*b)*x^2 + 25*(2*b*e*x^2*log(x^n) - ((e*n - 2*e*log(c))*b - 2*a*
e)*x^2)*k*log(f*sqrt(x) + e) - (10*b*f*k*x^3*log(x^n) + (10*a*f*k - (9*f*k*
n - 10*f*k*log(c))*b)*x^3)/sqrt(x))/e + integrate(1/8*(2*b*f^2*k*x^2*log(x^
n) + (2*a*f^2*k - (f^2*k*n - 2*f^2*k*log(c))*b)*x^2)/(e*f*sqrt(x) + e^2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\left(e+f\,\sqrt{x}\right)^{k}\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
[Out] int(x*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k),x)
[Out] Timed out
```

3.117 $\int \log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=209

$$x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^2k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}+\frac{ek\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{f}-\frac{1}{2}kx\left(a+\right.$$

```
[Out] b*k*n*x-1/2*k*x*(a+b*ln(c*x^n))+b*e^2*k*n*ln(e+f*x^(1/2))/f^2-e^2*k*(a+b*ln
(c*x^n))*ln(e+f*x^(1/2))/f^2+2*b*e^2*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f
^2-b*n*x*ln(d*(e+f*x^(1/2))^k)+x*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)+2*b*
e^2*k*n*polylog(2,1+f*x^(1/2)/e)/f^2-3*b*e*k*n*x^(1/2)/f+e*k*(a+b*ln(c*x^n)
)*x^(1/2)/f
```

Rubi [A] time = 0.15, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2448, 266, 43, 2370, 2454, 2394, 2315}

$$\frac{2be^2kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{f^2}+x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^2k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}+\frac{ek}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-3*b*e*k*n*Sqrt[x])/f + b*k*n*x + (b*e^2*k*n*Log[e + f*Sqrt[x]])/f^2 - b*n
*x*Log[d*(e + f*Sqrt[x])^k] + (2*b*e^2*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt
[x])/e)])/f^2 + (e*k*Sqrt[x]*(a + b*Log[c*x^n]))/f - (k*x*(a + b*Log[c*x^n]
))/2 - (e^2*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/f^2 + x*Log[d*(e + f*S
qrt[x])^k]*(a + b*Log[c*x^n]) + (2*b*e^2*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])
/f^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)dx = \frac{ek\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{f}-\frac{1}{2}kx\left(a+b\log\left(cx^n\right)\right)-\frac{e^2k\log\left(e+f\sqrt{x}\right.}{f}$$

$$= -\frac{2bekn\sqrt{x}}{f}+\frac{1}{2}bknx+\frac{ek\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{f}-\frac{1}{2}kx\left(a+b\log\left(cx\right.\right.$$

$$= -\frac{2bekn\sqrt{x}}{f}+\frac{1}{2}bknx-bnx\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{ek\sqrt{x}\left(a+b\log\left(\right.\right.}{f}$$

$$= -\frac{2bekn\sqrt{x}}{f}+\frac{1}{2}bknx-bnx\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2be^2kn\log\left(e+f\right.}{f}$$

$$= -\frac{2bekn\sqrt{x}}{f}+\frac{1}{2}bknx-bnx\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2be^2kn\log\left(e+f\right.}{f}$$

$$= -\frac{3bekn\sqrt{x}}{f}+bknx+\frac{be^2kn\log\left(e+f\sqrt{x}\right)}{f^2}-bnx\log\left(d\left(e+f\sqrt{x}\right)\right.$$

Mathematica [A] time = 0.23, size = 218, normalized size = 1.04

$$-\frac{e^2k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)-bn\log(x)-bn\right)}{f^2}+ax\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{aek\sqrt{x}}{f}-\frac{akx}{2}+bx\log\left(cx^n\right)\log\left(d\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (a*e*k*Sqrt[x])/f - (3*b*e*k*n*Sqrt[x])/f - (a*k*x)/2 + b*k*n*x + a*x*Log[d
*(e + f*Sqrt[x])^k] - b*n*x*Log[d*(e + f*Sqrt[x])^k] - (b*e^2*k*n*Log[1 + (
f*Sqrt[x])/e]*Log[x])/f^2 + (b*e*k*Sqrt[x]*Log[c*x^n])/f - (b*k*x*Log[c*x^n
])/2 + b*x*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n] - (e^2*k*Log[e + f*Sqrt[x]]*
```

```
(a - b*n - b*n*Log[x] + b*Log[c*x^n]))/f^2 - (2*b*e^2*k*n*PolyLog[2, -((f*S
qrt[x])/e)])/f^2
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)\ln\left(d\left(f\sqrt{x}+e\right)^k\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k),x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9\,bex\log(d)\log\left(x^n\right)+9\left(bex\log\left(x^n\right)-\left(\left(en-e\log(c)\right)b-ae\right)x\right)k\log\left(f\sqrt{x}+e\right)+9\left(ae\log(d)-\left(en\log(d)-\right.\right.}{9\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="maxima")
```

```
[Out] 1/9*(9*b*e*x*log(d)*log(x^n) + 9*(b*e*x*log(x^n) - ((e*n - e*log(c))*b - a*
e)*x)*k*log(f*sqrt(x) + e) + 9*(a*e*log(d) - (e*n*log(d) - e*log(c)*log(d))
*b)*x - (3*b*f*k*x^2*log(x^n) + (3*a*f*k - (5*f*k*n - 3*f*k*log(c))*b)*x^2)
/sqrt(x))/e + integrate(1/2*(b*f^2*k*x*log(x^n) + (a*f^2*k - (f^2*k*n - f^2
*k*log(c))*b)*x)/(e*f*sqrt(x) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\ln\left(d\left(e+f\,\sqrt{x}\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k),x)
```

```
[Out] Timed out
```

3.118 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=117

$$\frac{(a+b\log(cx^n))^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2bn}-2k\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))-\frac{k\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^2}{2bn}+4$$

```
[Out] 1/2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2))^k)/b/n-1/2*k*(a+b*ln(c*x^n))^2*ln(
1+f*x^(1/2)/e)/b/n-2*k*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)+4*b*k*n*poly
log(3,-f*x^(1/2)/e)
```

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2375, 2337, 2374, 6589}

$$-2k\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))+4bkn\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)+\frac{(a+b\log(cx^n))^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2bn}-$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n])^2)/(2*b*n) - (k*Log[1 + (f*Sqr
t[x])/e]*(a + b*Log[c*x^n])^2)/(2*b*n) - 2*k*(a + b*Log[c*x^n])*PolyLog[2,
-((f*Sqrt[x])/e)] + 4*b*k*n*PolyLog[3, -((f*Sqrt[x])/e)]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)^2}{2bn} - \frac{(fk)\int \frac{\left(a+b\log(cx^n)\right)^2}{\left(e+f\sqrt{x}\right)\sqrt{x}}\,dx}{4bn}$$
$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)^2}{2bn} - \frac{k\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{2bn}$$
$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)^2}{2bn} - \frac{k\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{2bn}$$
$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)^2}{2bn} - \frac{k\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{2bn}$$

Mathematica [A] time = 0.17, size = 186, normalized size = 1.59

$$\frac{1}{2}\left(4a\log\left(-\frac{f\sqrt{x}}{e}\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)+4ak\mathrm{Li}_2\left(\frac{\sqrt{x}f}{e}+1\right)+2b\log(x)\log\left(cx^n\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)-4bk\log(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (4*a*Log[d*(e + f*Sqrt[x])^k]*Log[-((f*Sqrt[x])/e)] - b*n*Log[d*(e + f*Sqrt
[x])^k]*Log[x]^2 + b*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 2*b*Log[d*(e + f
*Sqrt[x])^k]*Log[x]*Log[c*x^n] - 2*b*k*Log[1 + (f*Sqrt[x])/e]*Log[x]*Log[c*
x^n] + 4*a*k*PolyLog[2, 1 + (f*Sqrt[x])/e] - 4*b*k*Log[c*x^n]*PolyLog[2, -(
(f*Sqrt[x])/e)] + 8*b*k*n*PolyLog[3, -((f*Sqrt[x])/e)])/2
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right)+a\right)\ln\left(d\left(f\sqrt{x}+e\right)^k\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ben\log(d)\log(x)^2 - 2\,be\log(d)\log(x)\log(x^n) + \big(ben\log(x)^2 - 2\,be\log(x)\log(x^n) - 2\big(be\log(c) + ae\big)\log(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*e*n*log(d)*log(x)^2 - 2*b*e*log(d)*log(x)*log(x^n) + (b*e*n*log(x)^
2 - 2*b*e*log(x)*log(x^n) - 2*(b*e*log(c) + a*e)*log(x))*k*log(f*sqrt(x) +
e) - 2*(b*e*log(c)*log(d) + a*e*log(d))*log(x) - (b*f*k*n*x*log(x)^2 - 2*(b
*f*k*log(c) + a*f*k)*x*log(x) + 4*(a*f*k - (2*f*k*n - f*k*log(c))*b)*x - 2*
(b*f*k*x*log(x) - 2*b*f*k*x)*log(x^n))/sqrt(x))/e + integrate(-1/4*(b*f^2*k
*n*log(x)^2 - 2*b*f^2*k*log(x)*log(x^n) - 2*(b*f^2*k*log(c) + a*f^2*k)*log(
x))/(e*f*sqrt(x) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\,\sqrt{x}\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x,x)
```

```
[Out] Timed out
```

3.119 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=248

$$-\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x}+\frac{f^2k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e^2}-\frac{f^2k\log(x)(a+b\log(cx^n))}{2e^2}-\frac{fk(a}{}$$

```
[Out] -1/2*b*f^2*k*n*ln(x)/e^2+1/4*b*f^2*k*n*ln(x)^2/e^2-1/2*f^2*k*ln(x)*(a+b*ln(
c*x^n))/e^2+b*f^2*k*n*ln(e+f*x^(1/2))/e^2+f^2*k*(a+b*ln(c*x^n))*ln(e+f*x^(1
/2))/e^2-2*b*f^2*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/e^2-b*n*ln(d*(e+f*x^(
1/2))^k)/x-(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x-2*b*f^2*k*n*polylog(2,1+
f*x^(1/2)/e)/e^2-3*b*f*k*n/e/x^(1/2)-f*k*(a+b*ln(c*x^n))/e/x^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2395, 44, 2376, 2394, 2315, 2301}

$$-\frac{2bf^2kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{e^2}-\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x}+\frac{f^2k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e^2}-\frac{f^2k}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] (-3*b*f*k*n)/(e*Sqrt[x]) + (b*f^2*k*n*Log[e + f*Sqrt[x]])/e^2 - (b*n*Log[d*
(e + f*Sqrt[x])^k])/x - (2*b*f^2*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e
)])/e^2 - (b*f^2*k*n*Log[x])/(2*e^2) + (b*f^2*k*n*Log[x]^2)/(4*e^2) - (f*k*
(a + b*Log[c*x^n]))/(e*Sqrt[x]) + (f^2*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^
n]))/e^2 - (Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x - (f^2*k*Log[x]*
(a + b*Log[c*x^n]))/(2*e^2) - (2*b*f^2*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/e
^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = -\frac{fk\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}}+\frac{f^2k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{e^2}-\ldots$$

$$=-\frac{2bfkn}{e\sqrt{x}}-\frac{fk\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}}+\frac{f^2k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(\ldots\right.\right.}{e^2}$$

$$=-\frac{2bfkn}{e\sqrt{x}}+\frac{bf^2kn\log^2(x)}{4e^2}-\frac{fk\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}}+\frac{f^2k\log\left(e+\ldots\right.}{\ldots}$$

$$=-\frac{2bfkn}{e\sqrt{x}}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x}-\frac{2bf^2kn\log\left(e+f\sqrt{x}\right)\log\left(\ldots\right.}{e^2}$$

$$=-\frac{2bfkn}{e\sqrt{x}}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x}-\frac{2bf^2kn\log\left(e+f\sqrt{x}\right)\log\left(\ldots\right.}{e^2}$$

$$=-\frac{3bfkn}{e\sqrt{x}}+\frac{bf^2kn\log\left(e+f\sqrt{x}\right)}{e^2}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x}-\frac{2bf\ldots}{\ldots}$$

Mathematica [A] time = 0.32, size = 250, normalized size = 1.01

$$-\frac{-4f^2kx\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)-bn\log(x)+bn\right)+4ae^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)+4aefk\sqrt{x}+2af^2kx\log\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -1/4*(4*a*e*f*k*Sqrt[x] + 12*b*e*f*k*n*Sqrt[x] + 4*a*e^2*Log[d*(e + f*Sqrt[
x])^k] + 4*b*e^2*n*Log[d*(e + f*Sqrt[x])^k] + 2*a*f^2*k*x*Log[x] + 2*b*f^2*
```

```
k*n*x*Log[x] - 4*b*f^2*k*n*x*Log[1 + (f*Sqrt[x])/e]*Log[x] - b*f^2*k*n*x*Lo
g[x]^2 + 4*b*e*f*k*Sqrt[x]*Log[c*x^n] + 4*b*e^2*Log[d*(e + f*Sqrt[x])^k]*Lo
g[c*x^n] + 2*b*f^2*k*x*Log[x]*Log[c*x^n] - 4*f^2*k*x*Log[e + f*Sqrt[x]]*(a
+ b*n - b*n*Log[x] + b*Log[c*x^n]) - 8*b*f^2*k*n*x*PolyLog[2, -((f*Sqrt[x])
/e)])/(e^2*x)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^2, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^{n}\right)+a\right)\ln\left(d\left(f\sqrt{x}+e\right)^{k}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{be\log(d)\log\left(x^{n}\right)+\left(be\log\left(x^{n}\right)+\left(en+e\log(c)\right)b+ae\right)k\log\left(f\sqrt{x}+e\right)+ae\log(d)+\left(en\log(d)+e\log(c)\log\right.}{ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^2,x, algorithm="maxima"
)
```

```
[Out] -(b*e*log(d)*log(x^n) + (b*e*log(x^n) + (e*n + e*log(c))*b + a*e)*k*log(f*s
qrt(x) + e) + a*e*log(d) + (e*n*log(d) + e*log(c)*log(d))*b + (b*f*k*x*log(
x^n) + (a*f*k + (3*f*k*n + f*k*log(c))*b)*x)/sqrt(x))/(e*x) - integrate(1/2
*(b*f^2*k*log(x^n) + a*f^2*k + (f^2*k*n + f^2*k*log(c))*b)/(e*f*x^(3/2) + e
^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\ln\left(d\left(e+f\sqrt{x}\right)^{k}\right)\left(a+b\ln\left(c\,x^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**2,x)
```

```
[Out] Timed out
```

3.120 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log(cx^n)\right)}{x^3}\,dx$

Optimal. Leaf size=346

$$-\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2x^2}+\frac{f^4k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{2e^4}-\frac{f^4k\log(x)\left(a+b\log\left(cx^n\right)\right)}{4e^4}-\frac{f^3k\left(a\right.}{}$$

```
[Out] -7/36*b*f*k*n/e/x^(3/2)+3/8*b*f^2*k*n/e^2/x-1/8*b*f^4*k*n*ln(x)/e^4+1/8*b*f
^4*k*n*ln(x)^2/e^4-1/6*f*k*(a+b*ln(c*x^n))/e/x^(3/2)+1/4*f^2*k*(a+b*ln(c*x^
n))/e^2/x-1/4*f^4*k*ln(x)*(a+b*ln(c*x^n))/e^4+1/4*b*f^4*k*n*ln(e+f*x^(1/2))
/e^4+1/2*f^4*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e^4-b*f^4*k*n*ln(-f*x^(1/2)/
e)*ln(e+f*x^(1/2))/e^4-1/4*b*n*ln(d*(e+f*x^(1/2))^k)/x^2-1/2*(a+b*ln(c*x^n)
)*ln(d*(e+f*x^(1/2))^k)/x^2-b*f^4*k*n*polylog(2,1+f*x^(1/2)/e)/e^4-5/4*b*f^
3*k*n/e^3/x^(1/2)-1/2*f^3*k*(a+b*ln(c*x^n))/e^3/x^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2454, 2395, 44, 2376, 2394, 2315, 2301}

$$-\frac{bf^4kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{e^4}-\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2x^2}+\frac{f^4k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{2e^4}-\frac{f^4k\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] (-7*b*f*k*n)/(36*e*x^(3/2)) + (3*b*f^2*k*n)/(8*e^2*x) - (5*b*f^3*k*n)/(4*e^
3*Sqrt[x]) + (b*f^4*k*n*Log[e + f*Sqrt[x]])/(4*e^4) - (b*n*Log[d*(e + f*Sqr
t[x])^k])/(4*x^2) - (b*f^4*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/e^
4 - (b*f^4*k*n*Log[x])/(8*e^4) + (b*f^4*k*n*Log[x]^2)/(8*e^4) - (f*k*(a + b
*Log[c*x^n]))/(6*e*x^(3/2)) + (f^2*k*(a + b*Log[c*x^n]))/(4*e^2*x) - (f^3*k
*(a + b*Log[c*x^n]))/(2*e^3*Sqrt[x]) + (f^4*k*Log[e + f*Sqrt[x]]*(a + b*Log
[c*x^n]))/(2*e^4) - (Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/(2*x^2) -
 (f^4*k*Log[x]*(a + b*Log[c*x^n]))/(4*e^4) - (b*f^4*k*n*PolyLog[2, 1 + (f*S
qrt[x])/e])/e^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
```

```
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx = -\frac{fk\left(a+b\log\left(cx^n\right)\right)}{6ex^{3/2}}+\frac{f^2k\left(a+b\log\left(cx^n\right)\right)}{4e^2x}-\frac{f^3k\left(a+b\log\left(c\right.\right.}{2e^3\sqrt{x}}$$

$$= -\frac{bfkn}{9ex^{3/2}}+\frac{bf^2kn}{4e^2x}-\frac{bf^3kn}{e^3\sqrt{x}}-\frac{fk\left(a+b\log\left(cx^n\right)\right)}{6ex^{3/2}}+\frac{f^2k\left(a+b\log\right.}{4e^2x}$$

$$= -\frac{bfkn}{9ex^{3/2}}+\frac{bf^2kn}{4e^2x}-\frac{bf^3kn}{e^3\sqrt{x}}+\frac{bf^4kn\log^2(x)}{8e^4}-\frac{fk\left(a+b\log\left(cx^n\right.\right.}{6ex^{3/2}}$$

$$= -\frac{bfkn}{9ex^{3/2}}+\frac{bf^2kn}{4e^2x}-\frac{bf^3kn}{e^3\sqrt{x}}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{4x^2}-\frac{bf^4kn\log}{}$$

$$= -\frac{bfkn}{9ex^{3/2}}+\frac{bf^2kn}{4e^2x}-\frac{bf^3kn}{e^3\sqrt{x}}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{4x^2}-\frac{bf^4kn\log}{}$$

$$= -\frac{7bfkn}{36ex^{3/2}}+\frac{3bf^2kn}{8e^2x}-\frac{5bf^3kn}{4e^3\sqrt{x}}+\frac{bf^4kn\log\left(e+f\sqrt{x}\right)}{4e^4}-\frac{bn\log}{}$$

Mathematica [A] time = 0.39, size = 359, normalized size = 1.04

$$-\frac{-18f^4kx^2\log\left(e+f\sqrt{x}\right)\left(2a+2b\log\left(cx^n\right)-2bn\log(x)+bn\right)+36ae^4\log\left(d\left(e+f\sqrt{x}\right)^k\right)+12ae^3fk\sqrt{x}-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/72*(12*a*e^3*f*k*Sqrt[x] + 14*b*e^3*f*k*n*Sqrt[x] - 18*a*e^2*f^2*k*x - 2
7*b*e^2*f^2*k*n*x + 36*a*e*f^3*k*x^(3/2) + 90*b*e*f^3*k*n*x^(3/2) + 36*a*e^
4*Log[d*(e + f*Sqrt[x])^k] + 18*b*e^4*n*Log[d*(e + f*Sqrt[x])^k] + 18*a*f^4
*k*x^2*Log[x] + 9*b*f^4*k*n*x^2*Log[x] - 36*b*f^4*k*n*x^2*Log[1 + (f*Sqrt[x
])/e]*Log[x] - 9*b*f^4*k*n*x^2*Log[x]^2 + 12*b*e^3*f*k*Sqrt[x]*Log[c*x^n] -
 18*b*e^2*f^2*k*x*Log[c*x^n] + 36*b*e*f^3*k*x^(3/2)*Log[c*x^n] + 36*b*e^4*L
og[d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 18*b*f^4*k*x^2*Log[x]*Log[c*x^n] - 18*
f^4*k*x^2*Log[e + f*Sqrt[x]]*(2*a + b*n - 2*b*n*Log[x] + 2*b*Log[c*x^n]) -
72*b*f^4*k*n*x^2*PolyLog[2, -((f*Sqrt[x])/e)])/(e^4*x^2)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^3, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^{n}\right)+a\right)\ln\left(d\left(f\sqrt{x}+e\right)^{k}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{18\,be\log(d)\log\left(x^{n}\right)+9\left(2\,be\log\left(x^{n}\right)+\left(en+2\,e\log(c)\right)b+2\,ae\right)k\log\left(f\sqrt{x}+e\right)+18\,ae\log(d)+9\left(en\log(d)\right.}{36\,ex^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^3,x, algorithm="maxima"
)
```

```
[Out] -1/36*(18*b*e*log(d)*log(x^n) + 9*(2*b*e*log(x^n) + (e*n + 2*e*log(c))*b +
2*a*e)*k*log(f*sqrt(x) + e) + 18*a*e*log(d) + 9*(e*n*log(d) + 2*e*log(c)*lo
g(d))*b + (6*b*f*k*x*log(x^n) + (6*a*f*k + (7*f*k*n + 6*f*k*log(c))*b)*x)/s
```

```
qrt(x))/(e*x^2) - integrate(1/8*(2*b*f^2*k*log(x^n) + 2*a*f^2*k + (f^2*k*n
+ 2*f^2*k*log(c))*b)/(e*f*x^(5/2) + e^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^3,x)
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**3,x)
[Out] Timed out
```

3.121 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log(cx^n)\right)}{x^4}\,dx$

Optimal. Leaf size=434

$$-\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{3x^3}+\frac{f^6k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3e^6}-\frac{f^6k\log(x)(a+b\log(cx^n))}{6e^6}-\frac{f^5k(a}{}$$

```
[Out] -11/225*b*f*k*n/e/x^(5/2)+5/72*b*f^2*k*n/e^2/x^2-1/9*b*f^3*k*n/e^3/x^(3/2)+
2/9*b*f^4*k*n/e^4/x-1/18*b*f^6*k*n*ln(x)/e^6+1/12*b*f^6*k*n*ln(x)^2/e^6-1/1
5*f*k*(a+b*ln(c*x^n))/e/x^(5/2)+1/12*f^2*k*(a+b*ln(c*x^n))/e^2/x^2-1/9*f^3*
k*(a+b*ln(c*x^n))/e^3/x^(3/2)+1/6*f^4*k*(a+b*ln(c*x^n))/e^4/x-1/6*f^6*k*ln(
x)*(a+b*ln(c*x^n))/e^6+1/9*b*f^6*k*n*ln(e+f*x^(1/2))/e^6+1/3*f^6*k*(a+b*ln(
c*x^n))*ln(e+f*x^(1/2))/e^6-2/3*b*f^6*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/
e^6-1/9*b*n*ln(d*(e+f*x^(1/2))^k)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)
)^k)/x^3-2/3*b*f^6*k*n*polylog(2,1+f*x^(1/2)/e)/e^6-7/9*b*f^5*k*n/e^5/x^(1/
2)-1/3*f^5*k*(a+b*ln(c*x^n))/e^5/x^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2454, 2395, 44, 2376, 2394, 2315, 2301}

$$-\frac{2bf^6kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{3e^6}-\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{3x^3}+\frac{f^6k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3e^6}-\frac{f^6k}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] (-11*b*f*k*n)/(225*e*x^(5/2)) + (5*b*f^2*k*n)/(72*e^2*x^2) - (b*f^3*k*n)/(9
*e^3*x^(3/2)) + (2*b*f^4*k*n)/(9*e^4*x) - (7*b*f^5*k*n)/(9*e^5*Sqrt[x]) + (
b*f^6*k*n*Log[e + f*Sqrt[x]])/(9*e^6) - (b*n*Log[d*(e + f*Sqrt[x])^k])/(9*x
^3) - (2*b*f^6*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(3*e^6) - (b*f
^6*k*n*Log[x])/(18*e^6) + (b*f^6*k*n*Log[x]^2)/(12*e^6) - (f*k*(a + b*Log[c
*x^n]))/(15*e*x^(5/2)) + (f^2*k*(a + b*Log[c*x^n]))/(12*e^2*x^2) - (f^3*k*(
a + b*Log[c*x^n]))/(9*e^3*x^(3/2)) + (f^4*k*(a + b*Log[c*x^n]))/(6*e^4*x) -
 (f^5*k*(a + b*Log[c*x^n]))/(3*e^5*Sqrt[x]) + (f^6*k*Log[e + f*Sqrt[x]]*(a
+ b*Log[c*x^n]))/(3*e^6) - (Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/(3
*x^3) - (f^6*k*Log[x]*(a + b*Log[c*x^n]))/(6*e^6) - (2*b*f^6*k*n*PolyLog[2,
 1 + (f*Sqrt[x])/e])/(3*e^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^4}\,dx = -\frac{fk\left(a+b\log\left(cx^n\right)\right)}{15ex^{5/2}} + \frac{f^2k\left(a+b\log\left(cx^n\right)\right)}{12e^2x^2} - \frac{f^3k\left(a+b\log\left(c\right.\right.}{9e^3x^{3/2}}$$

$$= -\frac{2bfkn}{75ex^{5/2}} + \frac{bf^2kn}{24e^2x^2} - \frac{2bf^3kn}{27e^3x^{3/2}} + \frac{bf^4kn}{6e^4x} - \frac{2bf^5kn}{3e^5\sqrt{x}} - \frac{fk\left(a+b\log\right.}{15ex}$$

$$= -\frac{2bfkn}{75ex^{5/2}} + \frac{bf^2kn}{24e^2x^2} - \frac{2bf^3kn}{27e^3x^{3/2}} + \frac{bf^4kn}{6e^4x} - \frac{2bf^5kn}{3e^5\sqrt{x}} + \frac{bf^6kn\log^2}{12e^6}$$

$$= -\frac{2bfkn}{75ex^{5/2}} + \frac{bf^2kn}{24e^2x^2} - \frac{2bf^3kn}{27e^3x^{3/2}} + \frac{bf^4kn}{6e^4x} - \frac{2bf^5kn}{3e^5\sqrt{x}} - \frac{bn\log\left(d\left(\right.\right.}{9}$$

$$= -\frac{2bfkn}{75ex^{5/2}} + \frac{bf^2kn}{24e^2x^2} - \frac{2bf^3kn}{27e^3x^{3/2}} + \frac{bf^4kn}{6e^4x} - \frac{2bf^5kn}{3e^5\sqrt{x}} - \frac{bn\log\left(d\left(\right.\right.}{9}$$

$$= -\frac{11bfkn}{225ex^{5/2}} + \frac{5bf^2kn}{72e^2x^2} - \frac{bf^3kn}{9e^3x^{3/2}} + \frac{2bf^4kn}{9e^4x} - \frac{7bf^5kn}{9e^5\sqrt{x}} + \frac{bf^6kn\log}{}$$

Mathematica [A] time = 0.50, size = 457, normalized size = 1.05

$$-\frac{-200f^6kx^3\log\left(e+f\sqrt{x}\right)\left(3a+3b\log\left(cx^n\right)-3bn\log(x)+bn\right)+600ae^6\log\left(d\left(e+f\sqrt{x}\right)^k\right)+120ae^5fk\sqrt{x}-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/1800*(120*a*e^5*f*k*Sqrt[x] + 88*b*e^5*f*k*n*Sqrt[x] - 150*a*e^4*f^2*k*x
 - 125*b*e^4*f^2*k*n*x + 200*a*e^3*f^3*k*x^(3/2) + 200*b*e^3*f^3*k*n*x^(3/2
) - 300*a*e^2*f^4*k*x^2 - 400*b*e^2*f^4*k*n*x^2 + 600*a*e*f^5*k*x^(5/2) + 1
400*b*e*f^5*k*n*x^(5/2) + 600*a*e^6*Log[d*(e + f*Sqrt[x])^k] + 200*b*e^6*n*
Log[d*(e + f*Sqrt[x])^k] + 300*a*f^6*k*x^3*Log[x] + 100*b*f^6*k*n*x^3*Log[x
] - 600*b*f^6*k*n*x^3*Log[1 + (f*Sqrt[x])/e]*Log[x] - 150*b*f^6*k*n*x^3*Log
[x]^2 + 120*b*e^5*f*k*Sqrt[x]*Log[c*x^n] - 150*b*e^4*f^2*k*x*Log[c*x^n] + 2
00*b*e^3*f^3*k*x^(3/2)*Log[c*x^n] - 300*b*e^2*f^4*k*x^2*Log[c*x^n] + 600*b*
e*f^5*k*x^(5/2)*Log[c*x^n] + 600*b*e^6*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n]
+ 300*b*f^6*k*x^3*Log[x]*Log[c*x^n] - 200*f^6*k*x^3*Log[e + f*Sqrt[x]]*(3*a
 + b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n]) - 1200*b*f^6*k*n*x^3*PolyLog[2, -((
f*Sqrt[x])/e)])/(e^6*x^3)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^4,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^4, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^n\right)+a\right)\ln\left(d\left(f\sqrt{x}+e\right)^k\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^4,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{75\,be\log(d)\log\left(x^n\right)+25\left(3\,be\log\left(x^n\right)+\left(en+3\,e\log(c)\right)b+3\,ae\right)k\log\left(f\sqrt{x}+e\right)+75\,ae\log(d)+25\left(en\log(\right.}{225\,ex^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^4,x, algorithm="maxima"
)
```

```
[Out] -1/225*(75*b*e*log(d)*log(x^n) + 25*(3*b*e*log(x^n) + (e*n + 3*e*log(c))*b
+ 3*a*e)*k*log(f*sqrt(x) + e) + 75*a*e*log(d) + 25*(e*n*log(d) + 3*e*log(c)
*log(d))*b + (15*b*f*k*x*log(x^n) + (15*a*f*k + (11*f*k*n + 15*f*k*log(c))*
b)*x)/sqrt(x))/(e*x^3) - integrate(1/18*(3*b*f^2*k*log(x^n) + 3*a*f^2*k + (
f^2*k*n + 3*f^2*k*log(c))*b)/(e*f*x^(7/2) + e^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\ln(c\,x^n))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**4,x)
```

```
[Out] Timed out
```

3.122 $\int x^2 \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=750

$$\frac{1}{3}x^3\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{2}{9}bnx^3\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)-\frac{4be^6n\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{3f^6}$$

```
[Out] -13/27*b^2*e^4*n^2*x/f^4+14/81*b^2*e^3*n^2*x^(3/2)/f^3-19/216*b^2*e^2*n^2*x
^2/f^2+182/3375*b^2*e*n^2*x^(5/2)/f+2/27*b*n*x^3*(a+b*ln(c*x^n))-1/27*b^2*n
^2*x^3-1/18*x^3*(a+b*ln(c*x^n))^2+1/3*b^2*e^4*n*x*ln(c*x^n)/f^4+1/3*x^3*(a+
b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))-1/6*e^4*x*(a+b*ln(c*x^n))^2/f^4+1/9*e^3*
x^(3/2)*(a+b*ln(c*x^n))^2/f^3-1/12*e^2*x^2*(a+b*ln(c*x^n))^2/f^2+1/15*e*x^(
5/2)*(a+b*ln(c*x^n))^2/f+2/27*b^2*n^2*x^3*ln(d*(e+f*x^(1/2)))-1/3*e^6*(a+b*
ln(c*x^n))^2*ln(1+f*x^(1/2)/e)/f^6+1/3*e^5*(a+b*ln(c*x^n))^2*x^(1/2)/f^5+1/
3*a*b*e^4*n*x/f^4-2/27*b^2*e^6*n^2*ln(e+f*x^(1/2))/f^6-2/9*b*n*x^3*(a+b*ln(
c*x^n))*ln(d*(e+f*x^(1/2)))-4/9*b^2*e^6*n^2*polylog(2,1+f*x^(1/2)/e)/f^6+8/
3*b^2*e^6*n^2*polylog(3,-f*x^(1/2)/e)/f^6+86/27*b^2*e^5*n^2*x^(1/2)/f^5+1/9
*b*e^4*n*x*(a+b*ln(c*x^n))/f^4-2/9*b*e^3*n*x^(3/2)*(a+b*ln(c*x^n))/f^3+5/36
*b*e^2*n*x^2*(a+b*ln(c*x^n))/f^2-22/225*b*e*n*x^(5/2)*(a+b*ln(c*x^n))/f+2/9
*b*e^6*n*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/f^6-4/9*b^2*e^6*n^2*ln(-f*x^(1/2)/
e)*ln(e+f*x^(1/2))/f^6-4/3*b*e^6*n*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/
f^6-14/9*b*e^5*n*(a+b*ln(c*x^n))*x^(1/2)/f^5
```

Rubi [A] time = 0.85, antiderivative size = 750, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {2454, 2395, 43, 2377, 2295, 2304, 2375, 2337, 2374, 6589, 2376, 2394, 2315}

$$-\frac{4be^6n\mathrm{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{3f^6}-\frac{4b^2e^6n^2\mathrm{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{9f^6}+\frac{8b^2e^6n^2\mathrm{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{3f^6}+\frac{1}{3}x^3\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (86*b^2*e^5*n^2*Sqrt[x])/(27*f^5) + (a*b*e^4*n*x)/(3*f^4) - (13*b^2*e^4*n^2
*x)/(27*f^4) + (14*b^2*e^3*n^2*x^(3/2))/(81*f^3) - (19*b^2*e^2*n^2*x^2)/(21
6*f^2) + (182*b^2*e*n^2*x^(5/2))/(3375*f) - (b^2*n^2*x^3)/27 - (2*b^2*e^6*n
^2*Log[e + f*Sqrt[x]])/(27*f^6) + (2*b^2*n^2*x^3*Log[d*(e + f*Sqrt[x])])/27
 - (4*b^2*e^6*n^2*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(9*f^6) + (b^2*
e^4*n*x*Log[c*x^n])/(3*f^4) - (14*b*e^5*n*Sqrt[x]*(a + b*Log[c*x^n]))/(9*f^
5) + (b*e^4*n*x*(a + b*Log[c*x^n]))/(9*f^4) - (2*b*e^3*n*x^(3/2)*(a + b*Log
[c*x^n]))/(9*f^3) + (5*b*e^2*n*x^2*(a + b*Log[c*x^n]))/(36*f^2) - (22*b*e*n
*x^(5/2)*(a + b*Log[c*x^n]))/(225*f) + (2*b*n*x^3*(a + b*Log[c*x^n]))/27 +
(2*b*e^6*n*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(9*f^6) - (2*b*n*x^3*Log[
d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/9 + (e^5*Sqrt[x]*(a + b*Log[c*x^n])^
2)/(3*f^5) - (e^4*x*(a + b*Log[c*x^n])^2)/(6*f^4) + (e^3*x^(3/2)*(a + b*Log
[c*x^n])^2)/(9*f^3) - (e^2*x^2*(a + b*Log[c*x^n])^2)/(12*f^2) + (e*x^(5/2)*
(a + b*Log[c*x^n])^2)/(15*f) - (x^3*(a + b*Log[c*x^n])^2)/18 + (x^3*Log[d*(
e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/3 - (e^6*Log[1 + (f*Sqrt[x])/e]*(a +
b*Log[c*x^n])^2)/(3*f^6) - (4*b^2*e^6*n^2*PolyLog[2, 1 + (f*Sqrt[x])/e])/(9
*f^6) - (4*b*e^6*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/(3*f^6)
 + (8*b^2*e^6*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/(3*f^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
```

```
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{e^5\sqrt{x}\,\left(a+b\log\left(cx^n\right)\right)^2}{3f^5} - \frac{e^4x\left(a+b\log\left(cx^n\right)\right)^2}{6f^4} + \frac{e^3x^{3/2}\left(a+\right.}{9}$$

$$= \frac{e^5\sqrt{x}\,\left(a+b\log\left(cx^n\right)\right)^2}{3f^5} - \frac{e^4x\left(a+b\log\left(cx^n\right)\right)^2}{6f^4} + \frac{e^3x^{3/2}\left(a+\right.}{9}$$

$$= \frac{8b^2e^5n^2\sqrt{x}}{3f^5} + \frac{abe^4nx}{3f^4} + \frac{8b^2e^3n^2x^{3/2}}{81f^3} - \frac{b^2e^2n^2x^2}{24f^2} + \frac{8b^2en^2x^{5/2}}{375f}$$

$$= \frac{28b^2e^5n^2\sqrt{x}}{9f^5} + \frac{abe^4nx}{3f^4} - \frac{4b^2e^4n^2x}{9f^4} + \frac{4b^2e^3n^2x^{3/2}}{27f^3} - \frac{5b^2e^2n^2x}{72f^2}$$

$$= \frac{28b^2e^5n^2\sqrt{x}}{9f^5} + \frac{abe^4nx}{3f^4} - \frac{4b^2e^4n^2x}{9f^4} + \frac{4b^2e^3n^2x^{3/2}}{27f^3} - \frac{5b^2e^2n^2x}{72f^2}$$

$$= \frac{28b^2e^5n^2\sqrt{x}}{9f^5} + \frac{abe^4nx}{3f^4} - \frac{4b^2e^4n^2x}{9f^4} + \frac{4b^2e^3n^2x^{3/2}}{27f^3} - \frac{5b^2e^2n^2x}{72f^2}$$

$$= \frac{28b^2e^5n^2\sqrt{x}}{9f^5} + \frac{abe^4nx}{3f^4} - \frac{4b^2e^4n^2x}{9f^4} + \frac{4b^2e^3n^2x^{3/2}}{27f^3} - \frac{5b^2e^2n^2x}{72f^2}$$

$$= \frac{86b^2e^5n^2\sqrt{x}}{27f^5} + \frac{abe^4nx}{3f^4} - \frac{13b^2e^4n^2x}{27f^4} + \frac{14b^2e^3n^2x^{3/2}}{81f^3} - \frac{19b^2e^2}{216}$$

Mathematica [A] time = 0.89, size = 1319, normalized size = 1.76

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (a^2*e^5*Sqrt[x])/(3*f^5) - (14*a*b*e^5*n*Sqrt[x])/(9*f^5) + (86*b^2*e^5*n^
2*Sqrt[x])/(27*f^5) - (a^2*e^4*x)/(6*f^4) + (4*a*b*e^4*n*x)/(9*f^4) - (13*b
^2*e^4*n^2*x)/(27*f^4) + (a^2*e^3*x^(3/2))/(9*f^3) - (2*a*b*e^3*n*x^(3/2))/
(9*f^3) + (14*b^2*e^3*n^2*x^(3/2))/(81*f^3) - (a^2*e^2*x^2)/(12*f^2) + (5*a
*b*e^2*n*x^2)/(36*f^2) - (19*b^2*e^2*n^2*x^2)/(216*f^2) + (a^2*e*x^(5/2))/(
15*f) - (22*a*b*e*n*x^(5/2))/(225*f) + (182*b^2*e*n^2*x^(5/2))/(3375*f) - (
a^2*x^3)/18 + (2*a*b*n*x^3)/27 - (b^2*n^2*x^3)/27 - (a^2*e^6*Log[e + f*Sqrt
[x]])/(3*f^6) + (2*a*b*e^6*n*Log[e + f*Sqrt[x]])/(9*f^6) - (2*b^2*e^6*n^2*L
og[e + f*Sqrt[x]])/(27*f^6) + (a^2*x^3*Log[d*(e + f*Sqrt[x])])/3 - (2*a*b*n
*x^3*Log[d*(e + f*Sqrt[x])])/9 + (2*b^2*n^2*x^3*Log[d*(e + f*Sqrt[x])])/27
+ (2*a*b*e^6*n*Log[e + f*Sqrt[x]]*Log[x])/(3*f^6) - (2*b^2*e^6*n^2*Log[e +
f*Sqrt[x]]*Log[x])/(9*f^6) - (2*a*b*e^6*n*Log[1 + (f*Sqrt[x])/e]*Log[x])/(3
*f^6) + (2*b^2*e^6*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x])/(9*f^6) - (b^2*e^6*n^
2*Log[e + f*Sqrt[x]]*Log[x]^2)/(3*f^6) + (b^2*e^6*n^2*Log[1 + (f*Sqrt[x])/e
]*Log[x]^2)/(3*f^6) + (2*a*b*e^5*Sqrt[x]*Log[c*x^n])/(3*f^5) - (14*b^2*e^5*
n*Sqrt[x]*Log[c*x^n])/(9*f^5) - (a*b*e^4*x*Log[c*x^n])/(3*f^4) + (4*b^2*e^4
*n*x*Log[c*x^n])/(9*f^4) + (2*a*b*e^3*x^(3/2)*Log[c*x^n])/(9*f^3) - (2*b^2*
e^3*n*x^(3/2)*Log[c*x^n])/(9*f^3) - (a*b*e^2*x^2*Log[c*x^n])/(6*f^2) + (5*b
^2*e^2*n*x^2*Log[c*x^n])/(36*f^2) + (2*a*b*e*x^(5/2)*Log[c*x^n])/(15*f) - (
22*b^2*e*n*x^(5/2)*Log[c*x^n])/(225*f) - (a*b*x^3*Log[c*x^n])/9 + (2*b^2*n*
x^3*Log[c*x^n])/27 - (2*a*b*e^6*Log[e + f*Sqrt[x]]*Log[c*x^n])/(3*f^6) + (2
```

```
*b^2*e^6*n*Log[e + f*Sqrt[x]]*Log[c*x^n])/(9*f^6) + (2*a*b*x^3*Log[d*(e + f
*Sqrt[x])]*Log[c*x^n])/3 - (2*b^2*n*x^3*Log[d*(e + f*Sqrt[x])]*Log[c*x^n])/
9 + (2*b^2*e^6*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n])/(3*f^6) - (2*b^2*e^6
*n*Log[1 + (f*Sqrt[x])/e]*Log[x]*Log[c*x^n])/(3*f^6) + (b^2*e^5*Sqrt[x]*Log
[c*x^n]^2)/(3*f^5) - (b^2*e^4*x*Log[c*x^n]^2)/(6*f^4) + (b^2*e^3*x^(3/2)*Lo
g[c*x^n]^2)/(9*f^3) - (b^2*e^2*x^2*Log[c*x^n]^2)/(12*f^2) + (b^2*e*x^(5/2)*
Log[c*x^n]^2)/(15*f) - (b^2*x^3*Log[c*x^n]^2)/18 - (b^2*e^6*Log[e + f*Sqrt[
x]]*Log[c*x^n]^2)/(3*f^6) + (b^2*x^3*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^2)/3
 + (4*b*e^6*n*(-3*a + b*n - 3*b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/(
9*f^6) + (8*b^2*e^6*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/(3*f^6)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^2\log\left(cx^n\right)^2+2\,abx^2\log\left(cx^n\right)+a^2x^2\right)\log\left(df\sqrt{x}+de\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="fricas"
)
```

```
[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log(d*f*sq
rt(x) + d*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^2x^2\log\left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + e)*d), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^2x^2\ln\left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)^2*ln(d*(f*x^(1/2)+e)),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)^2*ln(d*(f*x^(1/2)+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^2x^2\log\left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + e)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,\left(e+f\,\sqrt{x}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^2*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2))),x)
[Out] Timed out
```

3.123 $\int x \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=598

$$\frac{1}{2}x^2\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{2}bnx^2\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)-\frac{2be^4n\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{f^4}$$

```
[Out] 1/2*a*b*e^2*n*x/f^2-7/8*b^2*e^2*n^2*x/f^2+37/108*b^2*e*n^2*x^(3/2)/f-3/16*b
^2*n^2*x^2+1/2*b^2*e^2*n*x*ln(c*x^n)/f^2+1/4*b*e^2*n*x*(a+b*ln(c*x^n))/f^2-
7/18*b*e*n*x^(3/2)*(a+b*ln(c*x^n))/f+1/4*b*n*x^2*(a+b*ln(c*x^n))-1/4*e^2*x*
(a+b*ln(c*x^n))^2/f^2+1/6*e*x^(3/2)*(a+b*ln(c*x^n))^2/f-1/8*x^2*(a+b*ln(c*x
^n))^2-1/4*b^2*e^4*n^2*ln(e+f*x^(1/2))/f^4+1/2*b*e^4*n*(a+b*ln(c*x^n))*ln(e
+f*x^(1/2))/f^4-b^2*e^4*n^2*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^4+1/4*b^2*n^
2*x^2*ln(d*(e+f*x^(1/2)))-1/2*b*n*x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)))+1
/2*x^2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))-1/2*e^4*(a+b*ln(c*x^n))^2*ln(1
+f*x^(1/2)/e)/f^4-2*b*e^4*n*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/f^4-b^2
*e^4*n^2*polylog(2,1+f*x^(1/2)/e)/f^4+4*b^2*e^4*n^2*polylog(3,-f*x^(1/2)/e)
/f^4+21/4*b^2*e^3*n^2*x^(1/2)/f^3-5/2*b*e^3*n*(a+b*ln(c*x^n))*x^(1/2)/f^3+1
/2*e^3*(a+b*ln(c*x^n))^2*x^(1/2)/f^3
```

Rubi [A] time = 0.66, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2395, 43, 2377, 2295, 2304, 2375, 2337, 2374, 6589, 2376, 2394, 2315}

$$-\frac{2be^4n\,\mathrm{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^4}-\frac{b^2e^4n^2\,\mathrm{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{f^4}+\frac{4b^2e^4n^2\,\mathrm{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{f^4}+\frac{1}{2}x^2\log$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (21*b^2*e^3*n^2*Sqrt[x])/(4*f^3) + (a*b*e^2*n*x)/(2*f^2) - (7*b^2*e^2*n^2*x
)/(8*f^2) + (37*b^2*e*n^2*x^(3/2))/(108*f) - (3*b^2*n^2*x^2)/16 - (b^2*e^4*
n^2*Log[e + f*Sqrt[x]])/(4*f^4) + (b^2*n^2*x^2*Log[d*(e + f*Sqrt[x])])/4 -
(b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/f^4 + (b^2*e^2*n*x*L
og[c*x^n])/(2*f^2) - (5*b*e^3*n*Sqrt[x]*(a + b*Log[c*x^n]))/(2*f^3) + (b*e^
2*n*x*(a + b*Log[c*x^n]))/(4*f^2) - (7*b*e*n*x^(3/2)*(a + b*Log[c*x^n]))/(1
8*f) + (b*n*x^2*(a + b*Log[c*x^n]))/4 + (b*e^4*n*Log[e + f*Sqrt[x]]*(a + b*
Log[c*x^n]))/(2*f^4) - (b*n*x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/
2 + (e^3*Sqrt[x]*(a + b*Log[c*x^n])^2)/(2*f^3) - (e^2*x*(a + b*Log[c*x^n])^
2)/(4*f^2) + (e*x^(3/2)*(a + b*Log[c*x^n])^2)/(6*f) - (x^2*(a + b*Log[c*x^n
])^2)/8 + (x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/2 - (e^4*Log[1
+ (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(2*f^4) - (b^2*e^4*n^2*PolyLog[2, 1
+ (f*Sqrt[x])/e])/f^4 - (2*b*e^4*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[
x])/e)])/f^4 + (4*b^2*e^4*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/f^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{e^3\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{2f^3}-\frac{e^2x\left(a+b\log\left(cx^n\right)\right)^2}{4f^2}+\frac{ex^{3/2}\left(a+b\log\right.}{6f}$$

$$=\frac{e^3\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{2f^3}-\frac{e^2x\left(a+b\log\left(cx^n\right)\right)^2}{4f^2}+\frac{ex^{3/2}\left(a+b\log\right.}{6f}$$

$$=\frac{4b^2e^3n^2\sqrt{x}}{f^3}+\frac{abe^2nx}{2f^2}+\frac{4b^2en^2x^{3/2}}{27f}-\frac{1}{16}b^2n^2x^2-\frac{5be^3n\sqrt{x}\left(a+\right.}{2f^3}$$

$$=\frac{5b^2e^3n^2\sqrt{x}}{f^3}+\frac{abe^2nx}{2f^2}-\frac{3b^2e^2n^2x}{4f^2}+\frac{7b^2en^2x^{3/2}}{27f}-\frac{1}{8}b^2n^2x^2+\frac{b^2e^2}{}$$

$$=\frac{5b^2e^3n^2\sqrt{x}}{f^3}+\frac{abe^2nx}{2f^2}-\frac{3b^2e^2n^2x}{4f^2}+\frac{7b^2en^2x^{3/2}}{27f}-\frac{1}{8}b^2n^2x^2+\frac{b^2e^2}{}$$

$$=\frac{5b^2e^3n^2\sqrt{x}}{f^3}+\frac{abe^2nx}{2f^2}-\frac{3b^2e^2n^2x}{4f^2}+\frac{7b^2en^2x^{3/2}}{27f}-\frac{1}{8}b^2n^2x^2+\frac{1}{4}b^2$$

$$=\frac{5b^2e^3n^2\sqrt{x}}{f^3}+\frac{abe^2nx}{2f^2}-\frac{3b^2e^2n^2x}{4f^2}+\frac{7b^2en^2x^{3/2}}{27f}-\frac{1}{8}b^2n^2x^2+\frac{1}{4}b^2$$

$$=\frac{21b^2e^3n^2\sqrt{x}}{4f^3}+\frac{abe^2nx}{2f^2}-\frac{7b^2e^2n^2x}{8f^2}+\frac{37b^2en^2x^{3/2}}{108f}-\frac{3}{16}b^2n^2x^2-$$

Mathematica [A] time = 0.51, size = 960, normalized size = 1.61

$$-216b^2n^2\log\left(e+f\sqrt{x}\right)\log^2(x)e^4+216b^2n^2\log\left(\frac{\sqrt{x}f}{e}+1\right)\log^2(x)e^4-216b^2\log\left(e+f\sqrt{x}\right)\log^2\left(cx^n\right)e^4-216$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (216*a^2*e^3*f*Sqrt[x] - 1080*a*b*e^3*f*n*Sqrt[x] + 2268*b^2*e^3*f*n^2*Sqrt
[x] - 108*a^2*e^2*f^2*x + 324*a*b*e^2*f^2*n*x - 378*b^2*e^2*f^2*n^2*x + 72*
a^2*e*f^3*x^(3/2) - 168*a*b*e*f^3*n*x^(3/2) + 148*b^2*e*f^3*n^2*x^(3/2) - 5
4*a^2*f^4*x^2 + 108*a*b*f^4*n*x^2 - 81*b^2*f^4*n^2*x^2 - 216*a^2*e^4*Log[e
+ f*Sqrt[x]] + 216*a*b*e^4*n*Log[e + f*Sqrt[x]] - 108*b^2*e^4*n^2*Log[e + f
*Sqrt[x]] + 216*a^2*f^4*x^2*Log[d*(e + f*Sqrt[x])] - 216*a*b*f^4*n*x^2*Log[
d*(e + f*Sqrt[x])] + 108*b^2*f^4*n^2*x^2*Log[d*(e + f*Sqrt[x])] + 432*a*b*e
^4*n*Log[e + f*Sqrt[x]]*Log[x] - 216*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x]
- 432*a*b*e^4*n*Log[1 + (f*Sqrt[x])/e]*Log[x] + 216*b^2*e^4*n^2*Log[1 + (f*
Sqrt[x])/e]*Log[x] - 216*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x]^2 + 216*b^2*
e^4*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 432*a*b*e^3*f*Sqrt[x]*Log[c*x^n]
- 1080*b^2*e^3*f*n*Sqrt[x]*Log[c*x^n] - 216*a*b*e^2*f^2*x*Log[c*x^n] + 324*
b^2*e^2*f^2*n*x*Log[c*x^n] + 144*a*b*e*f^3*x^(3/2)*Log[c*x^n] - 168*b^2*e*f
^3*n*x^(3/2)*Log[c*x^n] - 108*a*b*f^4*x^2*Log[c*x^n] + 108*b^2*f^4*n*x^2*Lo
g[c*x^n] - 432*a*b*e^4*Log[e + f*Sqrt[x]]*Log[c*x^n] + 216*b^2*e^4*n*Log[e
+ f*Sqrt[x]]*Log[c*x^n] + 432*a*b*f^4*x^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]
 - 216*b^2*f^4*n*x^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 432*b^2*e^4*n*Log[
e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 432*b^2*e^4*n*Log[1 + (f*Sqrt[x])/e]*Log
[x]*Log[c*x^n] + 216*b^2*e^3*f*Sqrt[x]*Log[c*x^n]^2 - 108*b^2*e^2*f^2*x*Log
[c*x^n]^2 + 72*b^2*e*f^3*x^(3/2)*Log[c*x^n]^2 - 54*b^2*f^4*x^2*Log[c*x^n]^2
 - 216*b^2*e^4*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 + 216*b^2*f^4*x^2*Log[d*(e +
 f*Sqrt[x])]*Log[c*x^n]^2 + 432*b*e^4*n*(-2*a + b*n - 2*b*Log[c*x^n])*PolyL
og[2, -((f*Sqrt[x])/e)] + 1728*b^2*e^4*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/(4
32*f^4)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 x \log\left(cx^n\right)^2 + 2\,abx \log\left(cx^n\right) + a^2 x\right) \log\left(df\sqrt{x} + de\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log(d*f*sqrt(x)
+ d*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^n\right) + a\right)^2 x \log\left(\left(f\sqrt{x} + e\right)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + e)*d), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\,x^n\right) + a\right)^2 x \ln\left(\left(f\sqrt{x} + e\right) d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+e)*d),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+e)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^n\right) + a\right)^2 x \log\left(\left(f\sqrt{x} + e\right)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + e)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \ln\left(d\left(e+f\sqrt{x}\right)\right)(a+b\ln(c\,x^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2,x)
[Out] int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2))),x)
[Out] Timed out
```

3.124 $\int \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=405

$$-2bnx\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)+x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{4be^2n\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{f^2}$$

```
[Out] a*b*n*x-3*b^2*n^2*x+b^2*n*x*ln(c*x^n)+b*n*x*(a+b*ln(c*x^n))-1/2*x*(a+b*ln(c
*x^n))^2-2*b^2*e^2*n^2*ln(e+f*x^(1/2))/f^2+2*b*e^2*n*(a+b*ln(c*x^n))*ln(e+f
*x^(1/2))/f^2-4*b^2*e^2*n^2*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^2+2*b^2*n^2*
x*ln(d*(e+f*x^(1/2)))-2*b*n*x*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)))+x*(a+b*ln
(c*x^n))^2*ln(d*(e+f*x^(1/2)))-e^2*(a+b*ln(c*x^n))^2*ln(1+f*x^(1/2)/e)/f^2-
4*b*e^2*n*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/f^2-4*b^2*e^2*n^2*polylog
(2,1+f*x^(1/2)/e)/f^2+8*b^2*e^2*n^2*polylog(3,-f*x^(1/2)/e)/f^2+14*b^2*e*n^
2*x^(1/2)/f-6*b*e*n*(a+b*ln(c*x^n))*x^(1/2)/f+e*(a+b*ln(c*x^n))^2*x^(1/2)/f
```

Rubi [A] time = 0.44, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2448, 266, 43, 2370, 2295, 2304, 2375, 2337, 2374, 6589, 2454, 2394, 2315}

$$-\frac{4be^2n\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}-\frac{4b^2e^2n^2\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{f^2}+\frac{8b^2e^2n^2\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{f^2}-2b$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (14*b^2*e*n^2*Sqrt[x])/f + a*b*n*x - 3*b^2*n^2*x - (2*b^2*e^2*n^2*Log[e + f
*Sqrt[x]])/f^2 + 2*b^2*n^2*x*Log[d*(e + f*Sqrt[x])] - (4*b^2*e^2*n^2*Log[e
+ f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/f^2 + b^2*n*x*Log[c*x^n] - (6*b*e*n*Sqr
t[x]*(a + b*Log[c*x^n]))/f + b*n*x*(a + b*Log[c*x^n]) + (2*b*e^2*n*Log[e +
f*Sqrt[x]]*(a + b*Log[c*x^n]))/f^2 - 2*b*n*x*Log[d*(e + f*Sqrt[x])]*(a + b*
Log[c*x^n]) + (e*Sqrt[x]*(a + b*Log[c*x^n])^2)/f - (x*(a + b*Log[c*x^n])^2)
/2 + x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2 - (e^2*Log[1 + (f*Sqrt[x
])/e]*(a + b*Log[c*x^n])^2)/f^2 - (4*b^2*e^2*n^2*PolyLog[2, 1 + (f*Sqrt[x])
/e])/f^2 - (4*b*e^2*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/f^2
+ (8*b^2*e^2*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/f^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2 dx = \frac{e\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{f} - \frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)^2 - \frac{e^2\log\left(e+f\sqrt{x}\right.}{}$$

$$= \frac{e\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{f} - \frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)^2 - \frac{e^2\log\left(e+f\sqrt{x}\right.}{}$$

$$= \frac{8b^2en^2\sqrt{x}}{f} + abnx - \frac{6ben\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{f} + bnx\left(a+b\log\right.$$

$$= \frac{12b^2en^2\sqrt{x}}{f} + abnx - 2b^2n^2x + b^2nx\log\left(cx^n\right) - \frac{6ben\sqrt{x}\left(a+b\right.}{f}$$

$$= \frac{12b^2en^2\sqrt{x}}{f} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right) + b^2n$$

$$= \frac{12b^2en^2\sqrt{x}}{f} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right) - \frac{4b^2}{}$$

$$= \frac{12b^2en^2\sqrt{x}}{f} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right) - \frac{4b^2}{}$$

$$= \frac{14b^2en^2\sqrt{x}}{f} + abnx - 3b^2n^2x - \frac{2b^2e^2n^2\log\left(e+f\sqrt{x}\right)}{f^2} + 2b^2n^2$$

Mathematica [A] time = 0.40, size = 718, normalized size = 1.77

$$-\frac{-2a^2f^2x\log\left(d\left(e+f\sqrt{x}\right)\right) + 2a^2e^2\log\left(e+f\sqrt{x}\right) - 2a^2ef\sqrt{x} + a^2f^2x - 4abf^2x\log\left(cx^n\right)\log\left(d\left(e+f\sqrt{x}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] -1/2*(-2*a^2*e*f*Sqrt[x] + 12*a*b*e*f*n*Sqrt[x] - 28*b^2*e*f*n^2*Sqrt[x] +
a^2*f^2*x - 4*a*b*f^2*n*x + 6*b^2*f^2*n^2*x + 2*a^2*e^2*Log[e + f*Sqrt[x]]
- 4*a*b*e^2*n*Log[e + f*Sqrt[x]] + 4*b^2*e^2*n^2*Log[e + f*Sqrt[x]] - 2*a^2
*f^2*x*Log[d*(e + f*Sqrt[x])] + 4*a*b*f^2*n*x*Log[d*(e + f*Sqrt[x])] - 4*b^
2*f^2*n^2*x*Log[d*(e + f*Sqrt[x])] - 4*a*b*e^2*n*Log[e + f*Sqrt[x]]*Log[x]
+ 4*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x] + 4*a*b*e^2*n*Log[1 + (f*Sqrt[x])
/e]*Log[x] - 4*b^2*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x] + 2*b^2*e^2*n^2*Lo
```

```
g[e + f*Sqrt[x]]*Log[x]^2 - 2*b^2*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 -
 4*a*b*e*f*Sqrt[x]*Log[c*x^n] + 12*b^2*e*f*n*Sqrt[x]*Log[c*x^n] + 2*a*b*f^2
*x*Log[c*x^n] - 4*b^2*f^2*n*x*Log[c*x^n] + 4*a*b*e^2*Log[e + f*Sqrt[x]]*Log
[c*x^n] - 4*b^2*e^2*n*Log[e + f*Sqrt[x]]*Log[c*x^n] - 4*a*b*f^2*x*Log[d*(e
+ f*Sqrt[x])]*Log[c*x^n] + 4*b^2*f^2*n*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]
- 4*b^2*e^2*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] + 4*b^2*e^2*n*Log[1 + (f
*Sqrt[x])/e]*Log[x]*Log[c*x^n] - 2*b^2*e*f*Sqrt[x]*Log[c*x^n]^2 + b^2*f^2*x
*Log[c*x^n]^2 + 2*b^2*e^2*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 - 2*b^2*f^2*x*Log
[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 + 8*b*e^2*n*(a - b*n + b*Log[c*x^n])*PolyL
og[2, -((f*Sqrt[x])/e)] - 16*b^2*e^2*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/f^2
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\log\left(cx^n\right)^2 + 2\,ab\log\left(cx^n\right) + a^2\right)\log\left(df\sqrt{x} + de\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right) + a\right)^2\log\left(\left(f\sqrt{x} + e\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right) + a\right)^2\ln\left(\left(f\sqrt{x} + e\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+e)*d),x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+e)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$27\,b^2ex\log(d)\log\left(x^n\right)^2 + 54\left(abe\log(d) - \left(en\log(d) - e\log(c)\log(d)\right)b^2\right)x\log\left(x^n\right) + 27\left(a^2e\log(d) - 2\left(en\log(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/27*(27*b^2*e*x*log(d)*log(x^n)^2 + 54*(a*b*e*log(d) - (e*n*log(d) - e*log
(c)*log(d))*b^2)*x*log(x^n) + 27*(a^2*e*log(d) - 2*(e*n*log(d) - e*log(c)*l
og(d))*a*b + (2*e*n^2*log(d) - 2*e*n*log(c)*log(d) + e*log(c)^2*log(d))*b^2
)*x + 27*(b^2*e*x*log(x^n)^2 - 2*((e*n - e*log(c))*b^2 - a*b*e)*x*log(x^n)
- (2*(e*n - e*log(c))*a*b - (2*e*n^2 - 2*e*n*log(c) + e*log(c)^2)*b^2 - a^2
*e)*x)*log(f*sqrt(x) + e) - (9*b^2*f*x^2*log(x^n)^2 - 6*((5*f*n - 3*f*log(c
))*b^2 - 3*a*b*f)*x^2*log(x^n) - (6*(5*f*n - 3*f*log(c))*a*b - (38*f*n^2 -
30*f*n*log(c) + 9*f*log(c)^2)*b^2 - 9*a^2*f)*x^2)/sqrt(x))/e + integrate(1/
2*(b^2*f^2*x*log(x^n)^2 + 2*(a*b*f^2 - (f^2*n - f^2*log(c))*b^2)*x*log(x^n)
```

```
 + (a^2*f^2 - 2*(f^2*n - f^2*log(c))*a*b + (2*f^2*n^2 - 2*f^2*n*log(c) + f^
2*log(c)^2)*b^2)*x)/(e*f*sqrt(x) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,\left(e+f\,\sqrt{x}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2,x)
[Out] int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2))),x)
[Out] Timed out
```

3.125 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=145

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn}-2\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2+8bn\mathrm{Li}_3\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{\log\left(\right.}{}$$

```
[Out] 1/3*(a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))/b/n-1/3*(a+b*ln(c*x^n))^3*ln(1+f*
x^(1/2)/e)/b/n-2*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)+8*b*n*(a+b*ln(c*
x^n))*polylog(3,-f*x^(1/2)/e)-16*b^2*n^2*polylog(4,-f*x^(1/2)/e)
```

Rubi [A] time = 0.19, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2375, 2337, 2374, 2383, 6589}

$$-2\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2+8bn\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)-16b^2n^2\text{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/(3*b*n) - (Log[1 + (f*Sqrt[x]
)/e]*(a + b*Log[c*x^n])^3)/(3*b*n) - 2*(a + b*Log[c*x^n])^2*PolyLog[2, -((f
*Sqrt[x])/e)] + 8*b*n*(a + b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)] - 16*
b^2*n^2*PolyLog[4, -((f*Sqrt[x])/e)]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - \frac{f\int \frac{\left(a+b\log(cx^n)\right)^3}{\left(e+f\sqrt{x}\right)\sqrt{x}}\,dx}{6bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{3bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{3bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{3bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{3bn}$$

Mathematica [A] time = 0.25, size = 263, normalized size = 1.81

$$\frac{1}{3}\left(\log(x)\log\left(d\left(e+f\sqrt{x}\right)\right)\left(-3bn\log(x)\left(a+b\log\left(cx^n\right)\right)+3\left(a+b\log\left(cx^n\right)\right)^2+b^2n^2\log^2(x)\right)-3bn\left(-8\mathrm{Li}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (Log[d*(e + f*Sqrt[x])]*Log[x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a + b*Log[
c*x^n]) + 3*(a + b*Log[c*x^n])^2) - 3*(a - b*n*Log[x] + b*Log[c*x^n])^2*(Lo
g[1 + (f*Sqrt[x])/e]*Log[x] + 2*PolyLog[2, -((f*Sqrt[x])/e)]) - 3*b*n*(a -
b*n*Log[x] + b*Log[c*x^n])*(Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 4*Log[x]*Poly
Log[2, -((f*Sqrt[x])/e)] - 8*PolyLog[3, -((f*Sqrt[x])/e)]) - b^2*n^2*(Log[1
 + (f*Sqrt[x])/e]*Log[x]^3 + 6*Log[x]^2*PolyLog[2, -((f*Sqrt[x])/e)] - 24*L
og[x]*PolyLog[3, -((f*Sqrt[x])/e)] + 48*PolyLog[4, -((f*Sqrt[x])/e)]))/3
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^2\log\left(cx^n\right)^2+2\,ab\log\left(cx^n\right)+a^2\right)\log\left(df\sqrt{x}+de\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e)
/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)^2\log\left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(b\ln(c\,x^n)+a)^2\ln\left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+e)*d)/x,x)
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+e)*d)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log(cx^n)+a\right)^2\log\left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,\left(e+f\,\sqrt{x}\right)\right)\,\left(a+b\,\ln(c\,x^n)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x,x)
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2)))/x,x)
[Out] Timed out
```

3.126 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^2}{x^2}\,dx$

Optimal. Leaf size=441

$$-\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}-\frac{2bn\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x}+\frac{4bf^2n\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\right.}{e^2}$$

```
[Out] -b^2*f^2*n^2*ln(x)/e^2+1/2*b^2*f^2*n^2*ln(x)^2/e^2-b*f^2*n*ln(x)*(a+b*ln(c*
x^n))/e^2-1/6*f^2*(a+b*ln(c*x^n))^3/b/e^2/n+2*b^2*f^2*n^2*ln(e+f*x^(1/2))/e
^2+2*b*f^2*n*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e^2-4*b^2*f^2*n^2*ln(-f*x^(1/2
)/e)*ln(e+f*x^(1/2))/e^2-2*b^2*n^2*ln(d*(e+f*x^(1/2)))/x-2*b*n*(a+b*ln(c*x^
n))*ln(d*(e+f*x^(1/2)))/x-(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x+f^2*(a+b*
ln(c*x^n))^2*ln(1+f*x^(1/2)/e)/e^2+4*b*f^2*n*(a+b*ln(c*x^n))*polylog(2,-f*x
^(1/2)/e)/e^2-4*b^2*f^2*n^2*polylog(2,1+f*x^(1/2)/e)/e^2-8*b^2*f^2*n^2*poly
log(3,-f*x^(1/2)/e)/e^2-14*b^2*f*n^2/e/x^(1/2)-6*b*f*n*(a+b*ln(c*x^n))/e/x^
(1/2)-f*(a+b*ln(c*x^n))^2/e/x^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.607, Rules used = {2454, 2395, 44, 2377, 2304, 2375, 2337, 2374, 6589, 2376, 2394, 2315, 2301, 2366, 12, 2302, 30}

$$\frac{4bf^2n\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e^2}-\frac{4b^2f^2n^2\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{e^2}-\frac{8b^2f^2n^2\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{e^2}-\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

```
[Out] (-14*b^2*f*n^2)/(e*Sqrt[x]) + (2*b^2*f^2*n^2*Log[e + f*Sqrt[x]])/e^2 - (2*b
^2*n^2*Log[d*(e + f*Sqrt[x])])/x - (4*b^2*f^2*n^2*Log[e + f*Sqrt[x]]*Log[-(
(f*Sqrt[x])/e)])/e^2 - (b^2*f^2*n^2*Log[x])/e^2 + (b^2*f^2*n^2*Log[x]^2)/(2
*e^2) - (6*b*f*n*(a + b*Log[c*x^n]))/(e*Sqrt[x]) + (2*b*f^2*n*Log[e + f*Sqr
t[x]]*(a + b*Log[c*x^n]))/e^2 - (2*b*n*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*
x^n]))/x - (b*f^2*n*Log[x]*(a + b*Log[c*x^n]))/e^2 - (f*(a + b*Log[c*x^n])^
2)/(e*Sqrt[x]) - (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x + (f^2*Log
[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/e^2 - (f^2*(a + b*Log[c*x^n])^3)/
(6*b*e^2*n) - (4*b^2*f^2*n^2*PolyLog[2, 1 + (f*Sqrt[x])/e])/e^2 + (4*b*f^2*
n*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/e^2 - (8*b^2*f^2*n^2*Pol
yLog[3, -((f*Sqrt[x])/e)])/e^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
```

```
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx = -\frac{f\left(a+b\log\left(cx^n\right)\right)^2}{e\sqrt{x}} + \frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{e^2} - \log\left(\right.$$

$$= -\frac{f\left(a+b\log\left(cx^n\right)\right)^2}{e\sqrt{x}} + \frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{e^2} - \log\left(\right.$$

$$= -\frac{8b^2fn^2}{e\sqrt{x}} - \frac{6bfn\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{2bf^2n\log\left(e+f\sqrt{x}\right)\left(a+b\log\right.}{e^2}$$

$$= -\frac{12b^2fn^2}{e\sqrt{x}} - \frac{6bfn\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{2bf^2n\log\left(e+f\sqrt{x}\right)\left(a+b\,l\right.}{e^2}$$

$$= -\frac{12b^2fn^2}{e\sqrt{x}} + \frac{b^2f^2n^2\log^2(x)}{2e^2} - \frac{6bfn\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{2bf^2n\log}{}$$

$$= -\frac{12b^2fn^2}{e\sqrt{x}} - \frac{2b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{x} - \frac{4b^2f^2n^2\log\left(e+f\sqrt{x}\right)\log}{e^2}$$

$$= -\frac{12b^2fn^2}{e\sqrt{x}} - \frac{2b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{x} - \frac{4b^2f^2n^2\log\left(e+f\sqrt{x}\right)\log}{e^2}$$

$$= -\frac{14b^2fn^2}{e\sqrt{x}} + \frac{2b^2f^2n^2\log\left(e+f\sqrt{x}\right)}{e^2} - \frac{2b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{x} -$$

Mathematica [A] time = 0.52, size = 821, normalized size = 1.86

$$-\frac{\frac{1}{2}b^2f^2n^2x\log^3(x) - \frac{3}{2}b^2f^2n^2x\log^2(x) - \frac{3}{2}abf^2nx\log^2(x) - 3b^2f^2n^2x\log\left(e+f\sqrt{x}\right)\log^2(x) + 3b^2f^2n^2x\log\left(\sqrt{\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

```
[Out] -1/3*(3*a^2*e*f*Sqrt[x] + 18*a*b*e*f*n*Sqrt[x] + 42*b^2*e*f*n^2*Sqrt[x] - 3
*a^2*f^2*x*Log[e + f*Sqrt[x]] - 6*a*b*f^2*n*x*Log[e + f*Sqrt[x]] - 6*b^2*f^
2*n^2*x*Log[e + f*Sqrt[x]] + 3*a^2*e^2*Log[d*(e + f*Sqrt[x])] + 6*a*b*e^2*n
*Log[d*(e + f*Sqrt[x])] + 6*b^2*e^2*n^2*Log[d*(e + f*Sqrt[x])] + (3*a^2*f^2
*x*Log[x])/2 + 3*a*b*f^2*n*x*Log[x] + 3*b^2*f^2*n^2*x*Log[x] + 6*a*b*f^2*n*
x*Log[e + f*Sqrt[x]]*Log[x] + 6*b^2*f^2*n^2*x*Log[e + f*Sqrt[x]]*Log[x] - 6
*a*b*f^2*n*x*Log[1 + (f*Sqrt[x])/e]*Log[x] - 6*b^2*f^2*n^2*x*Log[1 + (f*Sqr
t[x])/e]*Log[x] - (3*a*b*f^2*n*x*Log[x]^2)/2 - (3*b^2*f^2*n^2*x*Log[x]^2)/2
 - 3*b^2*f^2*n^2*x*Log[e + f*Sqrt[x]]*Log[x]^2 + 3*b^2*f^2*n^2*x*Log[1 + (f
*Sqrt[x])/e]*Log[x]^2 + (b^2*f^2*n^2*x*Log[x]^3)/2 + 6*a*b*e*f*Sqrt[x]*Log[
c*x^n] + 18*b^2*e*f*n*Sqrt[x]*Log[c*x^n] - 6*a*b*f^2*x*Log[e + f*Sqrt[x]]*L
og[c*x^n] - 6*b^2*f^2*n*x*Log[e + f*Sqrt[x]]*Log[c*x^n] + 6*a*b*e^2*Log[d*(
e + f*Sqrt[x])]*Log[c*x^n] + 6*b^2*e^2*n*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]
+ 3*a*b*f^2*x*Log[x]*Log[c*x^n] + 3*b^2*f^2*n*x*Log[x]*Log[c*x^n] + 6*b^2*f
^2*n*x*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 6*b^2*f^2*n*x*Log[1 + (f*Sqrt
[x])/e]*Log[x]*Log[c*x^n] - (3*b^2*f^2*n*x*Log[x]^2*Log[c*x^n])/2 + 3*b^2*e
*f*Sqrt[x]*Log[c*x^n]^2 - 3*b^2*f^2*x*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 + 3*b
^2*e^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 + (3*b^2*f^2*x*Log[x]*Log[c*x^n]
```

```
^2)/2 - 12*b*f^2*n*x*(a + b*n + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)]
+ 24*b^2*f^2*n^2*x*PolyLog[3, -((f*Sqrt[x])/e)])/(e^2*x)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2 \log\left(cx^n\right)^2 + 2\,ab\log\left(cx^n\right) + a^2\right)\log\left(df\sqrt{x} + de\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e)
/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right) + a\right)^2 \log\left(\left(f\sqrt{x} + e\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^2, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right) + a\right)^2 \ln\left(\left(f\sqrt{x} + e\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+e)*d)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+e)*d)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right) + a\right)^2 \log\left(\left(f\sqrt{x} + e\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(e + f\,\sqrt{x}\right)\right)\,\left(a + b\,\ln\left(c\,x^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^2,x)
```

```
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2)))/x**2,x)
```

```
[Out] Timed out
```

3.127 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^2}{x^3}\,dx$

Optimal. Leaf size=608

$$-\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{2x^2}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{2x^2}+\frac{2bf^4n\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(\right.\right.}{e^4}$$

```
[Out] -37/108*b^2*f*n^2/e/x^(3/2)+7/8*b^2*f^2*n^2/e^2/x-1/8*b^2*f^4*n^2*ln(x)/e^4
+1/8*b^2*f^4*n^2*ln(x)^2/e^4-7/18*b*f*n*(a+b*ln(c*x^n))/e/x^(3/2)+3/4*b*f^2
*n*(a+b*ln(c*x^n))/e^2/x-1/4*b*f^4*n*ln(x)*(a+b*ln(c*x^n))/e^4-1/6*f*(a+b*l
n(c*x^n))^2/e/x^(3/2)+1/4*f^2*(a+b*ln(c*x^n))^2/e^2/x-1/12*f^4*(a+b*ln(c*x^
n))^3/b/e^4/n+1/4*b^2*f^4*n^2*ln(e+f*x^(1/2))/e^4+1/2*b*f^4*n*(a+b*ln(c*x^n
))*ln(e+f*x^(1/2))/e^4-b^2*f^4*n^2*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/e^4-1/4
*b^2*n^2*ln(d*(e+f*x^(1/2)))/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))
)/x^2-1/2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x^2+1/2*f^4*(a+b*ln(c*x^n))
^2*ln(1+f*x^(1/2)/e)/e^4+2*b*f^4*n*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/
e^4-b^2*f^4*n^2*polylog(2,1+f*x^(1/2)/e)/e^4-4*b^2*f^4*n^2*polylog(3,-f*x^(
1/2)/e)/e^4-21/4*b^2*f^3*n^2/e^3/x^(1/2)-5/2*b*f^3*n*(a+b*ln(c*x^n))/e^3/x^
(1/2)-1/2*f^3*(a+b*ln(c*x^n))^2/e^3/x^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.607, Rules used = {2454, 2395, 44, 2377, 2304, 2375, 2337, 2374, 6589, 2376, 2394, 2315, 2301, 2366, 12, 2302, 30}

$$\frac{2bf^4n\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e^4}-\frac{b^2f^4n^2\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{e^4}-\frac{4b^2f^4n^2\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{e^4}-\log$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] (-37*b^2*f*n^2)/(108*e*x^(3/2)) + (7*b^2*f^2*n^2)/(8*e^2*x) - (21*b^2*f^3*n
^2)/(4*e^3*Sqrt[x]) + (b^2*f^4*n^2*Log[e + f*Sqrt[x]])/(4*e^4) - (b^2*n^2*L
og[d*(e + f*Sqrt[x])])/(4*x^2) - (b^2*f^4*n^2*Log[e + f*Sqrt[x]]*Log[-((f*S
qrt[x])/e)])/e^4 - (b^2*f^4*n^2*Log[x])/(8*e^4) + (b^2*f^4*n^2*Log[x]^2)/(8
*e^4) - (7*b*f*n*(a + b*Log[c*x^n]))/(18*e*x^(3/2)) + (3*b*f^2*n*(a + b*Log
[c*x^n]))/(4*e^2*x) - (5*b*f^3*n*(a + b*Log[c*x^n]))/(2*e^3*Sqrt[x]) + (b*f
^4*n*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*e^4) - (b*n*Log[d*(e + f*Sqr
t[x])]*(a + b*Log[c*x^n]))/(2*x^2) - (b*f^4*n*Log[x]*(a + b*Log[c*x^n]))/(4
*e^4) - (f*(a + b*Log[c*x^n])^2)/(6*e*x^(3/2)) + (f^2*(a + b*Log[c*x^n])^2)
/(4*e^2*x) - (f^3*(a + b*Log[c*x^n])^2)/(2*e^3*Sqrt[x]) - (Log[d*(e + f*Sqr
t[x])]*(a + b*Log[c*x^n])^2)/(2*x^2) + (f^4*Log[1 + (f*Sqrt[x])/e]*(a + b*L
og[c*x^n])^2)/(2*e^4) - (f^4*(a + b*Log[c*x^n])^3)/(12*b*e^4*n) - (b^2*f^4*
n^2*PolyLog[2, 1 + (f*Sqrt[x])/e])/e^4 + (2*b*f^4*n*(a + b*Log[c*x^n])*Poly
Log[2, -((f*Sqrt[x])/e)])/e^4 - (4*b^2*f^4*n^2*PolyLog[3, -((f*Sqrt[x])/e)]
)/e^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx = -\frac{f\left(a+b\log\left(cx^n\right)\right)^2}{6ex^{3/2}} + \frac{f^2\left(a+b\log\left(cx^n\right)\right)^2}{4e^2x} - \frac{f^3\left(a+b\log\left(cx^n\right)\right)^2}{2e^3\sqrt{x}}$$

$$= -\frac{f\left(a+b\log\left(cx^n\right)\right)^2}{6ex^{3/2}} + \frac{f^2\left(a+b\log\left(cx^n\right)\right)^2}{4e^2x} - \frac{f^3\left(a+b\log\left(cx^n\right)\right)^2}{2e^3\sqrt{x}}$$

$$= -\frac{4b^2fn^2}{27ex^{3/2}} + \frac{b^2f^2n^2}{2e^2x} - \frac{4b^2f^3n^2}{e^3\sqrt{x}} - \frac{7bfn\left(a+b\log\left(cx^n\right)\right)}{18ex^{3/2}} + \frac{3bf^2n\left(\right.}{}$$

$$= -\frac{7b^2fn^2}{27ex^{3/2}} + \frac{3b^2f^2n^2}{4e^2x} - \frac{5b^2f^3n^2}{e^3\sqrt{x}} - \frac{7bfn\left(a+b\log\left(cx^n\right)\right)}{18ex^{3/2}} + \frac{3bf^2n}{}$$

$$= -\frac{7b^2fn^2}{27ex^{3/2}} + \frac{3b^2f^2n^2}{4e^2x} - \frac{5b^2f^3n^2}{e^3\sqrt{x}} + \frac{b^2f^4n^2\log^2(x)}{8e^4} - \frac{7bfn\left(a+b\log\right.}{18ex^{3/2}}$$

$$= -\frac{7b^2fn^2}{27ex^{3/2}} + \frac{3b^2f^2n^2}{4e^2x} - \frac{5b^2f^3n^2}{e^3\sqrt{x}} - \frac{b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{4x^2} - \frac{b^2f^4n}{}$$

$$= -\frac{7b^2fn^2}{27ex^{3/2}} + \frac{3b^2f^2n^2}{4e^2x} - \frac{5b^2f^3n^2}{e^3\sqrt{x}} - \frac{b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{4x^2} - \frac{b^2f^4n}{}$$

$$= -\frac{37b^2fn^2}{108ex^{3/2}} + \frac{7b^2f^2n^2}{8e^2x} - \frac{21b^2f^3n^2}{4e^3\sqrt{x}} + \frac{b^2f^4n^2\log\left(e+f\sqrt{x}\right)}{4e^4} - \frac{b^2n^2}{}$$

Mathematica [A] time = 0.60, size = 1078, normalized size = 1.77

$$-\frac{108b^2\log\left(d\left(e+f\sqrt{x}\right)\right)\log^2\left(cx^n\right)e^4 + 108a^2\log\left(d\left(e+f\sqrt{x}\right)\right)e^4 + 54b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)e^4 + 108abn\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] -1/216*(36*a^2*e^3*f*Sqrt[x] + 84*a*b*e^3*f*n*Sqrt[x] + 74*b^2*e^3*f*n^2*Sq
rt[x] - 54*a^2*e^2*f^2*x - 162*a*b*e^2*f^2*n*x - 189*b^2*e^2*f^2*n^2*x + 10
8*a^2*e*f^3*x^(3/2) + 540*a*b*e*f^3*n*x^(3/2) + 1134*b^2*e*f^3*n^2*x^(3/2)
- 108*a^2*f^4*x^2*Log[e + f*Sqrt[x]] - 108*a*b*f^4*n*x^2*Log[e + f*Sqrt[x]]
 - 54*b^2*f^4*n^2*x^2*Log[e + f*Sqrt[x]] + 108*a^2*e^4*Log[d*(e + f*Sqrt[x]
)] + 108*a*b*e^4*n*Log[d*(e + f*Sqrt[x])] + 54*b^2*e^4*n^2*Log[d*(e + f*Sqr
t[x])] + 54*a^2*f^4*x^2*Log[x] + 54*a*b*f^4*n*x^2*Log[x] + 27*b^2*f^4*n^2*x
^2*Log[x] + 216*a*b*f^4*n*x^2*Log[e + f*Sqrt[x]]*Log[x] + 108*b^2*f^4*n^2*x
^2*Log[e + f*Sqrt[x]]*Log[x] - 216*a*b*f^4*n*x^2*Log[1 + (f*Sqrt[x])/e]*Log
[x] - 108*b^2*f^4*n^2*x^2*Log[1 + (f*Sqrt[x])/e]*Log[x] - 54*a*b*f^4*n*x^2*
Log[x]^2 - 27*b^2*f^4*n^2*x^2*Log[x]^2 - 108*b^2*f^4*n^2*x^2*Log[e + f*Sqrt
[x]]*Log[x]^2 + 108*b^2*f^4*n^2*x^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 18*b^
2*f^4*n^2*x^2*Log[x]^3 + 72*a*b*e^3*f*Sqrt[x]*Log[c*x^n] + 84*b^2*e^3*f*n*S
qrt[x]*Log[c*x^n] - 108*a*b*e^2*f^2*x*Log[c*x^n] - 162*b^2*e^2*f^2*n*x*Log[
c*x^n] + 216*a*b*e*f^3*x^(3/2)*Log[c*x^n] + 540*b^2*e*f^3*n*x^(3/2)*Log[c*x
^n] - 216*a*b*f^4*x^2*Log[e + f*Sqrt[x]]*Log[c*x^n] - 108*b^2*f^4*n*x^2*Log
[e + f*Sqrt[x]]*Log[c*x^n] + 216*a*b*e^4*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]
+ 108*b^2*e^4*n*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 108*a*b*f^4*x^2*Log[x]*
```

```
Log[c*x^n] + 54*b^2*f^4*n*x^2*Log[x]*Log[c*x^n] + 216*b^2*f^4*n*x^2*Log[e +
 f*Sqrt[x]]*Log[x]*Log[c*x^n] - 216*b^2*f^4*n*x^2*Log[1 + (f*Sqrt[x])/e]*Lo
g[x]*Log[c*x^n] - 54*b^2*f^4*n*x^2*Log[x]^2*Log[c*x^n] + 36*b^2*e^3*f*Sqrt[
x]*Log[c*x^n]^2 - 54*b^2*e^2*f^2*x*Log[c*x^n]^2 + 108*b^2*e*f^3*x^(3/2)*Log
[c*x^n]^2 - 108*b^2*f^4*x^2*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 + 108*b^2*e^4*L
og[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 + 54*b^2*f^4*x^2*Log[x]*Log[c*x^n]^2 - 2
16*b*f^4*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)] +
864*b^2*f^4*n^2*x^2*PolyLog[3, -((f*Sqrt[x])/e)])/(e^4*x^2)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2 \log\left(cx^n\right)^2 + 2\,ab\log\left(cx^n\right) + a^2\right)\log\left(df\sqrt{x} + de\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e)
/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right) + a\right)^2 \log\left(\left(f\sqrt{x} + e\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^3, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right) + a\right)^2 \ln\left(\left(f\sqrt{x} + e\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+e)*d)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln((f*x^(1/2)+e)*d)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right) + a\right)^2 \log\left(\left(f\sqrt{x} + e\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(e + f\,\sqrt{x}\right)\right)\,\left(a + b\,\ln\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^3,x)
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^3, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2)))/x**3,x)
[Out] Timed out
```

3.128 $\int x \log \left(d \left(e+f \sqrt{x}\right)\right) \left(a+b \log \left(c x^n\right)\right)^3 \, dx$

Optimal. Leaf size=907

$$-\frac{\log \left(\frac{\sqrt{x} f}{e}+1\right) \left(a+b \log \left(c x^n\right)\right)^3 e^4}{2 f^4}+\frac{3 b n \log \left(\frac{\sqrt{x} f}{e}+1\right) \left(a+b \log \left(c x^n\right)\right)^2 e^4}{4 f^4}+\frac{3 b^3 n^3 \log \left(e+f \sqrt{x}\right) e^4}{8 f^4}+\frac{3 b^3 n^3}{}$$

```
[Out] -9/16*b^2*n^2*x^2*(a+b*ln(c*x^n))+3/8*b*n*x^2*(a+b*ln(c*x^n))^2-1/8*x^2*(a+
b*ln(c*x^n))^3+3/8*b^3*n^3*x^2+3*b^2*e^4*n^2*(a+b*ln(c*x^n))*polylog(2,-f*x
^(1/2)/e)/f^4-3*b*e^4*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)/f^4+12*b^
2*e^4*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x^(1/2)/e)/f^4+63/4*b^2*e^3*n^2*(a+b
*ln(c*x^n))*x^(1/2)/f^3-15/4*b*e^3*n*(a+b*ln(c*x^n))^2*x^(1/2)/f^3+1/2*x^2*
(a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))-24*b^3*e^4*n^3*polylog(4,-f*x^(1/2)/e
)/f^4-255/8*b^3*e^3*n^3*x^(1/2)/f^3+1/2*e^3*(a+b*ln(c*x^n))^3*x^(1/2)/f^3-1
/4*e^2*x*(a+b*ln(c*x^n))^3/f^2+1/6*e*x^(3/2)*(a+b*ln(c*x^n))^3/f-3/8*b^3*n^
3*x^2*ln(d*(e+f*x^(1/2)))-1/2*e^4*(a+b*ln(c*x^n))^3*ln(1+f*x^(1/2)/e)/f^4-9
/4*a*b^2*e^2*n^2*x/f^2+3/8*b^3*e^4*n^3*ln(e+f*x^(1/2))/f^4+3/4*b^2*n^2*x^2*
(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)))-3/4*b*n*x^2*(a+b*ln(c*x^n))^2*ln(d*(e+f
*x^(1/2)))+3/2*b^3*e^4*n^3*polylog(2,1+f*x^(1/2)/e)/f^4-6*b^3*e^4*n^3*polyl
og(3,-f*x^(1/2)/e)/f^4-9/4*b^3*e^2*n^2*x*ln(c*x^n)/f^2-3/8*b^2*e^2*n^2*x*(a
+b*ln(c*x^n))/f^2+37/36*b^2*e*n^2*x^(3/2)*(a+b*ln(c*x^n))/f+9/8*b*e^2*n*x*(
a+b*ln(c*x^n))^2/f^2-7/12*b*e*n*x^(3/2)*(a+b*ln(c*x^n))^2/f-3/4*b^2*e^4*n^2
*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/f^4+3/2*b^3*e^4*n^3*ln(-f*x^(1/2)/e)*ln(e+
f*x^(1/2))/f^4+3/4*b*e^4*n*(a+b*ln(c*x^n))^2*ln(1+f*x^(1/2)/e)/f^4+45/16*b^
3*e^2*n^3*x/f^2-175/216*b^3*e*n^3*x^(3/2)/f
```

Rubi [A] time = 1.31, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 16, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2454, 2395, 43, 2377, 2296, 2295, 2305, 2304, 2375, 2337, 2374, 2383, 6589, 2376, 2394, 2315}

$$-\frac{\log \left(\frac{\sqrt{x} f}{e}+1\right) \left(a+b \log \left(c x^n\right)\right)^3 e^4}{2 f^4}+\frac{3 b n \log \left(\frac{\sqrt{x} f}{e}+1\right) \left(a+b \log \left(c x^n\right)\right)^2 e^4}{4 f^4}+\frac{3 b^3 n^3 \log \left(e+f \sqrt{x}\right) e^4}{8 f^4}+\frac{3 b^3 n^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-255*b^3*e^3*n^3*Sqrt[x])/(8*f^3) - (9*a*b^2*e^2*n^2*x)/(4*f^2) + (45*b^3*
e^2*n^3*x)/(16*f^2) - (175*b^3*e*n^3*x^(3/2))/(216*f) + (3*b^3*n^3*x^2)/8 +
 (3*b^3*e^4*n^3*Log[e + f*Sqrt[x]])/(8*f^4) - (3*b^3*n^3*x^2*Log[d*(e + f*S
qrt[x])])/8 + (3*b^3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(2*f
^4) - (9*b^3*e^2*n^2*x*Log[c*x^n])/(4*f^2) + (63*b^2*e^3*n^2*Sqrt[x]*(a + b
*Log[c*x^n]))/(4*f^3) - (3*b^2*e^2*n^2*x*(a + b*Log[c*x^n]))/(8*f^2) + (37*
b^2*e*n^2*x^(3/2)*(a + b*Log[c*x^n]))/(36*f) - (9*b^2*n^2*x^2*(a + b*Log[c*
x^n]))/16 - (3*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(4*f^4) +
 (3*b^2*n^2*x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/4 - (15*b*e^3*n*
Sqrt[x]*(a + b*Log[c*x^n])^2)/(4*f^3) + (9*b*e^2*n*x*(a + b*Log[c*x^n])^2)/
(8*f^2) - (7*b*e*n*x^(3/2)*(a + b*Log[c*x^n])^2)/(12*f) + (3*b*n*x^2*(a + b
*Log[c*x^n])^2)/8 - (3*b*n*x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)
/4 + (3*b*e^4*n*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(4*f^4) + (e^3
*Sqrt[x]*(a + b*Log[c*x^n])^3)/(2*f^3) - (e^2*x*(a + b*Log[c*x^n])^3)/(4*f^
2) + (e*x^(3/2)*(a + b*Log[c*x^n])^3)/(6*f) - (x^2*(a + b*Log[c*x^n])^3)/8
+ (x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/2 - (e^4*Log[1 + (f*Sqr
t[x])/e]*(a + b*Log[c*x^n])^3)/(2*f^4) + (3*b^3*e^4*n^3*PolyLog[2, 1 + (f*S
qrt[x])/e])/(2*f^4) + (3*b^2*e^4*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqr
t[x])/e)])/f^4 - (3*b*e^4*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*Sqrt[x])/e
)])/f^4 - (6*b^3*e^4*n^3*PolyLog[3, -((f*Sqrt[x])/e)])/f^4 + (12*b^2*e^4*n^
```

```
2*(a + b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)])/f^4 - (24*b^3*e^4*n^3*Po
lyLog[4, -((f*Sqrt[x])/e)])/f^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
```

```
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3\,dx = \frac{e^3\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^3}{2f^3} - \frac{e^2x\left(a+b\log\left(cx^n\right)\right)^3}{4f^2} + \frac{ex^{3/2}\left(a+b\log\right.}{6f}$$

$$= \frac{e^3\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^3}{2f^3} - \frac{e^2x\left(a+b\log\left(cx^n\right)\right)^3}{4f^2} + \frac{ex^{3/2}\left(a+b\log\right.}{6f}$$

$$= -\frac{15be^3n\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{4f^3} + \frac{9be^2nx\left(a+b\log\left(cx^n\right)\right)^2}{8f^2} - \frac{7ben}{}$$

$$= -\frac{24b^3e^3n^3\sqrt{x}}{f^3} - \frac{3ab^2e^2n^2x}{2f^2} - \frac{8b^3en^3x^{3/2}}{27f} + \frac{3}{32}b^3n^3x^2 + \frac{12b^2e^3n^2}{}$$

$$= -\frac{30b^3e^3n^3\sqrt{x}}{f^3} - \frac{9ab^2e^2n^2x}{4f^2} + \frac{3b^3e^2n^3x}{2f^2} - \frac{14b^3en^3x^{3/2}}{27f} + \frac{3}{16}b^3n^3$$

$$= -\frac{63b^3e^3n^3\sqrt{x}}{2f^3} - \frac{9ab^2e^2n^2x}{4f^2} + \frac{21b^3e^2n^3x}{8f^2} - \frac{37b^3en^3x^{3/2}}{54f} + \frac{9}{32}b^3n$$

$$= -\frac{63b^3e^3n^3\sqrt{x}}{2f^3} - \frac{9ab^2e^2n^2x}{4f^2} + \frac{21b^3e^2n^3x}{8f^2} - \frac{37b^3en^3x^{3/2}}{54f} + \frac{9}{32}b^3n$$

$$= -\frac{63b^3e^3n^3\sqrt{x}}{2f^3} - \frac{9ab^2e^2n^2x}{4f^2} + \frac{21b^3e^2n^3x}{8f^2} - \frac{37b^3en^3x^{3/2}}{54f} + \frac{9}{32}b^3n$$

$$= -\frac{63b^3e^3n^3\sqrt{x}}{2f^3} - \frac{9ab^2e^2n^2x}{4f^2} + \frac{21b^3e^2n^3x}{8f^2} - \frac{37b^3en^3x^{3/2}}{54f} + \frac{9}{32}b^3n$$

$$= -\frac{255b^3e^3n^3\sqrt{x}}{8f^3} - \frac{9ab^2e^2n^2x}{4f^2} + \frac{45b^3e^2n^3x}{16f^2} - \frac{175b^3en^3x^{3/2}}{216f} + \frac{3}{8}b^3$$

<u>**Mathematica**</u> **[B]** time = 0.88, size = 1968, normalized size = 2.17

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (216*a^3*e^3*f*Sqrt[x] - 1620*a^2*b*e^3*f*n*Sqrt[x] + 6804*a*b^2*e^3*f*n^2*
Sqrt[x] - 13770*b^3*e^3*f*n^3*Sqrt[x] - 108*a^3*e^2*f^2*x + 486*a^2*b*e^2*f
^2*n*x - 1134*a*b^2*e^2*f^2*n^2*x + 1215*b^3*e^2*f^2*n^3*x + 72*a^3*e*f^3*x
^(3/2) - 252*a^2*b*e*f^3*n*x^(3/2) + 444*a*b^2*e*f^3*n^2*x^(3/2) - 350*b^3*
e*f^3*n^3*x^(3/2) - 54*a^3*f^4*x^2 + 162*a^2*b*f^4*n*x^2 - 243*a*b^2*f^4*n^
2*x^2 + 162*b^3*f^4*n^3*x^2 - 216*a^3*e^4*Log[e + f*Sqrt[x]] + 324*a^2*b*e^
4*n*Log[e + f*Sqrt[x]] - 324*a*b^2*e^4*n^2*Log[e + f*Sqrt[x]] + 162*b^3*e^4
*n^3*Log[e + f*Sqrt[x]] + 216*a^3*f^4*x^2*Log[d*(e + f*Sqrt[x])] - 324*a^2*
b*f^4*n*x^2*Log[d*(e + f*Sqrt[x])] + 324*a*b^2*f^4*n^2*x^2*Log[d*(e + f*Sqr
t[x])] - 162*b^3*f^4*n^3*x^2*Log[d*(e + f*Sqrt[x])] + 648*a^2*b*e^4*n*Log[e
 + f*Sqrt[x]]*Log[x] - 648*a*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x] + 324*b^
3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[x] - 648*a^2*b*e^4*n*Log[1 + (f*Sqrt[x])/e
```

```
]*Log[x] + 648*a*b^2*e^4*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x] - 324*b^3*e^4*n^
3*Log[1 + (f*Sqrt[x])/e]*Log[x] - 648*a*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[
x]^2 + 324*b^3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[x]^2 + 648*a*b^2*e^4*n^2*Log[
1 + (f*Sqrt[x])/e]*Log[x]^2 - 324*b^3*e^4*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x]
^2 + 216*b^3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[x]^3 - 216*b^3*e^4*n^3*Log[1 +
(f*Sqrt[x])/e]*Log[x]^3 + 648*a^2*b*e^3*f*Sqrt[x]*Log[c*x^n] - 3240*a*b^2*e
^3*f*n*Sqrt[x]*Log[c*x^n] + 6804*b^3*e^3*f*n^2*Sqrt[x]*Log[c*x^n] - 324*a^2
*b*e^2*f^2*x*Log[c*x^n] + 972*a*b^2*e^2*f^2*n*x*Log[c*x^n] - 1134*b^3*e^2*f
^2*n^2*x*Log[c*x^n] + 216*a^2*b*e*f^3*x^(3/2)*Log[c*x^n] - 504*a*b^2*e*f^3*
n*x^(3/2)*Log[c*x^n] + 444*b^3*e*f^3*n^2*x^(3/2)*Log[c*x^n] - 162*a^2*b*f^4
*x^2*Log[c*x^n] + 324*a*b^2*f^4*n*x^2*Log[c*x^n] - 243*b^3*f^4*n^2*x^2*Log[
c*x^n] - 648*a^2*b*e^4*Log[e + f*Sqrt[x]]*Log[c*x^n] + 648*a*b^2*e^4*n*Log[
e + f*Sqrt[x]]*Log[c*x^n] - 324*b^3*e^4*n^2*Log[e + f*Sqrt[x]]*Log[c*x^n] +
 648*a^2*b*f^4*x^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] - 648*a*b^2*f^4*n*x^2*
Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 324*b^3*f^4*n^2*x^2*Log[d*(e + f*Sqrt[x
])]*Log[c*x^n] + 1296*a*b^2*e^4*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 64
8*b^3*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 1296*a*b^2*e^4*n*Log[1
 + (f*Sqrt[x])/e]*Log[x]*Log[c*x^n] + 648*b^3*e^4*n^2*Log[1 + (f*Sqrt[x])/e
]*Log[x]*Log[c*x^n] - 648*b^3*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x]^2*Log[c*x^n
] + 648*b^3*e^4*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2*Log[c*x^n] + 648*a*b^2*
e^3*f*Sqrt[x]*Log[c*x^n]^2 - 1620*b^3*e^3*f*n*Sqrt[x]*Log[c*x^n]^2 - 324*a*
b^2*e^2*f^2*x*Log[c*x^n]^2 + 486*b^3*e^2*f^2*n*x*Log[c*x^n]^2 + 216*a*b^2*e
*f^3*x^(3/2)*Log[c*x^n]^2 - 252*b^3*e*f^3*n*x^(3/2)*Log[c*x^n]^2 - 162*a*b^
2*f^4*x^2*Log[c*x^n]^2 + 162*b^3*f^4*n*x^2*Log[c*x^n]^2 - 648*a*b^2*e^4*Log
[e + f*Sqrt[x]]*Log[c*x^n]^2 + 324*b^3*e^4*n*Log[e + f*Sqrt[x]]*Log[c*x^n]^
2 + 648*a*b^2*f^4*x^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 - 324*b^3*f^4*n*x
^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 + 648*b^3*e^4*n*Log[e + f*Sqrt[x]]*L
og[x]*Log[c*x^n]^2 - 648*b^3*e^4*n*Log[1 + (f*Sqrt[x])/e]*Log[x]*Log[c*x^n]
^2 + 216*b^3*e^3*f*Sqrt[x]*Log[c*x^n]^3 - 108*b^3*e^2*f^2*x*Log[c*x^n]^3 +
72*b^3*e*f^3*x^(3/2)*Log[c*x^n]^3 - 54*b^3*f^4*x^2*Log[c*x^n]^3 - 216*b^3*e
^4*Log[e + f*Sqrt[x]]*Log[c*x^n]^3 + 216*b^3*f^4*x^2*Log[d*(e + f*Sqrt[x])]
*Log[c*x^n]^3 - 648*b*e^4*n*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*L
og[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, -((f*Sqrt[x])/e)] + 2592*b^2*e^4
*n^2*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)] - 10368*b^3*
e^4*n^3*PolyLog[4, -((f*Sqrt[x])/e)])/(432*f^4)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3x\log\left(cx^n\right)^3+3\,ab^2x\log\left(cx^n\right)^2+3\,a^2bx\log\left(cx^n\right)+a^3x\right)\log\left(df\sqrt{x}+de\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^n
) + a^3*x)*log(d*f*sqrt(x) + d*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)^3x\log\left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + e)*d), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^n\right)+a\right)^3x\ln\left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+e)*d),x)
[Out] int(x*(b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+e)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^{n}\right)+a\right)^{3}x\log\left(\left(f\sqrt{x}+e\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + e)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,\left(e+f\,\sqrt{x}\right)\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3,x)
[Out] int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(d*(e+f*x**(1/2))),x)
[Out] Timed out
```

3.129 $\int \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3\,dx$

Optimal. Leaf size=639

$$6b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)+\frac{12b^2e^2n^2\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}+\frac{24b^2e^2n^2\mathrm{Li}_3\left(-\frac{f\sqrt{x}}{e}\right)\left(a+\right.}{f^2}$$

```
[Out] -6*b^3*n^2*x*ln(c*x^n)+3*b*n*x*(a+b*ln(c*x^n))^2-6*a*b^2*n^2*x-1/2*x*(a+b*l
n(c*x^n))^3+12*b^3*n^3*x-6*b^2*e^2*n^2*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/f^2+
12*b^3*e^2*n^3*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^2+3*b*e^2*n*(a+b*ln(c*x^n
))^2*ln(1+f*x^(1/2)/e)/f^2+12*b^2*e^2*n^2*(a+b*ln(c*x^n))*polylog(2,-f*x^(1
/2)/e)/f^2-6*b*e^2*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)/f^2+24*b^2*e
^2*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x^(1/2)/e)/f^2+42*b^2*e*n^2*(a+b*ln(c*x
^n))*x^(1/2)/f-9*b*e*n*(a+b*ln(c*x^n))^2*x^(1/2)/f+x*(a+b*ln(c*x^n))^3*ln(d
*(e+f*x^(1/2)))+6*b^3*e^2*n^3*ln(e+f*x^(1/2))/f^2+6*b^2*n^2*x*(a+b*ln(c*x^n
))*ln(d*(e+f*x^(1/2)))-3*b*n*x*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))+12*b^3
*e^2*n^3*polylog(2,1+f*x^(1/2)/e)/f^2-24*b^3*e^2*n^3*polylog(3,-f*x^(1/2)/e
)/f^2-48*b^3*e^2*n^3*polylog(4,-f*x^(1/2)/e)/f^2-90*b^3*e*n^3*x^(1/2)/f-6*b
^3*n^3*x*ln(d*(e+f*x^(1/2)))-e^2*(a+b*ln(c*x^n))^3*ln(1+f*x^(1/2)/e)/f^2+e*
(a+b*ln(c*x^n))^3*x^(1/2)/f-3*b^2*n^2*x*(a+b*ln(c*x^n))
```

Rubi [A] time = 0.86, antiderivative size = 639, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {2448, 266, 43, 2370, 2296, 2295, 2305, 2304, 2375, 2337, 2374, 2383, 6589, 2454, 2394, 2315}

$$\frac{12b^2e^2n^2\mathrm{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}+\frac{24b^2e^2n^2\mathrm{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}-\frac{6be^2n\mathrm{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-90*b^3*e*n^3*Sqrt[x])/f - 6*a*b^2*n^2*x + 12*b^3*n^3*x + (6*b^3*e^2*n^3*L
og[e + f*Sqrt[x]])/f^2 - 6*b^3*n^3*x*Log[d*(e + f*Sqrt[x])] + (12*b^3*e^2*n
^3*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/f^2 - 6*b^3*n^2*x*Log[c*x^n] +
 (42*b^2*e*n^2*Sqrt[x]*(a + b*Log[c*x^n]))/f - 3*b^2*n^2*x*(a + b*Log[c*x^n
]) - (6*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/f^2 + 6*b^2*n^2*
x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]) - (9*b*e*n*Sqrt[x]*(a + b*Log[c
*x^n])^2)/f + 3*b*n*x*(a + b*Log[c*x^n])^2 - 3*b*n*x*Log[d*(e + f*Sqrt[x])]
*(a + b*Log[c*x^n])^2 + (3*b*e^2*n*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n]
)^2)/f^2 + (e*Sqrt[x]*(a + b*Log[c*x^n])^3)/f - (x*(a + b*Log[c*x^n])^3)/2
+ x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3 - (e^2*Log[1 + (f*Sqrt[x])/
e]*(a + b*Log[c*x^n])^3)/f^2 + (12*b^3*e^2*n^3*PolyLog[2, 1 + (f*Sqrt[x])/e
])/f^2 + (12*b^2*e^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/f
^2 - (6*b*e^2*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*Sqrt[x])/e)])/f^2 - (2
4*b^3*e^2*n^3*PolyLog[3, -((f*Sqrt[x])/e)])/f^2 + (24*b^2*e^2*n^2*(a + b*Lo
g[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)])/f^2 - (48*b^3*e^2*n^3*PolyLog[4, -(
(f*Sqrt[x])/e)])/f^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^{n}\right)\right)^{3}\,dx = \frac{e\sqrt{x}\,\left(a+b\log \left(cx^{n}\right)\right)^{3}}{f} - \frac{1}{2}x\left(a+b\log \left(cx^{n}\right)\right)^{3} - \frac{e^{2}\log \left(e+f\sqrt{x}\right)(}{f^{2}}$$

$$= \frac{e\sqrt{x}\,\left(a+b\log \left(cx^{n}\right)\right)^{3}}{f} - \frac{1}{2}x\left(a+b\log \left(cx^{n}\right)\right)^{3} - \frac{e^{2}\log \left(e+f\sqrt{x}\right)(}{f^{2}}$$

$$= -\frac{9ben\sqrt{x}\,\left(a+b\log \left(cx^{n}\right)\right)^{2}}{f} + 3bnx\left(a+b\log \left(cx^{n}\right)\right)^{2} + \frac{3be^{2}n\log (}{}$$

$$= -\frac{48b^{3}en^{3}\sqrt{x}}{f} - 3ab^{2}n^{2}x + \frac{24b^{2}en^{2}\sqrt{x}\,\left(a+b\log \left(cx^{n}\right)\right)}{f} - \frac{9ben\sqrt{x}}{}$$

$$= -\frac{72b^{3}en^{3}\sqrt{x}}{f} - 6ab^{2}n^{2}x + 3b^{3}n^{3}x - 3b^{3}n^{2}x\log \left(cx^{n}\right) + \frac{42b^{2}en^{2}\sqrt{x}}{}$$

$$= -\frac{84b^{3}en^{3}\sqrt{x}}{f} - 6ab^{2}n^{2}x + 9b^{3}n^{3}x - 6b^{3}n^{2}x\log \left(cx^{n}\right) + \frac{42b^{2}en^{2}\sqrt{x}}{}$$

$$= -\frac{84b^{3}en^{3}\sqrt{x}}{f} - 6ab^{2}n^{2}x + 9b^{3}n^{3}x - 6b^{3}n^{3}x\log \left(d\left(e+f\sqrt{x}\right)\right) - 6b$$

$$= -\frac{84b^{3}en^{3}\sqrt{x}}{f} - 6ab^{2}n^{2}x + 9b^{3}n^{3}x - 6b^{3}n^{3}x\log \left(d\left(e+f\sqrt{x}\right)\right) + \frac{12}{}$$

$$= -\frac{84b^{3}en^{3}\sqrt{x}}{f} - 6ab^{2}n^{2}x + 9b^{3}n^{3}x - 6b^{3}n^{3}x\log \left(d\left(e+f\sqrt{x}\right)\right) + \frac{12}{}$$

$$= -\frac{90b^{3}en^{3}\sqrt{x}}{f} - 6ab^{2}n^{2}x + 12b^{3}n^{3}x + \frac{6b^{3}e^{2}n^{3}\log \left(e+f\sqrt{x}\right)}{f^{2}} - 6b^{3}$$

Mathematica [B] time = 0.68, size = 1522, normalized size = 2.38

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] -1/2*(-2*a^3*e*f*Sqrt[x] + 18*a^2*b*e*f*n*Sqrt[x] - 84*a*b^2*e*f*n^2*Sqrt[x
] + 180*b^3*e*f*n^3*Sqrt[x] + a^3*f^2*x - 6*a^2*b*f^2*n*x + 18*a*b^2*f^2*n^
2*x - 24*b^3*f^2*n^3*x + 2*a^3*e^2*Log[e + f*Sqrt[x]] - 6*a^2*b*e^2*n*Log[e
 + f*Sqrt[x]] + 12*a*b^2*e^2*n^2*Log[e + f*Sqrt[x]] - 12*b^3*e^2*n^3*Log[e
+ f*Sqrt[x]] - 2*a^3*f^2*x*Log[d*(e + f*Sqrt[x])] + 6*a^2*b*f^2*n*x*Log[d*(
e + f*Sqrt[x])] - 12*a*b^2*f^2*n^2*x*Log[d*(e + f*Sqrt[x])] + 12*b^3*f^2*n^
3*x*Log[d*(e + f*Sqrt[x])] - 6*a^2*b*e^2*n*Log[e + f*Sqrt[x]]*Log[x] + 12*a
*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x] - 12*b^3*e^2*n^3*Log[e + f*Sqrt[x]]*
Log[x] + 6*a^2*b*e^2*n*Log[1 + (f*Sqrt[x])/e]*Log[x] - 12*a*b^2*e^2*n^2*Log
[1 + (f*Sqrt[x])/e]*Log[x] + 12*b^3*e^2*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x] +
 6*a*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x]^2 - 6*b^3*e^2*n^3*Log[e + f*Sqrt
[x]]*Log[x]^2 - 6*a*b^2*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 6*b^3*e^2
```

```
*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 - 2*b^3*e^2*n^3*Log[e + f*Sqrt[x]]*Log
[x]^3 + 2*b^3*e^2*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x]^3 - 6*a^2*b*e*f*Sqrt[x]
*Log[c*x^n] + 36*a*b^2*e*f*n*Sqrt[x]*Log[c*x^n] - 84*b^3*e*f*n^2*Sqrt[x]*Lo
g[c*x^n] + 3*a^2*b*f^2*x*Log[c*x^n] - 12*a*b^2*f^2*n*x*Log[c*x^n] + 18*b^3*
f^2*n^2*x*Log[c*x^n] + 6*a^2*b*e^2*Log[e + f*Sqrt[x]]*Log[c*x^n] - 12*a*b^2
*e^2*n*Log[e + f*Sqrt[x]]*Log[c*x^n] + 12*b^3*e^2*n^2*Log[e + f*Sqrt[x]]*Lo
g[c*x^n] - 6*a^2*b*f^2*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 12*a*b^2*f^2*n
*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] - 12*b^3*f^2*n^2*x*Log[d*(e + f*Sqrt[x
])]*Log[c*x^n] - 12*a*b^2*e^2*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] + 12*b
^3*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] + 12*a*b^2*e^2*n*Log[1 + (f
*Sqrt[x])/e]*Log[x]*Log[c*x^n] - 12*b^3*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[
x]*Log[c*x^n] + 6*b^3*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x]^2*Log[c*x^n] - 6*b^
3*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2*Log[c*x^n] - 6*a*b^2*e*f*Sqrt[x]*
Log[c*x^n]^2 + 18*b^3*e*f*n*Sqrt[x]*Log[c*x^n]^2 + 3*a*b^2*f^2*x*Log[c*x^n]
^2 - 6*b^3*f^2*n*x*Log[c*x^n]^2 + 6*a*b^2*e^2*Log[e + f*Sqrt[x]]*Log[c*x^n]
^2 - 6*b^3*e^2*n*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 - 6*a*b^2*f^2*x*Log[d*(e +
 f*Sqrt[x])]*Log[c*x^n]^2 + 6*b^3*f^2*n*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]
^2 - 6*b^3*e^2*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n]^2 + 6*b^3*e^2*n*Log[1
 + (f*Sqrt[x])/e]*Log[x]*Log[c*x^n]^2 - 2*b^3*e*f*Sqrt[x]*Log[c*x^n]^3 + b^
3*f^2*x*Log[c*x^n]^3 + 2*b^3*e^2*Log[e + f*Sqrt[x]]*Log[c*x^n]^3 - 2*b^3*f^
2*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^3 + 12*b*e^2*n*(a^2 - 2*a*b*n + 2*b^2
*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -((f*Sqrt[x]
)/e)] - 48*b^2*e^2*n^2*(a - b*n + b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)
] + 96*b^3*e^2*n^3*PolyLog[4, -((f*Sqrt[x])/e)])/f^2
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\log\left(cx^n\right)^3 + 3\,ab^2\log\left(cx^n\right)^2 + 3\,a^2b\log\left(cx^n\right) + a^3\right)\log\left(df\sqrt{x} + de\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + d*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right) + a\right)^3 \log\left(\left(f\sqrt{x} + e\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right) + a\right)^3 \ln\left(\left(f\sqrt{x} + e\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+e)*d),x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+e)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/27*(27*b^3*e*x*log(d)*log(x^n)^3 + 81*(a*b^2*e*log(d) - (e*n*log(d) - e*l
og(c)*log(d))*b^3)*x*log(x^n)^2 + 81*(a^2*b*e*log(d) - 2*(e*n*log(d) - e*lo
g(c)*log(d))*a*b^2 + (2*e*n^2*log(d) - 2*e*n*log(c)*log(d) + e*log(c)^2*log
(d))*b^3)*x*log(x^n) + 27*(a^3*e*log(d) - 3*(e*n*log(d) - e*log(c)*log(d))*
a^2*b + 3*(2*e*n^2*log(d) - 2*e*n*log(c)*log(d) + e*log(c)^2*log(d))*a*b^2
- (6*e*n^3*log(d) - 6*e*n^2*log(c)*log(d) + 3*e*n*log(c)^2*log(d) - e*log(c
)^3*log(d))*b^3)*x + 27*(b^3*e*x*log(x^n)^3 - 3*((e*n - e*log(c))*b^3 - a*b
^2*e)*x*log(x^n)^2 - 3*(2*(e*n - e*log(c))*a*b^2 - (2*e*n^2 - 2*e*n*log(c)
+ e*log(c)^2)*b^3 - a^2*b*e)*x*log(x^n) - (3*(e*n - e*log(c))*a^2*b - 3*(2*
e*n^2 - 2*e*n*log(c) + e*log(c)^2)*a*b^2 + (6*e*n^3 - 6*e*n^2*log(c) + 3*e*
n*log(c)^2 - e*log(c)^3)*b^3 - a^3*e)*x)*log(f*sqrt(x) + e) - (9*b^3*f*x^2*
log(x^n)^3 - 9*((5*f*n - 3*f*log(c))*b^3 - 3*a*b^2*f)*x^2*log(x^n)^2 - 3*(6
*(5*f*n - 3*f*log(c))*a*b^2 - (38*f*n^2 - 30*f*n*log(c) + 9*f*log(c)^2)*b^3
 - 9*a^2*b*f)*x^2*log(x^n) - (9*(5*f*n - 3*f*log(c))*a^2*b - 3*(38*f*n^2 -
30*f*n*log(c) + 9*f*log(c)^2)*a*b^2 + (130*f*n^3 - 114*f*n^2*log(c) + 45*f*
n*log(c)^2 - 9*f*log(c)^3)*b^3 - 9*a^3*f)*x^2)/sqrt(x))/e + integrate(1/2*(
b^3*f^2*x*log(x^n)^3 + 3*(a*b^2*f^2 - (f^2*n - f^2*log(c))*b^3)*x*log(x^n)^
2 + 3*(a^2*b*f^2 - 2*(f^2*n - f^2*log(c))*a*b^2 + (2*f^2*n^2 - 2*f^2*n*log(
c) + f^2*log(c)^2)*b^3)*x*log(x^n) + (a^3*f^2 - 3*(f^2*n - f^2*log(c))*a^2*
b + 3*(2*f^2*n^2 - 2*f^2*n*log(c) + f^2*log(c)^2)*a*b^2 - (6*f^2*n^3 - 6*f^
2*n^2*log(c) + 3*f^2*n*log(c)^2 - f^2*log(c)^3)*b^3)*x)/(e*f*sqrt(x) + e^2)
, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,\left(e+f\,\sqrt{x}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**(1/2))),x)
```

```
[Out] Timed out
```

3.130 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^3}{x}\,dx$

Optimal. Leaf size=178

$$-48b^2n^2\text{Li}_4\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn}-2\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^3$$

```
[Out] 1/4*(a+b*ln(c*x^n))^4*ln(d*(e+f*x^(1/2)))/b/n-1/4*(a+b*ln(c*x^n))^4*ln(1+f*
x^(1/2)/e)/b/n-2*(a+b*ln(c*x^n))^3*polylog(2,-f*x^(1/2)/e)+12*b*n*(a+b*ln(c
*x^n))^2*polylog(3,-f*x^(1/2)/e)-48*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-f*x^
(1/2)/e)+96*b^3*n^3*polylog(5,-f*x^(1/2)/e)
```

Rubi [A] time = 0.23, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2375, 2337, 2374, 2383, 6589}

$$-48b^2n^2\text{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)-2\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^3+12bn\text{PolyLog}\left(3,-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

```
[Out] (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^4)/(4*b*n) - (Log[1 + (f*Sqrt[x]
)/e]*(a + b*Log[c*x^n])^4)/(4*b*n) - 2*(a + b*Log[c*x^n])^3*PolyLog[2, -((f
*Sqrt[x])/e)] + 12*b*n*(a + b*Log[c*x^n])^2*PolyLog[3, -((f*Sqrt[x])/e)] -
48*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[4, -((f*Sqrt[x])/e)] + 96*b^3*n^3*Pol
yLog[5, -((f*Sqrt[x])/e)]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
```

```
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x}\,dx = \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{f\int\frac{\left(a+b\log\left(cx^n\right)\right)^4}{\left(e+f\sqrt{x}\right)\sqrt{x}}\,dx}{8bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx\right.\right.}{4bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx\right.\right.}{4bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx\right.\right.}{4bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx\right.\right.}{4bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx\right.\right.}{4bn}$$

Mathematica [B] time = 0.42, size = 403, normalized size = 2.26

$$\frac{1}{8}\left(-8b^2n^2\left(48\text{Li}_4\left(-\frac{f\sqrt{x}}{e}\right)+6\log^2(x)\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)-24\log(x)\text{Li}_3\left(-\frac{f\sqrt{x}}{e}\right)+\log^3(x)\log\left(\frac{f\sqrt{x}}{e}+1\right)\right)\left(a+b\,\text{lo}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

```
[Out] (-2*Log[d*(e + f*Sqrt[x])]*Log[x]*(b^3*n^3*Log[x]^3 - 4*b^2*n^2*Log[x]^2*(a
 + b*Log[c*x^n]) + 6*b*n*Log[x]*(a + b*Log[c*x^n])^2 - 4*(a + b*Log[c*x^n])
^3) - 8*(a - b*n*Log[x] + b*Log[c*x^n])^3*(Log[1 + (f*Sqrt[x])/e]*Log[x] +
2*PolyLog[2, -((f*Sqrt[x])/e)]) - 12*b*n*(a - b*n*Log[x] + b*Log[c*x^n])^2*
(Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 4*Log[x]*PolyLog[2, -((f*Sqrt[x])/e)] -
8*PolyLog[3, -((f*Sqrt[x])/e)]) - 8*b^2*n^2*(a - b*n*Log[x] + b*Log[c*x^n])
*(Log[1 + (f*Sqrt[x])/e]*Log[x]^3 + 6*Log[x]^2*PolyLog[2, -((f*Sqrt[x])/e)]
 - 24*Log[x]*PolyLog[3, -((f*Sqrt[x])/e)] + 48*PolyLog[4, -((f*Sqrt[x])/e)]
) - 2*b^3*n^3*(Log[1 + (f*Sqrt[x])/e]*Log[x]^4 + 8*Log[x]^3*PolyLog[2, -((f
*Sqrt[x])/e)] - 48*Log[x]^2*PolyLog[3, -((f*Sqrt[x])/e)] + 192*Log[x]*PolyL
og[4, -((f*Sqrt[x])/e)] - 384*PolyLog[5, -((f*Sqrt[x])/e)]))/8
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\log\left(cx^n\right)^3+3\,ab^2\log\left(cx^n\right)^2+3\,a^2b\log\left(cx^n\right)+a^3\right)\log\left(df\sqrt{x}+de\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + d*e)/x, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{3}\log\left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x, x)
```

maple **[F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^{n}\right)+a\right)^{3}\ln\left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+e)*d)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+e)*d)/x,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)^{3}\log\left(\left(f\sqrt{x}+e\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,\left(e+f\,\sqrt{x}\right)\right)\,{\left(a+b\,\ln\left(c\,x^{n}\right)\right)}^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**(1/2)))/x,x)
```

```
[Out] Timed out
```

3.131 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^3}{x^2}\,dx$

Optimal. Leaf size=673

$$-\frac{6b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x}+\frac{12b^2f^2n^2\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e^2}-\frac{24b^2f^2n^2\text{Li}_3\left(-\frac{f\sqrt{x}}{e}\right)\left(a+b\right.}{e^2}$$

```
[Out] -3*b^3*f^2*n^3*ln(x)/e^2+3/2*b^3*f^2*n^3*ln(x)^2/e^2-3*b^2*f^2*n^2*ln(x)*(a
+b*ln(c*x^n))/e^2-1/2*f^2*(a+b*ln(c*x^n))^3/e^2-1/8*f^2*(a+b*ln(c*x^n))^4/b
/e^2/n+6*b^3*f^2*n^3*ln(e+f*x^(1/2))/e^2+6*b^2*f^2*n^2*(a+b*ln(c*x^n))*ln(e
+f*x^(1/2))/e^2-12*b^3*f^2*n^3*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/e^2-6*b^3*n
^3*ln(d*(e+f*x^(1/2)))/x-6*b^2*n^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)))/x-3*
b*n*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x-(a+b*ln(c*x^n))^3*ln(d*(e+f*x^(
1/2)))/x+3*b*f^2*n*(a+b*ln(c*x^n))^2*ln(1+f*x^(1/2)/e)/e^2+f^2*(a+b*ln(c*x^
n))^3*ln(1+f*x^(1/2)/e)/e^2+12*b^2*f^2*n^2*(a+b*ln(c*x^n))*polylog(2,-f*x^(
1/2)/e)/e^2+6*b*f^2*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)/e^2-12*b^3*
f^2*n^3*polylog(2,1+f*x^(1/2)/e)/e^2-24*b^3*f^2*n^3*polylog(3,-f*x^(1/2)/e)
/e^2-24*b^2*f^2*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x^(1/2)/e)/e^2+48*b^3*f^2*
n^3*polylog(4,-f*x^(1/2)/e)/e^2-90*b^3*f*n^3/e/x^(1/2)-42*b^2*f*n^2*(a+b*ln
(c*x^n))/e/x^(1/2)-9*b*f*n*(a+b*ln(c*x^n))^2/e/x^(1/2)-f*(a+b*ln(c*x^n))^3/
e/x^(1/2)
```

Rubi [A] time = 1.18, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 19, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.679, Rules used = {2454, 2395, 44, 2377, 2305, 2304, 2375, 2337, 2374, 2383, 6589, 2376, 2394, 2315, 2301, 2366, 12, 2302, 30}

$$\frac{12b^2f^2n^2\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e^2}-\frac{24b^2f^2n^2\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{e^2}+\frac{6bf^2n\text{PolyLog}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

```
[Out] (-90*b^3*f*n^3)/(e*Sqrt[x]) + (6*b^3*f^2*n^3*Log[e + f*Sqrt[x]])/e^2 - (6*b
^3*n^3*Log[d*(e + f*Sqrt[x])])/x - (12*b^3*f^2*n^3*Log[e + f*Sqrt[x]]*Log[-
((f*Sqrt[x])/e)])/e^2 - (3*b^3*f^2*n^3*Log[x])/e^2 + (3*b^3*f^2*n^3*Log[x]^
2)/(2*e^2) - (42*b^2*f*n^2*(a + b*Log[c*x^n]))/(e*Sqrt[x]) + (6*b^2*f^2*n^2
*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/e^2 - (6*b^2*n^2*Log[d*(e + f*Sqrt[
x])]*(a + b*Log[c*x^n]))/x - (3*b^2*f^2*n^2*Log[x]*(a + b*Log[c*x^n]))/e^2
- (9*b*f*n*(a + b*Log[c*x^n])^2)/(e*Sqrt[x]) - (3*b*n*Log[d*(e + f*Sqrt[x])
]*(a + b*Log[c*x^n])^2)/x + (3*b*f^2*n*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*
x^n])^2)/e^2 - (f^2*(a + b*Log[c*x^n])^3)/(2*e^2) - (f*(a + b*Log[c*x^n])^3
)/(e*Sqrt[x]) - (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x + (f^2*Log[
1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^3)/e^2 - (f^2*(a + b*Log[c*x^n])^4)/(
8*b*e^2*n) - (12*b^3*f^2*n^3*PolyLog[2, 1 + (f*Sqrt[x])/e])/e^2 + (12*b^2*f
^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/e^2 + (6*b*f^2*n*(a
 + b*Log[c*x^n])^2*PolyLog[2, -((f*Sqrt[x])/e)])/e^2 - (24*b^3*f^2*n^3*Poly
Log[3, -((f*Sqrt[x])/e)])/e^2 - (24*b^2*f^2*n^2*(a + b*Log[c*x^n])*PolyLog[
3, -((f*Sqrt[x])/e)])/e^2 + (48*b^3*f^2*n^3*PolyLog[4, -((f*Sqrt[x])/e)])/e
^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^2}\,dx = -\frac{f\left(a+b\log\left(cx^n\right)\right)^3}{e\sqrt{x}} + \frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3}{e^2} - \frac{\text{lo}}{}$$

$$= -\frac{f\left(a+b\log\left(cx^n\right)\right)^3}{e\sqrt{x}} + \frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3}{e^2} - \frac{\text{lo}}{}$$

$$= -\frac{9bfn\left(a+b\log\left(cx^n\right)\right)^2}{e\sqrt{x}} + \frac{3bf^2n\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right.\right.}{e^2}$$

$$= -\frac{48b^3fn^3}{e\sqrt{x}} - \frac{24b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} - \frac{9bfn\left(a+b\log\left(cx^n\right)\right)^2}{e\sqrt{x}}$$

$$= -\frac{72b^3fn^3}{e\sqrt{x}} - \frac{42b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{6b^2f^2n^2\log\left(e+f\sqrt{x}\right.}{e^2}$$

$$= -\frac{84b^3fn^3}{e\sqrt{x}} - \frac{42b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{6b^2f^2n^2\log\left(e+f\sqrt{x}\right.}{e^2}$$

$$= -\frac{84b^3fn^3}{e\sqrt{x}} + \frac{3b^3f^2n^3\log^2(x)}{2e^2} - \frac{42b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{6b^2}{}$$

$$= -\frac{84b^3fn^3}{e\sqrt{x}} - \frac{6b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right)}{x} - \frac{12b^3f^2n^3\log\left(e+f\sqrt{x}\right.}{e^2}$$

$$= -\frac{84b^3fn^3}{e\sqrt{x}} - \frac{6b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right)}{x} - \frac{12b^3f^2n^3\log\left(e+f\sqrt{x}\right.}{e^2}$$

$$= -\frac{90b^3fn^3}{e\sqrt{x}} + \frac{6b^3f^2n^3\log\left(e+f\sqrt{x}\right)}{e^2} - \frac{6b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right)}{x}$$

Mathematica [A] time = 1.19, size = 976, normalized size = 1.45

$$-\frac{b^3\left(6f^2x\text{Li}_2\left(-\frac{e}{f\sqrt{x}}\right)\log^2(x)+f\sqrt{x}\left(e\log^3(x)-f\sqrt{x}\log\left(\frac{e}{f\sqrt{x}}+1\right)\log^3(x)+6e\log^2(x)+24e\log(x)+24f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

```
[Out] -((e^2*Log[d*(e + f*Sqrt[x])]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 +
3*b*(a^2 + 2*a*b*n + 2*b^2*n^2)*Log[c*x^n] + 3*b^2*(a + b*n)*Log[c*x^n]^2 +
 b^3*Log[c*x^n]^3) + e*f*Sqrt[x]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3
 + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]
) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n
])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n]
)^3) - f^2*x*Log[e + f*Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3
+ 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n])
 + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n]
)^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])
^3) + (f^2*x*Log[x]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3*a^2*b*(-
(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2
*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 3*b^3*
n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3))/2 + 3*b
*f*n*Sqrt[x]*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*a*b*(-(n*Log[x]) + Log[c*x^n])
+ 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x]) + Log[c*x^n])^2)*(2
*e + (e - f*Sqrt[x]*Log[1 + (f*Sqrt[x])/e])*Log[x] + (f*Sqrt[x]*Log[x]^2)/4
 - 2*f*Sqrt[x]*PolyLog[2, -((f*Sqrt[x])/e)]) + b^2*f*n^2*Sqrt[x]*(a + b*n -
 b*n*Log[x] + b*Log[c*x^n])*(24*e + 12*e*Log[x] + 3*e*Log[x]^2 - 3*f*Sqrt[x
]*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + (f*Sqrt[x]*Log[x]^3)/2 - 12*f*Sqrt[x]*L
og[x]*PolyLog[2, -((f*Sqrt[x])/e)] + 24*f*Sqrt[x]*PolyLog[3, -((f*Sqrt[x])/
e)]) + b^3*n^3*(6*f^2*x*Log[x]^2*PolyLog[2, -(e/(f*Sqrt[x]))] + f*Sqrt[x]*(
48*e + 24*e*Log[x] + 6*e*Log[x]^2 + e*Log[x]^3 - f*Sqrt[x]*Log[1 + e/(f*Sqr
t[x])]*Log[x]^3 + 24*f*Sqrt[x]*Log[x]*PolyLog[3, -(e/(f*Sqrt[x]))] + 48*f*S
qrt[x]*PolyLog[4, -(e/(f*Sqrt[x]))])))/(e^2*x))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3 \log\left(cx^n\right)^3 + 3\,ab^2 \log\left(cx^n\right)^2 + 3\,a^2 b \log\left(cx^n\right) + a^3\right) \log\left(df\sqrt{x} + de\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + d*e)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^n\right) + a\right)^3 \log\left(\left(f\sqrt{x} + e\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^2, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^n\right) + a\right)^3 \ln\left(\left(f\sqrt{x} + e\right) d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+e)*d)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+e)*d)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right) + a\right)^3 \log\left(\left(f\sqrt{x} + e\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,\left(e+f\,\sqrt{x}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^2,x)
```

```
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**(1/2)))/x**2,x)
```

```
[Out] Timed out
```

3.132 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log(cx^n)\right)^3}{x^3}\,dx$

Optimal. Leaf size=914

$$-\frac{\left(a+b\log\left(cx^n\right)\right)^4 f^4}{16be^4n}+\frac{\log\left(\frac{\sqrt{x}f}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^3 f^4}{2e^4}-\frac{\left(a+b\log\left(cx^n\right)\right)^3 f^4}{8e^4}+\frac{3b^3n^3\log^2(x)f^4}{16e^4}+\frac{3bn\log\left(\sqrt{\cdots}\right.}{\cdots}$$

```
[Out] -37/36*b^2*f*n^2*(a+b*ln(c*x^n))/e/x^(3/2)+21/8*b^2*f^2*n^2*(a+b*ln(c*x^n))
/e^2/x-3/8*b^2*f^4*n^2*ln(x)*(a+b*ln(c*x^n))/e^4-7/12*b*f*n*(a+b*ln(c*x^n))
^2/e/x^(3/2)+9/8*b*f^2*n*(a+b*ln(c*x^n))^2/e^2/x+3/4*b^2*f^4*n^2*(a+b*ln(c*
x^n))*ln(e+f*x^(1/2))/e^4-3/2*b^3*f^4*n^3*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/
e^4+3/4*b*f^4*n*(a+b*ln(c*x^n))^2*ln(1+f*x^(1/2)/e)/e^4+3*b^2*f^4*n^2*(a+b*
ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/e^4+3*b*f^4*n*(a+b*ln(c*x^n))^2*polylog(
2,-f*x^(1/2)/e)/e^4-12*b^2*f^4*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x^(1/2)/e)/
e^4-63/4*b^2*f^3*n^2*(a+b*ln(c*x^n))/e^3/x^(1/2)-15/4*b*f^3*n*(a+b*ln(c*x^n
))^2/e^3/x^(1/2)-1/2*(a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))/x^2-1/8*f^4*(a+b
*ln(c*x^n))^3/e^4-3/16*b^3*f^4*n^3*ln(x)/e^4+3/16*b^3*f^4*n^3*ln(x)^2/e^4-1
/16*f^4*(a+b*ln(c*x^n))^4/b/e^4/n+3/8*b^3*f^4*n^3*ln(e+f*x^(1/2))/e^4-3/4*b
^2*n^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)))/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln
(d*(e+f*x^(1/2)))/x^2-3/2*b^3*f^4*n^3*polylog(2,1+f*x^(1/2)/e)/e^4-6*b^3*f^
4*n^3*polylog(3,-f*x^(1/2)/e)/e^4+24*b^3*f^4*n^3*polylog(4,-f*x^(1/2)/e)/e^
4-255/8*b^3*f^3*n^3/e^3/x^(1/2)-1/6*f*(a+b*ln(c*x^n))^3/e/x^(3/2)+1/4*f^2*(
a+b*ln(c*x^n))^3/e^2/x-3/8*b^3*n^3*ln(d*(e+f*x^(1/2)))/x^2+1/2*f^4*(a+b*ln(
c*x^n))^3*ln(1+f*x^(1/2)/e)/e^4-1/2*f^3*(a+b*ln(c*x^n))^3/e^3/x^(1/2)-175/2
16*b^3*f*n^3/e/x^(3/2)+45/16*b^3*f^2*n^3/e^2/x
```

Rubi [A] time = 1.52, antiderivative size = 914, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 19, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.679, Rules used = {2454, 2395, 44, 2377, 2305, 2304, 2375, 2337, 2374, 2383, 6589, 2376, 2394, 2315, 2301, 2366, 12, 2302, 30}

$$-\frac{\left(a+b\log\left(cx^n\right)\right)^4 f^4}{16be^4n}+\frac{\log\left(\frac{\sqrt{x}f}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^3 f^4}{2e^4}-\frac{\left(a+b\log\left(cx^n\right)\right)^3 f^4}{8e^4}+\frac{3b^3n^3\log^2(x)f^4}{16e^4}+\frac{3bn\log\left(\sqrt{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

```
[Out] (-175*b^3*f*n^3)/(216*e*x^(3/2)) + (45*b^3*f^2*n^3)/(16*e^2*x) - (255*b^3*f
^3*n^3)/(8*e^3*Sqrt[x]) + (3*b^3*f^4*n^3*Log[e + f*Sqrt[x]])/(8*e^4) - (3*b
^3*n^3*Log[d*(e + f*Sqrt[x])])/(8*x^2) - (3*b^3*f^4*n^3*Log[e + f*Sqrt[x]]*
Log[-((f*Sqrt[x])/e)])/(2*e^4) - (3*b^3*f^4*n^3*Log[x])/(16*e^4) + (3*b^3*f
^4*n^3*Log[x]^2)/(16*e^4) - (37*b^2*f*n^2*(a + b*Log[c*x^n]))/(36*e*x^(3/2)
) + (21*b^2*f^2*n^2*(a + b*Log[c*x^n]))/(8*e^2*x) - (63*b^2*f^3*n^2*(a + b*
Log[c*x^n]))/(4*e^3*Sqrt[x]) + (3*b^2*f^4*n^2*Log[e + f*Sqrt[x]]*(a + b*Log
[c*x^n]))/(4*e^4) - (3*b^2*n^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/(
4*x^2) - (3*b^2*f^4*n^2*Log[x]*(a + b*Log[c*x^n]))/(8*e^4) - (7*b*f*n*(a +
b*Log[c*x^n])^2)/(12*e*x^(3/2)) + (9*b*f^2*n*(a + b*Log[c*x^n])^2)/(8*e^2*x
) - (15*b*f^3*n*(a + b*Log[c*x^n])^2)/(4*e^3*Sqrt[x]) - (3*b*n*Log[d*(e + f
*Sqrt[x])]*(a + b*Log[c*x^n])^2)/(4*x^2) + (3*b*f^4*n*Log[1 + (f*Sqrt[x])/e
]*(a + b*Log[c*x^n])^2)/(4*e^4) - (f^4*(a + b*Log[c*x^n])^3)/(8*e^4) - (f*(
a + b*Log[c*x^n])^3)/(6*e*x^(3/2)) + (f^2*(a + b*Log[c*x^n])^3)/(4*e^2*x) -
 (f^3*(a + b*Log[c*x^n])^3)/(2*e^3*Sqrt[x]) - (Log[d*(e + f*Sqrt[x])]*(a +
b*Log[c*x^n])^3)/(2*x^2) + (f^4*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^3
)/(2*e^4) - (f^4*(a + b*Log[c*x^n])^4)/(16*b*e^4*n) - (3*b^3*f^4*n^3*PolyLo
g[2, 1 + (f*Sqrt[x])/e])/(2*e^4) + (3*b^2*f^4*n^2*(a + b*Log[c*x^n])*PolyLo
g[2, -((f*Sqrt[x])/e)])/e^4 + (3*b*f^4*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(
```

```
(f*Sqrt[x])/e)])/e^4 - (6*b^3*f^4*n^3*PolyLog[3, -((f*Sqrt[x])/e)])/e^4 - (
12*b^2*f^4*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)])/e^4 + (24*b
^3*f^4*n^3*PolyLog[4, -((f*Sqrt[x])/e)])/e^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2377

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^3}\,dx = -\frac{f\left(a+b\log\left(cx^n\right)\right)^3}{6ex^{3/2}}+\frac{f^2\left(a+b\log\left(cx^n\right)\right)^3}{4e^2x}-\frac{f^3\left(a+b\log\left(cx\right.\right.}{2e^3\sqrt{x}}$$

$$= -\frac{f\left(a+b\log\left(cx^n\right)\right)^3}{6ex^{3/2}}+\frac{f^2\left(a+b\log\left(cx^n\right)\right)^3}{4e^2x}-\frac{f^3\left(a+b\log\left(cx\right.\right.}{2e^3\sqrt{x}}$$

$$= -\frac{7bfn\left(a+b\log\left(cx^n\right)\right)^2}{12ex^{3/2}}+\frac{9bf^2n\left(a+b\log\left(cx^n\right)\right)^2}{8e^2x}-\frac{15bf^3n\left(\right.}{}$$

$$= -\frac{8b^3fn^3}{27ex^{3/2}}+\frac{3b^3f^2n^3}{2e^2x}-\frac{24b^3f^3n^3}{e^3\sqrt{x}}-\frac{4b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{9ex^{3/2}}+$$

$$= -\frac{14b^3fn^3}{27ex^{3/2}}+\frac{9b^3f^2n^3}{4e^2x}-\frac{30b^3f^3n^3}{e^3\sqrt{x}}-\frac{37b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{36ex^{3/2}}$$

$$= -\frac{37b^3fn^3}{54ex^{3/2}}+\frac{21b^3f^2n^3}{8e^2x}-\frac{63b^3f^3n^3}{2e^3\sqrt{x}}-\frac{37b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{36ex^{3/2}}$$

$$= -\frac{37b^3fn^3}{54ex^{3/2}}+\frac{21b^3f^2n^3}{8e^2x}-\frac{63b^3f^3n^3}{2e^3\sqrt{x}}+\frac{3b^3f^4n^3\log^2(x)}{16e^4}-\frac{37b^2f}{}$$

$$= -\frac{37b^3fn^3}{54ex^{3/2}}+\frac{21b^3f^2n^3}{8e^2x}-\frac{63b^3f^3n^3}{2e^3\sqrt{x}}-\frac{3b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right)}{8x^2}$$

$$= -\frac{37b^3fn^3}{54ex^{3/2}}+\frac{21b^3f^2n^3}{8e^2x}-\frac{63b^3f^3n^3}{2e^3\sqrt{x}}-\frac{3b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right)}{8x^2}$$

$$= -\frac{175b^3fn^3}{216ex^{3/2}}+\frac{45b^3f^2n^3}{16e^2x}-\frac{255b^3f^3n^3}{8e^3\sqrt{x}}+\frac{3b^3f^4n^3\log\left(e+f\sqrt{x}\right)}{8e^4}$$

Mathematica [A] time = 2.30, size = 1549, normalized size = 1.69

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

```
[Out] -1/432*(54*e^4*Log[d*(e + f*Sqrt[x])]*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*
b^3*n^3 + 6*b*(2*a^2 + 2*a*b*n + b^2*n^2)*Log[c*x^n] + 6*b^2*(2*a + b*n)*Lo
g[c*x^n]^2 + 4*b^3*Log[c*x^n]^3) + 18*e^3*f*Sqrt[x]*(4*a^3 + 6*a^2*b*n + 6*
a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-
(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*
(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3
*(-(n*Log[x]) + Log[c*x^n])^3) - 27*e^2*f^2*x*(4*a^3 + 6*a^2*b*n + 6*a*b^2*
n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-(n*Log
[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*L
og[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*
Log[x]) + Log[c*x^n])^3) + 54*e*f^3*x^(3/2)*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^
2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-(n*Log[x
]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log
[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Lo
g[x]) + Log[c*x^n])^3) - 54*f^4*x^2*Log[e + f*Sqrt[x]]*(4*a^3 + 6*a^2*b*n +
 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n
*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b
^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*
b^3*(-(n*Log[x]) + Log[c*x^n])^3) + 27*f^4*x^2*Log[x]*(4*a^3 + 6*a^2*b*n +
6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*
(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^
2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b
^3*(-(n*Log[x]) + Log[c*x^n])^3) + 18*b*f*n*Sqrt[x]*(2*a^2 + 2*a*b*n + b^2*
n^2 + 4*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n])
 + 2*b^2*(-(n*Log[x]) + Log[c*x^n])^2)*(e*(4*e^2 - 9*e*f*Sqrt[x] + 36*f^2*x
) + 3*(2*e^3 - 3*e^2*f*Sqrt[x] + 6*e*f^2*x - 6*f^3*x^(3/2)*Log[1 + (f*Sqrt[
x])/e])*Log[x] + (9*f^3*x^(3/2)*Log[x]^2)/2 - 36*f^3*x^(3/2)*PolyLog[2, -((
f*Sqrt[x])/e)]) - 6*b^2*f*n^2*Sqrt[x]*(-2*a - b*n + 2*b*n*Log[x] - 2*b*Log[
c*x^n])*(16*e^3 - 54*e^2*f*Sqrt[x] + 432*e*f^2*x + 24*e^3*Log[x] - 54*e^2*f
*Sqrt[x]*Log[x] + 216*e*f^2*x*Log[x] + 18*e^3*Log[x]^2 - 27*e^2*f*Sqrt[x]*L
og[x]^2 + 54*e*f^2*x*Log[x]^2 - 54*f^3*x^(3/2)*Log[1 + (f*Sqrt[x])/e]*Log[x
]^2 + 9*f^3*x^(3/2)*Log[x]^3 - 216*f^3*x^(3/2)*Log[x]*PolyLog[2, -((f*Sqrt[
x])/e)] + 432*f^3*x^(3/2)*PolyLog[3, -((f*Sqrt[x])/e)]) + 4*b^3*n^3*(2*e*f*
Sqrt[x]*(16*e^2 - 81*e*f*Sqrt[x] + 1296*f^2*x) + 9*(e*f*Sqrt[x]*(2*e^2 - 3*
e*f*Sqrt[x] + 6*f^2*x) - 6*f^4*x^2*Log[1 + e/(f*Sqrt[x])])*Log[x]^3 + 9*f*S
qrt[x]*Log[x]^2*(e*(4*e^2 - 9*e*f*Sqrt[x] + 36*f^2*x) + 36*f^3*x^(3/2)*Poly
Log[2, -(e/(f*Sqrt[x]))]) + 6*f*Sqrt[x]*Log[x]*(e*(8*e^2 - 27*e*f*Sqrt[x] +
 216*f^2*x) + 216*f^3*x^(3/2)*PolyLog[3, -(e/(f*Sqrt[x]))]) + 2592*f^4*x^2*
PolyLog[4, -(e/(f*Sqrt[x]))]))/(e^4*x^2)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\log\left(cx^n\right)^3+3\,ab^2\log\left(cx^n\right)^2+3\,a^2b\log\left(cx^n\right)+a^3\right)\log\left(df\sqrt{x}+de\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + d*e)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(cx^{n}\right)+a\right)^{3} \log \left(\left(f \sqrt{x}+e\right) d\right)}{x^{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^3, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c\,x^{n}\right)+a\right)^{3} \ln \left(\left(f \sqrt{x}+e\right) d\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+e)*d)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln((f*x^(1/2)+e)*d)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(cx^{n}\right)+a\right)^{3} \log \left(\left(f \sqrt{x}+e\right) d\right)}{x^{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(d\,\left(e+f\,\sqrt{x}\right)\right)\,{\left(a+b\,\ln \left(c\,x^{n}\right)\right)}^{3}}{x^{3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^3,x)
```

```
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**(1/2)))/x**3,x)
```

```
[Out] Timed out
```

3.133 $\int x^{3/2} \log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=367

$$\frac{2}{5}x^{5/2}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2e^5k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{5f^5}-\frac{2e^4k\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{5f^4}+\frac{e^3k\cdots}{\cdots}$$

```
[Out] -7/25*b*e^3*k*n*x/f^3+32/225*b*e^2*k*n*x^(3/2)/f^2-9/100*b*e*k*n*x^2/f+8/12
5*b*k*n*x^(5/2)+1/5*e^3*k*x*(a+b*ln(c*x^n))/f^3-2/15*e^2*k*x^(3/2)*(a+b*ln(
c*x^n))/f^2+1/10*e*k*x^2*(a+b*ln(c*x^n))/f-2/25*k*x^(5/2)*(a+b*ln(c*x^n))-4
/25*b*e^5*k*n*ln(e+f*x^(1/2))/f^5+2/5*e^5*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))
/f^5-4/5*b*e^5*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^5-4/25*b*n*x^(5/2)*ln
(d*(e+f*x^(1/2))^k)+2/5*x^(5/2)*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)-4/5*b
*e^5*k*n*polylog(2,1+f*x^(1/2)/e)/f^5+24/25*b*e^4*k*n*x^(1/2)/f^4-2/5*e^4*k
*(a+b*ln(c*x^n))*x^(1/2)/f^4
```

Rubi [A] time = 0.30, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2454, 2395, 43, 2376, 2394, 2315}

$$-\frac{4be^5kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{5f^5}+\frac{2}{5}x^{5/2}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2e^5k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{5f^5}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (24*b*e^4*k*n*Sqrt[x])/(25*f^4) - (7*b*e^3*k*n*x)/(25*f^3) + (32*b*e^2*k*n*
x^(3/2))/(225*f^2) - (9*b*e*k*n*x^2)/(100*f) + (8*b*k*n*x^(5/2))/125 - (4*b
*e^5*k*n*Log[e + f*Sqrt[x]])/(25*f^5) - (4*b*n*x^(5/2)*Log[d*(e + f*Sqrt[x]
)^k])/25 - (4*b*e^5*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(5*f^5) -
 (2*e^4*k*Sqrt[x]*(a + b*Log[c*x^n]))/(5*f^4) + (e^3*k*x*(a + b*Log[c*x^n])
)/(5*f^3) - (2*e^2*k*x^(3/2)*(a + b*Log[c*x^n]))/(15*f^2) + (e*k*x^2*(a + b
*Log[c*x^n]))/(10*f) - (2*k*x^(5/2)*(a + b*Log[c*x^n]))/25 + (2*e^5*k*Log[e
 + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(5*f^5) + (2*x^(5/2)*Log[d*(e + f*Sqrt[x]
)^k]*(a + b*Log[c*x^n]))/5 - (4*b*e^5*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(5
*f^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)dx=-\frac{2e^4k\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{5f^4}+\frac{e^3kx\left(a+b\log\left(cx^n\right)\right)}{5f^3}-\frac{2e^2kx^{3/2}}{}$$

$$=\frac{4be^4kn\sqrt{x}}{5f^4}-\frac{be^3knx}{5f^3}+\frac{4be^2knx^{3/2}}{45f^2}-\frac{beknx^2}{20f}+\frac{4}{125}bknx^{5/2}$$

$$=\frac{4be^4kn\sqrt{x}}{5f^4}-\frac{be^3knx}{5f^3}+\frac{4be^2knx^{3/2}}{45f^2}-\frac{beknx^2}{20f}+\frac{4}{125}bknx^{5/2}$$

$$=\frac{4be^4kn\sqrt{x}}{5f^4}-\frac{be^3knx}{5f^3}+\frac{4be^2knx^{3/2}}{45f^2}-\frac{beknx^2}{20f}+\frac{4}{125}bknx^{5/2}$$

$$=\frac{4be^4kn\sqrt{x}}{5f^4}-\frac{be^3knx}{5f^3}+\frac{4be^2knx^{3/2}}{45f^2}-\frac{beknx^2}{20f}+\frac{4}{125}bknx^{5/2}$$

$$=\frac{24be^4kn\sqrt{x}}{25f^4}-\frac{7be^3knx}{25f^3}+\frac{32be^2knx^{3/2}}{225f^2}-\frac{9beknx^2}{100f}+\frac{8}{125}bkn$$

Mathematica [A] time = 0.42, size = 394, normalized size = 1.07

$$\frac{360e^5k\log\left(e+f\sqrt{x}\right)\left(5a+5b\log\left(cx^n\right)-5bn\log(x)-2bn\right)+1800af^5x^{5/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)-1800ae^4fk}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-1800*a*e^4*f*k*Sqrt[x] + 4320*b*e^4*f*k*n*Sqrt[x] + 900*a*e^3*f^2*k*x - 1
260*b*e^3*f^2*k*n*x - 600*a*e^2*f^3*k*x^(3/2) + 640*b*e^2*f^3*k*n*x^(3/2) +
 450*a*e*f^4*k*x^2 - 405*b*e*f^4*k*n*x^2 - 360*a*f^5*k*x^(5/2) + 288*b*f^5*
```

```
k*n*x^(5/2) + 1800*a*f^5*x^(5/2)*Log[d*(e + f*Sqrt[x])^k] - 720*b*f^5*n*x^(
5/2)*Log[d*(e + f*Sqrt[x])^k] + 1800*b*e^5*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x
] - 1800*b*e^4*f*k*Sqrt[x]*Log[c*x^n] + 900*b*e^3*f^2*k*x*Log[c*x^n] - 600*
b*e^2*f^3*k*x^(3/2)*Log[c*x^n] + 450*b*e*f^4*k*x^2*Log[c*x^n] - 360*b*f^5*k
*x^(5/2)*Log[c*x^n] + 1800*b*f^5*x^(5/2)*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n
] + 360*e^5*k*Log[e + f*Sqrt[x]]*(5*a - 2*b*n - 5*b*n*Log[x] + 5*b*Log[c*x^
n]) + 3600*b*e^5*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/(4500*f^5)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^{\frac{3}{2}}\log\left(cx^{n}\right)+ax^{\frac{3}{2}}\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="fri
cas")
```

```
[Out] integral((b*x^(3/2)*log(c*x^n) + a*x^(3/2))*log((f*sqrt(x) + e)^k*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^{n}\right)+a\right)x^{\frac{3}{2}}\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^(3/2)*log((f*sqrt(x) + e)^k*d), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\,x^{n}\right)+a\right)x^{\frac{3}{2}}\ln\left(d\left(f\sqrt{x}+e\right)^{k}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k),x)
```

```
[Out] int(x^(3/2)*(b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$50\,bekx^{2}\log\left(x^{n}\right)+40\left(5\,bfx\log\left(x^{n}\right)-\left(\left(2\,fn-5\,f\log(c)\right)b-5\,af\right)x\right)kx^{\frac{3}{2}}\log\left(f\sqrt{x}+e\right)+5\left(10\,aek-\left(9\,ekn-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="max
ima")
```

```
[Out] 1/500*(50*b*e*k*x^2*log(x^n) + 40*(5*b*f*x*log(x^n) - ((2*f*n - 5*f*log(c))
*b - 5*a*f)*x)*k*x^(3/2)*log(f*sqrt(x) + e) + 5*(10*a*e*k - (9*e*k*n - 10*e
*k*log(c))*b)*x^2 + 40*(5*b*f*x*log(d)*log(x^n) + (5*a*f*log(d) - (2*f*n*lo
g(d) - 5*f*log(c)*log(d))*b)*x)*x^(3/2) - 8*(5*b*f*k*x^2*log(x^n) + (5*a*f*
k - (4*f*k*n - 5*f*k*log(c))*b)*x^2)*sqrt(x))/f - integrate(1/25*(5*b*e^2*k
*x*log(x^n) + (5*a*e^2*k - (2*e^2*k*n - 5*e^2*k*log(c))*b)*x)/(f^2*sqrt(x)
+ e*f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\,\ln\left(d\,{\left(e+f\,\sqrt{x}\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
[Out] int(x^(3/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k),x)
[Out] Timed out
```

3.134 $\int \sqrt{x} \log \left(d \left(e+f \sqrt{x}\right)^k\right) \left(a+b \log \left(c x^n\right)\right) \, dx$

Optimal. Leaf size=283

$$\frac{2}{3} x^{3/2} \left(a+b \log \left(c x^n\right)\right) \log \left(d \left(e+f \sqrt{x}\right)^k\right)+\frac{2 e^3 k \log \left(e+f \sqrt{x}\right) \left(a+b \log \left(c x^n\right)\right)}{3 f^3}-\frac{2 e^2 k \sqrt{x} \left(a+b \log \left(c x^n\right)\right)}{3 f^2}+\frac{e k x}{}$$

```
[Out] -5/9*b*e*k*n*x/f+8/27*b*k*n*x^(3/2)+1/3*e*k*x*(a+b*ln(c*x^n))/f-2/9*k*x^(3/
2)*(a+b*ln(c*x^n))-4/9*b*e^3*k*n*ln(e+f*x^(1/2))/f^3+2/3*e^3*k*(a+b*ln(c*x^
n))*ln(e+f*x^(1/2))/f^3-4/3*b*e^3*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^3-
4/9*b*n*x^(3/2)*ln(d*(e+f*x^(1/2))^k)+2/3*x^(3/2)*(a+b*ln(c*x^n))*ln(d*(e+f
*x^(1/2))^k)-4/3*b*e^3*k*n*polylog(2,1+f*x^(1/2)/e)/f^3+16/9*b*e^2*k*n*x^(1
/2)/f^2-2/3*e^2*k*(a+b*ln(c*x^n))*x^(1/2)/f^2
```

Rubi [A] time = 0.23, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2454, 2395, 43, 2376, 2394, 2315}

$$-\frac{4 b e^3 k n \text{PolyLog}\left(2,\frac{f \sqrt{x}}{e}+1\right)}{3 f^3}+\frac{2}{3} x^{3/2} \left(a+b \log \left(c x^n\right)\right) \log \left(d \left(e+f \sqrt{x}\right)^k\right)+\frac{2 e^3 k \log \left(e+f \sqrt{x}\right) \left(a+b \log \left(c x^n\right)\right)}{3 f^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (16*b*e^2*k*n*Sqrt[x])/(9*f^2) - (5*b*e*k*n*x)/(9*f) + (8*b*k*n*x^(3/2))/27
 - (4*b*e^3*k*n*Log[e + f*Sqrt[x]])/(9*f^3) - (4*b*n*x^(3/2)*Log[d*(e + f*S
qrt[x])^k])/9 - (4*b*e^3*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(3*f
^3) - (2*e^2*k*Sqrt[x]*(a + b*Log[c*x^n]))/(3*f^2) + (e*k*x*(a + b*Log[c*x^
n]))/(3*f) - (2*k*x^(3/2)*(a + b*Log[c*x^n]))/9 + (2*e^3*k*Log[e + f*Sqrt[x
]]*(a + b*Log[c*x^n]))/(3*f^3) + (2*x^(3/2)*Log[d*(e + f*Sqrt[x])^k]*(a + b
*Log[c*x^n]))/3 - (4*b*e^3*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(3*f^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \sqrt{x}\,\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)dx = -\frac{2e^2k\sqrt{x}\,\left(a+b\log\left(cx^n\right)\right)}{3f^2}+\frac{ekx\left(a+b\log\left(cx^n\right)\right)}{3f}-\frac{2}{9}kx^{3/2}$$

$$=\frac{4be^2kn\sqrt{x}}{3f^2}-\frac{beknx}{3f}+\frac{4}{27}bknx^{3/2}-\frac{2e^2k\sqrt{x}\,\left(a+b\log\left(cx^n\right)\right)}{3f^2}$$

$$=\frac{4be^2kn\sqrt{x}}{3f^2}-\frac{beknx}{3f}+\frac{4}{27}bknx^{3/2}-\frac{2e^2k\sqrt{x}\,\left(a+b\log\left(cx^n\right)\right)}{3f^2}$$

$$=\frac{4be^2kn\sqrt{x}}{3f^2}-\frac{beknx}{3f}+\frac{4}{27}bknx^{3/2}-\frac{4}{9}bnx^{3/2}\log\left(d\left(e+f\sqrt{x}\right.\right.$$

$$=\frac{4be^2kn\sqrt{x}}{3f^2}-\frac{beknx}{3f}+\frac{4}{27}bknx^{3/2}-\frac{4}{9}bnx^{3/2}\log\left(d\left(e+f\sqrt{x}\right.\right.$$

$$=\frac{16be^2kn\sqrt{x}}{9f^2}-\frac{5beknx}{9f}+\frac{8}{27}bknx^{3/2}-\frac{4be^3kn\log\left(e+f\sqrt{x}\right)}{9f^3}$$

Mathematica [A] time = 0.31, size = 296, normalized size = 1.05

$$6e^3k\log\left(e+f\sqrt{x}\right)\left(3a+3b\log\left(cx^n\right)-3bn\log(x)-2bn\right)+18af^3x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)-18ae^2fk\sqrt{x}+9a$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-18*a*e^2*f*k*Sqrt[x] + 48*b*e^2*f*k*n*Sqrt[x] + 9*a*e*f^2*k*x - 15*b*e*f^
2*k*n*x - 6*a*f^3*k*x^(3/2) + 8*b*f^3*k*n*x^(3/2) + 18*a*f^3*x^(3/2)*Log[d*
(e + f*Sqrt[x])^k] - 12*b*f^3*n*x^(3/2)*Log[d*(e + f*Sqrt[x])^k] + 18*b*e^3
*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x] - 18*b*e^2*f*k*Sqrt[x]*Log[c*x^n] + 9*b*
e*f^2*k*x*Log[c*x^n] - 6*b*f^3*k*x^(3/2)*Log[c*x^n] + 18*b*f^3*x^(3/2)*Log[
d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 6*e^3*k*Log[e + f*Sqrt[x]]*(3*a - 2*b*n -
 3*b*n*Log[x] + 3*b*Log[c*x^n]) + 36*b*e^3*k*n*PolyLog[2, -((f*Sqrt[x])/e)]
)/(27*f^3)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sqrt{x}\log\left(cx^n\right)+a\sqrt{x}\right)\log\left(\left(f\sqrt{x}+e\right)^k d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="fri
cas")
```

```
[Out] integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)\sqrt{x}\log\left(\left(f\sqrt{x}+e\right)^k d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*sqrt(x)*log((f*sqrt(x) + e)^k*d), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)\sqrt{x}\,\ln\left(d\left(f\sqrt{x}+e\right)^k\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k),x)
```

```
[Out] int(x^(1/2)*(b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{9}\left(3bx\log\left(x^n\right)-\left(b\left(2n-3\log(c)\right)-3a\right)x\right)k\sqrt{x}\log\left(f\sqrt{x}+e\right)+\frac{2}{9}\left(3bx\log(d)\log\left(x^n\right)-\left(\left(2n\log(d)-3\log(c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="max
ima")
```

```
[Out] 2/9*(3*b*x*log(x^n) - (b*(2*n - 3*log(c)) - 3*a)*x)*k*sqrt(x)*log(f*sqrt(x)
 + e) + 2/9*(3*b*x*log(d)*log(x^n) - ((2*n*log(d) - 3*log(c)*log(d))*b - 3*
a*log(d))*x)*sqrt(x) - integrate(1/9*(3*b*f*k*x*log(x^n) + (3*a*f*k - (2*f*
k*n - 3*f*k*log(c))*b)*x)/(f*sqrt(x) + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x}\,\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\ln\left(c\,x^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^(1/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k),x)
```

```
[Out] Timed out
```

3.135 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{3/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}}+\frac{fk\log(x)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{2fk\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{e}-4b$$

```
[Out] 2*b*f*k*n*ln(x)/e-1/2*b*f*k*n*ln(x)^2/e+f*k*ln(x)*(a+b*ln(c*x^n))/e-4*b*f*k
*n*ln(e+f*x^(1/2))/e-2*f*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e+4*b*f*k*n*ln(-
f*x^(1/2)/e)*ln(e+f*x^(1/2))/e+4*b*f*k*n*polylog(2,1+f*x^(1/2)/e)/e-4*b*n*l
n(d*(e+f*x^(1/2))^k)/x^(1/2)-2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^(1/2
)
```

Rubi [A] time = 0.17, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2454, 2395, 36, 29, 31, 2376, 2394, 2315, 2301}

$$\frac{4bfkn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{e}-\frac{2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}}+\frac{fk\log(x)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{2fk\log\left(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(3/2),x]
```

```
[Out] (-4*b*f*k*n*Log[e + f*Sqrt[x]])/e - (4*b*n*Log[d*(e + f*Sqrt[x])^k])/Sqrt[x
] + (4*b*f*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/e + (2*b*f*k*n*Log
[x])/e - (b*f*k*n*Log[x]^2)/(2*e) - (2*f*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*
x^n]))/e - (2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/Sqrt[x] + (f*k*L
og[x]*(a + b*Log[c*x^n]))/e + (4*b*f*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/e
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^{3/2}}\,dx = -\frac{2fk\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{2\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+\ldots\right.}{\sqrt{x}}$$

$$= -\frac{2fk\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{2\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+\ldots\right.}{\sqrt{x}}$$

$$= -\frac{bfkn\log^2(x)}{2e} - \frac{2fk\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{2\log\left(d\left(e\ldots\right.\right.}{}$$

$$= -\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}} + \frac{4bfkn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e} - \ldots$$

$$= -\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}} + \frac{4bfkn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e} - \ldots$$

$$= -\frac{4bfkn\log\left(e+f\sqrt{x}\right)}{e} - \frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}} + \frac{4bfkn\log\left(e+\ldots\right.}{}$$

Mathematica [A] time = 0.41, size = 145, normalized size = 0.73

$$-\frac{2\left(a+b\log\left(cx^n\right)+2bn\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}} - \frac{2fk\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)-bn\log(x)+2bn\right)}{e} - \frac{fk\log(x\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(3/2),x]
```

```
[Out] (-2*Log[d*(e + f*Sqrt[x])^k]*(a + 2*b*n + b*Log[c*x^n]))/Sqrt[x] - (2*f*k*L
og[e + f*Sqrt[x]]*(a + 2*b*n - b*n*Log[x] + b*Log[c*x^n]))/e - (f*k*Log[x]*
(4*b*n*Log[1 + (f*Sqrt[x])/e] + b*n*Log[x] - 2*(a + 2*b*n + b*Log[c*x^n])))
/(2*e) - (4*b*f*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/e
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sqrt{x}\,\log\left(cx^{n}\right)+a\sqrt{x}\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d)/x^2, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^(3/2), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^{n}\right)+a\right)\ln\left(d\left(f\sqrt{x}+e\right)^{k}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^(3/2),x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,bfkn\log(x)+bfk\log(c)\log(x)+afk\log(x)+\frac{bfk\log(x^{n})^{2}}{2\,n}}{e}-\frac{\frac{18\left(be^{4}x\log(x^{n})+\left(ae^{4}+\left(2\,e^{4}n+e^{4}\log(c)\right)b\right)x\right)k\log\left(f\sqrt{x}+e\right)}{x^{\frac{3}{2}}}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*f*k*x*log(x^n) + (a*f*k + (2*f*k*n + f*k*log(c))*b)*x)/x^2, x)
/e - 1/9*(18*(b*e^4*x*log(x^n) + (a*e^4 + (2*e^4*n + e^4*log(c))*b)*x)*k*lo
g(f*sqrt(x) + e)/x^(3/2) + 2*(3*b*f^4*k*x^2*log(x^n) + (3*a*f^4*k + (4*f^4*
```

```
k*n + 3*f^4*k*log(c))*b)*x^2)/sqrt(x) - 9*(b*e*f^3*k*x^2*log(x^n) + (a*e*f^
3*k + (e*f^3*k*n + e*f^3*k*log(c))*b)*x^2)/x + 18*((b*e^2*f^2*k*log(c) + a*
e^2*f^2*k)*x^2 + (a*e^4*log(d) + (2*e^4*n*log(d) + e^4*log(c)*log(d))*b)*x
+ (b*e^2*f^2*k*x^2 + b*e^4*x*log(d))*log(x^n))/x^(3/2))/e^4 + integrate((b*
f^5*k*x*log(x^n) + (a*f^5*k + (2*f^5*k*n + f^5*k*log(c))*b)*x)/(e^4*f*sqrt(
x) + e^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(3/2),x)
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**(3/2),x)
[Out] Timed out
```

3.136 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{5/2}}\,dx$

Optimal. Leaf size=310

$$-\frac{2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{3x^{3/2}}-\frac{2f^3k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+\frac{f^3k\log(x)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+$$

```
[Out] -5/9*b*f*k*n/e/x+2/9*b*f^3*k*n*ln(x)/e^3-1/6*b*f^3*k*n*ln(x)^2/e^3-1/3*f*k*
(a+b*ln(c*x^n))/e/x+1/3*f^3*k*ln(x)*(a+b*ln(c*x^n))/e^3-4/9*b*f^3*k*n*ln(e+
f*x^(1/2))/e^3-2/3*f^3*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e^3+4/3*b*f^3*k*n*
ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/e^3-4/9*b*n*ln(d*(e+f*x^(1/2))^k)/x^(3/2)-
2/3*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^(3/2)+4/3*b*f^3*k*n*polylog(2,1
+f*x^(1/2)/e)/e^3+16/9*b*f^2*k*n/e^2/x^(1/2)+2/3*f^2*k*(a+b*ln(c*x^n))/e^2/
x^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2454, 2395, 44, 2376, 2394, 2315, 2301}

$$\frac{4bf^3kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{3e^3}-\frac{2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{3x^{3/2}}-\frac{2f^3k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{3e^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(5/2),x]
```

```
[Out] (-5*b*f*k*n)/(9*e*x) + (16*b*f^2*k*n)/(9*e^2*Sqrt[x]) - (4*b*f^3*k*n*Log[e
+ f*Sqrt[x]])/(9*e^3) - (4*b*n*Log[d*(e + f*Sqrt[x])^k])/(9*x^(3/2)) + (4*b
*f^3*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(3*e^3) + (2*b*f^3*k*n*L
og[x])/(9*e^3) - (b*f^3*k*n*Log[x]^2)/(6*e^3) - (f*k*(a + b*Log[c*x^n]))/(3
*e*x) + (2*f^2*k*(a + b*Log[c*x^n]))/(3*e^2*Sqrt[x]) - (2*f^3*k*Log[e + f*S
qrt[x]]*(a + b*Log[c*x^n]))/(3*e^3) - (2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Lo
g[c*x^n]))/(3*x^(3/2)) + (f^3*k*Log[x]*(a + b*Log[c*x^n]))/(3*e^3) + (4*b*f
^3*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(3*e^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
```

```
u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^{5/2}}\,dx = -\frac{fk\left(a+b\log\left(cx^n\right)\right)}{3ex}+\frac{2f^2k\left(a+b\log\left(cx^n\right)\right)}{3e^2\sqrt{x}}-\frac{2f^3k\log\left(e+f\sqrt{x}\right.}{3}$$

$$=-\frac{bfkn}{3ex}+\frac{4bf^2kn}{3e^2\sqrt{x}}-\frac{fk\left(a+b\log\left(cx^n\right)\right)}{3ex}+\frac{2f^2k\left(a+b\log\left(cx^n\right)\right)}{3e^2\sqrt{x}}-$$

$$=-\frac{bfkn}{3ex}+\frac{4bf^2kn}{3e^2\sqrt{x}}-\frac{bf^3kn\log^2(x)}{6e^3}-\frac{fk\left(a+b\log\left(cx^n\right)\right)}{3ex}+\frac{2f^2k\left(\right.}{}$$

$$=-\frac{bfkn}{3ex}+\frac{4bf^2kn}{3e^2\sqrt{x}}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^{3/2}}+\frac{4bf^3kn\log\left(e+f\sqrt{\right.}}{3e^3}$$

$$=-\frac{bfkn}{3ex}+\frac{4bf^2kn}{3e^2\sqrt{x}}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^{3/2}}+\frac{4bf^3kn\log\left(e+f\sqrt{\right.}}{3e^3}$$

$$=-\frac{5bfkn}{9ex}+\frac{16bf^2kn}{9e^2\sqrt{x}}-\frac{4bf^3kn\log\left(e+f\sqrt{x}\right)}{9e^3}-\frac{4bn\log\left(d\left(e+f\sqrt{\right.\right.}}{9x^{3/2}}$$

Mathematica [A] time = 0.40, size = 326, normalized size = 1.05

$$\frac{-2f^3kx^{3/2}\log\left(e+f\sqrt{x}\right)\left(3a+3b\log\left(cx^n\right)-3bn\log(x)+2bn\right)-6ae^3\log\left(d\left(e+f\sqrt{x}\right)^k\right)-3ae^2fk\sqrt{x}+6aef^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(5/2),x]
```

```
[Out] (-3*a*e^2*f*k*Sqrt[x] - 5*b*e^2*f*k*n*Sqrt[x] + 6*a*e*f^2*k*x + 16*b*e*f^2*
k*n*x - 6*a*e^3*Log[d*(e + f*Sqrt[x])^k] - 4*b*e^3*n*Log[d*(e + f*Sqrt[x])^
k] + 3*a*f^3*k*x^(3/2)*Log[x] + 2*b*f^3*k*n*x^(3/2)*Log[x] - 6*b*f^3*k*n*x^
(3/2)*Log[1 + (f*Sqrt[x])/e]*Log[x] - (3*b*f^3*k*n*x^(3/2)*Log[x]^2)/2 - 3*
b*e^2*f*k*Sqrt[x]*Log[c*x^n] + 6*b*e*f^2*k*x*Log[c*x^n] - 6*b*e^3*Log[d*(e
+ f*Sqrt[x])^k]*Log[c*x^n] + 3*b*f^3*k*x^(3/2)*Log[x]*Log[c*x^n] - 2*f^3*k*
x^(3/2)*Log[e + f*Sqrt[x]]*(3*a + 2*b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n]) -
12*b*f^3*k*n*x^(3/2)*PolyLog[2, -((f*Sqrt[x])/e)])/(9*e^3*x^(3/2))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sqrt{x}\,\log\left(cx^{n}\right)+a\sqrt{x}\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d)/x^3, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^(5/2), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^{n}\right)+a\right)\ln\left(d\left(f\sqrt{x}+e\right)^{k}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^(5/2),x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\frac{5\,bfkn}{x}-\frac{3\,bfk\log(c)}{x}-\frac{3\,bfk\log(x^{n})}{x}-\frac{3\,afk}{x}}{9\,e}+\frac{2\,bf^{3}kn\log(x)+3\,bf^{3}k\log(c)\log(x)+3\,af^{3}k\log(x)+\frac{3\,bf^{3}k\log(x^{n})^{2}}{2\,n}}{9\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(5/2),x, algorithm="max
ima")
```

```
[Out] 1/9*integrate((3*b*f*k*x*log(x^n) + (3*a*f*k + (2*f*k*n + 3*f*k*log(c))*b)*
x)/x^3, x)/e + 1/9*integrate((3*b*f^3*k*x*log(x^n) + (3*a*f^3*k + (2*f^3*k*
```

```
n + 3*f^3*k*log(c))*b)*x)/x^2, x)/e^3 - 1/9*(2*(b*f^6*k*x^2*log(x^n) + (b*f
^6*k*log(c) + a*f^6*k)*x^2)/sqrt(x) + 2*(3*b*e^6*x*log(x^n) + (3*a*e^6 + (2
*e^6*n + 3*e^6*log(c))*b)*x)*k*log(f*sqrt(x) + e)/x^(5/2) - (3*b*e*f^5*k*x^
2*log(x^n) + (3*a*e*f^5*k - (e*f^5*k*n - 3*e*f^5*k*log(c))*b)*x^2)/x + 2*(3
*b*e^2*f^4*k*x^2*log(x^n) + (3*a*e^2*f^4*k - (4*e^2*f^4*k*n - 3*e^2*f^4*k*l
og(c))*b)*x^2)/x^(3/2) - 2*((3*a*e^4*f^2*k + (8*e^4*f^2*k*n + 3*e^4*f^2*k*l
og(c))*b)*x^2 - (3*a*e^6*log(d) + (2*e^6*n*log(d) + 3*e^6*log(c)*log(d))*b)
*x + 3*(b*e^4*f^2*k*x^2 - b*e^6*x*log(d))*log(x^n))/x^(5/2))/e^6 + integrat
e(1/9*(3*b*f^7*k*x*log(x^n) + (3*a*f^7*k + (2*f^7*k*n + 3*f^7*k*log(c))*b)*
x)/(e^6*f*sqrt(x) + e^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(5/2),x)
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**(5/2),x)
[Out] Timed out
```

3.137 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{7/2}}\,dx$

Optimal. Leaf size=394

$$-\frac{2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{5x^{5/2}}-\frac{2f^5k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{5e^5}+\frac{f^5k\log(x)\left(a+b\log\left(cx^n\right)\right)}{5e^5}+$$

```
[Out] -9/100*b*f*k*n/e/x^2+32/225*b*f^2*k*n/e^2/x^(3/2)-7/25*b*f^3*k*n/e^3/x+2/25
*b*f^5*k*n*ln(x)/e^5-1/10*b*f^5*k*n*ln(x)^2/e^5-1/10*f*k*(a+b*ln(c*x^n))/e/
x^2+2/15*f^2*k*(a+b*ln(c*x^n))/e^2/x^(3/2)-1/5*f^3*k*(a+b*ln(c*x^n))/e^3/x+
1/5*f^5*k*ln(x)*(a+b*ln(c*x^n))/e^5-4/25*b*f^5*k*n*ln(e+f*x^(1/2))/e^5-2/5*
f^5*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e^5+4/5*b*f^5*k*n*ln(-f*x^(1/2)/e)*ln
(e+f*x^(1/2))/e^5-4/25*b*n*ln(d*(e+f*x^(1/2))^k)/x^(5/2)-2/5*(a+b*ln(c*x^n)
)*ln(d*(e+f*x^(1/2))^k)/x^(5/2)+4/5*b*f^5*k*n*polylog(2,1+f*x^(1/2)/e)/e^5+
24/25*b*f^4*k*n/e^4/x^(1/2)+2/5*f^4*k*(a+b*ln(c*x^n))/e^4/x^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2454, 2395, 44, 2376, 2394, 2315, 2301}

$$\frac{4bf^5kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{5e^5}-\frac{2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{5x^{5/2}}-\frac{2f^5k\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{5e^5}+$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(7/2),x]
```

```
[Out] (-9*b*f*k*n)/(100*e*x^2) + (32*b*f^2*k*n)/(225*e^2*x^(3/2)) - (7*b*f^3*k*n)
/(25*e^3*x) + (24*b*f^4*k*n)/(25*e^4*Sqrt[x]) - (4*b*f^5*k*n*Log[e + f*Sqrt
[x]])/(25*e^5) - (4*b*n*Log[d*(e + f*Sqrt[x])^k])/(25*x^(5/2)) + (4*b*f^5*k
*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(5*e^5) + (2*b*f^5*k*n*Log[x])
/(25*e^5) - (b*f^5*k*n*Log[x]^2)/(10*e^5) - (f*k*(a + b*Log[c*x^n]))/(10*e*
x^2) + (2*f^2*k*(a + b*Log[c*x^n]))/(15*e^2*x^(3/2)) - (f^3*k*(a + b*Log[c*
x^n]))/(5*e^3*x) + (2*f^4*k*(a + b*Log[c*x^n]))/(5*e^4*Sqrt[x]) - (2*f^5*k*
Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(5*e^5) - (2*Log[d*(e + f*Sqrt[x])^k
]*(a + b*Log[c*x^n]))/(5*x^(5/2)) + (f^5*k*Log[x]*(a + b*Log[c*x^n]))/(5*e^
5) + (4*b*f^5*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(5*e^5)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^{7/2}}\,dx = -\frac{fk\left(a+b\log\left(cx^n\right)\right)}{10ex^2}+\frac{2f^2k\left(a+b\log\left(cx^n\right)\right)}{15e^2x^{3/2}}-\frac{f^3k\left(a+b\log\left(cx^n\right)\right)}{5e^3x}$$

$$=-\frac{bfkn}{20ex^2}+\frac{4bf^2kn}{45e^2x^{3/2}}-\frac{bf^3kn}{5e^3x}+\frac{4bf^4kn}{5e^4\sqrt{x}}-\frac{fk\left(a+b\log\left(cx^n\right)\right)}{10ex^2}+\frac{2f}{}$$

$$=-\frac{bfkn}{20ex^2}+\frac{4bf^2kn}{45e^2x^{3/2}}-\frac{bf^3kn}{5e^3x}+\frac{4bf^4kn}{5e^4\sqrt{x}}-\frac{bf^5kn\log^2(x)}{10e^5}-\frac{fk\left(a+\right.}{1}$$

$$=-\frac{bfkn}{20ex^2}+\frac{4bf^2kn}{45e^2x^{3/2}}-\frac{bf^3kn}{5e^3x}+\frac{4bf^4kn}{5e^4\sqrt{x}}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{25x^{5/2}}+$$

$$=-\frac{bfkn}{20ex^2}+\frac{4bf^2kn}{45e^2x^{3/2}}-\frac{bf^3kn}{5e^3x}+\frac{4bf^4kn}{5e^4\sqrt{x}}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{25x^{5/2}}+$$

$$=-\frac{9bfkn}{100ex^2}+\frac{32bf^2kn}{225e^2x^{3/2}}-\frac{7bf^3kn}{25e^3x}+\frac{24bf^4kn}{25e^4\sqrt{x}}-\frac{4bf^5kn\log\left(e+f\sqrt{x}\right.}{25e^5}$$

Mathematica [A] time = 0.45, size = 422, normalized size = 1.07

$$\frac{-72f^5kx^{5/2}\log\left(e+f\sqrt{x}\right)\left(5a+5b\log\left(cx^n\right)-5bn\log(x)+2bn\right)-360ae^5\log\left(d\left(e+f\sqrt{x}\right)^k\right)-90ae^4fk\sqrt{x}+1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(7/2),x]
```

```
[Out] (-90*a*e^4*f*k*Sqrt[x] - 81*b*e^4*f*k*n*Sqrt[x] + 120*a*e^3*f^2*k*x + 128*b
*e^3*f^2*k*n*x - 180*a*e^2*f^3*k*x^(3/2) - 252*b*e^2*f^3*k*n*x^(3/2) + 360*
a*e*f^4*k*x^2 + 864*b*e*f^4*k*n*x^2 - 360*a*e^5*Log[d*(e + f*Sqrt[x])^k] -
144*b*e^5*n*Log[d*(e + f*Sqrt[x])^k] + 180*a*f^5*k*x^(5/2)*Log[x] + 72*b*f^
5*k*n*x^(5/2)*Log[x] - 360*b*f^5*k*n*x^(5/2)*Log[1 + (f*Sqrt[x])/e]*Log[x]
- 90*b*f^5*k*n*x^(5/2)*Log[x]^2 - 90*b*e^4*f*k*Sqrt[x]*Log[c*x^n] + 120*b*e
^3*f^2*k*x*Log[c*x^n] - 180*b*e^2*f^3*k*x^(3/2)*Log[c*x^n] + 360*b*e*f^4*k*
x^2*Log[c*x^n] - 360*b*e^5*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 180*b*f^5*
k*x^(5/2)*Log[x]*Log[c*x^n] - 72*f^5*k*x^(5/2)*Log[e + f*Sqrt[x]]*(5*a + 2*
b*n - 5*b*n*Log[x] + 5*b*Log[c*x^n]) - 720*b*f^5*k*n*x^(5/2)*PolyLog[2, -((
f*Sqrt[x])/e)])/(900*e^5*x^(5/2))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\sqrt{x}\,\log\left(cx^{n}\right)+a\sqrt{x}\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(7/2),x, algorithm="fri
cas")
```

```
[Out] integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d)/x^4, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)\log\left(\left(f\sqrt{x}+e\right)^{k}d\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(7/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^(7/2), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^{n}\right)+a\right)\ln\left(d\left(f\sqrt{x}+e\right)^{k}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^(7/2),x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^(1/2)+e)^k)/x^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\frac{9\,bfkn}{4\,x^{2}}-\frac{5\,bfk\log(c)}{2\,x^{2}}-\frac{5\,bfk\log(x^{n})}{2\,x^{2}}-\frac{5\,afk}{2\,x^{2}}}{25\,e}+\frac{-\frac{7\,bf^{3}kn}{x}-\frac{5\,bf^{3}k\log(c)}{x}-\frac{5\,bf^{3}k\log(x^{n})}{x}-\frac{5\,af^{3}k}{x}}{25\,e^{3}}+\frac{2\,bf^{5}kn\log(x)+5\,bf^{5}k}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(7/2),x, algorithm="max
ima")
```

```
[Out] 1/25*integrate((5*b*f*k*x*log(x^n) + (5*a*f*k + (2*f*k*n + 5*f*k*log(c))*b)
*x)/x^4, x)/e + 1/25*integrate((5*b*f^3*k*x*log(x^n) + (5*a*f^3*k + (2*f^3*
k*n + 5*f^3*k*log(c))*b)*x)/x^3, x)/e^3 + 1/25*integrate((5*b*f^5*k*x*log(x
^n) + (5*a*f^5*k + (2*f^5*k*n + 5*f^5*k*log(c))*b)*x)/x^2, x)/e^5 - 1/225*(
2*(15*b*f^8*k*x^2*log(x^n) + (15*a*f^8*k - (4*f^8*k*n - 15*f^8*k*log(c))*b)
*x^2)/sqrt(x) - 9*(5*b*e*f^7*k*x^2*log(x^n) + (5*a*e*f^7*k - (3*e*f^7*k*n -
 5*e*f^7*k*log(c))*b)*x^2)/x + 18*(5*b*e^2*f^6*k*x^2*log(x^n) + (5*a*e^2*f^
6*k - (8*e^2*f^6*k*n - 5*e^2*f^6*k*log(c))*b)*x^2)/x^(3/2) + 18*(5*b*e^8*x*
log(x^n) + (5*a*e^8 + (2*e^8*n + 5*e^8*log(c))*b)*x)*k*log(f*sqrt(x) + e)/x
^(7/2) - 18*(5*b*e^4*f^4*k*x^2*log(x^n) + (5*a*e^4*f^4*k + (12*e^4*f^4*k*n
+ 5*e^4*f^4*k*log(c))*b)*x^2)/x^(5/2) - 2*((15*a*e^6*f^2*k + (16*e^6*f^2*k*
n + 15*e^6*f^2*k*log(c))*b)*x^2 - 9*(5*a*e^8*log(d) + (2*e^8*n*log(d) + 5*e
^8*log(c)*log(d))*b)*x + 15*(b*e^6*f^2*k*x^2 - 3*b*e^8*x*log(d))*log(x^n))/
x^(7/2))/e^8 + integrate(1/25*(5*b*f^9*k*x*log(x^n) + (5*a*f^9*k + (2*f^9*k
*n + 5*f^9*k*log(c))*b)*x)/(e^8*f*sqrt(x) + e^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\ln\left(c\,x^n\right)\right)}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(7/2),x)
```

```
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**(7/2),x)
```

```
[Out] Timed out
```

3.138 $\int (gx)^q \left(a+b\log\left(cx^n\right)\right) \log\left(d\left(e+fx^m\right)^k\right) \, dx$

Optimal. Leaf size=31

$$\mathrm{Int}\left((gx)^q \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right),x\right)$$

[Out] Unintegrable((g*x)^q*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (gx)^q \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$$

Verification is Not applicable to the result.

[In] Int[(g*x)^q*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]

[Out] Defer[Int][(g*x)^q*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k], x]

Rubi steps

$$\int (gx)^q \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx = \int (gx)^q \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$$

Mathematica [A] time = 0.35, size = 304, normalized size = 9.81

$$x(gx)^q\left(-bkmn\,{}_3F_2\left(1,\frac{q}{m}+\frac{1}{m},\frac{q}{m}+\frac{1}{m};\frac{q}{m}+\frac{1}{m}+1,\frac{q}{m}+\frac{1}{m}+1;-\frac{fx^m}{e}\right)+km\,{}_2F_1\left(1,\frac{q+1}{m};\frac{m+q+1}{m};-\frac{fx^m}{e}\right)\left(aq+a+b\right.\right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[(g*x)^q*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]

[Out] (x*(g*x)^q*(-(a*k*m) + 2*b*k*m*n - a*k*m*q - b*k*m*n*HypergeometricPFQ[{1, m^(-1) + q/m, m^(-1) + q/m}, {1 + m^(-1) + q/m, 1 + m^(-1) + q/m}, -((f*x^m)/e)] - b*k*m*Log[c*x^n] - b*k*m*q*Log[c*x^n] + k*m*Hypergeometric2F1[1, (1 + q)/m, (1 + m + q)/m, -((f*x^m)/e)]*(a - b*n + a*q + b*(1 + q)*Log[c*x^n]) + a*Log[d*(e + f*x^m)^k] - b*n*Log[d*(e + f*x^m)^k] + 2*a*q*Log[d*(e + f*x^m)^k] - b*n*q*Log[d*(e + f*x^m)^k] + a*q^2*Log[d*(e + f*x^m)^k] + b*Log[c*x^n]*Log[d*(e + f*x^m)^k] + 2*b*q*Log[c*x^n]*Log[d*(e + f*x^m)^k] + b*q^2*Log[c*x^n]*Log[d*(e + f*x^m)^k]))/(1 + q)^3

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\left(gx\right)^q b\log\left(cx^n\right)+\left(gx\right)^q a\right)\log\left(\left(fx^m+e\right)^k d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x)^q*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas")

[Out] integral(((g*x)^q*b*log(c*x^n) + (g*x)^q*a)*log((f*x^m + e)^k*d), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)\left(gx\right)^q\log\left(\left(fx^m+e\right)^k d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^q*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^q*log((f*x^m + e)^k*d), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int (b \ln (c x^n) + a) \left(gx\right)^q \ln \left(d \left(f x^m + e\right)^k\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^q*(b*ln(c*x^n)+a)*ln(d*(e+f*x^m)^k),x)
```

```
[Out] int((g*x)^q*(b*ln(c*x^n)+a)*ln(d*(e+f*x^m)^k),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(bg^q\left(q+1\right)xx^q \log\left(x^n\right) + \left(ag^q\left(q+1\right) + \left(g^q\left(q+1\right)\log(c) - g^q n\right)b\right)xx^q\right)\log\left(\left(fx^m+e\right)^k\right)}{q^2+2q+1} + \int \frac{\left(\left(q^2+2q+1\right)beg^q\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^q*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima"
)
```

```
[Out] (b*g^q*(q + 1)*x*x^q*log(x^n) + (a*g^q*(q + 1) + (g^q*(q + 1)*log(c) - g^q*
n)*b)*x*x^q)*log((f*x^m + e)^k)/(q^2 + 2*q + 1) + integrate((((q^2 + 2*q +
1)*b*e*g^q*log(d) - (f*g^q*k*m*(q + 1) - (q^2 + 2*q + 1)*f*g^q*log(d))*b*x^
m)*x^q*log(x^n) + ((q^2 + 2*q + 1)*b*e*g^q*log(c)*log(d) + (q^2 + 2*q + 1)*
a*e*g^q*log(d) - ((f*g^q*k*m*(q + 1) - (q^2 + 2*q + 1)*f*g^q*log(d))*a - (f
*g^q*k*m*n - (f*g^q*k*m*(q + 1) - (q^2 + 2*q + 1)*f*g^q*log(d))*log(c))*b)*
x^m)*x^q)/((q^2 + 2*q + 1)*f*x^m + (q^2 + 2*q + 1)*e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(d\left(e+f\,x^m\right)^k\right)\,{\left(g\,x\right)}^q\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^k)*(g*x)^q*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^m)^k)*(g*x)^q*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**q*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Timed out
```

3.139 $\int \frac{\left(a+b\log(cx^n)\right)^3\log\left(d\left(e+fx^m\right)^r\right)}{x}\,dx$

Optimal. Leaf size=185

$$-\frac{6b^2n^2r\text{Li}_4\left(-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{m^3}+\frac{\left(a+b\log\left(cx^n\right)\right)^4\log\left(d\left(e+fx^m\right)^r\right)}{4bn}+\frac{3bnr\text{Li}_3\left(-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{m^2}$$

```
[Out] 1/4*(a+b*ln(c*x^n))^4*ln(d*(e+f*x^m)^r)/b/n-1/4*r*(a+b*ln(c*x^n))^4*ln(1+f*
x^m/e)/b/n-r*(a+b*ln(c*x^n))^3*polylog(2,-f*x^m/e)/m+3*b*n*r*(a+b*ln(c*x^n)
)^2*polylog(3,-f*x^m/e)/m^2-6*b^2*n^2*r*(a+b*ln(c*x^n))*polylog(4,-f*x^m/e)
/m^3+6*b^3*n^3*r*polylog(5,-f*x^m/e)/m^4
```

Rubi [A] time = 0.30, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2375, 2337, 2374, 2383, 6589}

$$-\frac{6b^2n^2r\text{PolyLog}\left(4,-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{m^3}+\frac{3bnr\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2}{m^2}-\frac{r\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^4*Log[d*(e + f*x^m)^r])/(4*b*n) - (r*(a + b*Log[c*x^n])
^4*Log[1 + (f*x^m)/e])/(4*b*n) - (r*(a + b*Log[c*x^n])^3*PolyLog[2, -((f*x^
m)/e)])/m + (3*b*n*r*(a + b*Log[c*x^n])^2*PolyLog[3, -((f*x^m)/e)])/m^2 - (
6*b^2*n^2*r*(a + b*Log[c*x^n])*PolyLog[4, -((f*x^m)/e)])/m^3 + (6*b^3*n^3*r
*PolyLog[5, -((f*x^m)/e)])/m^4
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
```

```
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^m\right)^r\right)}{x}dx = \frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^m\right)^r\right)}{4bn} - \frac{(fmr)\int\frac{x^{-1+m}(a+b\log(cx^n))^4}{e+fx^m}d}{4bn}$$
$$= \frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^m\right)^r\right)}{4bn} - \frac{r(a+b\log(cx^n))^4\log\left(1+\right.}{4bn}$$
$$= \frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^m\right)^r\right)}{4bn} - \frac{r(a+b\log(cx^n))^4\log\left(1+\right.}{4bn}$$
$$= \frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^m\right)^r\right)}{4bn} - \frac{r(a+b\log(cx^n))^4\log\left(1+\right.}{4bn}$$
$$= \frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^m\right)^r\right)}{4bn} - \frac{r(a+b\log(cx^n))^4\log\left(1+\right.}{4bn}$$
$$= \frac{(a+b\log(cx^n))^4\log\left(d\left(e+fx^m\right)^r\right)}{4bn} - \frac{r(a+b\log(cx^n))^4\log\left(1+\right.}{4bn}$$

Mathematica [B] time = 0.65, size = 1395, normalized size = 7.54

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] -1/2*(a^2*b*m*n*r*Log[x]^3) + (3*a*b^2*m*n^2*r*Log[x]^4)/4 - (3*b^3*m*n^3*r
*Log[x]^5)/10 - a*b^2*m*n*r*Log[x]^3*Log[c*x^n] + (3*b^3*m*n^2*r*Log[x]^4*L
og[c*x^n])/4 - (b^3*m*n*r*Log[x]^3*Log[c*x^n]^2)/2 - (3*a^2*b*n*r*Log[x]^2*
Log[1 + e/(f*x^m)])/2 + 2*a*b^2*n^2*r*Log[x]^3*Log[1 + e/(f*x^m)] - (3*b^3*
n^3*r*Log[x]^4*Log[1 + e/(f*x^m)])/4 - 3*a*b^2*n*r*Log[x]^2*Log[c*x^n]*Log[
1 + e/(f*x^m)] + 2*b^3*n^2*r*Log[x]^3*Log[c*x^n]*Log[1 + e/(f*x^m)] - (3*b^
3*n*r*Log[x]^2*Log[c*x^n]^2*Log[1 + e/(f*x^m)])/2 - a^3*r*Log[x]*Log[e + f*
x^m] + 3*a^2*b*n*r*Log[x]^2*Log[e + f*x^m] - 3*a*b^2*n^2*r*Log[x]^3*Log[e +
 f*x^m] + b^3*n^3*r*Log[x]^4*Log[e + f*x^m] + (a^3*r*Log[-((f*x^m)/e)]*Log[
e + f*x^m])/m - (3*a^2*b*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m + (
3*a*b^2*n^2*r*Log[x]^2*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m - (b^3*n^3*r*Log
[x]^3*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m - 3*a^2*b*r*Log[x]*Log[c*x^n]*Log
[e + f*x^m] + 6*a*b^2*n*r*Log[x]^2*Log[c*x^n]*Log[e + f*x^m] - 3*b^3*n^2*r*
Log[x]^3*Log[c*x^n]*Log[e + f*x^m] + (3*a^2*b*r*Log[-((f*x^m)/e)]*Log[c*x^n
]*Log[e + f*x^m])/m - (6*a*b^2*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[c*x^n]*Log[
e + f*x^m])/m + (3*b^3*n^2*r*Log[x]^2*Log[-((f*x^m)/e)]*Log[c*x^n]*Log[e +
f*x^m])/m - 3*a*b^2*r*Log[x]*Log[c*x^n]^2*Log[e + f*x^m] + 3*b^3*n*r*Log[x]
^2*Log[c*x^n]^2*Log[e + f*x^m] + (3*a*b^2*r*Log[-((f*x^m)/e)]*Log[c*x^n]^2*
Log[e + f*x^m])/m - (3*b^3*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[c*x^n]^2*Log[e
+ f*x^m])/m - b^3*r*Log[x]*Log[c*x^n]^3*Log[e + f*x^m] + (b^3*r*Log[-((f*x^
```

```
m)/e)]*Log[c*x^n]^3*Log[e + f*x^m])/m + a^3*Log[x]*Log[d*(e + f*x^m)^r] - (
3*a^2*b*n*Log[x]^2*Log[d*(e + f*x^m)^r])/2 + a*b^2*n^2*Log[x]^3*Log[d*(e +
f*x^m)^r] - (b^3*n^3*Log[x]^4*Log[d*(e + f*x^m)^r])/4 + 3*a^2*b*Log[x]*Log[
c*x^n]*Log[d*(e + f*x^m)^r] - 3*a*b^2*n*Log[x]^2*Log[c*x^n]*Log[d*(e + f*x^
m)^r] + b^3*n^2*Log[x]^3*Log[c*x^n]*Log[d*(e + f*x^m)^r] + 3*a*b^2*Log[x]*L
og[c*x^n]^2*Log[d*(e + f*x^m)^r] - (3*b^3*n*Log[x]^2*Log[c*x^n]^2*Log[d*(e
+ f*x^m)^r])/2 + b^3*Log[x]*Log[c*x^n]^3*Log[d*(e + f*x^m)^r] + (b*n*r*Log[
x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a + b*Log[c*x^n]) + 3*(a + b*Log[c*x^n
])^2)*PolyLog[2, -(e/(f*x^m))])/m + (r*(a - b*n*Log[x] + b*Log[c*x^n])^3*Po
lyLog[2, 1 + (f*x^m)/e])/m + (3*a^2*b*n*r*PolyLog[3, -(e/(f*x^m))])/m^2 + (
6*a*b^2*n*r*Log[c*x^n]*PolyLog[3, -(e/(f*x^m))])/m^2 + (3*b^3*n*r*Log[c*x^n
]^2*PolyLog[3, -(e/(f*x^m))])/m^2 + (6*a*b^2*n^2*r*PolyLog[4, -(e/(f*x^m))]
)/m^3 + (6*b^3*n^2*r*Log[c*x^n]*PolyLog[4, -(e/(f*x^m))])/m^3 + (6*b^3*n^3*
r*PolyLog[5, -(e/(f*x^m))])/m^4
```

fricas [C] time = 0.65, size = 765, normalized size = 4.14

$$b^3m^4n^3\log(d)\log(x)^4 + 24\,b^3n^3r\text{polylog}\left(5, -\frac{fx^m}{e}\right) + 4\left(b^3m^4n^2\log(c) + ab^2m^4n^2\right)\log(d)\log(x)^3 + 6\left(b^3m^4n\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^m)^r)/x,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*m^4*n^3*log(d)*log(x)^4 + 24*b^3*n^3*r*polylog(5, -f*x^m/e) + 4*(b
^3*m^4*n^2*log(c) + a*b^2*m^4*n^2)*log(d)*log(x)^3 + 6*(b^3*m^4*n*log(c)^2
+ 2*a*b^2*m^4*n*log(c) + a^2*b*m^4*n)*log(d)*log(x)^2 + 4*(b^3*m^4*log(c)^3
 + 3*a*b^2*m^4*log(c)^2 + 3*a^2*b*m^4*log(c) + a^3*m^4)*log(d)*log(x) - 4*(
b^3*m^3*n^3*r*log(x)^3 + b^3*m^3*r*log(c)^3 + 3*a*b^2*m^3*r*log(c)^2 + 3*a^
2*b*m^3*r*log(c) + a^3*m^3*r + 3*(b^3*m^3*n^2*r*log(c) + a*b^2*m^3*n^2*r)*l
og(x)^2 + 3*(b^3*m^3*n*r*log(c)^2 + 2*a*b^2*m^3*n*r*log(c) + a^2*b*m^3*n*r)
*log(x))*dilog(-(f*x^m + e)/e + 1) + (b^3*m^4*n^3*r*log(x)^4 + 4*(b^3*m^4*n
^2*r*log(c) + a*b^2*m^4*n^2*r)*log(x)^3 + 6*(b^3*m^4*n*r*log(c)^2 + 2*a*b^2
*m^4*n*r*log(c) + a^2*b*m^4*n*r)*log(x)^2 + 4*(b^3*m^4*r*log(c)^3 + 3*a*b^2
*m^4*r*log(c)^2 + 3*a^2*b*m^4*r*log(c) + a^3*m^4*r)*log(x))*log(f*x^m + e)
- (b^3*m^4*n^3*r*log(x)^4 + 4*(b^3*m^4*n^2*r*log(c) + a*b^2*m^4*n^2*r)*log(
x)^3 + 6*(b^3*m^4*n*r*log(c)^2 + 2*a*b^2*m^4*n*r*log(c) + a^2*b*m^4*n*r)*lo
g(x)^2 + 4*(b^3*m^4*r*log(c)^3 + 3*a*b^2*m^4*r*log(c)^2 + 3*a^2*b*m^4*r*log
(c) + a^3*m^4*r)*log(x))*log((f*x^m + e)/e) - 24*(b^3*m*n^3*r*log(x) + b^3*
m*n^2*r*log(c) + a*b^2*m*n^2*r)*polylog(4, -f*x^m/e) + 12*(b^3*m^2*n^3*r*lo
g(x)^2 + b^3*m^2*n*r*log(c)^2 + 2*a*b^2*m^2*n*r*log(c) + a^2*b*m^2*n*r + 2*
(b^3*m^2*n^2*r*log(c) + a*b^2*m^2*n^2*r)*log(x))*polylog(3, -f*x^m/e))/m^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right) + a\right)^3 \log\left(\left(fx^m + e\right)^r d\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^m)^r)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^m + e)^r*d)/x, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right) + a\right)^3 \ln\left(d\left(f\,x^m + e\right)^r\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^m+e)^r)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*ln(d*(f*x^m+e)^r)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^m)^r)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b^
2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^2
+ 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3*n^
2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*a*b^
2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3
*a^2*b*log(c) + a^3)*log(x))*log((f*x^m + e)^r) - integrate(-1/4*(4*b^3*e*l
og(c)^3*log(d) + 12*a*b^2*e*log(c)^2*log(d) + 12*a^2*b*e*log(c)*log(d) + 4*
a^3*e*log(d) + 4*(b^3*e*log(d) - (b^3*f*m*r*log(x) - b^3*f*log(d))*x^m)*log
(x^n)^3 + 6*(2*b^3*e*log(c)*log(d) + 2*a*b^2*e*log(d) + (b^3*f*m*n*r*log(x)
^2 + 2*b^3*f*log(c)*log(d) + 2*a*b^2*f*log(d) - 2*(b^3*f*m*r*log(c) + a*b^2
*f*m*r)*log(x))*x^m)*log(x^n)^2 + (b^3*f*m*n^3*r*log(x)^4 + 4*b^3*f*log(c)^
3*log(d) + 12*a*b^2*f*log(c)^2*log(d) + 12*a^2*b*f*log(c)*log(d) + 4*a^3*f*
log(d) - 4*(b^3*f*m*n^2*r*log(c) + a*b^2*f*m*n^2*r)*log(x)^3 + 6*(b^3*f*m*n
*r*log(c)^2 + 2*a*b^2*f*m*n*r*log(c) + a^2*b*f*m*n*r)*log(x)^2 - 4*(b^3*f*m
*r*log(c)^3 + 3*a*b^2*f*m*r*log(c)^2 + 3*a^2*b*f*m*r*log(c) + a^3*f*m*r)*lo
g(x))*x^m + 4*(3*b^3*e*log(c)^2*log(d) + 6*a*b^2*e*log(c)*log(d) + 3*a^2*b*
e*log(d) - (b^3*f*m*n^2*r*log(x)^3 - 3*b^3*f*log(c)^2*log(d) - 6*a*b^2*f*lo
g(c)*log(d) - 3*a^2*b*f*log(d) - 3*(b^3*f*m*n*r*log(c) + a*b^2*f*m*n*r)*log
(x)^2 + 3*(b^3*f*m*r*log(c)^2 + 2*a*b^2*f*m*r*log(c) + a^2*b*f*m*r)*log(x))
*x^m)*log(x^n))/(f*x*x^m + e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\,x^m\right)^r\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^3)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**m)**r)/x,x)
```

```
[Out] Timed out
```

3.140 $\int \frac{\left(a+b\log(cx^n)\right)^2\log\left(d\left(e+fx^m\right)^r\right)}{x}\,dx$

Optimal. Leaf size=150

$$\frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^m\right)^r\right)}{3bn}+\frac{2bnr\text{Li}_3\left(-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{m^2}-\frac{r\text{Li}_2\left(-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2}{m}-\frac{r\log}{}$$

```
[Out] 1/3*(a+b*ln(c*x^n))^3*ln(d*(e+f*x^m)^r)/b/n-1/3*r*(a+b*ln(c*x^n))^3*ln(1+f*
x^m/e)/b/n-r*(a+b*ln(c*x^n))^2*polylog(2,-f*x^m/e)/m+2*b*n*r*(a+b*ln(c*x^n)
)*polylog(3,-f*x^m/e)/m^2-2*b^2*n^2*r*polylog(4,-f*x^m/e)/m^3
```

Rubi [A] time = 0.25, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2375, 2337, 2374, 2383, 6589}

$$\frac{2bnr\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{m^2}-\frac{r\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2}{m}-\frac{2b^2n^2r\text{PolyLog}\left(4,-\frac{fx^m}{e}\right)}{m^3}-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^3*Log[d*(e + f*x^m)^r])/(3*b*n) - (r*(a + b*Log[c*x^n])
^3*Log[1 + (f*x^m)/e])/(3*b*n) - (r*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x^
m)/e)])/m + (2*b*n*r*(a + b*Log[c*x^n])*PolyLog[3, -((f*x^m)/e)])/m^2 - (2*
b^2*n^2*r*PolyLog[4, -((f*x^m)/e)])/m^3
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^m\right)^r\right)}{x}\,dx=\frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^m\right)^r\right)}{3bn}-\frac{(fmr)\int\frac{x^{-1+m}\left(a+b\log\left(cx^n\right)\right)^3}{e+fx^m}\,d}{3bn}$$

$$=\frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^m\right)^r\right)}{3bn}-\frac{r\left(a+b\log\left(cx^n\right)\right)^3\log\left(1+\right.}{3bn}$$

$$=\frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^m\right)^r\right)}{3bn}-\frac{r\left(a+b\log\left(cx^n\right)\right)^3\log\left(1+\right.}{3bn}$$

$$=\frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^m\right)^r\right)}{3bn}-\frac{r\left(a+b\log\left(cx^n\right)\right)^3\log\left(1+\right.}{3bn}$$

$$=\frac{\left(a+b\log\left(cx^n\right)\right)^3\log\left(d\left(e+fx^m\right)^r\right)}{3bn}-\frac{r\left(a+b\log\left(cx^n\right)\right)^3\log\left(1+\right.}{3bn}$$

Mathematica [B] time = 0.38, size = 741, normalized size = 4.94

$$a^2\log(x)\log\left(d\left(e+fx^m\right)^r\right)-a^2r\log(x)\log\left(e+fx^m\right)+\frac{a^2r\log\left(-\frac{fx^m}{e}\right)\log\left(e+fx^m\right)}{m}+2ab\log(x)\log\left(cx^n\right)\log\left(d\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] -1/3*(a*b*m*n*r*Log[x]^3) + (b^2*m*n^2*r*Log[x]^4)/4 - (b^2*m*n*r*Log[x]^3*
Log[c*x^n])/3 - a*b*n*r*Log[x]^2*Log[1 + e/(f*x^m)] + (2*b^2*n^2*r*Log[x]^3
*Log[1 + e/(f*x^m)])/3 - b^2*n*r*Log[x]^2*Log[c*x^n]*Log[1 + e/(f*x^m)] - a
^2*r*Log[x]*Log[e + f*x^m] + 2*a*b*n*r*Log[x]^2*Log[e + f*x^m] - b^2*n^2*r*
Log[x]^3*Log[e + f*x^m] + (a^2*r*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m - (2*a
*b*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m + (b^2*n^2*r*Log[x]^2*Log
[-((f*x^m)/e)]*Log[e + f*x^m])/m - 2*a*b*r*Log[x]*Log[c*x^n]*Log[e + f*x^m]
 + 2*b^2*n*r*Log[x]^2*Log[c*x^n]*Log[e + f*x^m] + (2*a*b*r*Log[-((f*x^m)/e)
]*Log[c*x^n]*Log[e + f*x^m])/m - (2*b^2*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[c*
x^n]*Log[e + f*x^m])/m - b^2*r*Log[x]*Log[c*x^n]^2*Log[e + f*x^m] + (b^2*r*
Log[-((f*x^m)/e)]*Log[c*x^n]^2*Log[e + f*x^m])/m + a^2*Log[x]*Log[d*(e + f*
x^m)^r] - a*b*n*Log[x]^2*Log[d*(e + f*x^m)^r] + (b^2*n^2*Log[x]^3*Log[d*(e
+ f*x^m)^r])/3 + 2*a*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^m)^r] - b^2*n*Log[x
]^2*Log[c*x^n]*Log[d*(e + f*x^m)^r] + b^2*Log[x]*Log[c*x^n]^2*Log[d*(e + f*
x^m)^r] + (b*n*r*Log[x]*(-(b*n*Log[x]) + 2*(a + b*Log[c*x^n]))*PolyLog[2, -
(e/(f*x^m))])/m + (r*(a - b*n*Log[x] + b*Log[c*x^n])^2*PolyLog[2, 1 + (f*x^
m)/e])/m + (2*a*b*n*r*PolyLog[3, -(e/(f*x^m))])/m^2 + (2*b^2*n*r*Log[c*x^n]
*PolyLog[3, -(e/(f*x^m))])/m^2 + (2*b^2*n^2*r*PolyLog[4, -(e/(f*x^m))])/m^3
```

fricas [C] time = 0.77, size = 406, normalized size = 2.71

$$b^2m^3n^2\log(d)\log(x)^3-6b^2n^2r\,\mathrm{polylog}\left(4,-\frac{fx^m}{e}\right)+3\left(b^2m^3n\log(c)+abm^3n\right)\log(d)\log(x)^2+3\left(b^2m^3\log(c)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^m)^r)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*m^3*n^2*log(d)*log(x)^3 - 6*b^2*n^2*r*polylog(4, -f*x^m/e) + 3*(b^
2*m^3*n*log(c) + a*b*m^3*n)*log(d)*log(x)^2 + 3*(b^2*m^3*log(c)^2 + 2*a*b*m
^3*log(c) + a^2*m^3)*log(d)*log(x) - 3*(b^2*m^2*n^2*r*log(x)^2 + b^2*m^2*r*
log(c)^2 + 2*a*b*m^2*r*log(c) + a^2*m^2*r + 2*(b^2*m^2*n*r*log(c) + a*b*m^2
*n*r)*log(x))*dilog(-(f*x^m + e)/e + 1) + (b^2*m^3*n^2*r*log(x)^3 + 3*(b^2*
m^3*n*r*log(c) + a*b*m^3*n*r)*log(x)^2 + 3*(b^2*m^3*r*log(c)^2 + 2*a*b*m^3*
r*log(c) + a^2*m^3*r)*log(x))*log(f*x^m + e) - (b^2*m^3*n^2*r*log(x)^3 + 3*
(b^2*m^3*n*r*log(c) + a*b*m^3*n*r)*log(x)^2 + 3*(b^2*m^3*r*log(c)^2 + 2*a*b
*m^3*r*log(c) + a^2*m^3*r)*log(x))*log((f*x^m + e)/e) + 6*(b^2*m*n^2*r*log(
x) + b^2*m*n*r*log(c) + a*b*m*n*r)*polylog(3, -f*x^m/e))/m^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right)^{2} \log\left(\left(fx^{m} + e\right)^{r} d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^m)^r)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^m + e)^r*d)/x, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^{n}\right) + a\right)^{2} \ln\left(d\left(f\,x^{m} + e\right)^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^m+e)^r)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*ln(d*(f*x^m+e)^r)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(b^{2}n^{2}\log(x)^{3} + 3\,b^{2}\log(x)\log\left(x^{n}\right)^{2} - 3\left(b^{2}n\log(c) + abn\right)\log(x)^{2} - 3\left(b^{2}n\log(x)^{2} - 2\left(b^{2}\log(c) + ab\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^m)^r)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)*
log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*(b
^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log((f*x^m + e)^r) - integrate(-1
/3*(3*b^2*e*log(c)^2*log(d) + 6*a*b*e*log(c)*log(d) + 3*a^2*e*log(d) + 3*(b
^2*e*log(d) - (b^2*f*m*r*log(x) - b^2*f*log(d))*x^m)*log(x^n)^2 - (b^2*f*m*
n^2*r*log(x)^3 - 3*b^2*f*log(c)^2*log(d) - 6*a*b*f*log(c)*log(d) - 3*a^2*f*
log(d) - 3*(b^2*f*m*n*r*log(c) + a*b*f*m*n*r)*log(x)^2 + 3*(b^2*f*m*r*log(c
)^2 + 2*a*b*f*m*r*log(c) + a^2*f*m*r)*log(x))*x^m + 3*(2*b^2*e*log(c)*log(d
) + 2*a*b*e*log(d) + (b^2*f*m*n*r*log(x)^2 + 2*b^2*f*log(c)*log(d) + 2*a*b*
f*log(d) - 2*(b^2*f*m*r*log(c) + a*b*f*m*r)*log(x))*x^m)*log(x^n))/(f*x*x^m
 + e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e + f\,x^{m}\right)^{r}\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**m)**r)/x,x)
```

```
[Out] Timed out
```

3.141 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^r\right)}{x}\,dx$

Optimal. Leaf size=114

$$\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^m\right)^r\right)}{2bn}-\frac{r\text{Li}_2\left(-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{m}-\frac{r\log\left(\frac{fx^m}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{2bn}+\frac{bnr\text{Li}$$

```
[Out] 1/2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^m)^r)/b/n-1/2*r*(a+b*ln(c*x^n))^2*ln(1+f*
x^m/e)/b/n-r*(a+b*ln(c*x^n))*polylog(2,-f*x^m/e)/m+b*n*r*polylog(3,-f*x^m/e
)/m^2
```

Rubi [A] time = 0.19, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2375, 2337, 2374, 6589}

$$-\frac{r\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{m}+\frac{bnr\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{m^2}+\frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^m\right)^r\right)}{2bn}-\frac{r\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^r])/(2*b*n) - (r*(a + b*Log[c*x^n])
^2*Log[1 + (f*x^m)/e])/(2*b*n) - (r*(a + b*Log[c*x^n])*PolyLog[2, -((f*x^m)
/e)])/m + (b*n*r*PolyLog[3, -((f*x^m)/e)])/m^2
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^r\right)}{x}dx = \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^m\right)^r\right)}{2bn} - \frac{(fmr)\int \frac{x^{-1+m}(a+b\log(cx^n))^2}{e+fx^m}dx}{2bn}$$
$$= \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^m\right)^r\right)}{2bn} - \frac{r(a+b\log(cx^n))^2\log\left(1+\right.}{2bn}$$
$$= \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^m\right)^r\right)}{2bn} - \frac{r(a+b\log(cx^n))^2\log\left(1+\right.}{2bn}$$
$$= \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^m\right)^r\right)}{2bn} - \frac{r(a+b\log(cx^n))^2\log\left(1+\right.}{2bn}$$

Mathematica [B] time = 0.17, size = 277, normalized size = 2.43

$$\frac{r\text{Li}_2\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n)-bn\log(x))}{m}+\frac{a\log\left(-\frac{fx^m}{e}\right)\log\left(d\left(e+fx^m\right)^r\right)}{m}+b\log(x)\log(cx^n)\log\left(d\left(e+fx^m\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] -1/6*(b*m*n*r*Log[x]^3) - (b*n*r*Log[x]^2*Log[1 + e/(f*x^m)])/2 + b*n*r*Log
[x]^2*Log[e + f*x^m] - (b*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m -
b*r*Log[x]*Log[c*x^n]*Log[e + f*x^m] + (b*r*Log[-((f*x^m)/e)]*Log[c*x^n]*Lo
g[e + f*x^m])/m - (b*n*Log[x]^2*Log[d*(e + f*x^m)^r])/2 + (a*Log[-((f*x^m)/
e)]*Log[d*(e + f*x^m)^r])/m + b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^m)^r] + (b
*n*r*Log[x]*PolyLog[2, -(e/(f*x^m))])/m + (r*(a - b*n*Log[x] + b*Log[c*x^n]
)*PolyLog[2, 1 + (f*x^m)/e])/m + (b*n*r*PolyLog[3, -(e/(f*x^m))])/m^2
```

fricas [C] time = 0.70, size = 173, normalized size = 1.52

$$\frac{bm^2n\log(d)\log(x)^2+2\,bnr\text{polylog}\left(3,-\frac{fx^m}{e}\right)+2\left(bm^2\log(c)+am^2\right)\log(d)\log(x)-2\left(bmnr\log(x)+bmr\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^r)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*m^2*n*log(d)*log(x)^2 + 2*b*n*r*polylog(3, -f*x^m/e) + 2*(b*m^2*log(
c) + a*m^2)*log(d)*log(x) - 2*(b*m*n*r*log(x) + b*m*r*log(c) + a*m*r)*dilog
(-(f*x^m + e)/e + 1) + (b*m^2*n*r*log(x)^2 + 2*(b*m^2*r*log(c) + a*m^2*r)*l
og(x))*log(f*x^m + e) - (b*m^2*n*r*log(x)^2 + 2*(b*m^2*r*log(c) + a*m^2*r)*
log(x))*log((f*x^m + e)/e))/m^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\log(cx^n)+a)\log\left(\left(fx^m+e\right)^r d\right)}{x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^r)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + e)^r*d)/x, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(b\ln(c\,x^{n})+a)\ln\left(d\left(f\,x^{m}+e\right)^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^r)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^r)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(bn\log(x)^{2}-2\,b\log(x)\log\left(x^{n}\right)-2\left(b\log(c)+a\right)\log(x)\right)\log\left(\left(fx^{m}+e\right)^{r}\right)-\int -\frac{2\,be\log(c)\log(d)+2\,ae\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^r)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log((f*
x^m + e)^r) - integrate(-1/2*(2*b*e*log(c)*log(d) + 2*a*e*log(d) + (b*f*m*n
*r*log(x)^2 + 2*b*f*log(c)*log(d) + 2*a*f*log(d) - 2*(b*f*m*r*log(c) + a*f*
m*r)*log(x))*x^m + 2*(b*e*log(d) - (b*f*m*r*log(x) - b*f*log(d))*x^m)*log(x
^n))/(f*x*x^m + e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\,x^{m}\right)^{r}\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**r)/x,x)
```

```
[Out] Timed out
```

3.142 $\int \frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)},x\right)$$

```
[Out] Unintegrable(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])),x]
[Out] Defer[Int][Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Mathematica [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])),x]
[Out] Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(fx^m+e\right)^r d\right)}{bx\log\left(cx^n\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(log((f*x^m + e)^r*d)/(b*x*log(c*x^n) + a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(fx^m+e\right)^r d\right)}{\left(b\log\left(cx^n\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(log((f*x^m + e)^r*d)/((b*log(c*x^n) + a)*x), x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(d \left(f\, x^{m}+e \right)^{r}\right)}{\left(b \ln \left(c\, x^{n}\right)+a \right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(f*x^m+e)^r)/x/(b*ln(c*x^n)+a),x)
[Out] int(ln(d*(f*x^m+e)^r)/x/(b*ln(c*x^n)+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left({\left(f x^{m} + e\right)}^{r} d\right)}{{\left(b \log \left(c x^{n}\right) + a\right)} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(log((f*x^m + e)^r*d)/((b*log(c*x^n) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^r\right)}{x\,\left(a+b\,\ln\left(c\,x^n\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))),x)
[Out] int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(e+f*x**m)**r)/x/(a+b*ln(c*x**n)),x)
[Out] Timed out
```

3.143 $\int \frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log(cx^n)\right)^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2),x]
[Out] Defer[Int][Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2), x]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Mathematica [A] time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d\left(e+fx^m\right)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2),x]
[Out] Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2), x]
```

fricas [A] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(fx^m+e\right)^r d\right)}{b^2x\log\left(cx^n\right)^2+2\,abx\log\left(cx^n\right)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] integral(log((f*x^m + e)^r*d)/(b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^
2*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(fx^m+e\right)^r d\right)}{\left(b\log\left(cx^n\right)+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(log((f*x^m + e)^r*d)/((b*log(c*x^n) + a)^2*x), x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d\left(f\,x^m+e\right)^r\right)}{\left(b\ln\left(c\,x^n\right)+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(f*x^m+e)^r)/x/(b*ln(c*x^n)+a)^2,x)
[Out] int(ln(d*(f*x^m+e)^r)/x/(b*ln(c*x^n)+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$fmr\int \frac{x^m}{\left(b^2fn\log(c)+abfn\right)xx^m+\left(b^2en\log(c)+aben\right)x+\left(b^2fnxx^m+b^2enx\right)\log\left(x^n\right)}\,dx-\frac{\log\left(\left(fx^m+e\right.\right.}{b^2n\log(c)+b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] f*m*r*integrate(x^m/((b^2*f*n*log(c) + a*b*f*n)*x*x^m + (b^2*e*n*log(c) + a
*b*e*n)*x + (b^2*f*n*x*x^m + b^2*e*n*x)*log(x^n)), x) - (log((f*x^m + e)^r)
 + log(d))/(b^2*n*log(c) + b^2*n*log(x^n) + a*b*n)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\left(e+f\,x^m\right)^r\right)}{x\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))^2),x)
[Out] int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(e+f*x**m)**r)/x/(a+b*ln(c*x**n))**2,x)
[Out] Timed out
```

3.144 $\int x^2 \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right),x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] Defer[Int][x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k], x]
```

Rubi steps

$$\int x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx=\int x^2\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$$

Mathematica [A] time = 0.19, size = 292, normalized size = 10.07

$$-\frac{x^3\left(bekm(m+3)n\,{}_3F_2\left(1,\frac{3}{m},\frac{3}{m};1+\frac{3}{m},1+\frac{3}{m};-\frac{fx^m}{e}\right)-27ae\log\left(d\left(e+fx^m\right)^k\right)-9aem\log\left(d\left(e+fx^m\right)^k\right)+9af\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] -1/27*(x^3*(-6*b*e*k*m*n - 2*b*e*k*m^2*n + 9*a*f*k*m*x^m*Hypergeometric2F1[
1, (3 + m)/m, 2 + 3/m, -((f*x^m)/e)] + b*e*k*m*(3 + m)*n*HypergeometricPFQ[
{1, 3/m, 3/m}, {1 + 3/m, 1 + 3/m}, -((f*x^m)/e)] + b*e*k*m*(3 + m)*Hypergeo
metric2F1[1, 3/m, (3 + m)/m, -((f*x^m)/e)]*(n - 3*Log[c*x^n]) + 9*b*e*k*m*L
og[c*x^n] + 3*b*e*k*m^2*Log[c*x^n] - 27*a*e*Log[d*(e + f*x^m)^k] - 9*a*e*m*
Log[d*(e + f*x^m)^k] + 9*b*e*n*Log[d*(e + f*x^m)^k] + 3*b*e*m*n*Log[d*(e +
f*x^m)^k] - 27*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 9*b*e*m*Log[c*x^n]*Log
[d*(e + f*x^m)^k]))/(e*(3 + m))
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2\log\left(cx^n\right)+ax^2\right)\log\left(\left(fx^m+e\right)^k d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)*log((f*x^m + e)^k*d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x^2\log\left(\left(fx^m+e\right)^k d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*x^m + e)^k*d), x)
```

maple [A] time = 0.15, size = 0, normalized size = 0.00

$$\int (b \ln (c x^n) + a) x^2 \ln \left(d \left(f x^m + e\right)^k\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{9}\left(3 bx^3 \log (x^n) - \left(b\left(n - 3 \log(c)\right) - 3 a\right)x^3\right) \log \left(\left(f x^m + e\right)^k\right) + \int -\frac{\left(3\left(fkm - 3 f \log(d)\right)a - \left(fkmn - 3\left(fk\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima")
```

```
[Out] 1/9*(3*b*x^3*log(x^n) - (b*(n - 3*log(c)) - 3*a)*x^3)*log((f*x^m + e)^k) +
integrate(-1/9*((3*(f*k*m - 3*f*log(d))*a - (f*k*m*n - 3*(f*k*m - 3*f*log(d
))*log(c))*b)*x^2*x^m - 9*(b*e*log(c)*log(d) + a*e*log(d))*x^2 + 3*((f*k*m
- 3*f*log(d))*b*x^2*x^m - 3*b*e*x^2*log(d))*log(x^n))/(f*x^m + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \ln \left(d \left(e + f x^m\right)^k\right) (a + b \ln (c x^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Timed out
```

3.145 $\int x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right),x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
[Out] Defer[Int][x*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k], x]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx=\int x\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$$

Mathematica [A] time = 0.18, size = 292, normalized size = 10.81

$$-\frac{x^2\left(bekm(m+2)n\,{}_3F_2\left(1,\frac{2}{m},\frac{2}{m};1+\frac{2}{m},1+\frac{2}{m};-\frac{fx^m}{e}\right)-8ae\log\left(d\left(e+fx^m\right)^k\right)-4aem\log\left(d\left(e+fx^m\right)^k\right)+4afk\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
[Out] -1/8*(x^2*(-4*b*e*k*m*n - 2*b*e*k*m^2*n + 4*a*f*k*m*x^m*Hypergeometric2F1[1
, (2 + m)/m, 2 + 2/m, -((f*x^m)/e)] + b*e*k*m*(2 + m)*n*HypergeometricPFQ[{
1, 2/m, 2/m}, {1 + 2/m, 1 + 2/m}, -((f*x^m)/e)] + b*e*k*m*(2 + m)*Hypergeom
etric2F1[1, 2/m, (2 + m)/m, -((f*x^m)/e)]*(n - 2*Log[c*x^n]) + 4*b*e*k*m*Lo
g[c*x^n] + 2*b*e*k*m^2*Log[c*x^n] - 8*a*e*Log[d*(e + f*x^m)^k] - 4*a*e*m*Lo
g[d*(e + f*x^m)^k] + 4*b*e*n*Log[d*(e + f*x^m)^k] + 2*b*e*m*n*Log[d*(e + f*
x^m)^k] - 8*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 4*b*e*m*Log[c*x^n]*Log[d*
(e + f*x^m)^k]))/(e*(2 + m))
```

fricas [A] time = 0.91, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(bx\log\left(cx^n\right)+ax\right)\log\left(\left(fx^m+e\right)^kd\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas")
[Out] integral((b*x*log(c*x^n) + a*x)*log((f*x^m + e)^k*d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x\log\left(\left(fx^m+e\right)^kd\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*log((f*x^m + e)^k*d), x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int (b\ln (c\,x^{n})+a)\,x\ln \left(d\left(f\,x^{m}+e\right)^{k}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(2\,bx^{2}\log\left(x^{n}\right)-\left(b\left(n-2\,\log(c)\right)-2\,a\right)x^{2}\right)\log\left(\left(fx^{m}+e\right)^{k}\right)+\int-\frac{\left(2\left(fkm-2\,f\log(d)\right)a-\left(fkmn-2\left(fk\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2*log(x^n) - (b*(n - 2*log(c)) - 2*a)*x^2)*log((f*x^m + e)^k) +
integrate(-1/4*((2*(f*k*m - 2*f*log(d))*a - (f*k*m*n - 2*(f*k*m - 2*f*log(d
))*log(c))*b)*x*x^m - 4*(b*e*log(c)*log(d) + a*e*log(d))*x + 2*((f*k*m - 2*
f*log(d))*b*x*x^m - 2*b*e*x*log(d))*log(x^n))/(f*x^m + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x\,\ln\left(d\left(e+f\,x^{m}\right)^{k}\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Timed out
```

3.146 $\int \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right),x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
[Out] Defer[Int][(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k], x]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx=\int \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$$

Mathematica [A] time = 0.18, size = 165, normalized size = 6.35

$$x\left(-bkmn\,{}_3F_2\left(1,\frac{1}{m},\frac{1}{m};1+\frac{1}{m},1+\frac{1}{m};-\frac{fx^m}{e}\right)+km\,{}_2F_1\left(1,\frac{1}{m};1+\frac{1}{m};-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)-bn\right)+a\log\left(d\left(e\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
[Out] b*k*m*n*x - k*m*x*(a + b*(-(n*Log[x]) + Log[c*x^n])) + x*(b*k*m*n - b*k*m*n
*HypergeometricPFQ[{1, m^(-1), m^(-1)}, {1 + m^(-1), 1 + m^(-1)}, -((f*x^m)
/e)] - b*k*m*n*Log[x] + k*m*Hypergeometric2F1[1, m^(-1), 1 + m^(-1), -((f*x
^m)/e)]*(a - b*n + b*Log[c*x^n]) + a*Log[d*(e + f*x^m)^k] - b*n*Log[d*(e +
f*x^m)^k] + b*Log[c*x^n]*Log[d*(e + f*x^m)^k])
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^m+e\right)^k d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*log((f*x^m + e)^k*d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^m+e\right)^k d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d), x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int (b \ln (c x^n) + a) \ln \left(d \left(f x^m + e\right)^k\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\left(bx \log (x^n) - \left(b\left(n - \log(c)\right) - a\right)x\right) \log \left(\left(fx^m + e\right)^k\right) + \int \frac{be \log(c) \log(d) + ae \log(d) - \left(\left(fkm - f \log(d)\right)a - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima")
[Out] (b*x*log(x^n) - (b*(n - log(c)) - a)*x)*log((f*x^m + e)^k) + integrate((b*e
*log(c)*log(d) + a*e*log(d) - ((f*k*m - f*log(d))*a - (f*k*m*n - (f*k*m - f
*log(d))*log(c))*b)*x^m - ((f*k*m - f*log(d))*b*x^m - b*e*log(d))*log(x^n))
/(f*x^m + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \ln \left(d \left(e + f x^m\right)^k\right) (a + b \ln (c x^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)),x)
[Out] int(log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log (cx^n)\right) \log \left(d \left(e + fx^m\right)^k\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
[Out] Integral((a + b*log(c*x**n))*log(d*(e + f*x**m)**k), x)
```

3.147 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^k\right)}{x}\,dx$

Optimal. Leaf size=114

$$\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^m\right)^k\right)}{2bn}-\frac{k\text{Li}_2\left(-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m}-\frac{k\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^2}{2bn}+\frac{bkn\text{Li}_3(}{m}$$

```
[Out] 1/2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^m)^k)/b/n-1/2*k*(a+b*ln(c*x^n))^2*ln(1+f*
x^m/e)/b/n-k*(a+b*ln(c*x^n))*polylog(2,-f*x^m/e)/m+b*k*n*polylog(3,-f*x^m/e
)/m^2
```

Rubi [A] time = 0.19, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2375, 2337, 2374, 6589}

$$-\frac{k\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m}+\frac{bkn\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{m^2}+\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^m\right)^k\right)}{2bn}-\frac{k\log\left(\frac{f}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^k])/(2*b*n) - (k*(a + b*Log[c*x^n])
^2*Log[1 + (f*x^m)/e])/(2*b*n) - (k*(a + b*Log[c*x^n])*PolyLog[2, -((f*x^m)
/e)])/m + (b*k*n*PolyLog[3, -((f*x^m)/e)])/m^2
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{x}dx = \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^m\right)^k\right)}{2bn} - \frac{(fkm)\int\frac{x^{-1+m}\left(a+b\log(cx^n)\right)^2}{e+fx^m}}{2bn}$$
$$= \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^m\right)^k\right)}{2bn} - \frac{k\left(a+b\log\left(cx^n\right)\right)^2\log\left(1\right.}{2bn}$$
$$= \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^m\right)^k\right)}{2bn} - \frac{k\left(a+b\log\left(cx^n\right)\right)^2\log\left(1\right.}{2bn}$$
$$= \frac{\left(a+b\log\left(cx^n\right)\right)^2\log\left(d\left(e+fx^m\right)^k\right)}{2bn} - \frac{k\left(a+b\log\left(cx^n\right)\right)^2\log\left(1\right.}{2bn}$$

Mathematica [B] time = 0.17, size = 277, normalized size = 2.43

$$\frac{k\text{Li}_2\left(\frac{fx^m}{e}+1\right)\left(a+b\log\left(cx^n\right)-bn\log(x)\right)}{m}+\frac{a\log\left(-\frac{fx^m}{e}\right)\log\left(d\left(e+fx^m\right)^k\right)}{m}+b\log(x)\log\left(cx^n\right)\log\left(d\left(e+f\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x,x]
```

```
[Out] -1/6*(b*k*m*n*Log[x]^3) - (b*k*n*Log[x]^2*Log[1 + e/(f*x^m)])/2 + b*k*n*Log
[x]^2*Log[e + f*x^m] - (b*k*n*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m -
b*k*Log[x]*Log[c*x^n]*Log[e + f*x^m] + (b*k*Log[-((f*x^m)/e)]*Log[c*x^n]*Lo
g[e + f*x^m])/m - (b*n*Log[x]^2*Log[d*(e + f*x^m)^k])/2 + (a*Log[-((f*x^m)/
e)]*Log[d*(e + f*x^m)^k])/m + b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^m)^k] + (b
*k*n*Log[x]*PolyLog[2, -(e/(f*x^m))])/m + (k*(a - b*n*Log[x] + b*Log[c*x^n]
)*PolyLog[2, 1 + (f*x^m)/e])/m + (b*k*n*PolyLog[3, -(e/(f*x^m))])/m^2
```

fricas [C] time = 0.69, size = 173, normalized size = 1.52

$$bm^2n\log(d)\log(x)^2+2\,bkn\text{polylog}\left(3,-\frac{fx^m}{e}\right)+2\left(bm^2\log(c)+am^2\right)\log(d)\log(x)-2\left(bkmn\log(x)+bkm\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*m^2*n*log(d)*log(x)^2 + 2*b*k*n*polylog(3, -f*x^m/e) + 2*(b*m^2*log(
c) + a*m^2)*log(d)*log(x) - 2*(b*k*m*n*log(x) + b*k*m*log(c) + a*k*m)*dilog
(-(f*x^m + e)/e + 1) + (b*k*m^2*n*log(x)^2 + 2*(b*k*m^2*log(c) + a*k*m^2)*l
og(x))*log(f*x^m + e) - (b*k*m^2*n*log(x)^2 + 2*(b*k*m^2*log(c) + a*k*m^2)*
log(x))*log((f*x^m + e)/e))/m^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^m+e\right)^k d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c\,x^{n}\right)+a\right) \ln \left(d \left(f\,x^{m}+e\right)^{k}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(bn\log(x)^2-2\,b\log(x)\log\left(x^n\right)-2\left(b\log(c)+a\right)\log(x)\right)\log\left(\left(fx^m+e\right)^k\right)-\int -\frac{2\,be\log(c)\log(d)+2\,ae\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log((f*
x^m + e)^k) - integrate(-1/2*(2*b*e*log(c)*log(d) + 2*a*e*log(d) + (b*f*k*m
*n*log(x)^2 + 2*b*f*log(c)*log(d) + 2*a*f*log(d) - 2*(b*f*k*m*log(c) + a*f*
k*m)*log(x))*x^m + 2*(b*e*log(d) - (b*f*k*m*log(x) - b*f*log(d))*x^m)*log(x
^n))/(f*x*x^m + e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\,x^m\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k)/x,x)
```

```
[Out] Timed out
```

3.148 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^k\right)}{x^2}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^2,x]
[Out] Defer[Int][((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{x^2}\,dx=\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{x^2}\,dx$$

Mathematica [A] time = 0.17, size = 282, normalized size = 9.72

$$bek(m-1)mn\,{}_3F_2\left(1,-\frac{1}{m},-\frac{1}{m};1-\frac{1}{m},1-\frac{1}{m};-\frac{fx^m}{e}\right)+ae\log\left(d\left(e+fx^m\right)^k\right)-aem\log\left(d\left(e+fx^m\right)^k\right)+afkmx$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^2,x]
[Out] (2*b*e*k*m*n - 2*b*e*k*m^2*n + a*f*k*m*x^m*Hypergeometric2F1[1, (-1 + m)/m,
 2 - m^(-1), -((f*x^m)/e)] + b*e*k*(-1 + m)*m*n*HypergeometricPFQ[{1, -m^(-
1), -m^(-1)}, {1 - m^(-1), 1 - m^(-1)}, -((f*x^m)/e)] + b*e*k*m*Log[c*x^n]
- b*e*k*m^2*Log[c*x^n] + b*e*k*(-1 + m)*m*Hypergeometric2F1[1, -m^(-1), (-1
 + m)/m, -((f*x^m)/e)]*(n + Log[c*x^n]) + a*e*Log[d*(e + f*x^m)^k] - a*e*m*
Log[d*(e + f*x^m)^k] + b*e*n*Log[d*(e + f*x^m)^k] - b*e*m*n*Log[d*(e + f*x^
m)^k] + b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - b*e*m*Log[c*x^n]*Log[d*(e + f
*x^m)^k])/(e*(-1 + m)*x)
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\log\left(\left(fx^m+e\right)^k d\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right) \log\left(\left(fx^{m} + e\right)^{k} d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^2, x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^{n}\right) + a\right) \ln\left(d \left(f\,x^{m} + e\right)^{k}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b\left(n + \log(c)\right) + b \log\left(x^{n}\right) + a\right) \log\left(\left(fx^{m} + e\right)^{k}\right)}{x} + \int \frac{be \log(c) \log(d) + ae \log(d) + \left(\left(fkm + f \log(d)\right)a + \left(fkn\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^2,x, algorithm="maxima")
```

```
[Out] -(b*(n + log(c)) + b*log(x^n) + a)*log((f*x^m + e)^k)/x + integrate((b*e*lo
g(c)*log(d) + a*e*log(d) + ((f*k*m + f*log(d))*a + (f*k*m*n + (f*k*m + f*lo
g(d))*log(c))*b)*x^m + ((f*k*m + f*log(d))*b*x^m + b*e*log(d))*log(x^n))/(f
*x^2*x^m + e*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d \left(e + f\,x^{m}\right)^{k}\right) \left(a + b\,\ln\left(c\,x^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k)/x**2,x)
```

```
[Out] Timed out
```

3.149 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^k\right)}{x^3}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^k\right)}{x^3},x\right)$$

[Out] Unintegrable((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^3,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^k\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^3,x]

[Out] Defer[Int][((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^3, x]

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^k\right)}{x^3}\,dx=\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^k\right)}{x^3}\,dx$$

Mathematica [A] time = 0.16, size = 292, normalized size = 10.07

$$bek(m-2)mn\,{}_3F_2\left(1,-\frac{2}{m},-\frac{2}{m};1-\frac{2}{m},1-\frac{2}{m};-\frac{fx^m}{e}\right)+8ae\log\left(d\left(e+fx^m\right)^k\right)-4aem\log\left(d\left(e+fx^m\right)^k\right)+4afk$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^3,x]

[Out] (4*b*e*k*m*n - 2*b*e*k*m^2*n + 4*a*f*k*m*x^m*Hypergeometric2F1[1, (-2 + m)/m, 2 - 2/m, -((f*x^m)/e)] + b*e*k*(-2 + m)*m*n*HypergeometricPFQ[{1, -2/m, -2/m}, {1 - 2/m, 1 - 2/m}, -((f*x^m)/e)] + 4*b*e*k*m*Log[c*x^n] - 2*b*e*k*m^2*Log[c*x^n] + b*e*k*(-2 + m)*m*Hypergeometric2F1[1, -2/m, (-2 + m)/m, -((f*x^m)/e)]*(n + 2*Log[c*x^n]) + 8*a*e*Log[d*(e + f*x^m)^k] - 4*a*e*m*Log[d*(e + f*x^m)^k] + 4*b*e*n*Log[d*(e + f*x^m)^k] - 2*b*e*m*n*Log[d*(e + f*x^m)^k] + 8*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 4*b*e*m*Log[c*x^n]*Log[d*(e + f*x^m)^k])/(8*e*(-2 + m)*x^2)

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\log(cx^n)+a)\log\left(\left(fx^m+e\right)^k d\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^3, x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right) \log\left(\left(fx^{m} + e\right)^{k} d\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^3, x)
```

maple [**A**] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^{n}\right) + a\right) \ln\left(d \left(f\,x^{m} + e\right)^{k}\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k)/x^3,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b\left(n + 2\,\log(c)\right) + 2\,b\log\left(x^{n}\right) + 2\,a\right)\log\left(\left(fx^{m} + e\right)^{k}\right)}{4\,x^{2}} + \int \frac{4\,be\log(c)\log(d) + 4\,ae\log(d) + \left(2\left(fkm + 2\,f\log\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*(n + 2*log(c)) + 2*b*log(x^n) + 2*a)*log((f*x^m + e)^k)/x^2 + integ
rate(1/4*(4*b*e*log(c)*log(d) + 4*a*e*log(d) + (2*(f*k*m + 2*f*log(d))*a +
(f*k*m*n + 2*(f*k*m + 2*f*log(d))*log(c))*b)*x^m + 2*((f*k*m + 2*f*log(d))*
b*x^m + 2*b*e*log(d))*log(x^n))/(f*x^3*x^m + e*x^3), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\left(e + f\,x^{m}\right)^{k}\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k)/x**3,x)
```

```
[Out] Timed out
```

3.150 $\int (gx)^{-1+3m} \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$

Optimal. Leaf size=433

$$\frac{(gx)^{3m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{3gm}+\frac{e^3kx^{-3m}(gx)^{3m}\log\left(e+fx^m\right)\left(a+b\log\left(cx^n\right)\right)}{3f^3gm}-\frac{e^2kx^{-2m}(gx)^{3m}\left(a\right.}{3f^2g}$$

```
[Out] 2/27*b*k*n*(g*x)^(3*m)/g/m^2+4/9*b*e^2*k*n*(g*x)^(3*m)/f^2/g/m^2/(x^(2*m))-
5/36*b*e*k*n*(g*x)^(3*m)/f/g/m^2/(x^m)-1/9*k*(g*x)^(3*m)*(a+b*ln(c*x^n))/g/
m-1/3*e^2*k*(g*x)^(3*m)*(a+b*ln(c*x^n))/f^2/g/m/(x^(2*m))+1/6*e*k*(g*x)^(3*
m)*(a+b*ln(c*x^n))/f/g/m/(x^m)-1/9*b*e^3*k*n*(g*x)^(3*m)*ln(e+f*x^m)/f^3/g/
m^2/(x^(3*m))-1/3*b*e^3*k*n*(g*x)^(3*m)*ln(-f*x^m/e)*ln(e+f*x^m)/f^3/g/m^2/
(x^(3*m))+1/3*e^3*k*(g*x)^(3*m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/f^3/g/m/(x^(3*m
))-1/9*b*n*(g*x)^(3*m)*ln(d*(e+f*x^m)^k)/g/m^2+1/3*(g*x)^(3*m)*(a+b*ln(c*x^
n))*ln(d*(e+f*x^m)^k)/g/m-1/3*b*e^3*k*n*(g*x)^(3*m)*polylog(2,1+f*x^m/e)/f^
3/g/m^2/(x^(3*m))
```

Rubi [A] time = 0.60, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2455, 20, 266, 43, 2376, 16, 32, 30, 19, 2454, 2394, 2315}

$$-\frac{be^3knx^{-3m}(gx)^{3m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{3f^3gm^2}+\frac{(gx)^{3m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{3gm}+\frac{e^3kx^{-3m}(gx)^{3m}\log\left(e+\right.}{3f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 + 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (2*b*k*n*(g*x)^(3*m))/(27*g*m^2) + (4*b*e^2*k*n*(g*x)^(3*m))/(9*f^2*g*m^2*x
^(2*m)) - (5*b*e*k*n*(g*x)^(3*m))/(36*f*g*m^2*x^m) - (k*(g*x)^(3*m)*(a + b*
Log[c*x^n]))/(9*g*m) - (e^2*k*(g*x)^(3*m)*(a + b*Log[c*x^n]))/(3*f^2*g*m*x^
(2*m)) + (e*k*(g*x)^(3*m)*(a + b*Log[c*x^n]))/(6*f*g*m*x^m) - (b*e^3*k*n*(g
*x)^(3*m)*Log[e + f*x^m])/(9*f^3*g*m^2*x^(3*m)) - (b*e^3*k*n*(g*x)^(3*m)*Lo
g[-((f*x^m)/e)]*Log[e + f*x^m])/(3*f^3*g*m^2*x^(3*m)) + (e^3*k*(g*x)^(3*m)*
(a + b*Log[c*x^n])*Log[e + f*x^m])/(3*f^3*g*m*x^(3*m)) - (b*n*(g*x)^(3*m)*L
og[d*(e + f*x^m)^k])/(9*g*m^2) + ((g*x)^(3*m)*(a + b*Log[c*x^n])*Log[d*(e +
 f*x^m)^k])/(3*g*m) - (b*e^3*k*n*(g*x)^(3*m)*PolyLog[2, 1 + (f*x^m)/e])/(3*
f^3*g*m^2*x^(3*m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !I
ntegerQ[m] && !IntegerQ[n] && IntegerQ[m + n]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1+3m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx = -\frac{k(gx)^{3m}\left(a+b\log\left(cx^n\right)\right)}{9gm} - \frac{e^2kx^{-2m}(gx)^{3m}\left(a+b\log\right.}{3f^2gm}$$

$$= -\frac{k(gx)^{3m}\left(a+b\log\left(cx^n\right)\right)}{9gm} - \frac{e^2kx^{-2m}(gx)^{3m}\left(a+b\log\right.}{3f^2gm}$$

$$= -\frac{k(gx)^{3m}\left(a+b\log\left(cx^n\right)\right)}{9gm} - \frac{e^2kx^{-2m}(gx)^{3m}\left(a+b\log\right.}{3f^2gm}$$

$$= \frac{bkn(gx)^{3m}}{27gm^2} + \frac{be^2knx^{-2m}(gx)^{3m}}{3f^2gm^2} - \frac{beknx^{-m}(gx)^{3m}}{12fgm^2} - \frac{k(g}{}$$

$$= \frac{bkn(gx)^{3m}}{27gm^2} + \frac{be^2knx^{-2m}(gx)^{3m}}{3f^2gm^2} - \frac{beknx^{-m}(gx)^{3m}}{12fgm^2} - \frac{k(g}{}$$

$$= \frac{bkn(gx)^{3m}}{27gm^2} + \frac{be^2knx^{-2m}(gx)^{3m}}{3f^2gm^2} - \frac{beknx^{-m}(gx)^{3m}}{12fgm^2} - \frac{k(g}{}$$

$$= \frac{bkn(gx)^{3m}}{27gm^2} + \frac{be^2knx^{-2m}(gx)^{3m}}{3f^2gm^2} - \frac{beknx^{-m}(gx)^{3m}}{12fgm^2} - \frac{k(g}{}$$

$$= \frac{2bkn(gx)^{3m}}{27gm^2} + \frac{4be^2knx^{-2m}(gx)^{3m}}{9f^2gm^2} - \frac{5beknx^{-m}(gx)^{3m}}{36fgm^2} -$$

Mathematica [A] time = 0.42, size = 410, normalized size = 0.95

$$x^{-3m}(gx)^{3m}\left(12e^3km\log(x)\left(3am+3bm\log\left(cx^n\right)+3bn\log\left(e+fx^m\right)-3bn\log\left(e-ex^m\right)-bn\right)+36af^3mx^{3m}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 + 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] ((g*x)^(3*m)*(-36*a*e^2*f*k*m*x^m + 48*b*e^2*f*k*n*x^m + 18*a*e*f^2*k*m*x^(
2*m) - 15*b*e*f^2*k*n*x^(2*m) - 12*a*f^3*k*m*x^(3*m) + 8*b*f^3*k*n*x^(3*m)
- 36*b*e^3*k*m^2*n*Log[x]^2 - 36*b*e^2*f*k*m*x^m*Log[c*x^n] + 18*b*e*f^2*k*
m*x^(2*m)*Log[c*x^n] - 12*b*f^3*k*m*x^(3*m)*Log[c*x^n] + 36*a*e^3*k*m*Log[e
 - e*x^m] - 12*b*e^3*k*n*Log[e - e*x^m] + 36*b*e^3*k*m*Log[c*x^n]*Log[e - e
*x^m] - 36*b*e^3*k*n*Log[-((f*x^m)/e)]*Log[e + f*x^m] + 12*e^3*k*m*Log[x]*(
3*a*m - b*n + 3*b*m*Log[c*x^n] - 3*b*n*Log[e - e*x^m] + 3*b*n*Log[e + f*x^m
]) + 36*a*f^3*m*x^(3*m)*Log[d*(e + f*x^m)^k] - 12*b*f^3*n*x^(3*m)*Log[d*(e
+ f*x^m)^k] + 36*b*f^3*m*x^(3*m)*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 36*b*e^3
*k*n*PolyLog[2, 1 + (f*x^m)/e]))/(108*f^3*g*m^2*x^(3*m))
```

fricas [A] time = 0.65, size = 368, normalized size = 0.85

$$36\,be^3g^{3\,m-1}kmn\log(x)\log\left(\frac{fx^m+e}{e}\right)+36\,be^3g^{3\,m-1}kn\mathrm{Li}_2\left(-\frac{fx^m+e}{e}+1\right)-4\left(3\,bf^3km\log(c)+3\,af^3km-2\,bf^3k\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
fricas")
```

```
[Out] 1/108*(36*b*e^3*g^(3*m - 1)*k*m*n*log(x)*log((f*x^m + e)/e) + 36*b*e^3*g^(3
*m - 1)*k*n*dilog(-(f*x^m + e)/e + 1) - 4*(3*b*f^3*k*m*log(c) + 3*a*f^3*k*m
 - 2*b*f^3*k*n - 3*(3*b*f^3*m*log(c) + 3*a*f^3*m - b*f^3*n)*log(d) + 3*(b*f
^3*k*m*n - 3*b*f^3*m*n*log(d))*log(x))*g^(3*m - 1)*x^(3*m) + 3*(6*b*e*f^2*k
*m*n*log(x) + 6*b*e*f^2*k*m*log(c) + 6*a*e*f^2*k*m - 5*b*e*f^2*k*n)*g^(3*m
- 1)*x^(2*m) - 12*(3*b*e^2*f*k*m*n*log(x) + 3*b*e^2*f*k*m*log(c) + 3*a*e^2*
f*k*m - 4*b*e^2*f*k*n)*g^(3*m - 1)*x^m + 12*((3*b*f^3*k*m*n*log(x) + 3*b*f^
3*k*m*log(c) + 3*a*f^3*k*m - b*f^3*k*n)*g^(3*m - 1)*x^(3*m) + (3*b*e^3*k*m*
log(c) + 3*a*e^3*k*m - b*e^3*k*n)*g^(3*m - 1))*log(f*x^m + e))/(f^3*m^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)\left(gx\right)^{3\,m-1}\log\left(\left(fx^m+e\right)^k d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(3*m - 1)*log((f*x^m + e)^k*d), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)\left(gx\right)^{3m-1}\ln\left(d\left(f\,x^m+e\right)^k\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-1+3*m)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

```
[Out] int((g*x)^(-1+3*m)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\,bg^{3\,m}mx^{3\,m}\log\left(x^n\right)+\left(3\,ag^{3\,m}m+\left(3\,g^{3\,m}m\log(c)-g^{3\,m}n\right)b\right)x^{3\,m}\right)\log\left(\left(fx^m+e\right)^k\right)}{9\,gm^2}+\int -\frac{\left(3\left(fg^{3\,m}km-3\,fg^{3}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
maxima")
```

```
[Out] 1/9*(3*b*g^(3*m)*m*x^(3*m)*log(x^n) + (3*a*g^(3*m)*m + (3*g^(3*m)*m*log(c)
- g^(3*m)*n)*b)*x^(3*m))*log((f*x^m + e)^k)/(g*m^2) + integrate(-1/9*((3*(f
*g^(3*m)*k*m - 3*f*g^(3*m)*m*log(d))*a - (f*g^(3*m)*k*n - 3*(f*g^(3*m)*k*m
- 3*f*g^(3*m)*m*log(d))*log(c))*b)*x^(4*m) - 9*(b*e*g^(3*m)*m*log(c)*log(d)
 + a*e*g^(3*m)*m*log(d))*x^(3*m) - 3*(3*b*e*g^(3*m)*m*x^(3*m)*log(d) - (f*g
^(3*m)*k*m - 3*f*g^(3*m)*m*log(d))*b*x^(4*m))*log(x^n))/(f*g*m*x*x^m + e*g*
m*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(e+f\,x^m\right)^k\right)\left(g\,x\right)^{3\,m-1}\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^k)*(g*x)^(3*m - 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^m)^k)*(g*x)^(3*m - 1)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1+3*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
[Out] Timed out
```

3.151 $\int (gx)^{-1+2m} \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)dx$

Optimal. Leaf size=363

$$\frac{(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{2gm}-\frac{e^2kx^{-2m}(gx)^{2m}\log\left(e+fx^m\right)\left(a+b\log\left(cx^n\right)\right)}{2f^2gm}+\frac{ekx^{-m}(gx)^{2m}\left(a+b\right.}{2fgm}$$

```
[Out] 1/4*b*k*n*(g*x)^(2*m)/g/m^2-3/4*b*e*k*n*(g*x)^(2*m)/f/g/m^2/(x^m)-1/4*k*(g*
x)^(2*m)*(a+b*ln(c*x^n))/g/m+1/2*e*k*(g*x)^(2*m)*(a+b*ln(c*x^n))/f/g/m/(x^m
)+1/4*b*e^2*k*n*(g*x)^(2*m)*ln(e+f*x^m)/f^2/g/m^2/(x^(2*m))+1/2*b*e^2*k*n*(
g*x)^(2*m)*ln(-f*x^m/e)*ln(e+f*x^m)/f^2/g/m^2/(x^(2*m))-1/2*e^2*k*(g*x)^(2*
m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/f^2/g/m/(x^(2*m))-1/4*b*n*(g*x)^(2*m)*ln(d*(
e+f*x^m)^k)/g/m^2+1/2*(g*x)^(2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m+1/2
*b*e^2*k*n*(g*x)^(2*m)*polylog(2,1+f*x^m/e)/f^2/g/m^2/(x^(2*m))
```

Rubi [A] time = 0.42, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2455, 20, 266, 43, 2376, 16, 32, 30, 19, 2454, 2394, 2315}

$$\frac{be^2knx^{-2m}(gx)^{2m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{2f^2gm^2}+\frac{(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{2gm}-\frac{e^2kx^{-2m}(gx)^{2m}\log\left(e+fx^m\right.}{2f^2gm}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 + 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (b*k*n*(g*x)^(2*m))/(4*g*m^2) - (3*b*e*k*n*(g*x)^(2*m))/(4*f*g*m^2*x^m) - (
k*(g*x)^(2*m)*(a + b*Log[c*x^n]))/(4*g*m) + (e*k*(g*x)^(2*m)*(a + b*Log[c*x
^n]))/(2*f*g*m*x^m) + (b*e^2*k*n*(g*x)^(2*m)*Log[e + f*x^m])/(4*f^2*g*m^2*x
^(2*m)) + (b*e^2*k*n*(g*x)^(2*m)*Log[-((f*x^m)/e)]*Log[e + f*x^m])/(2*f^2*g
*m^2*x^(2*m)) - (e^2*k*(g*x)^(2*m)*(a + b*Log[c*x^n])*Log[e + f*x^m])/(2*f^
2*g*m*x^(2*m)) - (b*n*(g*x)^(2*m)*Log[d*(e + f*x^m)^k])/(4*g*m^2) + ((g*x)^
(2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/(2*g*m) + (b*e^2*k*n*(g*x)^(
2*m)*PolyLog[2, 1 + (f*x^m)/e])/(2*f^2*g*m^2*x^(2*m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !I
ntegerQ[m] && !IntegerQ[n] && IntegerQ[m + n]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1+2m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)dx=-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}+\frac{ekx^{-m}(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{2fgm}$$

$$=-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}+\frac{ekx^{-m}(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{2fgm}$$

$$=-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}+\frac{ekx^{-m}(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{2fgm}$$

$$=\frac{bkn(gx)^{2m}}{8gm^2}-\frac{beknx^{-m}(gx)^{2m}}{2fgm^2}-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}+$$

$$=\frac{bkn(gx)^{2m}}{8gm^2}-\frac{beknx^{-m}(gx)^{2m}}{2fgm^2}-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}+$$

$$=\frac{bkn(gx)^{2m}}{8gm^2}-\frac{beknx^{-m}(gx)^{2m}}{2fgm^2}-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}+$$

$$=\frac{bkn(gx)^{2m}}{8gm^2}-\frac{beknx^{-m}(gx)^{2m}}{2fgm^2}-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}+$$

$$=\frac{bkn(gx)^{2m}}{4gm^2}-\frac{3beknx^{-m}(gx)^{2m}}{4fgm^2}-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}$$

Mathematica [A] time = 0.45, size = 352, normalized size = 0.97

$$x^{-2m}(gx)^{2m}\left(e^2km\log(x)\left(-2am-2bm\log\left(cx^n\right)-2bn\log\left(e+fx^m\right)+2bn\log\left(e-ex^m\right)+bn\right)+2af^2mx^{2m}\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 + 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] ((g*x)^(2*m)*(2*a*e*f*k*m*x^m - 3*b*e*f*k*n*x^m - a*f^2*k*m*x^(2*m) + b*f^2
*k*n*x^(2*m) + 2*b*e^2*k*m^2*n*Log[x]^2 + 2*b*e*f*k*m*x^m*Log[c*x^n] - b*f^
2*k*m*x^(2*m)*Log[c*x^n] - 2*a*e^2*k*m*Log[e - e*x^m] + b*e^2*k*n*Log[e - e
*x^m] - 2*b*e^2*k*m*Log[c*x^n]*Log[e - e*x^m] + 2*b*e^2*k*n*Log[-((f*x^m)/e
)]*Log[e + f*x^m] + e^2*k*m*Log[x]*(-2*a*m + b*n - 2*b*m*Log[c*x^n] + 2*b*n
*Log[e - e*x^m] - 2*b*n*Log[e + f*x^m]) + 2*a*f^2*m*x^(2*m)*Log[d*(e + f*x^
m)^k] - b*f^2*n*x^(2*m)*Log[d*(e + f*x^m)^k] + 2*b*f^2*m*x^(2*m)*Log[c*x^n]
*Log[d*(e + f*x^m)^k] + 2*b*e^2*k*n*PolyLog[2, 1 + (f*x^m)/e]))/(4*f^2*g*m^
2*x^(2*m))
```

fricas [A] time = 1.00, size = 301, normalized size = 0.83

$$-\frac{2\,be^2g^{2\,m-1}kmn\log(x)\log\left(\frac{fx^m+e}{e}\right)+2\,be^2g^{2\,m-1}kn\mathrm{Li}_2\left(-\frac{fx^m+e}{e}+1\right)+\left(bf^2km\log(c)+af^2km-bf^2kn-\left(2\,bf^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
fricas")
```

```
[Out] -1/4*(2*b*e^2*g^(2*m - 1)*k*m*n*log(x)*log((f*x^m + e)/e) + 2*b*e^2*g^(2*m
- 1)*k*n*dilog(-(f*x^m + e)/e + 1) + (b*f^2*k*m*log(c) + a*f^2*k*m - b*f^2*
k*n - (2*b*f^2*m*log(c) + 2*a*f^2*m - b*f^2*n)*log(d) + (b*f^2*k*m*n - 2*b*
f^2*m*n*log(d))*log(x))*g^(2*m - 1)*x^(2*m) - (2*b*e*f*k*m*n*log(x) + 2*b*e
*f*k*m*log(c) + 2*a*e*f*k*m - 3*b*e*f*k*n)*g^(2*m - 1)*x^m - ((2*b*f^2*k*m*
n*log(x) + 2*b*f^2*k*m*log(c) + 2*a*f^2*k*m - b*f^2*k*n)*g^(2*m - 1)*x^(2*m
) - (2*b*e^2*k*m*log(c) + 2*a*e^2*k*m - b*e^2*k*n)*g^(2*m - 1))*log(f*x^m +
 e))/(f^2*m^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(c x^{n}\right) + a\right) \left(g x\right)^{2 \, m - 1} \log \left(\left(f x^{m} + e\right)^{k} d\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(2*m - 1)*log((f*x^m + e)^k*d), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \,x^{n}\right)+a \right) \left(g x \right)^{2 m -1} \ln \left(d \left(f \,x^{m}+e \right)^{k}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-1+2*m)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

```
[Out] int((g*x)^(-1+2*m)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2 \, b g^{2 \, m} m x^{2 \, m} \log \left(x^{n}\right) + \left(2 \, a g^{2 \, m} m + \left(2 \, g^{2 \, m} m \log\left(c\right) - g^{2 \, m} n\right) b\right) x^{2 \, m}\right) \log \left(\left(f x^{m} + e\right)^{k}\right)}{4 \, g m^{2}} + \int -\frac{\left(2 \, \left(f g^{2 \, m} k m - 2 \,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
maxima")
```

```
[Out] 1/4*(2*b*g^(2*m)*m*x^(2*m)*log(x^n) + (2*a*g^(2*m)*m + (2*g^(2*m)*m*log(c)
- g^(2*m)*n)*b)*x^(2*m))*log((f*x^m + e)^k)/(g*m^2) + integrate(-1/4*((2*(f
*g^(2*m)*k*m - 2*f*g^(2*m)*m*log(d))*a - (f*g^(2*m)*k*n - 2*(f*g^(2*m)*k*m
- 2*f*g^(2*m)*m*log(d))*log(c))*b)*x^(3*m) - 4*(b*e*g^(2*m)*m*log(c)*log(d)
 + a*e*g^(2*m)*m*log(d))*x^(2*m) - 2*(2*b*e*g^(2*m)*m*x^(2*m)*log(d) - (f*g
^(2*m)*k*m - 2*f*g^(2*m)*m*log(d))*b*x^(3*m))*log(x^n))/(f*g*m*x*x^m + e*g*
m*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,{\left(g\,x\right)}^{2\,m-1}\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^k)*(g*x)^(2*m - 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^m)^k)*(g*x)^(2*m - 1)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1+2*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Timed out
```

3.152 $\int (gx)^{-1+m} \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$

Optimal. Leaf size=255

$$\frac{(gx)^m\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{gm}+\frac{ekx^{-m}(gx)^m\log\left(e+fx^m\right)\left(a+b\log\left(cx^n\right)\right)}{fgm}-\frac{k(gx)^m\left(a+b\log\left(cx\right.\right.}{gm}$$

```
[Out] 2*b*k*n*(g*x)^m/g/m^2-k*(g*x)^m*(a+b*ln(c*x^n))/g/m-b*e*k*n*(g*x)^m*ln(e+f*
x^m)/f/g/m^2/(x^m)-b*e*k*n*(g*x)^m*ln(-f*x^m/e)*ln(e+f*x^m)/f/g/m^2/(x^m)+e
*k*(g*x)^m*(a+b*ln(c*x^n))*ln(e+f*x^m)/f/g/m/(x^m)-b*n*(g*x)^m*ln(d*(e+f*x^
m)^k)/g/m^2+(g*x)^m*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m-b*e*k*n*(g*x)^m*p
olylog(2,1+f*x^m/e)/f/g/m^2/(x^m)
```

Rubi [A] time = 0.25, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {2455, 20, 266, 43, 2376, 16, 32, 19, 2454, 2394, 2315}

$$-\frac{beknx^{-m}(gx)^m\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{fgm^2}+\frac{(gx)^m\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{gm}+\frac{ekx^{-m}(gx)^m\log\left(e+fx^m\right)}{fgm}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 + m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (2*b*k*n*(g*x)^m)/(g*m^2) - (k*(g*x)^m*(a + b*Log[c*x^n]))/(g*m) - (b*e*k*n
*(g*x)^m*Log[e + f*x^m])/(f*g*m^2*x^m) - (b*e*k*n*(g*x)^m*Log[-((f*x^m)/e)]
*Log[e + f*x^m])/(f*g*m^2*x^m) + (e*k*(g*x)^m*(a + b*Log[c*x^n])*Log[e + f*
x^m])/(f*g*m*x^m) - (b*n*(g*x)^m*Log[d*(e + f*x^m)^k])/(g*m^2) + ((g*x)^m*(
a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/(g*m) - (b*e*k*n*(g*x)^m*PolyLog[2,
 1 + (f*x^m)/e])/(f*g*m^2*x^m)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx = -\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}+\frac{ekx^{-m}(gx)^m\left(a+b\log\left(cx^n\right)\right)}{fgm}$$

$$= -\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}+\frac{ekx^{-m}(gx)^m\left(a+b\log\left(cx^n\right)\right)}{fgm}$$

$$= -\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}+\frac{ekx^{-m}(gx)^m\left(a+b\log\left(cx^n\right)\right)}{fgm}$$

$$= \frac{bkn(gx)^m}{gm^2}-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}+\frac{ekx^{-m}(gx)^m\left(a+\right.}{}$$

$$= \frac{bkn(gx)^m}{gm^2}-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}-\frac{beknx^{-m}(gx)^m\,\mathrm{lo}}{}$$

$$= \frac{bkn(gx)^m}{gm^2}-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}-\frac{beknx^{-m}(gx)^m\,\mathrm{lo}}{}$$

$$= \frac{bkn(gx)^m}{gm^2}-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}-\frac{beknx^{-m}(gx)^m\,\mathrm{lo}}{}$$

$$= \frac{2bkn(gx)^m}{gm^2}-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}-\frac{beknx^{-m}(gx)^m\,\mathrm{l}}{fgn}$$

Mathematica [A] time = 0.23, size = 268, normalized size = 1.05

$$-\frac{x^{-m}(gx)^m\left(-ekm\log(x)\left(am+bm\log\left(cx^n\right)+bn\log\left(e+fx^m\right)-bn\log\left(e-ex^m\right)-bn\right)-afmx^m\log\left(d\left(e+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 + m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] -(((g*x)^m*(a*f*k*m*x^m - 2*b*f*k*n*x^m + b*e*k*m^2*n*Log[x]^2 + b*f*k*m*x^
m*Log[c*x^n] - a*e*k*m*Log[e - e*x^m] + b*e*k*n*Log[e - e*x^m] - b*e*k*m*Lo
g[c*x^n]*Log[e - e*x^m] + b*e*k*n*Log[-((f*x^m)/e)]*Log[e + f*x^m] - e*k*m*
Log[x]*(a*m - b*n + b*m*Log[c*x^n] - b*n*Log[e - e*x^m] + b*n*Log[e + f*x^m
]) - a*f*m*x^m*Log[d*(e + f*x^m)^k] + b*f*n*x^m*Log[d*(e + f*x^m)^k] - b*f*
m*x^m*Log[c*x^n]*Log[d*(e + f*x^m)^k] + b*e*k*n*PolyLog[2, 1 + (f*x^m)/e]))
/(f*g*m^2*x^m))
```

fricas [A] time = 0.75, size = 196, normalized size = 0.77

$$\frac{beg^{m-1}kmn\log(x)\log\left(\frac{fx^m+e}{e}\right)+beg^{m-1}kn\mathrm{Li}_2\left(-\frac{fx^m+e}{e}+1\right)-\left(bfkm\log(c)+afkm-2\,bfkn-\left(bfm\log(c)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fr
icas")
```

```
[Out] (b*e*g^(m - 1)*k*m*n*log(x)*log((f*x^m + e)/e) + b*e*g^(m - 1)*k*n*dilog(-(
f*x^m + e)/e + 1) - (b*f*k*m*log(c) + a*f*k*m - 2*b*f*k*n - (b*f*m*log(c) +
```

```
a*f*m - b*f*n)*log(d) + (b*f*k*m*n - b*f*m*n*log(d))*log(x))*g^(m - 1)*x^m
+ ((b*f*k*m*n*log(x) + b*f*k*m*log(c) + a*f*k*m - b*f*k*n)*g^(m - 1)*x^m +
(b*e*k*m*log(c) + a*e*k*m - b*e*k*n)*g^(m - 1))*log(f*x^m + e))/(f*m^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\log(cx^n) + a)(gx)^{m-1}\log\left((fx^m + e)^k d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="gi
ac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(m - 1)*log((f*x^m + e)^k*d), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (b\ln(cx^n) + a)(gx)^{m-1}\ln\left(d\left(fx^m + e\right)^k\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(m-1)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

```
[Out] int((g*x)^(m-1)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(bg^m mx^m\log(x^n) + \left(ag^m m + \left(g^m m\log(c) - g^m n\right)b\right)x^m\right)\log\left(\left(fx^m + e\right)^k\right)}{gm^2} + \int -\frac{\left(\left(fg^m km - fg^m m\log(d)\right)a - \left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="ma
xima")
```

```
[Out] (b*g^m*m*x^m*log(x^n) + (a*g^m*m + (g^m*m*log(c) - g^m*n)*b)*x^m)*log((f*x^
m + e)^k)/(g*m^2) + integrate(-(((f*g^m*k*m - f*g^m*m*log(d))*a - (f*g^m*k*
n - (f*g^m*k*m - f*g^m*m*log(d))*log(c))*b)*x^(2*m) - (b*e*g^m*m*log(c)*log
(d) + a*e*g^m*m*log(d))*x^m - (b*e*g^m*m*x^m*log(d) - (f*g^m*k*m - f*g^m*m*
log(d))*b*x^(2*m))*log(x^n))/(f*g*m*x*x^m + e*g*m*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(e + fx^m\right)^k\right)(gx)^{m-1}(a + b\ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^k)*(g*x)^(m - 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^m)^k)*(g*x)^(m - 1)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1+m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Timed out
```

3.153 $\int (gx)^{-1-m} \left(a + b\log\left(cx^n\right)\right) \log\left(d\left(e + fx^m\right)^k\right) dx$

Optimal. Leaf size=304

$$-\frac{(gx)^{-m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{gm}+\frac{fkx^m\log(x)(gx)^{-m}\left(a+b\log\left(cx^n\right)\right)}{eg}-\frac{fkx^m(gx)^{-m}\log\left(e+fx^m\right.}{egm}$$

```
[Out] b*f*k*n*x^m*ln(x)/e/g/m/((g*x)^m)-1/2*b*f*k*n*x^m*ln(x)^2/e/g/((g*x)^m)+f*k
*x^m*ln(x)*(a+b*ln(c*x^n))/e/g/((g*x)^m)-b*f*k*n*x^m*ln(e+f*x^m)/e/g/m^2/((
g*x)^m)+b*f*k*n*x^m*ln(-f*x^m/e)*ln(e+f*x^m)/e/g/m^2/((g*x)^m)-f*k*x^m*(a+b
*ln(c*x^n))*ln(e+f*x^m)/e/g/m/((g*x)^m)-b*n*ln(d*(e+f*x^m)^k)/g/m^2/((g*x)^
m)-(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m/((g*x)^m)+b*f*k*n*x^m*polylog(2,1+
f*x^m/e)/e/g/m^2/((g*x)^m)
```

Rubi [A] time = 0.31, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2455, 19, 266, 36, 29, 31, 2376, 2301, 2454, 2394, 2315, 16}

$$\frac{bfknx^m(gx)^{-m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{egm^2}-\frac{(gx)^{-m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{gm}+\frac{fkx^m\log(x)(gx)^{-m}\left(a+b\right.}{eg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 - m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (b*f*k*n*x^m*Log[x])/(e*g*m*(g*x)^m) - (b*f*k*n*x^m*Log[x]^2)/(2*e*g*(g*x)^
m) + (f*k*x^m*Log[x]*(a + b*Log[c*x^n]))/(e*g*(g*x)^m) - (b*f*k*n*x^m*Log[e
 + f*x^m])/(e*g*m^2*(g*x)^m) + (b*f*k*n*x^m*Log[-((f*x^m)/e)]*Log[e + f*x^m
])/(e*g*m^2*(g*x)^m) - (f*k*x^m*(a + b*Log[c*x^n])*Log[e + f*x^m])/(e*g*m*(
g*x)^m) - (b*n*Log[d*(e + f*x^m)^k])/(g*m^2*(g*x)^m) - ((a + b*Log[c*x^n])*
Log[d*(e + f*x^m)^k])/(g*m*(g*x)^m) + (b*f*k*n*x^m*PolyLog[2, 1 + (f*x^m)/e
])/(e*g*m^2*(g*x)^m)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !I
ntegerQ[m] && !IntegerQ[n] && IntegerQ[m + n]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1-m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx=\frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\left(cx^n\right)\right)}{eg}-\frac{fkx^m(gx)^{-m}\left(a+\right.}{}$$

$$=\frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\left(cx^n\right)\right)}{eg}-\frac{fkx^m(gx)^{-m}\left(a+\right.}{}$$

$$=\frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\left(cx^n\right)\right)}{eg}-\frac{fkx^m(gx)^{-m}\left(a+\right.}{}$$

$$=-\frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg}+\frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\right.}{eg}$$

$$=-\frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg}+\frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\right.}{eg}$$

$$=-\frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg}+\frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\right.}{eg}$$

$$=-\frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg}+\frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\right.}{eg}$$

$$=\frac{bfknx^m(gx)^{-m}\log(x)}{egm}-\frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg}+\frac{fkx^m}{}$$

Mathematica [A] time = 0.36, size = 162, normalized size = 0.53

$$\frac{(gx)^{-m}\left(-2\left(am+bm\log\left(cx^n\right)+bn\right)\left(e\log\left(d\left(e+fx^m\right)^k\right)+fkx^m\log\left(f-fx^{-m}\right)\right)+2fkmx^m\log(x)\left(am+\right.}{2egm^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 - m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (-(b*f*k*m^2*n*x^m*Log[x]^2) - 2*(a*m + b*n + b*m*Log[c*x^n])*(f*k*x^m*Log[
f - f/x^m] + e*Log[d*(e + f*x^m)^k]) + 2*f*k*m*x^m*Log[x]*(a*m + b*n + b*m*
Log[c*x^n] + b*n*Log[f - f/x^m] - b*n*Log[1 + (f*x^m)/e]) - 2*b*f*k*n*x^m*P
olyLog[2, -((f*x^m)/e)])/(2*e*g*m^2*(g*x)^m)
```

fricas [A] time = 0.93, size = 239, normalized size = 0.79

$$-\frac{2\,bfg^{-m-1}kmnx^m\log(x)\log\left(\frac{fx^m+e}{e}\right)+2\,bfg^{-m-1}knx^m\mathrm{Li}_2\left(-\frac{fx^m+e}{e}+1\right)-\left(bfkm^2n\log(x)^2+2\left(bfkm^2\log(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*b*f*g^(-m - 1)*k*m*n*x^m*log(x)*log((f*x^m + e)/e) + 2*b*f*g^(-m -
1)*k*n*x^m*dilog(-(f*x^m + e)/e + 1) - (b*f*k*m^2*n*log(x)^2 + 2*(b*f*k*m^2
*log(c) + a*f*k*m^2 + b*f*k*m*n)*log(x))*g^(-m - 1)*x^m + 2*(b*e*m*n*log(d)
*log(x) + (b*e*m*log(c) + a*e*m + b*e*n)*log(d))*g^(-m - 1) + 2*((b*f*k*m*l
```

```
og(c) + a*f*k*m + b*f*k*n)*g^(-m - 1)*x^m + (b*e*k*m*n*log(x) + b*e*k*m*log
(c) + a*e*k*m + b*e*k*n)*g^(-m - 1))*log(f*x^m + e))/(e*m^2*x^m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^{n}\right) + a\right) \left(gx\right)^{-m-1} \log\left(\left(fx^{m} + e\right)^{k} d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="gi
ac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(-m - 1)*log((f*x^m + e)^k*d), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\, x^{n}\right) + a\right) \left(gx\right)^{-m-1} \ln\left(d \left(f\, x^{m} + e\right)^{k}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-m-1)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

```
[Out] int((g*x)^(-m-1)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(bm \log\left(x^{n}\right) + \left(m \log(c) + n\right)b + am\right)g^{-m-1} \log\left(\left(fx^{m} + e\right)^{k}\right)}{m^{2}x^{m}} + \int \frac{bem \log(c) \log(d) + aem \log(d) + \left(\left(fkm + f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="ma
xima")
```

```
[Out] -(b*m*log(x^n) + (m*log(c) + n)*b + a*m)*g^(-m - 1)*log((f*x^m + e)^k)/(m^2
*x^m) + integrate((b*e*m*log(c)*log(d) + a*e*m*log(d) + ((f*k*m + f*m*log(d
))*a + (f*k*n + (f*k*m + f*m*log(d))*log(c))*b)*x^m + (b*e*m*log(d) + (f*k*
m + f*m*log(d))*b*x^m)*log(x^n))/(f*g^(m + 1)*m*x*x^(2*m) + e*g^(m + 1)*m*x
*x^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d \left(e + f\, x^{m}\right)^{k}\right) \left(a + b\, \ln\left(c\, x^{n}\right)\right)}{\left(g\, x\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(m + 1),x)
```

```
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1-m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Timed out
```

3.154 $\int (gx)^{-1-2m} \left(a+b\log\left(cx^n\right)\right) \log\left(d\left(e+fx^m\right)^k\right) dx$

Optimal. Leaf size=414

$$-\frac{(gx)^{-2m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{2gm}-\frac{f^2kx^{2m}\log(x)(gx)^{-2m}\left(a+b\log\left(cx^n\right)\right)}{2e^2g}+\frac{f^2kx^{2m}(gx)^{-2m}\log\left(e\right.}{2e}$$

```
[Out] -3/4*b*f*k*n*x^m/e/g/m^2/((g*x)^(2*m))-1/4*b*f^2*k*n*x^(2*m)*ln(x)/e^2/g/m/
((g*x)^(2*m))+1/4*b*f^2*k*n*x^(2*m)*ln(x)^2/e^2/g/((g*x)^(2*m))-1/2*f*k*x^m
*(a+b*ln(c*x^n))/e/g/m/((g*x)^(2*m))-1/2*f^2*k*x^(2*m)*ln(x)*(a+b*ln(c*x^n)
)/e^2/g/((g*x)^(2*m))+1/4*b*f^2*k*n*x^(2*m)*ln(e+f*x^m)/e^2/g/m^2/((g*x)^(2
*m))-1/2*b*f^2*k*n*x^(2*m)*ln(-f*x^m/e)*ln(e+f*x^m)/e^2/g/m^2/((g*x)^(2*m))
+1/2*f^2*k*x^(2*m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/e^2/g/m/((g*x)^(2*m))-1/4*b*
n*ln(d*(e+f*x^m)^k)/g/m^2/((g*x)^(2*m))-1/2*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^
k)/g/m/((g*x)^(2*m))-1/2*b*f^2*k*n*x^(2*m)*polylog(2,1+f*x^m/e)/e^2/g/m^2/(
(g*x)^(2*m))
```

Rubi [A] time = 0.52, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2455, 20, 266, 44, 2376, 30, 19, 2301, 2454, 2394, 2315, 16}

$$-\frac{bf^2knx^{2m}(gx)^{-2m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{2e^2gm^2}-\frac{(gx)^{-2m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{2gm}-\frac{f^2kx^{2m}\log(x)(gx)^{-2m}}{2e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 - 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (-3*b*f*k*n*x^m)/(4*e*g*m^2*(g*x)^(2*m)) - (b*f^2*k*n*x^(2*m)*Log[x])/(4*e^
2*g*m*(g*x)^(2*m)) + (b*f^2*k*n*x^(2*m)*Log[x]^2)/(4*e^2*g*(g*x)^(2*m)) - (
f*k*x^m*(a + b*Log[c*x^n]))/(2*e*g*m*(g*x)^(2*m)) - (f^2*k*x^(2*m)*Log[x]*(
a + b*Log[c*x^n]))/(2*e^2*g*(g*x)^(2*m)) + (b*f^2*k*n*x^(2*m)*Log[e + f*x^m
])/(4*e^2*g*m^2*(g*x)^(2*m)) - (b*f^2*k*n*x^(2*m)*Log[-((f*x^m)/e)]*Log[e +
 f*x^m])/(2*e^2*g*m^2*(g*x)^(2*m)) + (f^2*k*x^(2*m)*(a + b*Log[c*x^n])*Log[
e + f*x^m])/(2*e^2*g*m*(g*x)^(2*m)) - (b*n*Log[d*(e + f*x^m)^k])/(4*g*m^2*(
g*x)^(2*m)) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/(2*g*m*(g*x)^(2*m))
 - (b*f^2*k*n*x^(2*m)*PolyLog[2, 1 + (f*x^m)/e])/(2*e^2*g*m^2*(g*x)^(2*m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !I
ntegerQ[m] && !IntegerQ[n] && IntegerQ[m + n]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1-2m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)dx = -\frac{fkx^m(gx)^{-2m}\left(a+b\log\left(cx^n\right)\right)}{2egm} - \frac{f^2kx^{2m}(gx)^{-2m}\log(x}{2e^2}$$

$$= -\frac{fkx^m(gx)^{-2m}\left(a+b\log\left(cx^n\right)\right)}{2egm} - \frac{f^2kx^{2m}(gx)^{-2m}\log(x}{2e^2}$$

$$= -\frac{fkx^m(gx)^{-2m}\left(a+b\log\left(cx^n\right)\right)}{2egm} - \frac{f^2kx^{2m}(gx)^{-2m}\log(x}{2e^2}$$

$$= -\frac{bfknx^m(gx)^{-2m}}{2egm^2} + \frac{bf^2knx^{2m}(gx)^{-2m}\log^2(x)}{4e^2g} - \frac{fkx^m(}{}$$

$$= -\frac{bfknx^m(gx)^{-2m}}{2egm^2} + \frac{bf^2knx^{2m}(gx)^{-2m}\log^2(x)}{4e^2g} - \frac{fkx^m(}{}$$

$$= -\frac{bfknx^m(gx)^{-2m}}{2egm^2} + \frac{bf^2knx^{2m}(gx)^{-2m}\log^2(x)}{4e^2g} - \frac{fkx^m(}{}$$

$$= -\frac{bfknx^m(gx)^{-2m}}{2egm^2} + \frac{bf^2knx^{2m}(gx)^{-2m}\log^2(x)}{4e^2g} - \frac{fkx^m(}{}$$

$$= -\frac{3bfknx^m(gx)^{-2m}}{4egm^2} - \frac{bf^2knx^{2m}(gx)^{-2m}\log(x)}{4e^2gm} + \frac{bf^2kn}{}$$

Mathematica [A] time = 0.37, size = 302, normalized size = 0.73

$$\frac{(gx)^{-2m}\left(-f^2kmx^{2m}\log(x)\left(2am+2bm\log\left(cx^n\right)-2bn\log\left(\frac{fx^m}{e}+1\right)+2bn\log\left(f-fx^{-m}\right)+bn\right)-2ae^2m\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 - 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (-2*a*e*f*k*m*x^m - 3*b*e*f*k*n*x^m + b*f^2*k*m^2*n*x^(2*m)*Log[x]^2 - 2*b*
e*f*k*m*x^m*Log[c*x^n] + 2*a*f^2*k*m*x^(2*m)*Log[f - f/x^m] + b*f^2*k*n*x^(
2*m)*Log[f - f/x^m] + 2*b*f^2*k*m*x^(2*m)*Log[c*x^n]*Log[f - f/x^m] - 2*a*e
^2*m*Log[d*(e + f*x^m)^k] - b*e^2*n*Log[d*(e + f*x^m)^k] - 2*b*e^2*m*Log[c*
x^n]*Log[d*(e + f*x^m)^k] - f^2*k*m*x^(2*m)*Log[x]*(2*a*m + b*n + 2*b*m*Log
[c*x^n] + 2*b*n*Log[f - f/x^m] - 2*b*n*Log[1 + (f*x^m)/e]) + 2*b*f^2*k*n*x^
(2*m)*PolyLog[2, -((f*x^m)/e)])/(4*e^2*g*m^2*(g*x)^(2*m))
```

fricas [A] time = 0.67, size = 338, normalized size = 0.82

$$\frac{2\,bf^2g^{-2\,m-1}kmnx^{2\,m}\log(x)\log\left(\frac{fx^m+e}{e}\right)+2\,bf^2g^{-2\,m-1}knx^{2\,m}\mathrm{Li}_2\left(-\frac{fx^m+e}{e}+1\right)-\left(bf^2km^2n\log(x)^2+\left(2\,bf^2kr\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
fricas")
```

```
[Out] 1/4*(2*b*f^2*g^(-2*m - 1)*k*m*n*x^(2*m)*log(x)*log((f*x^m + e)/e) + 2*b*f^2
*g^(-2*m - 1)*k*n*x^(2*m)*dilog(-(f*x^m + e)/e + 1) - (b*f^2*k*m^2*n*log(x)
```

```
^2 + (2*b*f^2*k*m^2*log(c) + 2*a*f^2*k*m^2 + b*f^2*k*m*n)*log(x))*g^(-2*m -
 1)*x^(2*m) - (2*b*e*f*k*m*n*log(x) + 2*b*e*f*k*m*log(c) + 2*a*e*f*k*m + 3*
b*e*f*k*n)*g^(-2*m - 1)*x^m - (2*b*e^2*m*n*log(d)*log(x) + (2*b*e^2*m*log(c
) + 2*a*e^2*m + b*e^2*n)*log(d))*g^(-2*m - 1) + ((2*b*f^2*k*m*log(c) + 2*a*
f^2*k*m + b*f^2*k*n)*g^(-2*m - 1)*x^(2*m) - (2*b*e^2*k*m*n*log(x) + 2*b*e^2
*k*m*log(c) + 2*a*e^2*k*m + b*e^2*k*n)*g^(-2*m - 1))*log(f*x^m + e))/(e^2*m
^2*x^(2*m))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^{n}\right)+a\right)\left(gx\right)^{-2\,m-1}\log\left(\left(fx^{m}+e\right)^{k}d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(-2*m - 1)*log((f*x^m + e)^k*d), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^{n}\right)+a\right)\left(gx\right)^{-2m-1}\ln\left(d\left(f\,x^{m}+e\right)^{k}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-1-2*m)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

```
[Out] int((g*x)^(-1-2*m)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,bm\log\left(x^{n}\right)+\left(2\,m\log(c)+n\right)b+2\,am\right)g^{-2\,m-1}\log\left(\left(fx^{m}+e\right)^{k}\right)}{4\,m^{2}x^{2\,m}}+\int\frac{4\,bem\log(c)\log(d)+4\,aem\log(d)+(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
maxima")
```

```
[Out] -1/4*(2*b*m*log(x^n) + (2*m*log(c) + n)*b + 2*a*m)*g^(-2*m - 1)*log((f*x^m
+ e)^k)/(m^2*x^(2*m)) + integrate(1/4*(4*b*e*m*log(c)*log(d) + 4*a*e*m*log(
d) + (2*(f*k*m + 2*f*m*log(d))*a + (f*k*n + 2*(f*k*m + 2*f*m*log(d))*log(c)
)*b)*x^m + 2*(2*b*e*m*log(d) + (f*k*m + 2*f*m*log(d))*b*x^m)*log(x^n))/(f*g
^(2*m + 1)*m*x*x^(3*m) + e*g^(2*m + 1)*m*x*x^(2*m)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\,x^{m}\right)^{k}\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)}{{\left(g\,x\right)}^{2\,m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(2*m + 1),x)
```

```
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(2*m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1-2*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Timed out
```

3.155 $\int (gx)^{-1-3m} \left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$

Optimal. Leaf size=484

$$-\frac{(gx)^{-3m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{3gm}+\frac{f^3kx^{3m}\log(x)(gx)^{-3m}\left(a+b\log\left(cx^n\right)\right)}{3e^3g}-\frac{f^3kx^{3m}(gx)^{-3m}\log\left(e+\right.}{3e^3g}$$

```
[Out] -5/36*b*f*k*n*x^m/e/g/m^2/((g*x)^(3*m))+4/9*b*f^2*k*n*x^(2*m)/e^2/g/m^2/((g
*x)^(3*m))+1/9*b*f^3*k*n*x^(3*m)*ln(x)/e^3/g/m/((g*x)^(3*m))-1/6*b*f^3*k*n*
x^(3*m)*ln(x)^2/e^3/g/((g*x)^(3*m))-1/6*f*k*x^m*(a+b*ln(c*x^n))/e/g/m/((g*x
)^(3*m))+1/3*f^2*k*x^(2*m)*(a+b*ln(c*x^n))/e^2/g/m/((g*x)^(3*m))+1/3*f^3*k*
x^(3*m)*ln(x)*(a+b*ln(c*x^n))/e^3/g/((g*x)^(3*m))-1/9*b*f^3*k*n*x^(3*m)*ln(
e+f*x^m)/e^3/g/m^2/((g*x)^(3*m))+1/3*b*f^3*k*n*x^(3*m)*ln(-f*x^m/e)*ln(e+f*
x^m)/e^3/g/m^2/((g*x)^(3*m))-1/3*f^3*k*x^(3*m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/
e^3/g/m/((g*x)^(3*m))-1/9*b*n*ln(d*(e+f*x^m)^k)/g/m^2/((g*x)^(3*m))-1/3*(a+
b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m/((g*x)^(3*m))+1/3*b*f^3*k*n*x^(3*m)*poly
log(2,1+f*x^m/e)/e^3/g/m^2/((g*x)^(3*m))
```

Rubi [A] time = 0.70, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2455, 20, 266, 44, 2376, 30, 19, 2301, 2454, 2394, 2315, 16}

$$\frac{bf^3knx^{3m}(gx)^{-3m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{3e^3gm^2}-\frac{(gx)^{-3m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{3gm}+\frac{f^2kx^{2m}(gx)^{-3m}\left(a+b\log\right.}{3e^2gm}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 - 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (-5*b*f*k*n*x^m)/(36*e*g*m^2*(g*x)^(3*m)) + (4*b*f^2*k*n*x^(2*m))/(9*e^2*g*
m^2*(g*x)^(3*m)) + (b*f^3*k*n*x^(3*m)*Log[x])/(9*e^3*g*m*(g*x)^(3*m)) - (b*
f^3*k*n*x^(3*m)*Log[x]^2)/(6*e^3*g*(g*x)^(3*m)) - (f*k*x^m*(a + b*Log[c*x^n
]))/(6*e*g*m*(g*x)^(3*m)) + (f^2*k*x^(2*m)*(a + b*Log[c*x^n]))/(3*e^2*g*m*(
g*x)^(3*m)) + (f^3*k*x^(3*m)*Log[x]*(a + b*Log[c*x^n]))/(3*e^3*g*(g*x)^(3*m
)) - (b*f^3*k*n*x^(3*m)*Log[e + f*x^m])/(9*e^3*g*m^2*(g*x)^(3*m)) + (b*f^3*
k*n*x^(3*m)*Log[-((f*x^m)/e)]*Log[e + f*x^m])/(3*e^3*g*m^2*(g*x)^(3*m)) - (
f^3*k*x^(3*m)*(a + b*Log[c*x^n])*Log[e + f*x^m])/(3*e^3*g*m*(g*x)^(3*m)) -
(b*n*Log[d*(e + f*x^m)^k])/(9*g*m^2*(g*x)^(3*m)) - ((a + b*Log[c*x^n])*Log[
d*(e + f*x^m)^k])/(3*g*m*(g*x)^(3*m)) + (b*f^3*k*n*x^(3*m)*PolyLog[2, 1 + (
f*x^m)/e])/(3*e^3*g*m^2*(g*x)^(3*m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
```

```
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1-3m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)dx = -\frac{fkx^m(gx)^{-3m}\left(a+b\log\left(cx^n\right)\right)}{6egm}+\frac{f^2kx^{2m}(gx)^{-3m}\left(a+b\log\right.}{3e^2gm}$$

$$= -\frac{fkx^m(gx)^{-3m}\left(a+b\log\left(cx^n\right)\right)}{6egm}+\frac{f^2kx^{2m}(gx)^{-3m}\left(a+b\log\right.}{3e^2gm}$$

$$= -\frac{fkx^m(gx)^{-3m}\left(a+b\log\left(cx^n\right)\right)}{6egm}+\frac{f^2kx^{2m}(gx)^{-3m}\left(a+b\log\right.}{3e^2gm}$$

$$= -\frac{bfknx^m(gx)^{-3m}}{12egm^2}+\frac{bf^2knx^{2m}(gx)^{-3m}}{3e^2gm^2}-\frac{bf^3knx^{3m}(gx)^{-3m}}{6e^3g}$$

$$= -\frac{bfknx^m(gx)^{-3m}}{12egm^2}+\frac{bf^2knx^{2m}(gx)^{-3m}}{3e^2gm^2}-\frac{bf^3knx^{3m}(gx)^{-3m}}{6e^3g}$$

$$= -\frac{bfknx^m(gx)^{-3m}}{12egm^2}+\frac{bf^2knx^{2m}(gx)^{-3m}}{3e^2gm^2}-\frac{bf^3knx^{3m}(gx)^{-3m}}{6e^3g}$$

$$= -\frac{bfknx^m(gx)^{-3m}}{12egm^2}+\frac{bf^2knx^{2m}(gx)^{-3m}}{3e^2gm^2}-\frac{bf^3knx^{3m}(gx)^{-3m}}{6e^3g}$$

$$= -\frac{5bfknx^m(gx)^{-3m}}{36egm^2}+\frac{4bf^2knx^{2m}(gx)^{-3m}}{9e^2gm^2}+\frac{bf^3knx^{3m}(gx)}{9e^3gm}$$

Mathematica [A] time = 0.42, size = 358, normalized size = 0.74

$$\frac{(gx)^{-3m}\left(4f^3kmx^{3m}\log(x)\left(3am+3bm\log\left(cx^n\right)-3bn\log\left(\frac{fx^m}{e}+1\right)+3bn\log\left(f-fx^{-m}\right)+bn\right)-12ae^3m\log\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 - 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (-6*a*e^2*f*k*m*x^m - 5*b*e^2*f*k*n*x^m + 12*a*e*f^2*k*m*x^(2*m) + 16*b*e*f
^2*k*n*x^(2*m) - 6*b*f^3*k*m^2*n*x^(3*m)*Log[x]^2 - 6*b*e^2*f*k*m*x^m*Log[c
*x^n] + 12*b*e*f^2*k*m*x^(2*m)*Log[c*x^n] - 12*a*f^3*k*m*x^(3*m)*Log[f - f/
x^m] - 4*b*f^3*k*n*x^(3*m)*Log[f - f/x^m] - 12*b*f^3*k*m*x^(3*m)*Log[c*x^n]
*Log[f - f/x^m] - 12*a*e^3*m*Log[d*(e + f*x^m)^k] - 4*b*e^3*n*Log[d*(e + f*
x^m)^k] - 12*b*e^3*m*Log[c*x^n]*Log[d*(e + f*x^m)^k] + 4*f^3*k*m*x^(3*m)*Lo
g[x]*(3*a*m + b*n + 3*b*m*Log[c*x^n] + 3*b*n*Log[f - f/x^m] - 3*b*n*Log[1 +
 (f*x^m)/e]) - 12*b*f^3*k*n*x^(3*m)*PolyLog[2, -((f*x^m)/e)])/(36*e^3*g*m^2
*(g*x)^(3*m))
```

fricas [A] time = 0.99, size = 403, normalized size = 0.83

$$-\frac{12\,bf^3g^{-3\,m-1}kmnx^{3\,m}\log(x)\log\left(\frac{fx^m+e}{e}\right)+12\,bf^3g^{-3\,m-1}knx^{3\,m}\mathrm{Li}_2\left(-\frac{fx^m+e}{e}+1\right)-2\left(3\,bf^3km^2n\log(x)^2+2\left(3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
fricas")
```

```
[Out] -1/36*(12*b*f^3*g^(-3*m - 1)*k*m*n*x^(3*m)*log(x)*log((f*x^m + e)/e) + 12*b
*f^3*g^(-3*m - 1)*k*n*x^(3*m)*dilog(-(f*x^m + e)/e + 1) - 2*(3*b*f^3*k*m^2*
n*log(x)^2 + 2*(3*b*f^3*k*m^2*log(c) + 3*a*f^3*k*m^2 + b*f^3*k*m*n)*log(x))
*g^(-3*m - 1)*x^(3*m) - 4*(3*b*e*f^2*k*m*n*log(x) + 3*b*e*f^2*k*m*log(c) +
3*a*e*f^2*k*m + 4*b*e*f^2*k*n)*g^(-3*m - 1)*x^(2*m) + (6*b*e^2*f*k*m*n*log(
x) + 6*b*e^2*f*k*m*log(c) + 6*a*e^2*f*k*m + 5*b*e^2*f*k*n)*g^(-3*m - 1)*x^m
 + 4*(3*b*e^3*m*n*log(d)*log(x) + (3*b*e^3*m*log(c) + 3*a*e^3*m + b*e^3*n)*
log(d))*g^(-3*m - 1) + 4*((3*b*f^3*k*m*log(c) + 3*a*f^3*k*m + b*f^3*k*n)*g^
(-3*m - 1)*x^(3*m) + (3*b*e^3*k*m*n*log(x) + 3*b*e^3*k*m*log(c) + 3*a*e^3*k
*m + b*e^3*k*n)*g^(-3*m - 1))*log(f*x^m + e))/(e^3*m^2*x^(3*m))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(cx^{n}\right) + a\right) \left(gx\right)^{-3 \, m - 1} \log \left(\left(fx^{m} + e\right)^{k} d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(-3*m - 1)*log((f*x^m + e)^k*d), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \,x^{n}\right)+a \right) \left(g x \right)^{-3 m -1} \ln \left(d \left(f \,x^{m}+e \right)^{k}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-1-3*m)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

```
[Out] int((g*x)^(-1-3*m)*(b*ln(c*x^n)+a)*ln(d*(f*x^m+e)^k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3 \, bm \log \left(x^{n}\right) + \left(3 \, m \log(c) + n\right)b + 3 \, am\right)g^{-3 \, m - 1} \log \left(\left(fx^{m} + e\right)^{k}\right)}{9 \, m^{2}x^{3 \, m}} + \int \frac{9 \, bem \log(c) \log(d) + 9 \, aem \log(d) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
maxima")
```

```
[Out] -1/9*(3*b*m*log(x^n) + (3*m*log(c) + n)*b + 3*a*m)*g^(-3*m - 1)*log((f*x^m
+ e)^k)/(m^2*x^(3*m)) + integrate(1/9*(9*b*e*m*log(c)*log(d) + 9*a*e*m*log(
d) + (3*(f*k*m + 3*f*m*log(d))*a + (f*k*n + 3*(f*k*m + 3*f*m*log(d))*log(c)
)*b)*x^m + 3*(3*b*e*m*log(d) + (f*k*m + 3*f*m*log(d))*b*x^m)*log(x^n))/(f*g
^(3*m + 1)*m*x*x^(4*m) + e*g^(3*m + 1)*m*x*x^(3*m)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{{\left(g\,x\right)}^{3\,m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(3*m + 1),x)
```

```
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(3*m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1-3*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
[Out] Timed out
```

3.156 $\int x^2 \left(a+b\log \left(cx^n\right)\right) \left(d+e\log \left(fx^r\right)\right) \, dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)-\frac{1}{27}erx^3\left(3a+3b\log\left(cx^n\right)-bn\right)-\frac{1}{9}bnx^3\left(d+e\log\left(fx^r\right)\right)+\frac{1}{27}benrx^3$$

```
[Out] 1/27*b*e*n*r*x^3-1/27*e*r*x^3*(3*a-b*n+3*b*ln(c*x^n))-1/9*b*n*x^3*(d+e*ln(f
*x^r))+1/3*x^3*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304, 2366, 12}

$$\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)-\frac{1}{27}erx^3\left(3a+3b\log\left(cx^n\right)-bn\right)-\frac{1}{9}bnx^3\left(d+e\log\left(fx^r\right)\right)+\frac{1}{27}benrx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] (b*e*n*r*x^3)/27 - (e*r*x^3*(3*a - b*n + 3*b*Log[c*x^n]))/27 - (b*n*x^3*(d
+ e*Log[f*x^r]))/9 + (x^3*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)\,dx &= -\frac{1}{9}bnx^3\left(d+e\log\left(fx^r\right)\right)+\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right.\right.\\ &= -\frac{1}{9}bnx^3\left(d+e\log\left(fx^r\right)\right)+\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right.\right.\\ &= \frac{1}{27}benrx^3-\frac{1}{27}erx^3\left(3a-bn+3b\log\left(cx^n\right)\right)-\frac{1}{9}bnx^3\left(d+e\log\right.\end{aligned}$$

Mathematica [A] time = 0.08, size = 71, normalized size = 0.85

$$\frac{1}{27}x^3\left((9ae-3ben)\log\left(fx^r\right)+9ad-3aer+3b\log\left(cx^n\right)\left(3d+3e\log\left(fx^r\right)-er\right)-3bdn+2benr\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] (x^3*(9*a*d - 3*b*d*n - 3*a*e*r + 2*b*e*n*r + (9*a*e - 3*b*e*n)*Log[f*x^r]
+ 3*b*Log[c*x^n]*(3*d - e*r + 3*e*Log[f*x^r])))/27
```

fricas [A] time = 0.75, size = 134, normalized size = 1.60

$$\frac{1}{3}benrx^3\log(x)^2-\frac{1}{9}(ber-3bd)x^3\log(c)-\frac{1}{27}(3bdn-9ad-(2ben-3ae)r)x^3+\frac{1}{9}\left(3bex^3\log(c)-(ben-3ae)x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] 1/3*b*e*n*r*x^3*log(x)^2 - 1/9*(b*e*r - 3*b*d)*x^3*log(c) - 1/27*(3*b*d*n -
 9*a*d - (2*b*e*n - 3*a*e)*r)*x^3 + 1/9*(3*b*e*x^3*log(c) - (b*e*n - 3*a*e)
*x^3)*log(f) + 1/9*(3*b*e*r*x^3*log(c) + 3*b*e*n*x^3*log(f) + (3*b*d*n - (2
*b*e*n - 3*a*e)*r)*x^3)*log(x)
```

giac [B] time = 0.35, size = 161, normalized size = 1.92

$$\frac{1}{3}bnrx^3e\log(x)^2-\frac{2}{9}bnrx^3e\log(x)+\frac{1}{3}brx^3e\log(c)\log(x)+\frac{1}{3}bnx^3e\log(f)\log(x)+\frac{2}{27}bnrx^3e-\frac{1}{9}brx^3e\log(c)-\frac{1}{9}br$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] 1/3*b*n*r*x^3*e*log(x)^2 - 2/9*b*n*r*x^3*e*log(x) + 1/3*b*r*x^3*e*log(c)*lo
g(x) + 1/3*b*n*x^3*e*log(f)*log(x) + 2/27*b*n*r*x^3*e - 1/9*b*r*x^3*e*log(c
) - 1/9*b*n*x^3*e*log(f) + 1/3*b*x^3*e*log(c)*log(f) + 1/3*b*d*n*x^3*log(x)
 + 1/3*a*r*x^3*e*log(x) - 1/9*b*d*n*x^3 - 1/9*a*r*x^3*e + 1/3*b*d*x^3*log(c
) + 1/3*a*x^3*e*log(f) + 1/3*a*d*x^3
```

maple [C] time = 0.53, size = 1640, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)*(d+e*ln(f*x^r)),x)
```

```
[Out] (1/3*b*e*x^3*ln(x^n)+1/6*I*Pi*b*e*x^3*csgn(I*x^n)*csgn(I*c*x^n)^2-1/6*I*Pi*
b*e*x^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/6*I*Pi*b*e*x^3*csgn(I*c*x^n)^
3+1/6*I*Pi*b*e*x^3*csgn(I*c*x^n)^2*csgn(I*c)+1/3*b*e*x^3*ln(c)-1/9*b*e*n*x^
3+1/3*a*e*x^3)*ln(x^r)+1/3*ln(f)*a*e*x^3-1/9*a*e*r*x^3+1/3*a*d*x^3+1/3*b*d*
x^3*ln(x^n)+1/3*b*d*x^3*ln(c)-1/18*I*Pi*b*e*n*x^3*csgn(I*f)*csgn(I*f*x^r)^2
+1/6*I*Pi*b*e*x^3*csgn(I*x^r)*csgn(I*f*x^r)^2*ln(x^n)+1/6*I*Pi*b*d*x^3*csgn
(I*c*x^n)^2*csgn(I*c)-1/6*I*ln(c)*Pi*b*e*x^3*csgn(I*f*x^r)^3-1/6*I*Pi*ln(f)
*b*e*x^3*csgn(I*c*x^n)^3+1/6*I*Pi*a*e*x^3*csgn(I*x^r)*csgn(I*f*x^r)^2+2/27*
b*e*n*r*x^3-1/9*b*d*n*x^3-1/6*I*Pi*b*d*x^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I
*c)+1/12*Pi^2*b*e*x^3*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*c
sgn(I*c)+1/6*I*ln(c)*Pi*b*e*x^3*csgn(I*x^r)*csgn(I*f*x^r)^2-1/18*I*Pi*b*e*r
*x^3*csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*Pi^2*b*e*x^3*csgn(I*c*x^n)^3*csgn(I*f
)*csgn(I*x^r)*csgn(I*f*x^r)-1/12*Pi^2*b*e*x^3*csgn(I*c*x^n)^2*csgn(I*f)*csg
n(I*f*x^r)^2*csgn(I*c)-1/12*Pi^2*b*e*x^3*csgn(I*c*x^n)^2*csgn(I*x^r)*csgn(I
*f*x^r)^2*csgn(I*c)-1/18*I*Pi*b*e*r*x^3*csgn(I*c*x^n)^2*csgn(I*c)-1/18*I*Pi
*b*e*n*x^3*csgn(I*x^r)*csgn(I*f*x^r)^2-1/6*I*Pi*a*e*x^3*csgn(I*f)*csgn(I*x^
r)*csgn(I*f*x^r)+1/6*I*Pi*ln(f)*b*e*x^3*csgn(I*c*x^n)^2*csgn(I*c)+1/6*I*ln(
c)*Pi*b*e*x^3*csgn(I*f)*csgn(I*f*x^r)^2-1/12*Pi^2*b*e*x^3*csgn(I*x^n)*csgn(
I*c*x^n)^2*csgn(I*f)*csgn(I*f*x^r)^2-1/12*Pi^2*b*e*x^3*csgn(I*x^n)*csgn(I*c
*x^n)^2*csgn(I*x^r)*csgn(I*f*x^r)^2-1/12*Pi^2*b*e*x^3*csgn(I*x^n)*csgn(I*c*
x^n)*csgn(I*f*x^r)^3*csgn(I*c)+1/6*I*Pi*ln(f)*b*e*x^3*csgn(I*x^n)*csgn(I*c*
```

```
x^n)^2+1/6*I*Pi*b*e*x^3*csgn(I*f)*csgn(I*f*x^r)^2*ln(x^n)-1/6*I*Pi*b*d*x^3*
csgn(I*c*x^n)^3+1/18*I*Pi*b*e*r*x^3*csgn(I*c*x^n)^3+1/6*I*Pi*a*e*x^3*csgn(I
*f)*csgn(I*f*x^r)^2-1/12*Pi^2*b*e*x^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f)*c
sgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)+1/12*Pi^2*b*e*x^3*csgn(I*x^n)*csgn(I*c*x
^n)*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)+1/12*Pi^2*b*e*x^3*csgn(I*c*x^n)^3
*csgn(I*f)*csgn(I*f*x^r)^2+1/12*Pi^2*b*e*x^3*csgn(I*x^n)*csgn(I*c*x^n)^2*cs
gn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-1/6*I*Pi*b*e*x^3*csgn(I*f)*csgn(I*x^r)*cs
gn(I*f*x^r)*ln(x^n)-1/6*I*ln(c)*Pi*b*e*x^3*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x
^r)-1/12*Pi^2*b*e*x^3*csgn(I*c*x^n)^3*csgn(I*f*x^r)^3-1/6*I*Pi*a*e*x^3*csgn
(I*f*x^r)^3+1/12*Pi^2*b*e*x^3*csgn(I*c*x^n)^3*csgn(I*x^r)*csgn(I*f*x^r)^2+1
/12*Pi^2*b*e*x^3*csgn(I*c*x^n)^2*csgn(I*f*x^r)^3*csgn(I*c)+1/3*ln(f)*b*e*x^
3*ln(x^n)-1/9*b*e*r*x^3*ln(x^n)+1/3*ln(c)*ln(f)*b*e*x^3-1/9*ln(c)*b*e*r*x^3
-1/9*ln(f)*b*e*n*x^3+1/18*I*Pi*b*e*n*x^3*csgn(I*f*x^r)^3-1/6*I*Pi*b*e*x^3*c
sgn(I*f*x^r)^3*ln(x^n)+1/18*I*Pi*b*e*n*x^3*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x
^r)+1/18*I*Pi*b*e*r*x^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/12*Pi^2*b*e*x
^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)+1/6*I*Pi*b
*d*x^3*csgn(I*x^n)*csgn(I*c*x^n)^2+1/12*Pi^2*b*e*x^3*csgn(I*x^n)*csgn(I*c*x
^n)^2*csgn(I*f*x^r)^3-1/6*I*Pi*ln(f)*b*e*x^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn
(I*c)
```

maxima [A] time = 0.59, size = 104, normalized size = 1.24

$$-\frac{1}{9}\,bdnx^3-\frac{1}{9}\,aerx^3+\frac{1}{3}\,bdx^3\log\left(cx^n\right)+\frac{1}{3}\,aex^3\log\left(fx^r\right)+\frac{1}{3}\,adx^3+\frac{1}{27}\,\left(\left(2\,r-3\,\log(f)\right)x^3-3\,x^3\log\left(x^r\right)\right)ben-\frac{1}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] -1/9*b*d*n*x^3 - 1/9*a*e*r*x^3 + 1/3*b*d*x^3*log(c*x^n) + 1/3*a*e*x^3*log(f
*x^r) + 1/3*a*d*x^3 + 1/27*((2*r - 3*log(f))*x^3 - 3*x^3*log(x^r))*b*e*n -
1/9*(r*x^3 - 3*x^3*log(f*x^r))*b*e*log(c*x^n)
```

mupad [B] time = 4.04, size = 82, normalized size = 0.98

$$\ln\left(f\,x^r\right)\,\left(\frac{a\,e\,x^3}{3}-\frac{b\,e\,n\,x^3}{9}+\frac{b\,e\,x^3\,\ln\left(c\,x^n\right)}{3}\right)+x^3\,\left(\frac{a\,d}{3}-\frac{b\,d\,n}{9}-\frac{a\,e\,r}{9}+\frac{2\,b\,e\,n\,r}{27}\right)+\frac{b\,x^3\,\ln\left(c\,x^n\right)\,\left(3\,d-e\,r\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n)),x)
```

```
[Out] log(f*x^r)*((a*e*x^3)/3 - (b*e*n*x^3)/9 + (b*e*x^3*log(c*x^n))/3) + x^3*((a
*d)/3 - (b*d*n)/9 - (a*e*r)/9 + (2*b*e*n*r)/27) + (b*x^3*log(c*x^n)*(3*d -
e*r))/9
```

sympy [B] time = 23.75, size = 202, normalized size = 2.40

$$\frac{adx^3}{3}+\frac{aerx^3\log\left(x\right)}{3}-\frac{aerx^3}{9}+\frac{aex^3\log\left(f\right)}{3}+\frac{bdnx^3\log\left(x\right)}{3}-\frac{bdnx^3}{9}+\frac{bdx^3\log\left(c\right)}{3}+\frac{benrx^3\log\left(x\right)^2}{3}-\frac{2benrx^3\log}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*(d+e*ln(f*x**r)),x)
```

```
[Out] a*d*x**3/3 + a*e*r*x**3*log(x)/3 - a*e*r*x**3/9 + a*e*x**3*log(f)/3 + b*d*n
*x**3*log(x)/3 - b*d*n*x**3/9 + b*d*x**3*log(c)/3 + b*e*n*r*x**3*log(x)**2/
3 - 2*b*e*n*r*x**3*log(x)/9 + 2*b*e*n*r*x**3/27 + b*e*n*x**3*log(f)*log(x)/
3 - b*e*n*x**3*log(f)/9 + b*e*r*x**3*log(c)*log(x)/3 - b*e*r*x**3*log(c)/9
+ b*e*x**3*log(c)*log(f)/3
```

3.157 $\int x\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)\,dx$

Optimal. Leaf size=84

$$\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)-\frac{1}{8}erx^2\left(2a+2b\log\left(cx^n\right)-bn\right)-\frac{1}{4}bnx^2\left(d+e\log\left(fx^r\right)\right)+\frac{1}{8}benrx^2$$

```
[Out] 1/8*b*e*n*r*x^2-1/8*e*r*x^2*(2*a-b*n+2*b*ln(c*x^n))-1/4*b*n*x^2*(d+e*ln(f*x
^r))+1/2*x^2*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2304, 2366, 12}

$$\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)-\frac{1}{8}erx^2\left(2a+2b\log\left(cx^n\right)-bn\right)-\frac{1}{4}bnx^2\left(d+e\log\left(fx^r\right)\right)+\frac{1}{8}benrx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] (b*e*n*r*x^2)/8 - (e*r*x^2*(2*a - b*n + 2*b*Log[c*x^n]))/8 - (b*n*x^2*(d +
e*Log[f*x^r]))/4 + (x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)\,dx &= -\frac{1}{4}bnx^2\left(d+e\log\left(fx^r\right)\right)+\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)-\\ &= -\frac{1}{4}bnx^2\left(d+e\log\left(fx^r\right)\right)+\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)-\\ &= \frac{1}{8}benrx^2-\frac{1}{8}erx^2\left(2a-bn+2b\log\left(cx^n\right)\right)-\frac{1}{4}bnx^2\left(d+e\log\left(fx^r\right)\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 68, normalized size = 0.81

$$\frac{1}{4}x^2\left(e(2a-bn)\log\left(fx^r\right)+2ad-aer+b\log\left(cx^n\right)\left(2d+2e\log\left(fx^r\right)-er\right)-bdn+benr\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] (x^2*(2*a*d - b*d*n - a*e*r + b*e*n*r + e*(2*a - b*n)*Log[f*x^r] + b*Log[c*
x^n]*(2*d - e*r + 2*e*Log[f*x^r])))/4
```

fricas [A] time = 0.70, size = 128, normalized size = 1.52

$$\frac{1}{2}benrx^2\log(x)^2 - \frac{1}{4}(ber - 2bd)x^2\log(c) - \frac{1}{4}(bdn - 2ad - (ben - ae)r)x^2 + \frac{1}{4}\left(2bex^2\log(c) - (ben - 2ae)x^2\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] 1/2*b*e*n*r*x^2*log(x)^2 - 1/4*(b*e*r - 2*b*d)*x^2*log(c) - 1/4*(b*d*n - 2*
a*d - (b*e*n - a*e)*r)*x^2 + 1/4*(2*b*e*x^2*log(c) - (b*e*n - 2*a*e)*x^2)*l
og(f) + 1/2*(b*e*r*x^2*log(c) + b*e*n*x^2*log(f) + (b*d*n - (b*e*n - a*e)*r
)*x^2)*log(x)
```

giac [B] time = 0.25, size = 161, normalized size = 1.92

$$\frac{1}{2}bnrx^2e\log(x)^2 - \frac{1}{2}bnrx^2e\log(x) + \frac{1}{2}brx^2e\log(c)\log(x) + \frac{1}{2}bnx^2e\log(f)\log(x) + \frac{1}{4}bnrx^2e - \frac{1}{4}brx^2e\log(c) - \frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] 1/2*b*n*r*x^2*e*log(x)^2 - 1/2*b*n*r*x^2*e*log(x) + 1/2*b*r*x^2*e*log(c)*lo
g(x) + 1/2*b*n*x^2*e*log(f)*log(x) + 1/4*b*n*r*x^2*e - 1/4*b*r*x^2*e*log(c)
 - 1/4*b*n*x^2*e*log(f) + 1/2*b*x^2*e*log(c)*log(f) + 1/2*b*d*n*x^2*log(x)
+ 1/2*a*r*x^2*e*log(x) - 1/4*b*d*n*x^2 - 1/4*a*r*x^2*e + 1/2*b*d*x^2*log(c)
 + 1/2*a*x^2*e*log(f) + 1/2*a*d*x^2
```

maple [C] time = 0.49, size = 1640, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)*(d+e*ln(f*x^r)),x)
```

```
[Out] 1/2*a*d*x^2+(1/2*b*e*x^2*ln(x^n)+1/4*I*Pi*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)
^2-1/4*I*Pi*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/4*I*Pi*b*e*x^2*cs
gn(I*c*x^n)^3+1/4*I*Pi*b*e*x^2*csgn(I*c*x^n)^2*csgn(I*c)+1/2*b*e*x^2*ln(c)-
1/4*b*e*n*x^2+1/2*a*e*x^2)*ln(x^r)+1/2*ln(f)*a*e*x^2+1/2*b*d*x^2*ln(x^n)+1/
2*b*d*x^2*ln(c)-1/4*r*a*e*x^2-1/4*I*Pi*b*e*x^2*csgn(I*f*x^r)^3*ln(x^n)+1/4*
b*e*n*r*x^2-1/4*I*Pi*b*d*x^2*csgn(I*c*x^n)^3-1/4*b*d*n*x^2+1/4*I*Pi*b*d*x^2
*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I*Pi*b*d*x^2*csgn(I*c*x^n)^2*csgn(I*c)+1/8
*Pi^2*b*e*x^2*csgn(I*c*x^n)^3*csgn(I*x^r)*csgn(I*f*x^r)^2+1/8*Pi^2*b*e*x^2*
csgn(I*c*x^n)^2*csgn(I*f*x^r)^3*csgn(I*c)-1/4*I*ln(c)*Pi*b*e*x^2*csgn(I*f)*
csgn(I*x^r)*csgn(I*f*x^r)-1/8*I*Pi*b*e*n*x^2*csgn(I*x^r)*csgn(I*f*x^r)^2-1/
8*I*Pi*b*e*r*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I*Pi*b*e*r*x^2*csgn(I*c*x^
n)^2*csgn(I*c)-1/8*Pi^2*b*e*x^2*csgn(I*c*x^n)^3*csgn(I*f*x^r)^3-1/4*I*Pi*a*
e*x^2*csgn(I*f*x^r)^3-1/8*I*Pi*b*e*n*x^2*csgn(I*f)*csgn(I*f*x^r)^2-1/4*I*ln
(c)*Pi*b*e*x^2*csgn(I*f*x^r)^3-1/4*I*Pi*ln(f)*b*e*x^2*csgn(I*c*x^n)^3+1/8*I
*Pi*b*e*n*x^2*csgn(I*f*x^r)^3-1/4*ln(f)*b*e*n*x^2+1/2*ln(c)*ln(f)*b*e*x^2-1
/4*ln(c)*b*e*r*x^2+1/4*I*ln(c)*Pi*b*e*x^2*csgn(I*x^r)*csgn(I*f*x^r)^2+1/4*I
*Pi*ln(f)*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I*Pi*ln(f)*b*e*x^2*csgn(I
*c*x^n)^2*csgn(I*c)+1/2*ln(f)*b*e*x^2*ln(x^n)-1/4*b*e*r*x^2*ln(x^n)-1/4*I*P
i*b*d*x^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+1/4*I*ln(c)*Pi*b*e*x^2*csgn(I
*f)*csgn(I*f*x^r)^2-1/4*I*Pi*a*e*x^2*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-1/
8*Pi^2*b*e*x^2*csgn(I*c*x^n)^2*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)+1/4*I*
```

```
Pi*b*e*x^2*csgn(I*f)*csgn(I*f*x^r)^2*ln(x^n)+1/4*I*Pi*b*e*x^2*csgn(I*x^r)*c
sgn(I*f*x^r)^2*ln(x^n)-1/8*Pi^2*b*e*x^2*csgn(I*c*x^n)^3*csgn(I*f)*csgn(I*x^
r)*csgn(I*f*x^r)-1/8*Pi^2*b*e*x^2*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*f*x^r)^2
*csgn(I*c)-1/8*Pi^2*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*f*
x^r)^2-1/8*Pi^2*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*x^r)*csgn(I*f*x^
r)^2-1/8*Pi^2*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f*x^r)^3*csgn(I*c)+1
/8*Pi^2*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(
I*c)+1/8*Pi^2*b*e*x^2*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*c
sgn(I*c)-1/4*I*Pi*b*e*x^2*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(x^n)+1/8*P
i^2*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*f*x^r)^3+1/8*Pi^2*b*e*x^2*cs
gn(I*c*x^n)^3*csgn(I*f)*csgn(I*f*x^r)^2-1/4*I*Pi*ln(f)*b*e*x^2*csgn(I*x^n)*
csgn(I*c*x^n)*csgn(I*c)+1/8*I*Pi*b*e*n*x^2*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x
^r)+1/8*Pi^2*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*x^r)*csgn
(I*f*x^r)+1/8*Pi^2*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f)*csgn(I*f*x^r
)^2*csgn(I*c)+1/8*I*Pi*b*e*r*x^2*csgn(I*c*x^n)^3+1/4*I*Pi*a*e*x^2*csgn(I*f)
*csgn(I*f*x^r)^2+1/4*I*Pi*a*e*x^2*csgn(I*x^r)*csgn(I*f*x^r)^2-1/8*Pi^2*b*e*
x^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)
+1/8*I*Pi*b*e*r*x^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)
```

maxima [A] time = 0.54, size = 102, normalized size = 1.21

$$-\frac{1}{4}\,bdnx^2-\frac{1}{4}\,aerx^2+\frac{1}{2}\,bdx^2\log\left(cx^n\right)+\frac{1}{2}\,aex^2\log\left(fx^r\right)+\frac{1}{4}\,\left(\left(r-\log(f)\right)x^2-x^2\log\left(x^r\right)\right)ben+\frac{1}{2}\,adx^2-\frac{1}{4}\,\left(rx^2-2\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*n*x^2 - 1/4*a*e*r*x^2 + 1/2*b*d*x^2*log(c*x^n) + 1/2*a*e*x^2*log(f
*x^r) + 1/4*((r - log(f))*x^2 - x^2*log(x^r))*b*e*n + 1/2*a*d*x^2 - 1/4*(r*
x^2 - 2*x^2*log(f*x^r))*b*e*log(c*x^n)
```

mupad [B] time = 4.00, size = 82, normalized size = 0.98

$$\ln\left(f\,x^r\right)\,\left(\frac{a\,e\,x^2}{2}-\frac{b\,e\,n\,x^2}{4}+\frac{b\,e\,x^2\,\ln\left(c\,x^n\right)}{2}\right)+x^2\,\left(\frac{a\,d}{2}-\frac{b\,d\,n}{4}-\frac{a\,e\,r}{4}+\frac{b\,e\,n\,r}{4}\right)+\frac{b\,x^2\,\ln\left(c\,x^n\right)\,\left(2\,d-e\,r\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n)),x)
```

```
[Out] log(f*x^r)*((a*e*x^2)/2 - (b*e*n*x^2)/4 + (b*e*x^2*log(c*x^n))/2) + x^2*((a
*d)/2 - (b*d*n)/4 - (a*e*r)/4 + (b*e*n*r)/4) + (b*x^2*log(c*x^n)*(2*d - e*r
))/4
```

sympy [B] time = 8.22, size = 199, normalized size = 2.37

$$\frac{adx^2}{2}+\frac{aerx^2\log\left(x\right)}{2}-\frac{aerx^2}{4}+\frac{aex^2\log\left(f\right)}{2}+\frac{bdnx^2\log\left(x\right)}{2}-\frac{bdnx^2}{4}+\frac{bdx^2\log\left(c\right)}{2}+\frac{benrx^2\log\left(x\right)^2}{2}-\frac{benrx^2\log\left(x\right)}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*(d+e*ln(f*x**r)),x)
```

```
[Out] a*d*x**2/2 + a*e*r*x**2*log(x)/2 - a*e*r*x**2/4 + a*e*x**2*log(f)/2 + b*d*n
*x**2*log(x)/2 - b*d*n*x**2/4 + b*d*x**2*log(c)/2 + b*e*n*r*x**2*log(x)**2/
2 - b*e*n*r*x**2*log(x)/2 + b*e*n*r*x**2/4 + b*e*n*x**2*log(f)*log(x)/2 - b
*e*n*x**2*log(f)/4 + b*e*r*x**2*log(c)*log(x)/2 - b*e*r*x**2*log(c)/4 + b*e
*x**2*log(c)*log(f)/2
```

3.158 $\int \left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)\,dx$

Optimal. Leaf size=77

$$-erx(a-bn)+ax\left(d+e\log\left(fx^r\right)\right)+bx\log\left(cx^n\right)\left(d+e\log\left(fx^r\right)\right)-berx\log\left(cx^n\right)-bnx\left(d+e\log\left(fx^r\right)\right)+benrx$$

```
[Out] b*e*n*r*x-e*(-b*n+a)*r*x-b*e*r*x*ln(c*x^n)+a*x*(d+e*ln(f*x^r))-b*n*x*(d+e*l
n(f*x^r))+b*x*ln(c*x^n)*(d+e*ln(f*x^r))
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2295, 2361}

$$-erx(a-bn)+ax\left(d+e\log\left(fx^r\right)\right)+bx\log\left(cx^n\right)\left(d+e\log\left(fx^r\right)\right)-berx\log\left(cx^n\right)-bnx\left(d+e\log\left(fx^r\right)\right)+benrx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] b*e*n*r*x - e*(a - b*n)*r*x - b*e*r*x*Log[c*x^n] + a*x*(d + e*Log[f*x^r]) -
 b*n*x*(d + e*Log[f*x^r]) + b*x*Log[c*x^n]*(d + e*Log[f*x^r])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Dist[
d + e*Log[f*x^r], u, x] - Dist[e*r, Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)\,dx &= ax\left(d+e\log\left(fx^r\right)\right)-bnx\left(d+e\log\left(fx^r\right)\right)+bx\log\left(cx^n\right)\left(d+e\log\right.\\ &= -e(a-bn)rx+ax\left(d+e\log\left(fx^r\right)\right)-bnx\left(d+e\log\left(fx^r\right)\right)+bx\log\\ &= benrx-e(a-bn)rx-berx\log\left(cx^n\right)+ax\left(d+e\log\left(fx^r\right)\right)-bnx\left(d\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.75

$$x\left(e(a-bn)\log\left(fx^r\right)+ad-aer+b\log\left(cx^n\right)\left(d+e\log\left(fx^r\right)-er\right)-bdn+2benr\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] x*(a*d - b*d*n - a*e*r + 2*b*e*n*r + e*(a - b*n)*Log[f*x^r] + b*Log[c*x^n]*
(d - e*r + e*Log[f*x^r]))
```

fricas [A] time = 0.68, size = 110, normalized size = 1.43

$$benrx\log(x)^2-(ber-bd)x\log(c)-(bdn-ad-(2\,ben-ae)r)x+\left(bex\log(c)-(ben-ae)x\right)\log(f)+\left(berx\log(c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] b*e*n*r*x*log(x)^2 - (b*e*r - b*d)*x*log(c) - (b*d*n - a*d - (2*b*e*n - a*e
)*r)*x + (b*e*x*log(c) - (b*e*n - a*e)*x)*log(f) + (b*e*r*x*log(c) + b*e*n*
x*log(f) + (b*d*n - (2*b*e*n - a*e)*r)*x)*log(x)
```

giac [A] time = 0.27, size = 122, normalized size = 1.58

$$bnrxe\log(x)^2 - 2\,bnrxe\log(x) + brxe\log(c)\log(x) + bnxe\log(f)\log(x) + 2\,bnrxe - brxe\log(c) - bnxe\log(f) + bxe\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] b*n*r*x*e*log(x)^2 - 2*b*n*r*x*e*log(x) + b*r*x*e*log(c)*log(x) + b*n*x*e*l
og(f)*log(x) + 2*b*n*r*x*e - b*r*x*e*log(c) - b*n*x*e*log(f) + b*x*e*log(c)
*log(f) + b*d*n*x*log(x) + a*r*x*e*log(x) - b*d*n*x - a*r*x*e + b*d*x*log(c
) + a*x*e*log(f) + a*d*x
```

maple [C] time = 0.45, size = 1503, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*(d+e*ln(f*x^r)),x)
```

```
[Out] (x*b*e*ln(x^n)+1/2*I*Pi*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*e*x*cs
gn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I*Pi*b*e*x*csgn(I*c*x^n)^3+1/2*I*Pi*b
*e*x*csgn(I*c*x^n)^2*csgn(I*c)+b*e*x*ln(c)-b*e*n*x+a*e*x)*ln(x^r)+a*d*x+b*d
*x*ln(x^n)+ln(f)*a*e*x-r*a*e*x+1/2*I*Pi*b*e*n*x*csgn(I*f)*csgn(I*x^r)*csgn(
I*f*x^r)+b*d*x*ln(c)-1/2*I*ln(c)*Pi*b*e*x*csgn(I*f*x^r)^3-1/2*I*Pi*b*e*x*cs
gn(I*f*x^r)^3*ln(x^n)+1/2*I*Pi*b*d*csgn(I*c*x^n)^2*csgn(I*c)*x-1/2*I*Pi*a*e
*x*csgn(I*f*x^r)^3-1/4*Pi^2*b*e*x*csgn(I*c*x^n)^3*csgn(I*f*x^r)^3-1/2*I*Pi*
b*d*csgn(I*c*x^n)^3*x+1/2*I*Pi*b*e*r*x*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-
1/2*I*Pi*b*e*x*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(x^n)+1/4*Pi^2*b*e*x*c
sgn(I*c*x^n)^2*csgn(I*f*x^r)^3*csgn(I*c)+1/2*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*
x^n)^2*x-1/2*I*Pi*ln(f)*b*e*x*csgn(I*c*x^n)^3+1/2*I*Pi*b*e*n*x*csgn(I*f*x^r
)^3+1/2*I*Pi*b*e*r*x*csgn(I*c*x^n)^3-1/4*Pi^2*b*e*x*csgn(I*x^n)*csgn(I*c*x^
n)*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)+ln(c)*ln(f)*b*e*x-ln(c)*b*
e*r*x-ln(f)*b*e*n*x-1/2*I*Pi*ln(f)*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c
)+2*b*e*n*r*x+1/4*Pi^2*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f)*csgn(I*f*x
^r)^2*csgn(I*c)+1/4*Pi^2*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*f*x^r)^3+
1/4*Pi^2*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I
*c)+1/4*Pi^2*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*x^r)*csgn(I
*f*x^r)+1/2*I*Pi*a*e*x*csgn(I*f)*csgn(I*f*x^r)^2+1/2*I*Pi*a*e*x*csgn(I*x^r)
*csgn(I*f*x^r)^2-1/2*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*x-b*e*r*x
*ln(x^n)+ln(f)*b*e*x*ln(x^n)-1/2*I*Pi*b*e*n*x*csgn(I*x^r)*csgn(I*f*x^r)^2-1
/2*I*Pi*b*e*r*x*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*e*r*x*csgn(I*c*x^n)^
2*csgn(I*c)+1/2*I*ln(c)*Pi*b*e*x*csgn(I*x^r)*csgn(I*f*x^r)^2-1/2*I*Pi*b*e*n
*x*csgn(I*f)*csgn(I*f*x^r)^2+1/2*I*ln(c)*Pi*b*e*x*csgn(I*f)*csgn(I*f*x^r)^2
-1/4*Pi^2*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*f*x^r)^2-1/4*P
i^2*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*x^r)*csgn(I*f*x^r)^2-1/4*Pi^2*
b*e*x*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f*x^r)^3*csgn(I*c)-1/4*Pi^2*b*e*x*cs
gn(I*c*x^n)^3*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-1/4*Pi^2*b*e*x*csgn(I*c*x
^n)^2*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)-1/4*Pi^2*b*e*x*csgn(I*c*x^n)^2*cs
gn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)-1/2*I*Pi*a*e*x*csgn(I*f)*csgn(I*x^r)*cs
gn(I*f*x^r)+1/2*I*Pi*b*e*x*csgn(I*f)*csgn(I*f*x^r)^2*ln(x^n)+1/2*I*Pi*b*e*x
*csgn(I*x^r)*csgn(I*f*x^r)^2*ln(x^n)+1/2*I*Pi*ln(f)*b*e*x*csgn(I*c*x^n)^2*c
sgn(I*c)+1/2*I*Pi*ln(f)*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*Pi^2*b*e*x*cs
gn(I*c*x^n)^3*csgn(I*f)*csgn(I*f*x^r)^2+1/4*Pi^2*b*e*x*csgn(I*c*x^n)^3*csgn
(I*x^r)*csgn(I*f*x^r)^2+1/4*Pi^2*b*e*x*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*x^r
```

```
)*csgn(I*f*x^r)*csgn(I*c)-b*d*n*x-1/2*I*ln(c)*Pi*b*e*x*csgn(I*f)*csgn(I*x^r
)*csgn(I*f*x^r)
```

maxima [A] time = 0.64, size = 82, normalized size = 1.06

$$\left(\left(2\,r-\log(f)\right)x-x\log\left(x^{r}\right)\right)ben-bdnx-aerx-\left(rx-x\log\left(fx^{r}\right)\right)be\log\left(cx^{n}\right)+bdx\log\left(cx^{n}\right)+aex\log\left(fx^{r}\right)+ad$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] ((2*r - log(f))*x - x*log(x^r))*b*e*n - b*d*n*x - a*e*r*x - (r*x - x*log(f*
x^r))*b*e*log(c*x^n) + b*d*x*log(c*x^n) + a*e*x*log(f*x^r) + a*d*x
```

mupad [B] time = 3.78, size = 66, normalized size = 0.86

$$x\,\left(a\,d-b\,d\,n-a\,e\,r+2\,b\,e\,n\,r\right)+\ln\left(f\,x^r\right)\,\left(a\,e\,x-b\,e\,n\,x+b\,e\,x\,\ln\left(c\,x^n\right)\right)+b\,x\,\ln\left(c\,x^n\right)\,\left(d-e\,r\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*log(f*x^r))*(a + b*log(c*x^n)),x)
```

```
[Out] x*(a*d - b*d*n - a*e*r + 2*b*e*n*r) + log(f*x^r)*(a*e*x - b*e*n*x + b*e*x*l
og(c*x^n)) + b*x*log(c*x^n)*(d - e*r)
```

sympy [A] time = 2.83, size = 151, normalized size = 1.96

$$adx+aerx\log\left(x\right)-aerx+aex\log\left(f\right)+bdnx\log\left(x\right)-bdnx+bdx\log\left(c\right)+benrx\log\left(x\right)^{2}-2benrx\log\left(x\right)+2benrx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r)),x)
```

```
[Out] a*d*x + a*e*r*x*log(x) - a*e*r*x + a*e*x*log(f) + b*d*n*x*log(x) - b*d*n*x
+ b*d*x*log(c) + b*e*n*r*x*log(x)**2 - 2*b*e*n*r*x*log(x) + 2*b*e*n*r*x + b
*e*n*x*log(f)*log(x) - b*e*n*x*log(f) + b*e*r*x*log(c)*log(x) - b*e*r*x*log
(c) + b*e*x*log(c)*log(f)
```

3.159 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx$

Optimal. Leaf size=57

$$\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn}-\frac{er(a+b\log(cx^n))^3}{6b^2n^2}$$

```
[Out] -1/6*e*r*(a+b*ln(c*x^n))^3/b^2/n^2+1/2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/b/
n
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2301, 2366, 12, 2302, 30}

$$\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn}-\frac{er(a+b\log(cx^n))^3}{6b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] -(e*r*(a + b*Log[c*x^n])^3)/(6*b^2*n^2) + ((a + b*Log[c*x^n])^2*(d + e*Log[
f*x^r]))/(2*b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)}{x}\,dx = \frac{\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)}{2bn} - (er)\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{2bnx}\,dx$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)}{2bn} - \frac{(er)\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx}{2bn}$$

$$= \frac{\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)}{2bn} - \frac{(er)\,\text{Subst}\left(\int x^2\,dx, x, a+b\,\mathrm{l}\right.}{2b^2n^2}$$

$$= -\frac{er\left(a+b\log\left(cx^n\right)\right)^3}{6b^2n^2} + \frac{\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)}{2bn}$$

Mathematica [A] time = 0.06, size = 72, normalized size = 1.26

$$\frac{1}{6}\log(x)\left(-3\log(x)\left(aer+ber\log\left(cx^n\right)+bdn+ben\log\left(fx^r\right)\right)+6\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)+2benr\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] (Log[x]*(2*b*e*n*r*Log[x]^2 + 6*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]) - 3*L
og[x]*(b*d*n + a*e*r + b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/6
```

fricas [A] time = 0.68, size = 62, normalized size = 1.09

$$\frac{1}{3}benr\log(x)^3+\frac{1}{2}\left(ber\log(c)+ben\log(f)+bdn+aer\right)\log(x)^2+\left(bd\log(c)+ad+\left(be\log(c)+ae\right)\log(f)\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*e*n*r*log(x)^3 + 1/2*(b*e*r*log(c) + b*e*n*log(f) + b*d*n + a*e*r)*lo
g(x)^2 + (b*d*log(c) + a*d + (b*e*log(c) + a*e)*log(f))*log(x)
```

giac [A] time = 0.29, size = 85, normalized size = 1.49

$$\frac{1}{3}bnre\log(x)^3+\frac{1}{2}bre\log(c)\log(x)^2+\frac{1}{2}bne\log(f)\log(x)^2+be\log(c)\log(f)\log(x)+\frac{1}{2}bdn\log(x)^2+\frac{1}{2}are\log(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x,x, algorithm="giac")
```

```
[Out] 1/3*b*n*r*e*log(x)^3 + 1/2*b*r*e*log(c)*log(x)^2 + 1/2*b*n*e*log(f)*log(x)^
2 + b*e*log(c)*log(f)*log(x) + 1/2*b*d*n*log(x)^2 + 1/2*a*r*e*log(x)^2 + b*
d*log(c)*log(x) + a*e*log(f)*log(x) + a*d*log(x)
```

maple [C] time = 0.62, size = 1597, normalized size = 28.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*(d+e*ln(f*x^r))/x,x)
```

```
[Out] -1/4*Pi^2*ln(x)*b*e*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f)*csgn(I*x^r)*csgn(I*
f*x^r)*csgn(I*c)+(b*e*ln(x)*ln(x^n)-1/2*b*e*n*ln(x)^2+1/2*I*Pi*ln(x)*b*e*cs
gn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*ln(x)*b*e*csgn(I*x^n)*csgn(I*c*x^n)*csgn
```

```
(I*c)-1/2*I*Pi*ln(x)*b*e*csgn(I*c*x^n)^3+1/2*I*Pi*ln(x)*b*e*csgn(I*c*x^n)^2
*csgn(I*c)+ln(c)*ln(x)*b*e+ln(x)*a*e)*ln(x^r)+ln(f)*ln(x)*a*e-1/2*ln(x)^2*a
*e*r+a*d*ln(x)+b*d*ln(c)*ln(x)-1/2*b*d*n*ln(x)^2+1/2*I*ln(x)*Pi*b*d*csgn(I*
x^n)*csgn(I*c*x^n)^2+ln(x^n)*b*d*ln(x)+ln(x^n)*ln(f)*b*e*ln(x)-1/2*ln(x^n)*
r*b*e*ln(x)^2+ln(c)*ln(f)*ln(x)*b*e+1/3*b*e*n*r*ln(x)^3-1/2*ln(x)^2*ln(c)*b
*e*r-1/2*ln(x)^2*ln(f)*b*e*n-1/2*I*ln(x)*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)*c
sgn(I*c)-1/2*I*ln(c)*Pi*ln(x)*b*e*csgn(I*f*x^r)^3-1/2*I*Pi*ln(f)*ln(x)*b*e*
csgn(I*c*x^n)^3+1/2*I*Pi*ln(x)*a*e*csgn(I*f)*csgn(I*f*x^r)^2+1/2*I*Pi*ln(x)
*a*e*csgn(I*x^r)*csgn(I*f*x^r)^2-1/2*I*ln(c)*Pi*ln(x)*b*e*csgn(I*f)*csgn(I*
x^r)*csgn(I*f*x^r)+1/4*I*ln(x)^2*Pi*b*e*r*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*
c)+1/4*Pi^2*ln(x)*b*e*csgn(I*c*x^n)^3*csgn(I*f)*csgn(I*f*x^r)^2+1/4*Pi^2*ln
(x)*b*e*csgn(I*c*x^n)^3*csgn(I*x^r)*csgn(I*f*x^r)^2+1/4*Pi^2*ln(x)*b*e*csgn
(I*c*x^n)^2*csgn(I*f*x^r)^3*csgn(I*c)-1/2*I*Pi*ln(x)*a*e*csgn(I*f*x^r)^3-1/
4*Pi^2*ln(x)*b*e*csgn(I*c*x^n)^3*csgn(I*f*x^r)^3-1/4*Pi^2*ln(x)*b*e*csgn(I*
x^n)*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*f*x^r)^2-1/4*Pi^2*ln(x)*b*e*csgn(I*x^
n)*csgn(I*c*x^n)^2*csgn(I*x^r)*csgn(I*f*x^r)^2-1/4*Pi^2*ln(x)*b*e*csgn(I*x^
n)*csgn(I*c*x^n)*csgn(I*f*x^r)^3*csgn(I*c)+1/4*Pi^2*ln(x)*b*e*csgn(I*x^n)*c
sgn(I*c*x^n)^2*csgn(I*f*x^r)^3-1/2*I*ln(x^n)*Pi*b*e*csgn(I*f*x^r)^3*ln(x)+1
/4*I*ln(x)^2*Pi*b*e*n*csgn(I*f*x^r)^3-1/2*I*Pi*ln(f)*ln(x)*b*e*csgn(I*x^n)*
csgn(I*c*x^n)*csgn(I*c)+1/4*Pi^2*ln(x)*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn
(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+1/4*Pi^2*ln(x)*b*e*csgn(I*x^n)*csgn(I*c*x^n
)*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)+1/4*Pi^2*ln(x)*b*e*csgn(I*x^n)*csgn(I
*c*x^n)*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)+1/2*I*ln(x)*Pi*b*d*csgn(I*c*x
^n)^2*csgn(I*c)-1/2*I*ln(x)*Pi*b*d*csgn(I*c*x^n)^3+1/2*I*ln(c)*Pi*ln(x)*b*e
*csgn(I*x^r)*csgn(I*f*x^r)^2+1/2*I*Pi*ln(f)*ln(x)*b*e*csgn(I*x^n)*csgn(I*c*
x^n)^2+1/2*I*Pi*ln(f)*ln(x)*b*e*csgn(I*c*x^n)^2*csgn(I*c)-1/2*I*Pi*ln(x)*a*
e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+1/4*Pi^2*ln(x)*b*e*csgn(I*c*x^n)^2*cs
gn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)+1/2*I*ln(x^n)*Pi*b*e*csgn(I*f)*
csgn(I*f*x^r)^2*ln(x)+1/2*I*ln(x^n)*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*ln(x
)-1/4*I*ln(x)^2*Pi*b*e*n*csgn(I*f)*csgn(I*f*x^r)^2+1/4*I*ln(x)^2*Pi*b*e*r*c
sgn(I*c*x^n)^3+1/4*I*ln(x)^2*Pi*b*e*n*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-1
/2*I*ln(x^n)*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(x)-1/4*Pi^2*ln(x
)*b*e*csgn(I*c*x^n)^3*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-1/4*Pi^2*ln(x)*b*
e*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)-1/4*Pi^2*ln(x)*b*e*cs
gn(I*c*x^n)^2*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)-1/4*I*ln(x)^2*Pi*b*e*r*
csgn(I*c*x^n)^2*csgn(I*c)+1/2*I*ln(c)*Pi*ln(x)*b*e*csgn(I*f)*csgn(I*f*x^r)^
2-1/4*I*ln(x)^2*Pi*b*e*n*csgn(I*x^r)*csgn(I*f*x^r)^2-1/4*I*ln(x)^2*Pi*b*e*r
*csgn(I*x^n)*csgn(I*c*x^n)^2
```

maxima [A] time = 0.62, size = 73, normalized size = 1.28

$$\frac{be\log(cx^n)\log\left(fx^r\right)^2}{2r}-\frac{ben\log\left(fx^r\right)^3}{6r^2}+\frac{bd\log(cx^n)^2}{2n}+\frac{ae\log\left(fx^r\right)^2}{2r}+ad\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*log(c*x^n)*log(f*x^r)^2/r - 1/6*b*e*n*log(f*x^r)^3/r^2 + 1/2*b*d*lo
g(c*x^n)^2/n + 1/2*a*e*log(f*x^r)^2/r + a*d*log(x)
```

mupad [B] time = 3.87, size = 73, normalized size = 1.28

$$a\,d\,\ln(x)+\frac{b\,d\ln(c\,x^n)^2}{2\,n}+\frac{a\,e\ln\left(f\,x^r\right)^2}{2\,r}-\frac{b\,e\,r\ln(c\,x^n)^3}{6\,n^2}+\frac{b\,e\ln(c\,x^n)^2\,\ln\left(f\,x^r\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x,x)
```

```
[Out] a*d*log(x) + (b*d*log(c*x^n)^2)/(2*n) + (a*e*log(f*x^r)^2)/(2*r) - (b*e*r*l
og(c*x^n)^3)/(6*n^2) + (b*e*log(c*x^n)^2*log(f*x^r))/(2*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(cx^{n}\right)\right)\left(d + e\log\left(fx^{r}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))*(d + e*log(f*x**r))/x, x)
```

3.160 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}-\frac{er(a+b\log(cx^n)+bn)}{x}-\frac{bn(d+e\log(fx^r))}{x}-\frac{benr}{x}$$

```
[Out] -b*e*n*r/x-e*r*(a+b*n+b*ln(c*x^n))/x-b*n*(d+e*ln(f*x^r))/x-(a+b*ln(c*x^n))*
(d+e*ln(f*x^r))/x
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2304, 2366}

$$-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}-\frac{er(a+b\log(cx^n)+bn)}{x}-\frac{bn(d+e\log(fx^r))}{x}-\frac{benr}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] -((b*e*n*r)/x) - (e*r*(a + b*n + b*Log[c*x^n]))/x - (b*n*(d + e*Log[f*x^r])
)/x - ((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx=-\frac{bn(d+e\log(fx^r))}{x}-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}-(er)\int -$$
$$=-\frac{benr}{x}-\frac{er(a+bn+b\log(cx^n))}{x}-\frac{bn(d+e\log(fx^r))}{x}-\frac{(a+b\log}{}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.79

$$-\frac{e(a+bn)\log(fx^r)+ad+aer+b\log(cx^n)(d+e\log(fx^r)+er)+bdn+2benr}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] -((a*d + b*d*n + a*e*r + 2*b*e*n*r + e*(a + b*n)*Log[f*x^r] + b*Log[c*x^n]*
(d + e*r + e*Log[f*x^r]))/x)
```

fricas [A] time = 0.60, size = 91, normalized size = 1.26

$$-\frac{benr\log(x)^2 + bdn + ad + (2\,ben + ae)r + (ber + bd)\log(c) + \left(ben + be\log(c) + ae\right)\log(f) + \left(ber\log(c) + b\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*e*n*r*log(x)^2 + b*d*n + a*d + (2*b*e*n + a*e)*r + (b*e*r + b*d)*log(c)
 + (b*e*n + b*e*log(c) + a*e)*log(f) + (b*e*r*log(c) + b*e*n*log(f) + b*d*n
 + (2*b*e*n + a*e)*r)*log(x))/x
```

giac [A] time = 0.32, size = 108, normalized size = 1.50

$$-\frac{bnre\log(x)^2 + 2\,bnre\log(x) + bre\log(c)\log(x) + bne\log(f)\log(x) + 2\,bnre + bre\log(c) + bne\log(f) + be\,\mathrm{l}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^2,x, algorithm="giac")
```

```
[Out] -(b*n*r*e*log(x)^2 + 2*b*n*r*e*log(x) + b*r*e*log(c)*log(x) + b*n*e*log(f)*
log(x) + 2*b*n*r*e + b*r*e*log(c) + b*n*e*log(f) + b*e*log(c)*log(f) + b*d*
n*log(x) + a*r*e*log(x) + b*d*n + a*r*e + b*d*log(c) + a*e*log(f) + a*d)/x
```

maple [C] time = 0.35, size = 1443, normalized size = 20.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*(d+e*ln(f*x^r))/x^2,x)
```

```
[Out] -1/2*e*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*c)*csgn(I*x^n)*csg
n(I*c*x^n)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+2*b*ln(c
)+2*b*n+2*b*ln(x^n)+2*a)/x*ln(x^r)-1/4*(4*b*d*n+4*a*e*r+4*b*e*r*ln(x^n)+4*l
n(f)*b*e*ln(x^n)+4*n*ln(f)*b*e+4*ln(c)*b*e*r+4*ln(c)*ln(f)*b*e+4*b*d*ln(x^n
)+2*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)^2+4*ln(f)*a*e+4*a*d+8*b*e*n*r+4*b*d*
ln(c)-Pi^2*b*e*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r
)*csgn(I*c)-Pi^2*b*e*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)-Pi
^2*b*e*csgn(I*c*x^n)^2*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)+2*I*Pi*a*e*csg
n(I*f)*csgn(I*f*x^r)^2+2*I*Pi*a*e*csgn(I*x^r)*csgn(I*f*x^r)^2-2*I*Pi*b*e*r*
csgn(I*c*x^n)^3-2*I*n*Pi*b*e*csgn(I*f*x^r)^3-2*I*Pi*b*e*csgn(I*f*x^r)^3*ln(
x^n)+2*I*Pi*b*d*csgn(I*c*x^n)^2*csgn(I*c)+Pi^2*b*e*csgn(I*x^n)*csgn(I*c*x^n
)^2*csgn(I*f*x^r)^3-2*I*ln(c)*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-2*
I*Pi*ln(f)*b*e*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-2*I*Pi*b*e*r*csgn(I*x^n)
*csgn(I*c*x^n)*csgn(I*c)-2*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+Pi^
2*b*e*csgn(I*c*x^n)^3*csgn(I*f)*csgn(I*f*x^r)^2+Pi^2*b*e*csgn(I*c*x^n)^3*cs
gn(I*x^r)*csgn(I*f*x^r)^2+Pi^2*b*e*csgn(I*c*x^n)^2*csgn(I*f*x^r)^3*csgn(I*c
)-2*I*ln(c)*Pi*b*e*csgn(I*f*x^r)^3-2*I*Pi*ln(f)*b*e*csgn(I*c*x^n)^3+2*I*ln(
c)*Pi*b*e*csgn(I*f)*csgn(I*f*x^r)^2-Pi^2*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2*cs
gn(I*f)*csgn(I*f*x^r)^2-2*I*Pi*b*d*csgn(I*c*x^n)^3-2*I*n*Pi*b*e*csgn(I*f)*c
sgn(I*x^r)*csgn(I*f*x^r)-2*I*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(
x^n)+Pi^2*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^
r)+Pi^2*b*e*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)+2
*I*n*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2+2*I*Pi*b*e*csgn(I*f)*csgn(I*f*x^r)^
2*ln(x^n)+2*I*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*ln(x^n)-Pi^2*b*e*csgn(I*c*
x^n)^3*csgn(I*f*x^r)^3-2*I*Pi*a*e*csgn(I*f*x^r)^3+Pi^2*b*e*csgn(I*x^n)*csgn
(I*c*x^n)*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)+Pi^2*b*e*csgn(I*c*x^n)^2*cs
gn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)-Pi^2*b*e*csgn(I*c*x^n)^3*csgn(I
*f)*csgn(I*x^r)*csgn(I*f*x^r)+2*I*Pi*b*e*r*csgn(I*x^n)*csgn(I*c*x^n)^2+2*I*
```

```
Pi*b*e*r*csgn(I*c*x^n)^2*csgn(I*c)+2*I*n*Pi*b*e*csgn(I*f)*csgn(I*f*x^r)^2+2
*I*ln(c)*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2+2*I*Pi*ln(f)*b*e*csgn(I*x^n)*cs
gn(I*c*x^n)^2+2*I*Pi*ln(f)*b*e*csgn(I*c*x^n)^2*csgn(I*c)-2*I*Pi*a*e*csgn(I*
f)*csgn(I*x^r)*csgn(I*f*x^r)-Pi^2*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*x^
r)*csgn(I*f*x^r)^2-Pi^2*b*e*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f*x^r)^3*csgn(
I*c))/x
```

maxima [A] time = 0.75, size = 94, normalized size = 1.31

$$-be\left(\frac{r}{x}+\frac{\log\left(fx^r\right)}{x}\right)\log\left(cx^n\right)-\frac{ben\left(2\,r+\log(f)+\log\left(x^r\right)\right)}{x}-\frac{bdn}{x}-\frac{aer}{x}-\frac{bd\log\left(cx^n\right)}{x}-\frac{ae\log\left(fx^r\right)}{x}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^2,x, algorithm="maxima")
```

```
[Out] -b*e*(r/x + log(f*x^r)/x)*log(c*x^n) - b*e*n*(2*r + log(f) + log(x^r))/x -
b*d*n/x - a*e*r/x - b*d*log(c*x^n)/x - a*e*log(f*x^r)/x - a*d/x
```

mupad [B] time = 3.81, size = 75, normalized size = 1.04

$$-\ln\left(f\,x^r\right)\,\left(\frac{a\,e}{x}+\frac{b\,e\,n}{x}+\frac{b\,e\,\ln\left(c\,x^n\right)}{x}\right)-\frac{a\,d+b\,d\,n+a\,e\,r+2\,b\,e\,n\,r}{x}-\frac{b\,\ln\left(c\,x^n\right)\,\left(d+e\,r\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] - log(f*x^r)*((a*e)/x + (b*e*n)/x + (b*e*log(c*x^n))/x) - (a*d + b*d*n + a*
e*r + 2*b*e*n*r)/x - (b*log(c*x^n)*(d + e*r))/x
```

sympy [B] time = 2.60, size = 153, normalized size = 2.12

$$-\frac{ad}{x}-\frac{aer\log(x)}{x}-\frac{aer}{x}-\frac{ae\log(f)}{x}-\frac{bdn\log(x)}{x}-\frac{bdn}{x}-\frac{bd\log(c)}{x}-\frac{benr\log(x)^2}{x}-\frac{2benr\log(x)}{x}-\frac{2benr}{x}-\frac{ben\log(f}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x**2,x)
```

```
[Out] -a*d/x - a*e*r*log(x)/x - a*e*r/x - a*e*log(f)/x - b*d*n*log(x)/x - b*d*n/x
 - b*d*log(c)/x - b*e*n*r*log(x)**2/x - 2*b*e*n*r*log(x)/x - 2*b*e*n*r/x -
b*e*n*log(f)*log(x)/x - b*e*n*log(f)/x - b*e*r*log(c)*log(x)/x - b*e*r*log(
c)/x - b*e*log(c)*log(f)/x
```

3.161 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx$

Optimal. Leaf size=83

$$-\frac{(a+b\log(cx^n))\left(d+e\log\left(fx^r\right)\right)}{2x^2}-\frac{er\left(2a+2b\log(cx^n)+bn\right)}{8x^2}-\frac{bn\left(d+e\log\left(fx^r\right)\right)}{4x^2}-\frac{benr}{8x^2}$$

```
[Out] -1/8*b*e*n*r/x^2-1/8*e*r*(2*a+b*n+2*b*ln(c*x^n))/x^2-1/4*b*n*(d+e*ln(f*x^r)
)/x^2-1/2*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^2
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304, 2366, 12}

$$-\frac{(a+b\log(cx^n))\left(d+e\log\left(fx^r\right)\right)}{2x^2}-\frac{er\left(2a+2b\log(cx^n)+bn\right)}{8x^2}-\frac{bn\left(d+e\log\left(fx^r\right)\right)}{4x^2}-\frac{benr}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -(b*e*n*r)/(8*x^2) - (e*r*(2*a + b*n + 2*b*Log[c*x^n]))/(8*x^2) - (b*n*(d +
 e*Log[f*x^r]))/(4*x^2) - ((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\left(d+e\log\left(fx^r\right)\right)}{x^3}\,dx &= -\frac{bn\left(d+e\log\left(fx^r\right)\right)}{4x^2}-\frac{(a+b\log(cx^n))\left(d+e\log\left(fx^r\right)\right)}{2x^2}-(er)\int\\ &= -\frac{bn\left(d+e\log\left(fx^r\right)\right)}{4x^2}-\frac{(a+b\log(cx^n))\left(d+e\log\left(fx^r\right)\right)}{2x^2}-\frac{1}{4}(er)\\ &= -\frac{benr}{8x^2}-\frac{er\left(2a+bn+2b\log(cx^n)\right)}{8x^2}-\frac{bn\left(d+e\log\left(fx^r\right)\right)}{4x^2}-\frac{(a+\end{aligned}$$

Mathematica [A] time = 0.07, size = 64, normalized size = 0.77

$$-\frac{e(2a+bn)\log\left(fx^r\right)+2ad+aer+b\log(cx^n)\left(2d+2e\log\left(fx^r\right)+er\right)+bdn+benr}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -1/4*(2*a*d + b*d*n + a*e*r + b*e*n*r + e*(2*a + b*n)*Log[f*x^r] + b*Log[c*
x^n]*(2*d + e*r + 2*e*Log[f*x^r]))/x^2
```

fricas [A] time = 0.85, size = 95, normalized size = 1.14

$$-\frac{2\,benr\log(x)^{2}+bdn+2\,ad+(ben+ae)r+(ber+2\,bd)\log(c)+\left(ben+2\,be\log(c)+2\,ae\right)\log(f)+2\left(ber\log(c\right.}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*e*n*r*log(x)^2 + b*d*n + 2*a*d + (b*e*n + a*e)*r + (b*e*r + 2*b*d
)*log(c) + (b*e*n + 2*b*e*log(c) + 2*a*e)*log(f) + 2*(b*e*r*log(c) + b*e*n*
log(f) + b*d*n + (b*e*n + a*e)*r)*log(x))/x^2
```

giac [A] time = 0.24, size = 116, normalized size = 1.40

$$-\frac{2\,bnre\log(x)^{2}+2\,bnre\log(x)+2\,bre\log(c)\log(x)+2\,bne\log(f)\log(x)+bnre+bre\log(c)+bne\log(f)+2\,b}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*n*r*e*log(x)^2 + 2*b*n*r*e*log(x) + 2*b*r*e*log(c)*log(x) + 2*b*n
*e*log(f)*log(x) + b*n*r*e + b*r*e*log(c) + b*n*e*log(f) + 2*b*e*log(c)*log
(f) + 2*b*d*n*log(x) + 2*a*r*e*log(x) + b*d*n + a*r*e + 2*b*d*log(c) + 2*a*
e*log(f) + 2*a*d)/x^2
```

maple [C] time = 0.34, size = 1442, normalized size = 17.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*(d+e*ln(f*x^r))/x^3,x)
```

```
[Out] -1/4*e*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*c)*csgn(I*x^n)*csg
n(I*c*x^n)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+2*b*ln(c
)+b*n+2*b*ln(x^n)+2*a)/x^2*ln(x^r)-1/8*(2*b*d*n+2*a*e*r+2*b*e*r*ln(x^n)+4*b
*e*ln(f)*ln(x^n)+2*b*e*n*ln(f)+2*b*e*r*ln(c)+4*b*e*ln(c)*ln(f)+4*b*d*ln(x^n
)+2*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)^2+4*a*e*ln(f)+4*a*d+2*b*e*n*r+4*b*d*
ln(c)-Pi^2*b*e*csgn(I*c)*csgn(I*f)*csgn(I*x^n)*csgn(I*x^r)*csgn(I*c*x^n)*cs
gn(I*f*x^r)-Pi^2*b*e*csgn(I*c)*csgn(I*f)*csgn(I*c*x^n)^2*csgn(I*f*x^r)^2-Pi
^2*b*e*csgn(I*c)*csgn(I*x^r)*csgn(I*c*x^n)^2*csgn(I*f*x^r)^2+2*I*Pi*a*e*csg
n(I*f)*csgn(I*f*x^r)^2+2*I*Pi*a*e*csgn(I*x^r)*csgn(I*f*x^r)^2-2*I*Pi*b*e*cs
gn(I*f*x^r)^3*ln(x^n)+I*Pi*b*e*r*csgn(I*x^n)*csgn(I*c*x^n)^2+I*Pi*b*e*r*csg
n(I*c*x^n)^2*csgn(I*c)+I*n*Pi*b*e*csgn(I*f)*csgn(I*f*x^r)^2+I*n*Pi*b*e*csgn
(I*x^r)*csgn(I*f*x^r)^2-I*Pi*b*e*r*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+2*I*
Pi*b*d*csgn(I*c)*csgn(I*c*x^n)^2+Pi^2*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(
I*f*x^r)^3-2*I*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(c)-2*I*Pi*b*e*
csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(f)-2*I*Pi*b*d*csgn(I*c)*csgn(I*x^n)*
csgn(I*c*x^n)+Pi^2*b*e*csgn(I*f)*csgn(I*c*x^n)^3*csgn(I*f*x^r)^2+Pi^2*b*e*c
sgn(I*x^r)*csgn(I*c*x^n)^3*csgn(I*f*x^r)^2+Pi^2*b*e*csgn(I*c)*csgn(I*c*x^n)
^2*csgn(I*f*x^r)^3-2*I*Pi*b*e*csgn(I*f*x^r)^3*ln(c)-2*I*Pi*b*e*csgn(I*c*x^n
)^3*ln(f)-I*Pi*b*e*r*csgn(I*c*x^n)^3+2*I*Pi*b*e*csgn(I*f)*csgn(I*f*x^r)^2*l
n(c)-Pi^2*b*e*csgn(I*f)*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*f*x^r)^2-2*I*Pi*
b*d*csgn(I*c*x^n)^3-2*I*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(x^n)+
```

```
Pi^2*b*e*csgn(I*f)*csgn(I*x^n)*csgn(I*x^r)*csgn(I*c*x^n)^2*csgn(I*f*x^r)+Pi
^2*b*e*csgn(I*c)*csgn(I*f)*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f*x^r)^2+2*I*Pi
*b*e*csgn(I*f)*csgn(I*f*x^r)^2*ln(x^n)+2*I*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)
^2*ln(x^n)-Pi^2*b*e*csgn(I*c*x^n)^3*csgn(I*f*x^r)^3-2*I*Pi*a*e*csgn(I*f*x^r
)^3+Pi^2*b*e*csgn(I*c)*csgn(I*x^n)*csgn(I*x^r)*csgn(I*c*x^n)*csgn(I*f*x^r)^
2+Pi^2*b*e*csgn(I*c)*csgn(I*f)*csgn(I*x^r)*csgn(I*c*x^n)^2*csgn(I*f*x^r)-I*
n*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-Pi^2*b*e*csgn(I*f)*csgn(I*x^r)
*csgn(I*c*x^n)^3*csgn(I*f*x^r)-I*n*Pi*b*e*csgn(I*f*x^r)^3+2*I*Pi*b*e*csgn(I
*x^r)*csgn(I*f*x^r)^2*ln(c)+2*I*Pi*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(f)+2*
I*Pi*b*e*csgn(I*c)*csgn(I*c*x^n)^2*ln(f)-2*I*Pi*a*e*csgn(I*f)*csgn(I*x^r)*c
sgn(I*f*x^r)-Pi^2*b*e*csgn(I*x^n)*csgn(I*x^r)*csgn(I*c*x^n)^2*csgn(I*f*x^r)
^2-Pi^2*b*e*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f*x^r)^3)/x^2
```

maxima [A] time = 0.75, size = 93, normalized size = 1.12

$$-\frac{1}{4}\,be\left(\frac{r}{x^2}+\frac{2\,\log\left(fx^r\right)}{x^2}\right)\log\left(cx^n\right)-\frac{ben\left(r+\log(f)+\log\left(x^r\right)\right)}{4\,x^2}-\frac{bdn}{4\,x^2}-\frac{aer}{4\,x^2}-\frac{bd\log\left(cx^n\right)}{2\,x^2}-\frac{ae\log\left(fx^r\right)}{2\,x^2}-\frac{ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*e*(r/x^2 + 2*log(f*x^r)/x^2)*log(c*x^n) - 1/4*b*e*n*(r + log(f) + lo
g(x^r))/x^2 - 1/4*b*d*n/x^2 - 1/4*a*e*r/x^2 - 1/2*b*d*log(c*x^n)/x^2 - 1/2*
a*e*log(f*x^r)/x^2 - 1/2*a*d/x^2
```

mupad [B] time = 3.94, size = 83, normalized size = 1.00

$$-\ln\left(f\,x^r\right)\,\left(\frac{a\,e}{2\,x^2}+\frac{b\,e\,n}{4\,x^2}+\frac{b\,e\,\ln\left(c\,x^n\right)}{2\,x^2}\right)-\frac{\frac{a\,d}{2}+\frac{b\,d\,n}{4}+\frac{a\,e\,r}{4}+\frac{b\,e\,n\,r}{4}}{x^2}-\frac{b\,\ln\left(c\,x^n\right)\,\left(2\,d+e\,r\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] - log(f*x^r)*((a*e)/(2*x^2) + (b*e*n)/(4*x^2) + (b*e*log(c*x^n))/(2*x^2)) -
 ((a*d)/2 + (b*d*n)/4 + (a*e*r)/4 + (b*e*n*r)/4)/x^2 - (b*log(c*x^n)*(2*d +
 e*r))/(4*x^2)
```

sympy [B] time = 7.58, size = 201, normalized size = 2.42

$$-\frac{ad}{2x^2}-\frac{aer\log(x)}{2x^2}-\frac{aer}{4x^2}-\frac{ae\log(f)}{2x^2}-\frac{bdn\log(x)}{2x^2}-\frac{bdn}{4x^2}-\frac{bd\log(c)}{2x^2}-\frac{benr\log(x)^2}{2x^2}-\frac{benr\log(x)}{2x^2}-\frac{benr}{4x^2}-\frac{ben\log}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x**3,x)
```

```
[Out] -a*d/(2*x**2) - a*e*r*log(x)/(2*x**2) - a*e*r/(4*x**2) - a*e*log(f)/(2*x**2
) - b*d*n*log(x)/(2*x**2) - b*d*n/(4*x**2) - b*d*log(c)/(2*x**2) - b*e*n*r*
log(x)**2/(2*x**2) - b*e*n*r*log(x)/(2*x**2) - b*e*n*r/(4*x**2) - b*e*n*log
(f)*log(x)/(2*x**2) - b*e*n*log(f)/(4*x**2) - b*e*r*log(c)*log(x)/(2*x**2)
- b*e*r*log(c)/(4*x**2) - b*e*log(c)*log(f)/(2*x**2)
```

3.162 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx$

Optimal. Leaf size=83

$$-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3}-\frac{er(3a+3b\log(cx^n)+bn)}{27x^3}-\frac{bn(d+e\log(fx^r))}{9x^3}-\frac{benr}{27x^3}$$

```
[Out] -1/27*b*e*n*r/x^3-1/27*e*r*(3*a+b*n+3*b*ln(c*x^n))/x^3-1/9*b*n*(d+e*ln(f*x^
r))/x^3-1/3*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^3
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304, 2366, 12}

$$-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3}-\frac{er(3a+3b\log(cx^n)+bn)}{27x^3}-\frac{bn(d+e\log(fx^r))}{9x^3}-\frac{benr}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] -(b*e*n*r)/(27*x^3) - (e*r*(3*a + b*n + 3*b*Log[c*x^n]))/(27*x^3) - (b*n*(d
 + e*Log[f*x^r]))/(9*x^3) - ((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx=-\frac{bn(d+e\log(fx^r))}{9x^3}-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3}-(er)\int -$$

$$=-\frac{bn(d+e\log(fx^r))}{9x^3}-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3}-\frac{1}{9}(er)\int$$

$$=-\frac{benr}{27x^3}-\frac{er(3a+bn+3b\log(cx^n))}{27x^3}-\frac{bn(d+e\log(fx^r))}{9x^3}-\frac{(a+b\,\mathrm{l}}{}$$

Mathematica [A] time = 0.08, size = 69, normalized size = 0.83

$$-\frac{3e(3a+bn)\log(fx^r)+9ad+3aer+3b\log(cx^n)(3d+3e\log(fx^r)+er)+3bdn+2benr}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] -1/27*(9*a*d + 3*b*d*n + 3*a*e*r + 2*b*e*n*r + 3*e*(3*a + b*n)*Log[f*x^r] +
 3*b*Log[c*x^n]*(3*d + e*r + 3*e*Log[f*x^r]))/x^3
```

fricas [A] time = 0.68, size = 105, normalized size = 1.27

$$-\frac{9\,benr\log(x)^2 + 3\,bdn + 9\,ad + (2\,ben + 3\,ae)r + 3\,(ber + 3\,bd)\log(c) + 3\left(ben + 3\,be\log(c) + 3\,ae\right)\log(f) + \cdots}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(9*b*e*n*r*log(x)^2 + 3*b*d*n + 9*a*d + (2*b*e*n + 3*a*e)*r + 3*(b*e*
r + 3*b*d)*log(c) + 3*(b*e*n + 3*b*e*log(c) + 3*a*e)*log(f) + 3*(3*b*e*r*lo
g(c) + 3*b*e*n*log(f) + 3*b*d*n + (2*b*e*n + 3*a*e)*r)*log(x))/x^3
```

giac [A] time = 0.25, size = 121, normalized size = 1.46

$$-\frac{9\,bnre\log(x)^2 + 6\,bnre\log(x) + 9\,bre\log(c)\log(x) + 9\,bne\log(f)\log(x) + 2\,bnre + 3\,bre\log(c) + 3\,bne\log(\cdots}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^4,x, algorithm="giac")
```

```
[Out] -1/27*(9*b*n*r*e*log(x)^2 + 6*b*n*r*e*log(x) + 9*b*r*e*log(c)*log(x) + 9*b*
n*e*log(f)*log(x) + 2*b*n*r*e + 3*b*r*e*log(c) + 3*b*n*e*log(f) + 9*b*e*log
(c)*log(f) + 9*b*d*n*log(x) + 9*a*r*e*log(x) + 3*b*d*n + 3*a*r*e + 9*b*d*lo
g(c) + 9*a*e*log(f) + 9*a*d)/x^3
```

maple [C] time = 0.35, size = 1451, normalized size = 17.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*(d+e*ln(f*x^r))/x^4,x)
```

```
[Out] -1/18*e*(3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*x^n)*csgn(I*c
*x^n)*csgn(I*c)-3*I*b*Pi*csgn(I*c*x^n)^3+3*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)
+6*b*ln(c)+2*b*n+6*b*ln(x^n)+6*a)/x^3*ln(x^r)-1/108*(12*b*d*n+12*a*e*r+12*b
*e*r*ln(x^n)+36*b*e*ln(f)*ln(x^n)+12*b*e*n*ln(f)+12*b*e*r*ln(c)+36*b*e*ln(c
)*ln(f)+36*b*d*ln(x^n)+36*a*e*ln(f)+36*a*d+8*b*e*n*r+6*I*Pi*b*e*r*csgn(I*x^
n)*csgn(I*c*x^n)^2+6*I*Pi*b*e*r*csgn(I*c*x^n)^2*csgn(I*c)+6*I*n*Pi*b*e*csgn
(I*f)*csgn(I*f*x^r)^2+36*b*d*ln(c)-9*Pi^2*b*e*csgn(I*c)*csgn(I*f)*csgn(I*x^
n)*csgn(I*x^r)*csgn(I*c*x^n)*csgn(I*f*x^r)-9*Pi^2*b*e*csgn(I*c)*csgn(I*f)*c
sgn(I*c*x^n)^2*csgn(I*f*x^r)^2-9*Pi^2*b*e*csgn(I*c)*csgn(I*x^r)*csgn(I*c*x^
n)^2*csgn(I*f*x^r)^2+18*I*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*ln(x^n)+18*I*l
n(c)*Pi*b*e*csgn(I*f)*csgn(I*f*x^r)^2+18*I*Pi*ln(f)*b*e*csgn(I*x^n)*csgn(I*
c*x^n)^2+18*I*Pi*ln(f)*b*e*csgn(I*c*x^n)^2*csgn(I*c)-18*I*Pi*a*e*csgn(I*f)*
csgn(I*x^r)*csgn(I*f*x^r)-18*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+9
*Pi^2*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*f*x^r)^3+9*Pi^2*b*e*csgn(I*f)*
csgn(I*c*x^n)^3*csgn(I*f*x^r)^2+9*Pi^2*b*e*csgn(I*x^r)*csgn(I*c*x^n)^3*csgn
(I*f*x^r)^2+9*Pi^2*b*e*csgn(I*c)*csgn(I*c*x^n)^2*csgn(I*f*x^r)^3-9*Pi^2*b*e
*csgn(I*f)*csgn(I*x^n)*csgn(I*c*x^n)^2*csgn(I*f*x^r)^2+9*Pi^2*b*e*csgn(I*f)
*csgn(I*x^n)*csgn(I*x^r)*csgn(I*c*x^n)^2*csgn(I*f*x^r)+9*Pi^2*b*e*csgn(I*c)
*csgn(I*f)*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*f*x^r)^2-9*Pi^2*b*e*csgn(I*c*x^
n)^3*csgn(I*f*x^r)^3+9*Pi^2*b*e*csgn(I*c)*csgn(I*x^n)*csgn(I*x^r)*csgn(I*c*
x^n)*csgn(I*f*x^r)^2+9*Pi^2*b*e*csgn(I*c)*csgn(I*f)*csgn(I*x^r)*csgn(I*c*x^
```

```
n)^2*csgn(I*f*x^r)-18*I*ln(c)*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-18
*I*Pi*ln(f)*b*e*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-6*I*Pi*b*e*r*csgn(I*x^n
)*csgn(I*c*x^n)*csgn(I*c)-6*I*n*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-
18*I*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(x^n)-18*I*Pi*a*e*csgn(I*
f*x^r)^3-18*I*Pi*b*d*csgn(I*c*x^n)^3+18*I*ln(c)*Pi*b*e*csgn(I*x^r)*csgn(I*f
*x^r)^2+6*I*n*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2+18*I*Pi*b*e*csgn(I*f)*csgn
(I*f*x^r)^2*ln(x^n)-9*Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*c*x^n)^3*csgn(I
*f*x^r)+18*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)^2-6*I*Pi*b*e*r*csgn(I*c*x^n)^
3+18*I*Pi*b*d*csgn(I*c*x^n)^2*csgn(I*c)-6*I*n*Pi*b*e*csgn(I*f*x^r)^3-18*I*P
i*b*e*csgn(I*f*x^r)^3*ln(x^n)-9*Pi^2*b*e*csgn(I*x^n)*csgn(I*x^r)*csgn(I*c*x
^n)^2*csgn(I*f*x^r)^2-9*Pi^2*b*e*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I
*f*x^r)^3-18*I*ln(c)*Pi*b*e*csgn(I*f*x^r)^3-18*I*Pi*ln(f)*b*e*csgn(I*c*x^n)
^3+18*I*Pi*a*e*csgn(I*f)*csgn(I*f*x^r)^2+18*I*Pi*a*e*csgn(I*x^r)*csgn(I*f*x
^r)^2)/x^3
```

maxima [A] time = 0.66, size = 99, normalized size = 1.19

$$-\frac{1}{9}\,be\left(\frac{r}{x^3}+\frac{3\,\log\left(fx^r\right)}{x^3}\right)\log\left(cx^n\right)-\frac{ben\left(2\,r+3\,\log(f)+3\,\log\left(x^r\right)\right)}{27\,x^3}-\frac{bdn}{9\,x^3}-\frac{aer}{9\,x^3}-\frac{bd\log\left(cx^n\right)}{3\,x^3}-\frac{ae\log\left(fx^r\right)}{3\,x^3}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*b*e*(r/x^3 + 3*log(f*x^r)/x^3)*log(c*x^n) - 1/27*b*e*n*(2*r + 3*log(f)
 + 3*log(x^r))/x^3 - 1/9*b*d*n/x^3 - 1/9*a*e*r/x^3 - 1/3*b*d*log(c*x^n)/x^3
 - 1/3*a*e*log(f*x^r)/x^3 - 1/3*a*d/x^3
```

mupad [B] time = 3.93, size = 83, normalized size = 1.00

$$-\ln\left(f\,x^r\right)\,\left(\frac{a\,e}{3\,x^3}+\frac{b\,e\,n}{9\,x^3}+\frac{b\,e\,\ln\left(c\,x^n\right)}{3\,x^3}\right)-\frac{\frac{a\,d}{3}+\frac{b\,d\,n}{9}+\frac{a\,e\,r}{9}+\frac{2\,b\,e\,n\,r}{27}}{x^3}-\frac{b\,\ln\left(c\,x^n\right)\,\left(3\,d+e\,r\right)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] - log(f*x^r)*((a*e)/(3*x^3) + (b*e*n)/(9*x^3) + (b*e*log(c*x^n))/(3*x^3)) -
 ((a*d)/3 + (b*d*n)/9 + (a*e*r)/9 + (2*b*e*n*r)/27)/x^3 - (b*log(c*x^n)*(3*
d + e*r))/(9*x^3)
```

sympy [B] time = 22.20, size = 204, normalized size = 2.46

$$-\frac{ad}{3x^3}-\frac{aer\log(x)}{3x^3}-\frac{aer}{9x^3}-\frac{ae\log(f)}{3x^3}-\frac{bdn\log(x)}{3x^3}-\frac{bdn}{9x^3}-\frac{bd\log(c)}{3x^3}-\frac{benr\log(x)^2}{3x^3}-\frac{2benr\log(x)}{9x^3}-\frac{2benr}{27x^3}-\frac{ben\log(}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x**4,x)
```

```
[Out] -a*d/(3*x**3) - a*e*r*log(x)/(3*x**3) - a*e*r/(9*x**3) - a*e*log(f)/(3*x**3
) - b*d*n*log(x)/(3*x**3) - b*d*n/(9*x**3) - b*d*log(c)/(3*x**3) - b*e*n*r*
log(x)**2/(3*x**3) - 2*b*e*n*r*log(x)/(9*x**3) - 2*b*e*n*r/(27*x**3) - b*e*
n*log(f)*log(x)/(3*x**3) - b*e*n*log(f)/(9*x**3) - b*e*r*log(c)*log(x)/(3*x
**3) - b*e*r*log(c)/(9*x**3) - b*e*log(c)*log(f)/(3*x**3)
```

3.163 $\int x^2 \left(a+b\log\left(cx^n\right)\right)^2 \left(d+e\log\left(fx^r\right)\right)\,dx$

Optimal. Leaf size=207

$$-\frac{1}{81}erx^3\left(9a^2-6abn+2b^2n^2\right)+\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(f\right.\right.$$

```
[Out] -2/81*b^2*e*n^2*r*x^3+2/81*b*e*n*(-b*n+3*a)*r*x^3-1/81*e*(2*b^2*n^2-6*a*b*n
+9*a^2)*r*x^3+2/27*b^2*e*n*r*x^3*ln(c*x^n)-2/27*b*e*(-b*n+3*a)*r*x^3*ln(c*x
^n)-1/9*b^2*e*r*x^3*ln(c*x^n)^2+2/27*b^2*n^2*x^3*(d+e*ln(f*x^r))-2/9*b*n*x^
3*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))+1/3*x^3*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))
```

Rubi [A] time = 0.20, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2305, 2304, 2366, 12, 14}

$$-\frac{1}{81}erx^3\left(9a^2-6abn+2b^2n^2\right)+\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(f\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] (-2*b^2*e*n^2*r*x^3)/81 + (2*b*e*n*(3*a - b*n)*r*x^3)/81 - (e*(9*a^2 - 6*a*
b*n + 2*b^2*n^2)*r*x^3)/81 + (2*b^2*e*n*r*x^3*Log[c*x^n])/27 - (2*b*e*(3*a
- b*n)*r*x^3*Log[c*x^n])/27 - (b^2*e*r*x^3*Log[c*x^n]^2)/9 + (2*b^2*n^2*x^3
*(d + e*Log[f*x^r]))/27 - (2*b*n*x^3*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))
/9 + (x^3*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)dx=\frac{2}{27}b^2n^2x^3\left(d+e\log\left(fx^r\right)\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(f\right.\right.$$
$$=\frac{2}{27}b^2n^2x^3\left(d+e\log\left(fx^r\right)\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(f\right.\right.$$
$$=\frac{2}{27}b^2n^2x^3\left(d+e\log\left(fx^r\right)\right)-\frac{2}{9}bnx^3\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(f\right.\right.$$
$$=-\frac{1}{81}e\left(9a^2-6abn+2b^2n^2\right)rx^3+\frac{2}{27}b^2n^2x^3\left(d+e\log\left(fx^r\right)\right)-\frac{2}{9}b$$
$$=\frac{2}{81}ben(3a-bn)rx^3-\frac{1}{81}e\left(9a^2-6abn+2b^2n^2\right)rx^3-\frac{2}{27}be(3a-b$$
$$=-\frac{2}{81}b^2en^2rx^3+\frac{2}{81}ben(3a-bn)rx^3-\frac{1}{81}e\left(9a^2-6abn+2b^2n^2\right)rx$$

Mathematica [A] time = 0.16, size = 157, normalized size = 0.76

$$\frac{1}{27}x^3\left(e\left(9a^2-6abn+2b^2n^2\right)\log\left(fx^r\right)+9a^2d-3a^2er+2b\log\left(cx^n\right)\left((9ae-3ben)\log\left(fx^r\right)+9ad-3aer-3bd\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] (x^3*(9*a^2*d - 6*a*b*d*n + 2*b^2*d*n^2 - 3*a^2*e*r + 4*a*b*e*n*r - 2*b^2*e
*n^2*r + e*(9*a^2 - 6*a*b*n + 2*b^2*n^2)*Log[f*x^r] + 3*b^2*Log[c*x^n]^2*(3
*d - e*r + 3*e*Log[f*x^r]) + 2*b*Log[c*x^n]*(9*a*d - 3*b*d*n - 3*a*e*r + 2*
b*e*n*r + (9*a*e - 3*b*e*n)*Log[f*x^r])))/27
```

fricas [B] time = 0.84, size = 388, normalized size = 1.87

$$\frac{1}{3}b^2en^2rx^3\log(x)^3-\frac{1}{9}\left(b^2er-3b^2d\right)x^3\log(c)^2-\frac{2}{27}\left(3b^2dn-9abd-\left(2b^2en-3abe\right)r\right)x^3\log(c)+\frac{1}{27}\left(2b^2dn^2-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] 1/3*b^2*e*n^2*r*x^3*log(x)^3 - 1/9*(b^2*e*r - 3*b^2*d)*x^3*log(c)^2 - 2/27*
(3*b^2*d*n - 9*a*b*d - (2*b^2*e*n - 3*a*b*e)*r)*x^3*log(c) + 1/27*(2*b^2*d*
n^2 - 6*a*b*d*n + 9*a^2*d - (2*b^2*e*n^2 - 4*a*b*e*n + 3*a^2*e)*r)*x^3 + 1/
3*(2*b^2*e*n*r*x^3*log(c) + b^2*e*n^2*x^3*log(f) + (b^2*d*n^2 - (b^2*e*n^2
- 2*a*b*e*n)*r)*x^3)*log(x)^2 + 1/27*(9*b^2*e*x^3*log(c)^2 - 6*(b^2*e*n - 3
*a*b*e)*x^3*log(c) + (2*b^2*e*n^2 - 6*a*b*e*n + 9*a^2*e)*x^3)*log(f) + 1/9*
(3*b^2*e*r*x^3*log(c)^2 + 2*(3*b^2*d*n - (2*b^2*e*n - 3*a*b*e)*r)*x^3*log(c
) - (2*b^2*d*n^2 - 6*a*b*d*n - (2*b^2*e*n^2 - 4*a*b*e*n + 3*a^2*e)*r)*x^3 +
 2*(3*b^2*e*n*x^3*log(c) - (b^2*e*n^2 - 3*a*b*e*n)*x^3)*log(f))*log(x)
```

giac [B] time = 0.41, size = 506, normalized size = 2.44

$$\frac{1}{3}b^2n^2rx^3e\log(x)^3-\frac{1}{3}b^2n^2rx^3e\log(x)^2+\frac{2}{3}b^2nrx^3e\log(c)\log(x)^2+\frac{1}{3}b^2n^2x^3e\log(f)\log(x)^2+\frac{2}{9}b^2n^2rx^3e\log(x)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] 1/3*b^2*n^2*r*x^3*e*log(x)^3 - 1/3*b^2*n^2*r*x^3*e*log(x)^2 + 2/3*b^2*n*r*x
^3*e*log(c)*log(x)^2 + 1/3*b^2*n^2*x^3*e*log(f)*log(x)^2 + 2/9*b^2*n^2*r*x^
3*e*log(x) - 4/9*b^2*n*r*x^3*e*log(c)*log(x) + 1/3*b^2*r*x^3*e*log(c)^2*log
```

```
(x) - 2/9*b^2*n^2*x^3*e*log(f)*log(x) + 2/3*b^2*n*x^3*e*log(c)*log(f)*log(x
) + 1/3*b^2*d*n^2*x^3*log(x)^2 + 2/3*a*b*n*r*x^3*e*log(x)^2 - 2/27*b^2*n^2*
r*x^3*e + 4/27*b^2*n*r*x^3*e*log(c) - 1/9*b^2*r*x^3*e*log(c)^2 + 2/27*b^2*n
^2*x^3*e*log(f) - 2/9*b^2*n*x^3*e*log(c)*log(f) + 1/3*b^2*x^3*e*log(c)^2*lo
g(f) - 2/9*b^2*d*n^2*x^3*log(x) - 4/9*a*b*n*r*x^3*e*log(x) + 2/3*b^2*d*n*x^
3*log(c)*log(x) + 2/3*a*b*r*x^3*e*log(c)*log(x) + 2/3*a*b*n*x^3*e*log(f)*lo
g(x) + 2/27*b^2*d*n^2*x^3 + 4/27*a*b*n*r*x^3*e - 2/9*b^2*d*n*x^3*log(c) - 2
/9*a*b*r*x^3*e*log(c) + 1/3*b^2*d*x^3*log(c)^2 - 2/9*a*b*n*x^3*e*log(f) + 2
/3*a*b*x^3*e*log(c)*log(f) + 2/3*a*b*d*n*x^3*log(x) + 1/3*a^2*r*x^3*e*log(x
) - 2/9*a*b*d*n*x^3 - 1/9*a^2*r*x^3*e + 2/3*a*b*d*x^3*log(c) + 1/3*a^2*x^3*
e*log(f) + 1/3*a^2*d*x^3
```

maple [C] time = 0.86, size = 9271, normalized size = 44.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)^2*(d+e*ln(f*x^r)),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.83, size = 250, normalized size = 1.21

$$\frac{1}{3}\,b^2dx^3\log\left(cx^n\right)^2-\frac{2}{9}\,abdnx^3-\frac{1}{9}\,a^2erx^3+\frac{2}{3}\,abdx^3\log\left(cx^n\right)+\frac{1}{3}\,a^2ex^3\log\left(fx^r\right)+\frac{1}{3}\,a^2dx^3-\frac{1}{9}\left(rx^3-3\,x^3\log\left(fx^r\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*d*x^3*log(c*x^n)^2 - 2/9*a*b*d*n*x^3 - 1/9*a^2*e*r*x^3 + 2/3*a*b*d*
x^3*log(c*x^n) + 1/3*a^2*e*x^3*log(f*x^r) + 1/3*a^2*d*x^3 - 1/9*(r*x^3 - 3*
x^3*log(f*x^r))*b^2*e*log(c*x^n)^2 + 2/27*((2*r - 3*log(f))*x^3 - 3*x^3*log
(x^r))*a*b*e*n - 2/9*(r*x^3 - 3*x^3*log(f*x^r))*a*b*e*log(c*x^n) + 2/27*(n^
2*x^3 - 3*n*x^3*log(c*x^n))*b^2*d - 2/27*(((r - log(f))*x^3 - x^3*log(x^r))
*n^2 - ((2*r - 3*log(f))*x^3 - 3*x^3*log(x^r))*n*log(c*x^n))*b^2*e
```

mupad [B] time = 3.99, size = 189, normalized size = 0.91

$$\ln\left(f\,x^r\right)\,\left(\ln\left(c\,x^n\right)\,\left(\frac{2\,a\,b\,e\,x^3}{3}-\frac{2\,b^2\,e\,n\,x^3}{9}\right)+\frac{a^2\,e\,x^3}{3}+\frac{2\,b^2\,e\,n^2\,x^3}{27}+\frac{b^2\,e\,x^3\,\ln\left(c\,x^n\right)^2}{3}-\frac{2\,a\,b\,e\,n\,x^3}{9}\right)+x^3\,\left(\frac{a^2\,d}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n))^2,x)
```

```
[Out] log(f*x^r)*(log(c*x^n)*((2*a*b*e*x^3)/3 - (2*b^2*e*n*x^3)/9) + (a^2*e*x^3)/
3 + (2*b^2*e*n^2*x^3)/27 + (b^2*e*x^3*log(c*x^n)^2)/3 - (2*a*b*e*n*x^3)/9)
+ x^3*((a^2*d)/3 + (2*b^2*d*n^2)/27 - (a^2*e*r)/9 - (2*b^2*e*n^2*r)/27 - (2
*a*b*d*n)/9 + (4*a*b*e*n*r)/27) + (b^2*x^3*log(c*x^n)^2*(3*d - e*r))/9 + (2
*b*x^3*log(c*x^n)*(9*a*d - 3*b*d*n - 3*a*e*r + 2*b*e*n*r))/27
```

sympy [B] time = 70.85, size = 654, normalized size = 3.16

$$\frac{a^2dx^3}{3}+\frac{a^2erx^3\log\left(x\right)}{3}-\frac{a^2erx^3}{9}+\frac{a^2ex^3\log\left(f\right)}{3}+\frac{2abdnx^3\log\left(x\right)}{3}-\frac{2abdnx^3}{9}+\frac{2abdx^3\log\left(c\right)}{3}+\frac{2abenrx^3\log\left(x\right)^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*(d+e*ln(f*x**r)),x)
```

```
[Out] a**2*d*x**3/3 + a**2*e*r*x**3*log(x)/3 - a**2*e*r*x**3/9 + a**2*e*x**3*log(
f)/3 + 2*a*b*d*n*x**3*log(x)/3 - 2*a*b*d*n*x**3/9 + 2*a*b*d*x**3*log(c)/3 +
 2*a*b*e*n*r*x**3*log(x)**2/3 - 4*a*b*e*n*r*x**3*log(x)/9 + 4*a*b*e*n*r*x**
3/27 + 2*a*b*e*n*x**3*log(f)*log(x)/3 - 2*a*b*e*n*x**3*log(f)/9 + 2*a*b*e*r
*x**3*log(c)*log(x)/3 - 2*a*b*e*r*x**3*log(c)/9 + 2*a*b*e*x**3*log(c)*log(f
)/3 + b**2*d*n**2*x**3*log(x)**2/3 - 2*b**2*d*n**2*x**3*log(x)/9 + 2*b**2*d
*n**2*x**3/27 + 2*b**2*d*n*x**3*log(c)*log(x)/3 - 2*b**2*d*n*x**3*log(c)/9
+ b**2*d*x**3*log(c)**2/3 + b**2*e*n**2*r*x**3*log(x)**3/3 - b**2*e*n**2*r*
x**3*log(x)**2/3 + 2*b**2*e*n**2*r*x**3*log(x)/9 - 2*b**2*e*n**2*r*x**3/27
+ b**2*e*n**2*x**3*log(f)*log(x)**2/3 - 2*b**2*e*n**2*x**3*log(f)*log(x)/9
+ 2*b**2*e*n**2*x**3*log(f)/27 + 2*b**2*e*n*r*x**3*log(c)*log(x)**2/3 - 4*b
**2*e*n*r*x**3*log(c)*log(x)/9 + 4*b**2*e*n*r*x**3*log(c)/27 + 2*b**2*e*n*x
**3*log(c)*log(f)*log(x)/3 - 2*b**2*e*n*x**3*log(c)*log(f)/9 + b**2*e*r*x**
3*log(c)**2*log(x)/3 - b**2*e*r*x**3*log(c)**2/9 + b**2*e*x**3*log(c)**2*lo
g(f)/3
```

3.164 $\int x\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)dx$

Optimal. Leaf size=206

$$-\frac{1}{8}erx^2\left(2a^2-2abn+b^2n^2\right)+\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)$$

```
[Out] -1/8*b^2*e*n^2*r*x^2+1/8*b*e*n*(-b*n+2*a)*r*x^2-1/8*e*(b^2*n^2-2*a*b*n+2*a^
2)*r*x^2+1/4*b^2*e*n*r*x^2*ln(c*x^n)-1/4*b*e*(-b*n+2*a)*r*x^2*ln(c*x^n)-1/4
*b^2*e*r*x^2*ln(c*x^n)^2+1/4*b^2*n^2*x^2*(d+e*ln(f*x^r))-1/2*b*n*x^2*(a+b*l
n(c*x^n))*(d+e*ln(f*x^r))+1/2*x^2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))
```

Rubi [A] time = 0.17, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2305, 2304, 2366, 12, 14}

$$-\frac{1}{8}erx^2\left(2a^2-2abn+b^2n^2\right)+\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] -(b^2*e*n^2*r*x^2)/8 + (b*e*n*(2*a - b*n)*r*x^2)/8 - (e*(2*a^2 - 2*a*b*n +
b^2*n^2)*r*x^2)/8 + (b^2*e*n*r*x^2*Log[c*x^n])/4 - (b*e*(2*a - b*n)*r*x^2*L
og[c*x^n])/4 - (b^2*e*r*x^2*Log[c*x^n]^2)/4 + (b^2*n^2*x^2*(d + e*Log[f*x^r
]))/4 - (b*n*x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/2 + (x^2*(a + b*Log
[c*x^n])^2*(d + e*Log[f*x^r]))/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int x \left(a+b\log\left(cx^n\right)\right)^2 \left(d+e\log\left(fx^r\right)\right) dx = \frac{1}{4}b^2n^2x^2\left(d+e\log\left(fx^r\right)\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right.\right.$$
$$= \frac{1}{4}b^2n^2x^2\left(d+e\log\left(fx^r\right)\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right.\right.$$
$$= \frac{1}{4}b^2n^2x^2\left(d+e\log\left(fx^r\right)\right)-\frac{1}{2}bnx^2\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right.\right.$$
$$= -\frac{1}{8}e\left(2a^2-2abn+b^2n^2\right)rx^2+\frac{1}{4}b^2n^2x^2\left(d+e\log\left(fx^r\right)\right)-\frac{1}{2}bnx^2$$
$$= \frac{1}{8}ben(2a-bn)rx^2-\frac{1}{8}e\left(2a^2-2abn+b^2n^2\right)rx^2-\frac{1}{4}be(2a-bn)rx^2$$
$$= -\frac{1}{8}b^2en^2rx^2+\frac{1}{8}ben(2a-bn)rx^2-\frac{1}{8}e\left(2a^2-2abn+b^2n^2\right)rx^2+\frac{1}{4}$$

Mathematica [A] time = 0.15, size = 154, normalized size = 0.75

$$\frac{1}{8}x^2\left(2e\left(2a^2-2abn+b^2n^2\right)\log\left(fx^r\right)+4a^2d-2a^2er-4b\log\left(cx^n\right)\left(\left(ben-2ae\right)\log\left(fx^r\right)-2ad+aer+bdn-b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] (x^2*(4*a^2*d - 4*a*b*d*n + 2*b^2*d*n^2 - 2*a^2*e*r + 4*a*b*e*n*r - 3*b^2*e
*n^2*r + 2*e*(2*a^2 - 2*a*b*n + b^2*n^2)*Log[f*x^r] + 2*b^2*Log[c*x^n]^2*(2
*d - e*r + 2*e*Log[f*x^r]) - 4*b*Log[c*x^n]*(-2*a*d + b*d*n + a*e*r - b*e*n
*r + (-2*a*e + b*e*n)*Log[f*x^r])))/8
```

fricas [B] time = 0.76, size = 386, normalized size = 1.87

$$\frac{1}{2}b^2en^2rx^2\log(x)^3-\frac{1}{4}\left(b^2er-2b^2d\right)x^2\log(c)^2-\frac{1}{2}\left(b^2dn-2abd-\left(b^2en-abe\right)r\right)x^2\log(c)+\frac{1}{8}\left(2b^2dn^2-4abdn+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] 1/2*b^2*e*n^2*r*x^2*log(x)^3 - 1/4*(b^2*e*r - 2*b^2*d)*x^2*log(c)^2 - 1/2*(
b^2*d*n - 2*a*b*d - (b^2*e*n - a*b*e)*r)*x^2*log(c) + 1/8*(2*b^2*d*n^2 - 4*
a*b*d*n + 4*a^2*d - (3*b^2*e*n^2 - 4*a*b*e*n + 2*a^2*e)*r)*x^2 + 1/4*(4*b^2
*e*n*r*x^2*log(c) + 2*b^2*e*n^2*x^2*log(f) + (2*b^2*d*n^2 - (3*b^2*e*n^2 -
4*a*b*e*n)*r)*x^2)*log(x)^2 + 1/4*(2*b^2*e*x^2*log(c)^2 - 2*(b^2*e*n - 2*a*
b*e)*x^2*log(c) + (b^2*e*n^2 - 2*a*b*e*n + 2*a^2*e)*x^2)*log(f) + 1/4*(2*b^
2*e*r*x^2*log(c)^2 + 4*(b^2*d*n - (b^2*e*n - a*b*e)*r)*x^2*log(c) - (2*b^2*
d*n^2 - 4*a*b*d*n - (3*b^2*e*n^2 - 4*a*b*e*n + 2*a^2*e)*r)*x^2 + 2*(2*b^2*e
*n*x^2*log(c) - (b^2*e*n^2 - 2*a*b*e*n)*x^2)*log(f))*log(x)
```

giac [B] time = 0.36, size = 497, normalized size = 2.41

$$\frac{1}{2}b^2n^2rx^2e\log(x)^3-\frac{3}{4}b^2n^2rx^2e\log(x)^2+b^2nrx^2e\log(c)\log(x)^2+\frac{1}{2}b^2n^2x^2e\log(f)\log(x)^2+\frac{3}{4}b^2n^2rx^2e\log(x)-b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] 1/2*b^2*n^2*r*x^2*e*log(x)^3 - 3/4*b^2*n^2*r*x^2*e*log(x)^2 + b^2*n*r*x^2*e
*log(c)*log(x)^2 + 1/2*b^2*n^2*x^2*e*log(f)*log(x)^2 + 3/4*b^2*n^2*r*x^2*e*
```

```
log(x) - b^2*n*r*x^2*e*log(c)*log(x) + 1/2*b^2*r*x^2*e*log(c)^2*log(x) - 1/
2*b^2*n^2*x^2*e*log(f)*log(x) + b^2*n*x^2*e*log(c)*log(f)*log(x) + 1/2*b^2*
d*n^2*x^2*log(x)^2 + a*b*n*r*x^2*e*log(x)^2 - 3/8*b^2*n^2*r*x^2*e + 1/2*b^2
*n*r*x^2*e*log(c) - 1/4*b^2*r*x^2*e*log(c)^2 + 1/4*b^2*n^2*x^2*e*log(f) - 1
/2*b^2*n*x^2*e*log(c)*log(f) + 1/2*b^2*x^2*e*log(c)^2*log(f) - 1/2*b^2*d*n^
2*x^2*log(x) - a*b*n*r*x^2*e*log(x) + b^2*d*n*x^2*log(c)*log(x) + a*b*r*x^2
*e*log(c)*log(x) + a*b*n*x^2*e*log(f)*log(x) + 1/4*b^2*d*n^2*x^2 + 1/2*a*b*
n*r*x^2*e - 1/2*b^2*d*n*x^2*log(c) - 1/2*a*b*r*x^2*e*log(c) + 1/2*b^2*d*x^2
*log(c)^2 - 1/2*a*b*n*x^2*e*log(f) + a*b*x^2*e*log(c)*log(f) + a*b*d*n*x^2*
log(x) + 1/2*a^2*r*x^2*e*log(x) - 1/2*a*b*d*n*x^2 - 1/4*a^2*r*x^2*e + a*b*d
*x^2*log(c) + 1/2*a^2*x^2*e*log(f) + 1/2*a^2*d*x^2
```

maple [C] time = 0.86, size = 9262, normalized size = 44.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^2*(d+e*ln(f*x^r)),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.61, size = 247, normalized size = 1.20

$$\frac{1}{2}b^2dx^2\log(cx^n)^2-\frac{1}{2}abdnx^2-\frac{1}{4}a^2erx^2+abdx^2\log(cx^n)-\frac{1}{4}\left(rx^2-2\,x^2\log\left(fx^r\right)\right)b^2e\log(cx^n)^2+\frac{1}{2}a^2ex^2\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*d*x^2*log(c*x^n)^2 - 1/2*a*b*d*n*x^2 - 1/4*a^2*e*r*x^2 + a*b*d*x^2*
log(c*x^n) - 1/4*(r*x^2 - 2*x^2*log(f*x^r))*b^2*e*log(c*x^n)^2 + 1/2*a^2*e*
x^2*log(f*x^r) + 1/2*((r - log(f))*x^2 - x^2*log(x^r))*a*b*e*n + 1/2*a^2*d*
x^2 - 1/2*(r*x^2 - 2*x^2*log(f*x^r))*a*b*e*log(c*x^n) + 1/4*(n^2*x^2 - 2*n*
x^2*log(c*x^n))*b^2*d - 1/8*(((3*r - 2*log(f))*x^2 - 2*x^2*log(x^r))*n^2 -
4*((r - log(f))*x^2 - x^2*log(x^r))*n*log(c*x^n))*b^2*e
```

mupad [B] time = 4.15, size = 187, normalized size = 0.91

$$\ln\left(f\,x^r\right)\,\left(\ln\left(c\,x^n\right)\,\left(a\,b\,e\,x^2-\frac{b^2\,e\,n\,x^2}{2}\right)+\frac{a^2\,e\,x^2}{2}+\frac{b^2\,e\,n^2\,x^2}{4}+\frac{b^2\,e\,x^2\ln\left(c\,x^n\right)^2}{2}-\frac{a\,b\,e\,n\,x^2}{2}\right)+x^2\,\left(\frac{a^2\,d}{2}+\frac{b^2\,d}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n))^2,x)
```

```
[Out] log(f*x^r)*(log(c*x^n)*(a*b*e*x^2 - (b^2*e*n*x^2)/2) + (a^2*e*x^2)/2 + (b^2
*e*n^2*x^2)/4 + (b^2*e*x^2*log(c*x^n)^2)/2 - (a*b*e*n*x^2)/2) + x^2*((a^2*d
)/2 + (b^2*d*n^2)/4 - (a^2*e*r)/4 - (3*b^2*e*n^2*r)/8 - (a*b*d*n)/2 + (a*b*
e*n*r)/2) + (b^2*x^2*log(c*x^n)^2*(2*d - e*r))/4 + (b*x^2*log(c*x^n)*(2*a*d
 - b*d*n - a*e*r + b*e*n*r))/2
```

sympy [B] time = 26.50, size = 600, normalized size = 2.91

$$\frac{a^2dx^2}{2}+\frac{a^2erx^2\log(x)}{2}-\frac{a^2erx^2}{4}+\frac{a^2ex^2\log(f)}{2}+abdnx^2\log(x)-\frac{abdnx^2}{2}+abdx^2\log(c)+abenrx^2\log(x)^2-abenr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*(d+e*ln(f*x**r)),x)
```

```
[Out] a**2*d*x**2/2 + a**2*e*r*x**2*log(x)/2 - a**2*e*r*x**2/4 + a**2*e*x**2*log(
f)/2 + a*b*d*n*x**2*log(x) - a*b*d*n*x**2/2 + a*b*d*x**2*log(c) + a*b*e*n*r
*x**2*log(x)**2 - a*b*e*n*r*x**2*log(x) + a*b*e*n*r*x**2/2 + a*b*e*n*x**2*l
og(f)*log(x) - a*b*e*n*x**2*log(f)/2 + a*b*e*r*x**2*log(c)*log(x) - a*b*e*r
*x**2*log(c)/2 + a*b*e*x**2*log(c)*log(f) + b**2*d*n**2*x**2*log(x)**2/2 -
b**2*d*n**2*x**2*log(x)/2 + b**2*d*n**2*x**2/4 + b**2*d*n*x**2*log(c)*log(x
) - b**2*d*n*x**2*log(c)/2 + b**2*d*x**2*log(c)**2/2 + b**2*e*n**2*r*x**2*l
og(x)**3/2 - 3*b**2*e*n**2*r*x**2*log(x)**2/4 + 3*b**2*e*n**2*r*x**2*log(x)
/4 - 3*b**2*e*n**2*r*x**2/8 + b**2*e*n**2*x**2*log(f)*log(x)**2/2 - b**2*e*
n**2*x**2*log(f)*log(x)/2 + b**2*e*n**2*x**2*log(f)/4 + b**2*e*n*r*x**2*log
(c)*log(x)**2 - b**2*e*n*r*x**2*log(c)*log(x) + b**2*e*n*r*x**2*log(c)/2 +
b**2*e*n*x**2*log(c)*log(f)*log(x) - b**2*e*n*x**2*log(c)*log(f)/2 + b**2*e
*r*x**2*log(c)**2*log(x)/2 - b**2*e*r*x**2*log(c)**2/4 + b**2*e*x**2*log(c)
**2*log(f)/2
```

3.165 $\int (a+b\log(cx^n))^2 (d+e\log(fx^r))\,dx$

Optimal. Leaf size=147

$$x(a+b\log(cx^n))^2(d+e\log(fx^r)) - erx(a+b\log(cx^n))^2 - 2abnx(d+e\log(fx^r)) + 2abenrx + 2benrx(a-bn)\cdots$$

```
[Out] 2*a*b*e*n*r*x-4*b^2*e*n^2*r*x+2*b*e*n*(-b*n+a)*r*x+4*b^2*e*n*r*x*ln(c*x^n)-
e*r*x*(a+b*ln(c*x^n))^2-2*a*b*n*x*(d+e*ln(f*x^r))+2*b^2*n^2*x*(d+e*ln(f*x^r
))-2*b^2*n*x*ln(c*x^n)*(d+e*ln(f*x^r))+x*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))
```

Rubi [A] time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2296, 2295, 2361}

$$x(a+b\log(cx^n))^2(d+e\log(fx^r)) - erx(a+b\log(cx^n))^2 - 2abnx(d+e\log(fx^r)) + 2abenrx + 2benrx(a-bn)\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] 2*a*b*e*n*r*x - 4*b^2*e*n^2*r*x + 2*b*e*n*(a - b*n)*r*x + 4*b^2*e*n*r*x*Log
[c*x^n] - e*r*x*(a + b*Log[c*x^n])^2 - 2*a*b*n*x*(d + e*Log[f*x^r]) + 2*b^2
*n^2*x*(d + e*Log[f*x^r]) - 2*b^2*n*x*Log[c*x^n]*(d + e*Log[f*x^r]) + x*(a
+ b*Log[c*x^n])^2*(d + e*Log[f*x^r])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Dist[
d + e*Log[f*x^r], u, x] - Dist[e*r, Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+b\log(cx^n))^2(d+e\log(fx^r))\,dx &= -2abnx(d+e\log(fx^r)) + 2b^2n^2x(d+e\log(fx^r)) - 2b^2nx\log(\cdots \\
&= 2ben(a-bn)rx - 2abnx(d+e\log(fx^r)) + 2b^2n^2x(d+e\log(fx^\cdots \\
&= -2b^2en^2rx + 2ben(a-bn)rx + 2b^2enrx\log(cx^n) - erx(a+b\log(\cdots \\
&= 2abenrx - 2b^2en^2rx + 2ben(a-bn)rx + 2b^2enrx\log(cx^n) - erx(a\cdots \\
&= 2abenrx - 4b^2en^2rx + 2ben(a-bn)rx + 4b^2enrx\log(cx^n) - erx(a\cdots
\end{aligned}$$

Mathematica [A] time = 0.11, size = 141, normalized size = 0.96

$$x\left(e\left(a^2-2abn+2b^2n^2\right)\log(fx^r) + a^2d - a^2er + 2b\log(cx^n)\left(e(a-bn)\log(fx^r) + ad - aer - bdn + 2benr\right)\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] x*(a^2*d - 2*a*b*d*n + 2*b^2*d*n^2 - a^2*e*r + 4*a*b*e*n*r - 6*b^2*e*n^2*r
+ e*(a^2 - 2*a*b*n + 2*b^2*n^2)*Log[f*x^r] + b^2*Log[c*x^n]^2*(d - e*r + e*
Log[f*x^r]) + 2*b*Log[c*x^n]*(a*d - b*d*n - a*e*r + 2*b*e*n*r + e*(a - b*n)
*Log[f*x^r]))
```

fricas [B] time = 0.89, size = 345, normalized size = 2.35

$$b^2en^2rx\log(x)^3-(b^2er-b^2d)x\log(c)^2-2(b^2dn-abd-(2b^2en-abe)r)x\log(c)+(2b^2enrx\log(c)+b^2en^2x\log(f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] b^2*e*n^2*r*x*log(x)^3 - (b^2*e*r - b^2*d)*x*log(c)^2 - 2*(b^2*d*n - a*b*d
- (2*b^2*e*n - a*b*e)*r)*x*log(c) + (2*b^2*e*n*r*x*log(c) + b^2*e*n^2*x*log
(f) + (b^2*d*n^2 - (3*b^2*e*n^2 - 2*a*b*e*n)*r)*x)*log(x)^2 + (2*b^2*d*n^2
- 2*a*b*d*n + a^2*d - (6*b^2*e*n^2 - 4*a*b*e*n + a^2*e)*r)*x + (b^2*e*x*log
(c)^2 - 2*(b^2*e*n - a*b*e)*x*log(c) + (2*b^2*e*n^2 - 2*a*b*e*n + a^2*e)*x)
*log(f) + (b^2*e*r*x*log(c)^2 + 2*(b^2*d*n - (2*b^2*e*n - a*b*e)*r)*x*log(c
) - (2*b^2*d*n^2 - 2*a*b*d*n - (6*b^2*e*n^2 - 4*a*b*e*n + a^2*e)*r)*x + 2*(
b^2*e*n*x*log(c) - (b^2*e*n^2 - a*b*e*n)*x)*log(f))*log(x)
```

giac [B] time = 0.34, size = 425, normalized size = 2.89

$$b^2n^2rxe\log(x)^3-3b^2n^2rxe\log(x)^2+2b^2nrxe\log(c)\log(x)^2+b^2n^2xe\log(f)\log(x)^2+6b^2n^2rxe\log(x)-4b^2nrxe\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] b^2*n^2*r*x*e*log(x)^3 - 3*b^2*n^2*r*x*e*log(x)^2 + 2*b^2*n*r*x*e*log(c)*lo
g(x)^2 + b^2*n^2*x*e*log(f)*log(x)^2 + 6*b^2*n^2*r*x*e*log(x) - 4*b^2*n*r*x
*e*log(c)*log(x) + b^2*r*x*e*log(c)^2*log(x) - 2*b^2*n^2*x*e*log(f)*log(x)
+ 2*b^2*n*x*e*log(c)*log(f)*log(x) + b^2*d*n^2*x*log(x)^2 + 2*a*b*n*r*x*e*l
og(x)^2 - 6*b^2*n^2*r*x*e + 4*b^2*n*r*x*e*log(c) - b^2*r*x*e*log(c)^2 + 2*b
^2*n^2*x*e*log(f) - 2*b^2*n*x*e*log(c)*log(f) + b^2*x*e*log(c)^2*log(f) - 2
*b^2*d*n^2*x*log(x) - 4*a*b*n*r*x*e*log(x) + 2*b^2*d*n*x*log(c)*log(x) + 2*
a*b*r*x*e*log(c)*log(x) + 2*a*b*n*x*e*log(f)*log(x) + 2*b^2*d*n^2*x + 4*a*b
*n*r*x*e - 2*b^2*d*n*x*log(c) - 2*a*b*r*x*e*log(c) + b^2*d*x*log(c)^2 - 2*a
*b*n*x*e*log(f) + 2*a*b*x*e*log(c)*log(f) + 2*a*b*d*n*x*log(x) + a^2*r*x*e*
log(x) - 2*a*b*d*n*x - a^2*r*x*e + 2*a*b*d*x*log(c) + a^2*x*e*log(f) + a^2*
d*x
```

maple [C] time = 0.83, size = 8701, normalized size = 59.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*(d+e*ln(f*x^r)),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.77, size = 213, normalized size = 1.45

$$-(rx-x\log(fx^r))b^2e\log(cx^n)^2+b^2dx\log(cx^n)^2+2((2r-\log(f))x-x\log(x^r))aben-2abdnx-a^2erx-2(rx-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] -(r*x - x*log(f*x^r))*b^2*e*log(c*x^n)^2 + b^2*d*x*log(c*x^n)^2 + 2*((2*r -
 log(f))*x - x*log(x^r))*a*b*e*n - 2*a*b*d*n*x - a^2*e*r*x - 2*(r*x - x*log
(f*x^r))*a*b*e*log(c*x^n) + 2*a*b*d*x*log(c*x^n) + a^2*e*x*log(f*x^r) + 2*(
n^2*x - n*x*log(c*x^n))*b^2*d - 2*(((3*r - log(f))*x - x*log(x^r))*n^2 - ((
2*r - log(f))*x - x*log(x^r))*n*log(c*x^n))*b^2*e + a^2*d*x
```

mupad [B] time = 3.88, size = 165, normalized size = 1.12

$$x\left(a^2 d+2 b^2 d n^2-a^2 e r-6 b^2 e n^2 r-2 a b d n+4 a b e n r\right)+\ln\left(f x^r\right)\left(a^2 e x-\ln\left(c x^n\right)\left(2 b^2 e n x-2 a b e x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*log(f*x^r))*(a + b*log(c*x^n))^2,x)
```

```
[Out] x*(a^2*d + 2*b^2*d*n^2 - a^2*e*r - 6*b^2*e*n^2*r - 2*a*b*d*n + 4*a*b*e*n*r)
 + log(f*x^r)*(a^2*e*x - log(c*x^n)*(2*b^2*e*n*x - 2*a*b*e*x) + 2*b^2*e*n^2
*x + b^2*e*x*log(c*x^n)^2 - 2*a*b*e*n*x) + 2*b*x*log(c*x^n)*(a*d - b*d*n -
a*e*r + 2*b*e*n*r) + b^2*x*log(c*x^n)^2*(d - e*r)
```

sympy [B] time = 9.26, size = 534, normalized size = 3.63

$$a^2dx+a^2erx\log(x)-a^2erx+a^2ex\log(f)+2abdnx\log(x)-2abdnx+2abdx\log(c)+2abenrx\log(x)^2-4abenrx\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r)),x)
```

```
[Out] a**2*d*x + a**2*e*r*x*log(x) - a**2*e*r*x + a**2*e*x*log(f) + 2*a*b*d*n*x*l
og(x) - 2*a*b*d*n*x + 2*a*b*d*x*log(c) + 2*a*b*e*n*r*x*log(x)**2 - 4*a*b*e*
n*r*x*log(x) + 4*a*b*e*n*r*x + 2*a*b*e*n*x*log(f)*log(x) - 2*a*b*e*n*x*log(
f) + 2*a*b*e*r*x*log(c)*log(x) - 2*a*b*e*r*x*log(c) + 2*a*b*e*x*log(c)*log(
f) + b**2*d*n**2*x*log(x)**2 - 2*b**2*d*n**2*x*log(x) + 2*b**2*d*n**2*x + 2
*b**2*d*n*x*log(c)*log(x) - 2*b**2*d*n*x*log(c) + b**2*d*x*log(c)**2 + b**2
*e*n**2*r*x*log(x)**3 - 3*b**2*e*n**2*r*x*log(x)**2 + 6*b**2*e*n**2*r*x*log
(x) - 6*b**2*e*n**2*r*x + b**2*e*n**2*x*log(f)*log(x)**2 - 2*b**2*e*n**2*x*
log(f)*log(x) + 2*b**2*e*n**2*x*log(f) + 2*b**2*e*n*r*x*log(c)*log(x)**2 -
4*b**2*e*n*r*x*log(c)*log(x) + 4*b**2*e*n*r*x*log(c) + 2*b**2*e*n*x*log(c)*
log(f)*log(x) - 2*b**2*e*n*x*log(c)*log(f) + b**2*e*r*x*log(c)**2*log(x) -
b**2*e*r*x*log(c)**2 + b**2*e*x*log(c)**2*log(f)
```

3.166 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}\,dx$

Optimal. Leaf size=57

$$\frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}-\frac{er(a+b\log(cx^n))^4}{12b^2n^2}$$

```
[Out] -1/12*e*r*(a+b*ln(c*x^n))^4/b^2/n^2+1/3*(a+b*ln(c*x^n))^3*(d+e*ln(f*x^r))/b
/n
```

Rubi [A] time = 0.10, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2302, 30, 2366, 12}

$$\frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}-\frac{er(a+b\log(cx^n))^4}{12b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] -(e*r*(a + b*Log[c*x^n])^4)/(12*b^2*n^2) + ((a + b*Log[c*x^n])^3*(d + e*Log
[f*x^r]))/(3*b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)}{x}\,dx=\frac{\left(a+b\log\left(cx^n\right)\right)^3\left(d+e\log\left(fx^r\right)\right)}{3bn}-(er)\int\frac{\left(a+b\log\left(cx^n\right)\right)^3}{3bnx}\,dx$$
$$=\frac{\left(a+b\log\left(cx^n\right)\right)^3\left(d+e\log\left(fx^r\right)\right)}{3bn}-\frac{(er)\int\frac{\left(a+b\log\left(cx^n\right)\right)^3}{x}\,dx}{3bn}$$
$$=\frac{\left(a+b\log\left(cx^n\right)\right)^3\left(d+e\log\left(fx^r\right)\right)}{3bn}-\frac{(er)\,\text{Subst}\left(\int x^3\,dx,x,a+b$$
$$=-\frac{er\left(a+b\log\left(cx^n\right)\right)^4}{12b^2n^2}+\frac{\left(a+b\log\left(cx^n\right)\right)^3\left(d+e\log\left(fx^r\right)\right)}{3bn}$$

Mathematica [B] time = 0.14, size = 129, normalized size = 2.26

$$\frac{1}{12}\log(x)\left(4bn\log^2(x)\left(2aer+2ber\log\left(cx^n\right)+bdn+ben\log\left(fx^r\right)\right)-6\log(x)\left(a+b\log\left(cx^n\right)\right)\left(aer+ber\log$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] (Log[x]*(-3*b^2*e*n^2*r*Log[x]^3 + 12*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r
]) + 4*b*n*Log[x]^2*(b*d*n + 2*a*e*r + 2*b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r
]) - 6*Log[x]*(a + b*Log[c*x^n])*(2*b*d*n + a*e*r + b*e*r*Log[c*x^n] + 2*b*
e*n*Log[f*x^r])))/12
```

fricas [B] time = 0.85, size = 170, normalized size = 2.98

$$\frac{1}{4}b^2en^2r\log(x)^4+\frac{1}{3}\left(2b^2enr\log(c)+b^2en^2\log(f)+b^2dn^2+2abenr\right)\log(x)^3+\frac{1}{2}\left(b^2er\log(c)^2+2abdn+a^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*e*n^2*r*log(x)^4 + 1/3*(2*b^2*e*n*r*log(c) + b^2*e*n^2*log(f) + b^2
*d*n^2 + 2*a*b*e*n*r)*log(x)^3 + 1/2*(b^2*e*r*log(c)^2 + 2*a*b*d*n + a^2*e*
r + 2*(b^2*d*n + a*b*e*r)*log(c) + 2*(b^2*e*n*log(c) + a*b*e*n)*log(f))*log
(x)^2 + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d + (b^2*e*log(c)^2 + 2*a*b*
e*log(c) + a^2*e)*log(f))*log(x)
```

giac [B] time = 0.36, size = 223, normalized size = 3.91

$$\frac{1}{4}b^2n^2re\log(x)^4+\frac{2}{3}b^2nre\log(c)\log(x)^3+\frac{1}{3}b^2n^2e\log(f)\log(x)^3+\frac{1}{2}b^2re\log(c)^2\log(x)^2+b^2ne\log(c)\log(f)\,\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x,x, algorithm="giac")
```

```
[Out] 1/4*b^2*n^2*r*e*log(x)^4 + 2/3*b^2*n*r*e*log(c)*log(x)^3 + 1/3*b^2*n^2*e*lo
g(f)*log(x)^3 + 1/2*b^2*r*e*log(c)^2*log(x)^2 + b^2*n*e*log(c)*log(f)*log(x
)^2 + 1/3*b^2*d*n^2*log(x)^3 + 2/3*a*b*n*r*e*log(x)^3 + b^2*e*log(c)^2*log(
f)*log(x) + b^2*d*n*log(c)*log(x)^2 + a*b*r*e*log(c)*log(x)^2 + a*b*n*e*log
(f)*log(x)^2 + b^2*d*log(c)^2*log(x) + 2*a*b*e*log(c)*log(f)*log(x) + a*b*d
*n*log(x)^2 + 1/2*a^2*r*e*log(x)^2 + 2*a*b*d*log(c)*log(x) + a^2*e*log(f)*l
og(x) + a^2*d*log(x)
```

maple [C] time = 1.55, size = 9164, normalized size = 160.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*(d+e*ln(f*x^r))/x,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.66, size = 163, normalized size = 2.86

$$\frac{b^2 e \log(cx^n)^2 \log\left(fx^r\right)^2}{2r} + \frac{b^2 d \log(cx^n)^3}{3n} + \frac{abe \log(cx^n) \log\left(fx^r\right)^2}{r} - \frac{aben \log\left(fx^r\right)^3}{3r^2} - \frac{1}{12}\left(\frac{4n \log(cx^n) \log\left(fx^r\right)^3}{r^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*e*log(c*x^n)^2*log(f*x^r)^2/r + 1/3*b^2*d*log(c*x^n)^3/n + a*b*e*lo
g(c*x^n)*log(f*x^r)^2/r - 1/3*a*b*e*n*log(f*x^r)^3/r^2 - 1/12*(4*n*log(c*x^
n)*log(f*x^r)^3/r^2 - n^2*log(f*x^r)^4/r^3)*b^2*e + a*b*d*log(c*x^n)^2/n +
1/2*a^2*e*log(f*x^r)^2/r + a^2*d*log(x)
```

mupad [B] time = 3.99, size = 124, normalized size = 2.18

$$\ln\left(f\,x^r\right)\,\left(\frac{b^2\,e\ln\left(c\,x^n\right)^3}{3\,n}+\frac{a\,b\,e\ln\left(c\,x^n\right)^2}{n}\right)+\frac{\ln\left(c\,x^n\right)^3\,\left(b^2\,d\,n-a\,b\,e\,r\right)}{3\,n^2}+a^2\,d\,\ln(x)+\frac{a^2\,e\ln\left(f\,x^r\right)^2}{2\,r}+\frac{a\,b\,d\ln\left(c\,x^n\right)^2}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] log(f*x^r)*((b^2*e*log(c*x^n)^3)/(3*n) + (a*b*e*log(c*x^n)^2)/n) + (log(c*x
^n)^3*(b^2*d*n - a*b*e*r))/(3*n^2) + a^2*d*log(x) + (a^2*e*log(f*x^r)^2)/(2
*r) + (a*b*d*log(c*x^n)^2)/n - (b^2*e*r*log(c*x^n)^4)/(12*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(cx^n\right)\right)^2\left(d + e\log\left(fx^r\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))**2*(d + e*log(f*x**r))/x, x)
```

3.167 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx$

Optimal. Leaf size=181

$$-\frac{er\left(a^2+2abn+2b^2n^2\right)}{x}-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)}{x}-\frac{\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)}{x}-\frac{2ber(a$$

```
[Out] -2*b^2*e*n^2*r/x-2*b*e*n*(b*n+a)*r/x-e*(2*b^2*n^2+2*a*b*n+a^2)*r/x-2*b^2*e*
n*r*ln(c*x^n)/x-2*b*e*(b*n+a)*r*ln(c*x^n)/x-b^2*e*r*ln(c*x^n)^2/x-2*b^2*n^2
*(d+e*ln(f*x^r))/x-2*b*n*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x-(a+b*ln(c*x^n))^
2*(d+e*ln(f*x^r))/x
```

Rubi [A] time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2305, 2304, 2366, 14}

$$-\frac{er\left(a^2+2abn+2b^2n^2\right)}{x}-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)}{x}-\frac{\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)}{x}-\frac{2ber(a$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] (-2*b^2*e*n^2*r)/x - (2*b*e*n*(a + b*n)*r)/x - (e*(a^2 + 2*a*b*n + 2*b^2*n^
2)*r)/x - (2*b^2*e*n*r*Log[c*x^n])/x - (2*b*e*(a + b*n)*r*Log[c*x^n])/x - (
b^2*e*r*Log[c*x^n]^2)/x - (2*b^2*n^2*(d + e*Log[f*x^r]))/x - (2*b*n*(a + b*
Log[c*x^n])*(d + e*Log[f*x^r]))/x - ((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]
))/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\left(d+e\log\left(fx^r\right)\right)}{x^2}\,dx = -\frac{2b^2n^2\left(d+e\log\left(fx^r\right)\right)}{x} - \frac{2bn\left(a+b\log(cx^n)\right)\left(d+e\log\left(fx^r\right)\right)}{x} -$$

$$= -\frac{2b^2n^2\left(d+e\log\left(fx^r\right)\right)}{x} - \frac{2bn\left(a+b\log(cx^n)\right)\left(d+e\log\left(fx^r\right)\right)}{x} -$$

$$= -\frac{e\left(a^2+2abn+2b^2n^2\right)r}{x} - \frac{2b^2n^2\left(d+e\log\left(fx^r\right)\right)}{x} - \frac{2bn\left(a+b\log\right.}{}$$

$$= -\frac{2ben(a+bn)r}{x} - \frac{e\left(a^2+2abn+2b^2n^2\right)r}{x} - \frac{2be(a+bn)r\log(cx^n)}{x} -$$

$$= -\frac{2b^2en^2r}{x} - \frac{2ben(a+bn)r}{x} - \frac{e\left(a^2+2abn+2b^2n^2\right)r}{x} - \frac{2b^2enr\log(c}{x}$$

Mathematica [A] time = 0.16, size = 138, normalized size = 0.76

$$-\frac{e\left(a^2+2abn+2b^2n^2\right)\log\left(fx^r\right)+a^2d+a^2er+2b\log(cx^n)\left(e(a+bn)\log\left(fx^r\right)+a(d+er)+bn(d+2er)\right)+2a}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] -((a^2*d + 2*a*b*d*n + 2*b^2*d*n^2 + a^2*e*r + 4*a*b*e*n*r + 6*b^2*e*n^2*r
+ e*(a^2 + 2*a*b*n + 2*b^2*n^2)*Log[f*x^r] + b^2*Log[c*x^n]^2*(d + e*r + e*
Log[f*x^r]) + 2*b*Log[c*x^n]*(a*(d + e*r) + b*n*(d + 2*e*r) + e*(a + b*n)*L
og[f*x^r]))/x)
```

fricas [A] time = 0.65, size = 311, normalized size = 1.72

$$-\frac{b^2en^2r\log(x)^3+2\,b^2dn^2+2\,abdn+a^2d+\left(b^2er+b^2d\right)\log(c)^2+\left(2\,b^2enr\log(c)+b^2en^2\log(f)+b^2dn^2+\left(3\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^2,x, algorithm="fricas")
```

```
[Out] -(b^2*e*n^2*r*log(x)^3 + 2*b^2*d*n^2 + 2*a*b*d*n + a^2*d + (b^2*e*r + b^2*d
)*log(c)^2 + (2*b^2*e*n*r*log(c) + b^2*e*n^2*log(f) + b^2*d*n^2 + (3*b^2*e*
n^2 + 2*a*b*e*n)*r)*log(x)^2 + (6*b^2*e*n^2 + 4*a*b*e*n + a^2*e)*r + 2*(b^2
*d*n + a*b*d + (2*b^2*e*n + a*b*e)*r)*log(c) + (2*b^2*e*n^2 + b^2*e*log(c)^
2 + 2*a*b*e*n + a^2*e + 2*(b^2*e*n + a*b*e)*log(c))*log(f) + (b^2*e*r*log(c
)^2 + 2*b^2*d*n^2 + 2*a*b*d*n + (6*b^2*e*n^2 + 4*a*b*e*n + a^2*e)*r + 2*(b^
2*d*n + (2*b^2*e*n + a*b*e)*r)*log(c) + 2*(b^2*e*n^2 + b^2*e*n*log(c) + a*b
*e*n)*log(f))*log(x))/x
```

giac [B] time = 0.36, size = 392, normalized size = 2.17

$$-\frac{b^2n^2re\log(x)^3+3\,b^2n^2re\log(x)^2+2\,b^2nre\log(c)\log(x)^2+b^2n^2e\log(f)\log(x)^2+6\,b^2n^2re\log(x)+4\,b^2nre\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^2,x, algorithm="giac")
```

```
[Out] -(b^2*n^2*r*e*log(x)^3 + 3*b^2*n^2*r*e*log(x)^2 + 2*b^2*n*r*e*log(c)*log(x)
^2 + b^2*n^2*e*log(f)*log(x)^2 + 6*b^2*n^2*r*e*log(x) + 4*b^2*n*r*e*log(c)*
log(x) + b^2*r*e*log(c)^2*log(x) + 2*b^2*n^2*e*log(f)*log(x) + 2*b^2*n*e*lo
```

```
g(c)*log(f)*log(x) + b^2*d*n^2*log(x)^2 + 2*a*b*n*r*e*log(x)^2 + 6*b^2*n^2*
r*e + 4*b^2*n*r*e*log(c) + b^2*r*e*log(c)^2 + 2*b^2*n^2*e*log(f) + 2*b^2*n*
e*log(c)*log(f) + b^2*e*log(c)^2*log(f) + 2*b^2*d*n^2*log(x) + 4*a*b*n*r*e*
log(x) + 2*b^2*d*n*log(c)*log(x) + 2*a*b*r*e*log(c)*log(x) + 2*a*b*n*e*log(
f)*log(x) + 2*b^2*d*n^2 + 4*a*b*n*r*e + 2*b^2*d*n*log(c) + 2*a*b*r*e*log(c)
 + b^2*d*log(c)^2 + 2*a*b*n*e*log(f) + 2*a*b*e*log(c)*log(f) + 2*a*b*d*n*lo
g(x) + a^2*r*e*log(x) + 2*a*b*d*n + a^2*r*e + 2*a*b*d*log(c) + a^2*e*log(f)
 + a^2*d)/x
```

maple [C] time = 0.85, size = 8407, normalized size = 46.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*(d+e*ln(f*x^r))/x^2,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.98, size = 221, normalized size = 1.22

$$-b^2e\left(\frac{r}{x}+\frac{\log\left(fx^r\right)}{x}\right)\log\left(cx^n\right)^2-2\,abe\left(\frac{r}{x}+\frac{\log\left(fx^r\right)}{x}\right)\log\left(cx^n\right)-2\left(\frac{\left(r\log(x)+3\,r+\log(f)\right)n^2}{x}+\frac{n\left(2\,r+\log\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^2,x, algorithm="maxima")
```

```
[Out] -b^2*e*(r/x + log(f*x^r)/x)*log(c*x^n)^2 - 2*a*b*e*(r/x + log(f*x^r)/x)*log
(c*x^n) - 2*((r*log(x) + 3*r + log(f))*n^2/x + n*(2*r + log(f) + log(x^r))*
log(c*x^n)/x)*b^2*e - 2*b^2*d*(n^2/x + n*log(c*x^n)/x) - 2*a*b*e*n*(2*r + l
og(f) + log(x^r))/x - b^2*d*log(c*x^n)^2/x - 2*a*b*d*n/x - a^2*e*r/x - 2*a*
b*d*log(c*x^n)/x - a^2*e*log(f*x^r)/x - a^2*d/x
```

mupad [B] time = 4.01, size = 181, normalized size = 1.00

$$-\ln\left(f\,x^r\right)\left(\ln\left(c\,x^n\right)\left(\frac{2\,a\,b\,e}{x}+\frac{2\,b^2\,e\,n}{x}\right)+\frac{a^2\,e}{x}+\frac{2\,b^2\,e\,n^2}{x}+\frac{b^2\,e\ln\left(c\,x^n\right)^2}{x}+\frac{2\,a\,b\,e\,n}{x}\right)-\frac{a^2\,d+2\,b^2\,d\,n^2+a^2\,e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x^2,x)
```

```
[Out] - log(f*x^r)*(log(c*x^n)*((2*a*b*e)/x + (2*b^2*e*n)/x) + (a^2*e)/x + (2*b^2
*e*n^2)/x + (b^2*e*log(c*x^n)^2)/x + (2*a*b*e*n)/x) - (a^2*d + 2*b^2*d*n^2
+ a^2*e*r + 6*b^2*e*n^2*r + 2*a*b*d*n + 4*a*b*e*n*r)/x - (2*b*log(c*x^n)*(a
*d + b*d*n + a*e*r + 2*b*e*n*r))/x - (b^2*log(c*x^n)^2*(d + e*r))/x
```

sympy [B] time = 9.40, size = 536, normalized size = 2.96

$$-\frac{a^2d}{x}-\frac{a^2er\log\left(x\right)}{x}-\frac{a^2er}{x}-\frac{a^2e\log\left(f\right)}{x}-\frac{2abdn\log\left(x\right)}{x}-\frac{2abdn}{x}-\frac{2abd\log\left(c\right)}{x}-\frac{2abenr\log\left(x\right)^2}{x}-\frac{4abenr\log\left(x\right)}{x}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x**2,x)
```

```
[Out] -a**2*d/x - a**2*e*r*log(x)/x - a**2*e*r/x - a**2*e*log(f)/x - 2*a*b*d*n*lo
g(x)/x - 2*a*b*d*n/x - 2*a*b*d*log(c)/x - 2*a*b*e*n*r*log(x)**2/x - 4*a*b*e
*n*r*log(x)/x - 4*a*b*e*n*r/x - 2*a*b*e*n*log(f)*log(x)/x - 2*a*b*e*n*log(f
)/x - 2*a*b*e*r*log(c)*log(x)/x - 2*a*b*e*r*log(c)/x - 2*a*b*e*log(c)*log(f
)/x - b**2*d*n**2*log(x)**2/x - 2*b**2*d*n**2*log(x)/x - 2*b**2*d*n**2/x -
```

```
2*b**2*d*n*log(c)*log(x)/x - 2*b**2*d*n*log(c)/x - b**2*d*log(c)**2/x - b**
2*e*n**2*r*log(x)**3/x - 3*b**2*e*n**2*r*log(x)**2/x - 6*b**2*e*n**2*r*log(
x)/x - 6*b**2*e*n**2*r/x - b**2*e*n**2*log(f)*log(x)**2/x - 2*b**2*e*n**2*l
og(f)*log(x)/x - 2*b**2*e*n**2*log(f)/x - 2*b**2*e*n*r*log(c)*log(x)**2/x -
 4*b**2*e*n*r*log(c)*log(x)/x - 4*b**2*e*n*r*log(c)/x - 2*b**2*e*n*log(c)*l
og(f)*log(x)/x - 2*b**2*e*n*log(c)*log(f)/x - b**2*e*r*log(c)**2*log(x)/x -
 b**2*e*r*log(c)**2/x - b**2*e*log(c)**2*log(f)/x
```

3.168 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx$

Optimal. Leaf size=204

$$-\frac{er(2a^2+2abn+b^2n^2)}{8x^2}-\frac{bn(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2x^2}-\frac{ber(2a+$$

```
[Out] -1/8*b^2*e*n^2*r/x^2-1/8*b*e*n*(b*n+2*a)*r/x^2-1/8*e*(b^2*n^2+2*a*b*n+2*a^2
)*r/x^2-1/4*b^2*e*n*r*ln(c*x^n)/x^2-1/4*b*e*(b*n+2*a)*r*ln(c*x^n)/x^2-1/4*b
^2*e*r*ln(c*x^n)^2/x^2-1/4*b^2*n^2*(d+e*ln(f*x^r))/x^2-1/2*b*n*(a+b*ln(c*x^
n))*(d+e*ln(f*x^r))/x^2-1/2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^2
```

Rubi [A] time = 0.21, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2305, 2304, 2366, 12, 14}

$$-\frac{er(2a^2+2abn+b^2n^2)}{8x^2}-\frac{bn(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2x^2}-\frac{ber(2a+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -(b^2*e*n^2*r)/(8*x^2) - (b*e*n*(2*a + b*n)*r)/(8*x^2) - (e*(2*a^2 + 2*a*b*
n + b^2*n^2)*r)/(8*x^2) - (b^2*e*n*r*Log[c*x^n])/(4*x^2) - (b*e*(2*a + b*n)
*r*Log[c*x^n])/(4*x^2) - (b^2*e*r*Log[c*x^n]^2)/(4*x^2) - (b^2*n^2*(d + e*L
og[f*x^r]))/(4*x^2) - (b*n*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/(2*x^2) -
 ((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
```

```
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)}{x^3}\,dx = -\frac{b^2n^2\left(d+e\log\left(fx^r\right)\right)}{4x^2}-\frac{bn\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)}{2x^2}-\frac{(a}{}$$

$$= -\frac{b^2n^2\left(d+e\log\left(fx^r\right)\right)}{4x^2}-\frac{bn\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)}{2x^2}-\frac{(a}{}$$

$$= -\frac{b^2n^2\left(d+e\log\left(fx^r\right)\right)}{4x^2}-\frac{bn\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)}{2x^2}-\frac{(a}{}$$

$$= -\frac{e\left(2a^2+2abn+b^2n^2\right)r}{8x^2}-\frac{b^2n^2\left(d+e\log\left(fx^r\right)\right)}{4x^2}-\frac{bn\left(a+b\log(c\right.}{}$$

$$= -\frac{ben(2a+bn)r}{8x^2}-\frac{e\left(2a^2+2abn+b^2n^2\right)r}{8x^2}-\frac{be(2a+bn)r\log\left(cx^n\right)}{4x^2}-$$

$$= -\frac{b^2en^2r}{8x^2}-\frac{ben(2a+bn)r}{8x^2}-\frac{e\left(2a^2+2abn+b^2n^2\right)r}{8x^2}-\frac{b^2enr\log\left(cx^n\right.}{4x^2}$$

Mathematica [A] time = 0.18, size = 151, normalized size = 0.74

$$-\frac{2e\left(2a^2+2abn+b^2n^2\right)\log\left(fx^r\right)+4a^2d+2a^2er+4b\log\left(cx^n\right)\left(e(2a+bn)\log\left(fx^r\right)+2ad+aer+bdn+benr\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -1/8*(4*a^2*d + 4*a*b*d*n + 2*b^2*d*n^2 + 2*a^2*e*r + 4*a*b*e*n*r + 3*b^2*e
*n^2*r + 2*e*(2*a^2 + 2*a*b*n + b^2*n^2)*Log[f*x^r] + 2*b^2*Log[c*x^n]^2*(2
*d + e*r + 2*e*Log[f*x^r]) + 4*b*Log[c*x^n]*(2*a*d + b*d*n + a*e*r + b*e*n*
r + e*(2*a + b*n)*Log[f*x^r]))/x^2
```

fricas [A] time = 0.68, size = 326, normalized size = 1.60

$$-\frac{4b^2en^2r\log(x)^3+2b^2dn^2+4abdn+4a^2d+2\left(b^2er+2b^2d\right)\log(c)^2+2\left(4b^2enr\log(c)+2b^2en^2\log(f)+2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^2*e*n^2*r*log(x)^3 + 2*b^2*d*n^2 + 4*a*b*d*n + 4*a^2*d + 2*(b^2*e
*r + 2*b^2*d)*log(c)^2 + 2*(4*b^2*e*n*r*log(c) + 2*b^2*e*n^2*log(f) + 2*b^2
*d*n^2 + (3*b^2*e*n^2 + 4*a*b*e*n)*r)*log(x)^2 + (3*b^2*e*n^2 + 4*a*b*e*n +
 2*a^2*e)*r + 4*(b^2*d*n + 2*a*b*d + (b^2*e*n + a*b*e)*r)*log(c) + 2*(b^2*e
*n^2 + 2*b^2*e*log(c)^2 + 2*a*b*e*n + 2*a^2*e + 2*(b^2*e*n + 2*a*b*e)*log(c
))*log(f) + 2*(2*b^2*e*r*log(c)^2 + 2*b^2*d*n^2 + 4*a*b*d*n + (3*b^2*e*n^2
+ 4*a*b*e*n + 2*a^2*e)*r + 4*(b^2*d*n + (b^2*e*n + a*b*e)*r)*log(c) + 2*(b^
2*e*n^2 + 2*b^2*e*n*log(c) + 2*a*b*e*n)*log(f))*log(x))/x^2
```

giac [B] time = 0.43, size = 403, normalized size = 1.98

$$-\frac{4b^2n^2re\log(x)^3+6b^2n^2re\log(x)^2+8b^2nre\log(c)\log(x)^2+4b^2n^2e\log(f)\log(x)^2+6b^2n^2re\log(x)+8b^2nre}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*n^2*r*e*log(x)^3 + 6*b^2*n^2*r*e*log(x)^2 + 8*b^2*n*r*e*log(c)*
log(x)^2 + 4*b^2*n^2*e*log(f)*log(x)^2 + 6*b^2*n^2*r*e*log(x) + 8*b^2*n*r*e
*log(c)*log(x) + 4*b^2*r*e*log(c)^2*log(x) + 4*b^2*n^2*e*log(f)*log(x) + 8*
b^2*n*e*log(c)*log(f)*log(x) + 4*b^2*d*n^2*log(x)^2 + 8*a*b*n*r*e*log(x)^2
+ 3*b^2*n^2*r*e + 4*b^2*n*r*e*log(c) + 2*b^2*r*e*log(c)^2 + 2*b^2*n^2*e*log
(f) + 4*b^2*n*e*log(c)*log(f) + 4*b^2*e*log(c)^2*log(f) + 4*b^2*d*n^2*log(x
) + 8*a*b*n*r*e*log(x) + 8*b^2*d*n*log(c)*log(x) + 8*a*b*r*e*log(c)*log(x)
+ 8*a*b*n*e*log(f)*log(x) + 2*b^2*d*n^2 + 4*a*b*n*r*e + 4*b^2*d*n*log(c) +
4*a*b*r*e*log(c) + 4*b^2*d*log(c)^2 + 4*a*b*n*e*log(f) + 8*a*b*e*log(c)*log
(f) + 8*a*b*d*n*log(x) + 4*a^2*r*e*log(x) + 4*a*b*d*n + 2*a^2*r*e + 8*a*b*d
*log(c) + 4*a^2*e*log(f) + 4*a^2*d)/x^2
```

maple [C] time = 0.84, size = 8407, normalized size = 41.21

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*(d+e*ln(f*x^r))/x^3,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.68, size = 224, normalized size = 1.10

$$-\frac{1}{4}\,b^2e\left(\frac{r}{x^2}+\frac{2\,\log\left(fx^r\right)}{x^2}\right)\log\left(cx^n\right)^2-\frac{1}{2}\,abe\left(\frac{r}{x^2}+\frac{2\,\log\left(fx^r\right)}{x^2}\right)\log\left(cx^n\right)-\frac{1}{8}\,b^2e\left(\frac{\left(2\,r\log(x)+3\,r+2\,\log(f)\right)r}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*e*(r/x^2 + 2*log(f*x^r)/x^2)*log(c*x^n)^2 - 1/2*a*b*e*(r/x^2 + 2*l
og(f*x^r)/x^2)*log(c*x^n) - 1/8*b^2*e*((2*r*log(x) + 3*r + 2*log(f))*n^2/x^
2 + 4*n*(r + log(f) + log(x^r))*log(c*x^n)/x^2) - 1/4*b^2*d*(n^2/x^2 + 2*n*
log(c*x^n)/x^2) - 1/2*a*b*e*n*(r + log(f) + log(x^r))/x^2 - 1/2*b^2*d*log(c
*x^n)^2/x^2 - 1/2*a*b*d*n/x^2 - 1/4*a^2*e*r/x^2 - a*b*d*log(c*x^n)/x^2 - 1/
2*a^2*e*log(f*x^r)/x^2 - 1/2*a^2*d/x^2
```

mupad [B] time = 4.08, size = 186, normalized size = 0.91

$$-\ln\left(f\,x^r\right)\,\left(\ln\left(c\,x^n\right)\,\left(\frac{a\,b\,e}{x^2}+\frac{b^2\,e\,n}{2\,x^2}\right)+\frac{a^2\,e}{2\,x^2}+\frac{b^2\,e\,n^2}{4\,x^2}+\frac{b^2\,e\,\ln\left(c\,x^n\right)^2}{2\,x^2}+\frac{a\,b\,e\,n}{2\,x^2}\right)-\frac{\frac{a^2\,d}{2}+\frac{b^2\,d\,n^2}{4}+\frac{a^2\,e\,r}{4}+\frac{3\,b^2\,e\,n^2}{8}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] - log(f*x^r)*(log(c*x^n)*((a*b*e)/x^2 + (b^2*e*n)/(2*x^2)) + (a^2*e)/(2*x^2
) + (b^2*e*n^2)/(4*x^2) + (b^2*e*log(c*x^n)^2)/(2*x^2) + (a*b*e*n)/(2*x^2))
 - ((a^2*d)/2 + (b^2*d*n^2)/4 + (a^2*e*r)/4 + (3*b^2*e*n^2*r)/8 + (a*b*d*n)
/2 + (a*b*e*n*r)/2)/x^2 - (b^2*log(c*x^n)^2*(2*d + e*r))/(4*x^2) - (b*log(c
*x^n)*(2*a*d + b*d*n + a*e*r + b*e*n*r))/(2*x^2)
```

sympy [B] time = 9.01, size = 602, normalized size = 2.95

$$-\frac{a^2d}{2x^2}-\frac{a^2er\log(x)}{2x^2}-\frac{a^2er}{4x^2}-\frac{a^2e\log(f)}{2x^2}-\frac{abdn\log(x)}{x^2}-\frac{abdn}{2x^2}-\frac{abd\log(c)}{x^2}-\frac{abenr\log(x)^2}{x^2}-\frac{abenr\log(x)}{x^2}-\frac{abenr}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x**3,x)
```

```
[Out] -a**2*d/(2*x**2) - a**2*e*r*log(x)/(2*x**2) - a**2*e*r/(4*x**2) - a**2*e*lo
g(f)/(2*x**2) - a*b*d*n*log(x)/x**2 - a*b*d*n/(2*x**2) - a*b*d*log(c)/x**2
- a*b*e*n*r*log(x)**2/x**2 - a*b*e*n*r*log(x)/x**2 - a*b*e*n*r/(2*x**2) - a
*b*e*n*log(f)*log(x)/x**2 - a*b*e*n*log(f)/(2*x**2) - a*b*e*r*log(c)*log(x)
/x**2 - a*b*e*r*log(c)/(2*x**2) - a*b*e*log(c)*log(f)/x**2 - b**2*d*n**2*lo
g(x)**2/(2*x**2) - b**2*d*n**2*log(x)/(2*x**2) - b**2*d*n**2/(4*x**2) - b**
2*d*n*log(c)*log(x)/x**2 - b**2*d*n*log(c)/(2*x**2) - b**2*d*log(c)**2/(2*x
**2) - b**2*e*n**2*r*log(x)**3/(2*x**2) - 3*b**2*e*n**2*r*log(x)**2/(4*x**2
) - 3*b**2*e*n**2*r*log(x)/(4*x**2) - 3*b**2*e*n**2*r/(8*x**2) - b**2*e*n**
2*log(f)*log(x)**2/(2*x**2) - b**2*e*n**2*log(f)*log(x)/(2*x**2) - b**2*e*n
**2*log(f)/(4*x**2) - b**2*e*n*r*log(c)*log(x)**2/x**2 - b**2*e*n*r*log(c)*
log(x)/x**2 - b**2*e*n*r*log(c)/(2*x**2) - b**2*e*n*log(c)*log(f)*log(x)/x*
*2 - b**2*e*n*log(c)*log(f)/(2*x**2) - b**2*e*r*log(c)**2*log(x)/(2*x**2) -
 b**2*e*r*log(c)**2/(4*x**2) - b**2*e*log(c)**2*log(f)/(2*x**2)
```

3.169 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx$

Optimal. Leaf size=205

$$-\frac{er(9a^2+6abn+2b^2n^2)}{81x^3}-\frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{9x^3}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{3x^3}-\frac{2ber(\cdots}{}$$

```
[Out] -2/81*b^2*e*n^2*r/x^3-2/81*b*e*n*(b*n+3*a)*r/x^3-1/81*e*(2*b^2*n^2+6*a*b*n+
9*a^2)*r/x^3-2/27*b^2*e*n*r*ln(c*x^n)/x^3-2/27*b*e*(b*n+3*a)*r*ln(c*x^n)/x^
3-1/9*b^2*e*r*ln(c*x^n)^2/x^3-2/27*b^2*n^2*(d+e*ln(f*x^r))/x^3-2/9*b*n*(a+b
*ln(c*x^n))*(d+e*ln(f*x^r))/x^3-1/3*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^3
```

Rubi [A] time = 0.21, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2305, 2304, 2366, 12, 14}

$$-\frac{er(9a^2+6abn+2b^2n^2)}{81x^3}-\frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{9x^3}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{3x^3}-\frac{2ber(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] (-2*b^2*e*n^2*r)/(81*x^3) - (2*b*e*n*(3*a + b*n)*r)/(81*x^3) - (e*(9*a^2 +
6*a*b*n + 2*b^2*n^2)*r)/(81*x^3) - (2*b^2*e*n*r*Log[c*x^n])/(27*x^3) - (2*b
*e*(3*a + b*n)*r*Log[c*x^n])/(27*x^3) - (b^2*e*r*Log[c*x^n]^2)/(9*x^3) - (2
*b^2*n^2*(d + e*Log[f*x^r]))/(27*x^3) - (2*b*n*(a + b*Log[c*x^n])*(d + e*Lo
g[f*x^r]))/(9*x^3) - ((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
```

```
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)}{x^4}\,dx=-\frac{2b^2n^2\left(d+e\log\left(fx^r\right)\right)}{27x^3}-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)}{9x^3}-$$

$$=-\frac{2b^2n^2\left(d+e\log\left(fx^r\right)\right)}{27x^3}-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)}{9x^3}-$$

$$=-\frac{2b^2n^2\left(d+e\log\left(fx^r\right)\right)}{27x^3}-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\left(d+e\log\left(fx^r\right)\right)}{9x^3}-$$

$$=-\frac{e\left(9a^2+6abn+2b^2n^2\right)r}{81x^3}-\frac{2b^2n^2\left(d+e\log\left(fx^r\right)\right)}{27x^3}-\frac{2bn\left(a+b\log\right.}{}$$

$$=-\frac{2ben(3a+bn)r}{81x^3}-\frac{e\left(9a^2+6abn+2b^2n^2\right)r}{81x^3}-\frac{2be(3a+bn)r\log\left(cx\right.}{27x^3}$$

$$=-\frac{2b^2en^2r}{81x^3}-\frac{2ben(3a+bn)r}{81x^3}-\frac{e\left(9a^2+6abn+2b^2n^2\right)r}{81x^3}-\frac{2b^2enr\log}{27x^3}$$

Mathematica [A] time = 0.18, size = 155, normalized size = 0.76

$$-\frac{e\left(9a^2+6abn+2b^2n^2\right)\log\left(fx^r\right)+9a^2d+3a^2er+2b\log\left(cx^n\right)\left(3e(3a+bn)\log\left(fx^r\right)+9ad+3aer+3bdn+2\right.}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] -1/27*(9*a^2*d + 6*a*b*d*n + 2*b^2*d*n^2 + 3*a^2*e*r + 4*a*b*e*n*r + 2*b^2*
e*n^2*r + e*(9*a^2 + 6*a*b*n + 2*b^2*n^2)*Log[f*x^r] + 3*b^2*Log[c*x^n]^2*(
3*d + e*r + 3*e*Log[f*x^r]) + 2*b*Log[c*x^n]*(9*a*d + 3*b*d*n + 3*a*e*r + 2
*b*e*n*r + 3*e*(3*a + b*n)*Log[f*x^r]))/x^3
```

fricas [A] time = 0.69, size = 329, normalized size = 1.60

$$-\frac{9\,b^2en^2r\log(x)^3+2\,b^2dn^2+6\,abdn+9\,a^2d+3\left(b^2er+3\,b^2d\right)\log(c)^2+9\left(2\,b^2enr\log(c)+b^2en^2\log(f)+b^2dn\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(9*b^2*e*n^2*r*log(x)^3 + 2*b^2*d*n^2 + 6*a*b*d*n + 9*a^2*d + 3*(b^2*
e*r + 3*b^2*d)*log(c)^2 + 9*(2*b^2*e*n*r*log(c) + b^2*e*n^2*log(f) + b^2*d*
n^2 + (b^2*e*n^2 + 2*a*b*e*n)*r)*log(x)^2 + (2*b^2*e*n^2 + 4*a*b*e*n + 3*a^
2*e)*r + 2*(3*b^2*d*n + 9*a*b*d + (2*b^2*e*n + 3*a*b*e)*r)*log(c) + (2*b^2*
e*n^2 + 9*b^2*e*log(c)^2 + 6*a*b*e*n + 9*a^2*e + 6*(b^2*e*n + 3*a*b*e)*log(
c))*log(f) + 3*(3*b^2*e*r*log(c)^2 + 2*b^2*d*n^2 + 6*a*b*d*n + (2*b^2*e*n^2
 + 4*a*b*e*n + 3*a^2*e)*r + 2*(3*b^2*d*n + (2*b^2*e*n + 3*a*b*e)*r)*log(c)
+ 2*(b^2*e*n^2 + 3*b^2*e*n*log(c) + 3*a*b*e*n)*log(f))*log(x))/x^3
```

giac [B] time = 0.39, size = 403, normalized size = 1.97

$$-\frac{9\,b^2n^2re\log(x)^3+9\,b^2n^2re\log(x)^2+18\,b^2nre\log(c)\log(x)^2+9\,b^2n^2e\log(f)\log(x)^2+6\,b^2n^2re\log(x)+12\,b^2n}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^4,x, algorithm="giac")
```

```
[Out] -1/27*(9*b^2*n^2*r*e*log(x)^3 + 9*b^2*n^2*r*e*log(x)^2 + 18*b^2*n*r*e*log(c
)*log(x)^2 + 9*b^2*n^2*e*log(f)*log(x)^2 + 6*b^2*n^2*r*e*log(x) + 12*b^2*n*
r*e*log(c)*log(x) + 9*b^2*r*e*log(c)^2*log(x) + 6*b^2*n^2*e*log(f)*log(x) +
 18*b^2*n*e*log(c)*log(f)*log(x) + 9*b^2*d*n^2*log(x)^2 + 18*a*b*n*r*e*log(
x)^2 + 2*b^2*n^2*r*e + 4*b^2*n*r*e*log(c) + 3*b^2*r*e*log(c)^2 + 2*b^2*n^2*
e*log(f) + 6*b^2*n*e*log(c)*log(f) + 9*b^2*e*log(c)^2*log(f) + 6*b^2*d*n^2*
log(x) + 12*a*b*n*r*e*log(x) + 18*b^2*d*n*log(c)*log(x) + 18*a*b*r*e*log(c)
*log(x) + 18*a*b*n*e*log(f)*log(x) + 2*b^2*d*n^2 + 4*a*b*n*r*e + 6*b^2*d*n*
log(c) + 6*a*b*r*e*log(c) + 9*b^2*d*log(c)^2 + 6*a*b*n*e*log(f) + 18*a*b*e*
log(c)*log(f) + 18*a*b*d*n*log(x) + 9*a^2*r*e*log(x) + 6*a*b*d*n + 3*a^2*r*
e + 18*a*b*d*log(c) + 9*a^2*e*log(f) + 9*a^2*d)/x^3
```

maple [C] time = 0.90, size = 8407, normalized size = 41.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*(d+e*ln(f*x^r))/x^4,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.94, size = 230, normalized size = 1.12

$$-\frac{1}{9}b^2e\left(\frac{r}{x^3}+\frac{3\log\left(fx^r\right)}{x^3}\right)\log\left(cx^n\right)^2-\frac{2}{9}abe\left(\frac{r}{x^3}+\frac{3\log\left(fx^r\right)}{x^3}\right)\log\left(cx^n\right)-\frac{2}{27}b^2e\left(\frac{\left(r\log(x)+r+\log(f)\right)n^2}{x^3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*b^2*e*(r/x^3 + 3*log(f*x^r)/x^3)*log(c*x^n)^2 - 2/9*a*b*e*(r/x^3 + 3*l
og(f*x^r)/x^3)*log(c*x^n) - 2/27*b^2*e*((r*log(x) + r + log(f))*n^2/x^3 + n
*(2*r + 3*log(f) + 3*log(x^r))*log(c*x^n)/x^3) - 2/27*b^2*d*(n^2/x^3 + 3*n*
log(c*x^n)/x^3) - 2/27*a*b*e*n*(2*r + 3*log(f) + 3*log(x^r))/x^3 - 1/3*b^2*
d*log(c*x^n)^2/x^3 - 2/9*a*b*d*n/x^3 - 1/9*a^2*e*r/x^3 - 2/3*a*b*d*log(c*x^
n)/x^3 - 1/3*a^2*e*log(f*x^r)/x^3 - 1/3*a^2*d/x^3
```

mupad [B] time = 4.20, size = 190, normalized size = 0.93

$$-\ln\left(f\,x^r\right)\,\left(\ln\left(c\,x^n\right)\,\left(\frac{2\,a\,b\,e}{3\,x^3}+\frac{2\,b^2\,e\,n}{9\,x^3}\right)+\frac{a^2\,e}{3\,x^3}+\frac{2\,b^2\,e\,n^2}{27\,x^3}+\frac{b^2\,e\ln\left(c\,x^n\right)^2}{3\,x^3}+\frac{2\,a\,b\,e\,n}{9\,x^3}\right)-\frac{\frac{a^2\,d}{3}+\frac{2\,b^2\,d\,n^2}{27}+\frac{a^2\,e\,r}{9}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x^4,x)
```

```
[Out] - log(f*x^r)*(log(c*x^n)*((2*a*b*e)/(3*x^3) + (2*b^2*e*n)/(9*x^3)) + (a^2*e
)/(3*x^3) + (2*b^2*e*n^2)/(27*x^3) + (b^2*e*log(c*x^n)^2)/(3*x^3) + (2*a*b*
e*n)/(9*x^3)) - ((a^2*d)/3 + (2*b^2*d*n^2)/27 + (a^2*e*r)/9 + (2*b^2*e*n^2*
r)/27 + (2*a*b*d*n)/9 + (4*a*b*e*n*r)/27)/x^3 - (b^2*log(c*x^n)^2*(3*d + e*
r))/(9*x^3) - (2*b*log(c*x^n)*(9*a*d + 3*b*d*n + 3*a*e*r + 2*b*e*n*r))/(27*
x^3)
```

sympy [B] time = 24.96, size = 656, normalized size = 3.20

$$-\frac{a^2d}{3x^3}-\frac{a^2er\log(x)}{3x^3}-\frac{a^2er}{9x^3}-\frac{a^2e\log(f)}{3x^3}-\frac{2abdn\log(x)}{3x^3}-\frac{2abdn}{9x^3}-\frac{2abd\log(c)}{3x^3}-\frac{2abenr\log(x)^2}{3x^3}-\frac{4abenr\log(x)}{9x^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x**4,x)
```

```
[Out] -a**2*d/(3*x**3) - a**2*e*r*log(x)/(3*x**3) - a**2*e*r/(9*x**3) - a**2*e*lo
g(f)/(3*x**3) - 2*a*b*d*n*log(x)/(3*x**3) - 2*a*b*d*n/(9*x**3) - 2*a*b*d*lo
g(c)/(3*x**3) - 2*a*b*e*n*r*log(x)**2/(3*x**3) - 4*a*b*e*n*r*log(x)/(9*x**3
) - 4*a*b*e*n*r/(27*x**3) - 2*a*b*e*n*log(f)*log(x)/(3*x**3) - 2*a*b*e*n*lo
g(f)/(9*x**3) - 2*a*b*e*r*log(c)*log(x)/(3*x**3) - 2*a*b*e*r*log(c)/(9*x**3
) - 2*a*b*e*log(c)*log(f)/(3*x**3) - b**2*d*n**2*log(x)**2/(3*x**3) - 2*b**
2*d*n**2*log(x)/(9*x**3) - 2*b**2*d*n**2/(27*x**3) - 2*b**2*d*n*log(c)*log(
x)/(3*x**3) - 2*b**2*d*n*log(c)/(9*x**3) - b**2*d*log(c)**2/(3*x**3) - b**2
*e*n**2*r*log(x)**3/(3*x**3) - b**2*e*n**2*r*log(x)**2/(3*x**3) - 2*b**2*e*
n**2*r*log(x)/(9*x**3) - 2*b**2*e*n**2*r/(27*x**3) - b**2*e*n**2*log(f)*log
(x)**2/(3*x**3) - 2*b**2*e*n**2*log(f)*log(x)/(9*x**3) - 2*b**2*e*n**2*log(
f)/(27*x**3) - 2*b**2*e*n*r*log(c)*log(x)**2/(3*x**3) - 4*b**2*e*n*r*log(c)
*log(x)/(9*x**3) - 4*b**2*e*n*r*log(c)/(27*x**3) - 2*b**2*e*n*log(c)*log(f)
*log(x)/(3*x**3) - 2*b**2*e*n*log(c)*log(f)/(9*x**3) - b**2*e*r*log(c)**2*l
og(x)/(3*x**3) - b**2*e*r*log(c)**2/(9*x**3) - b**2*e*log(c)**2*log(f)/(3*x
**3)
```

3.170 $\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx$

Optimal. Leaf size=141

$$\frac{x^3e^{-\frac{3d}{em}}\left(fx^m\right)^{-3/m}\left(a+b\log\left(cx^n\right)\right)\text{Ei}\left(\frac{3\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{em}-\frac{bnx^3e^{-\frac{3d}{em}}\left(fx^m\right)^{-3/m}\left(d+e\log\left(fx^m\right)\right)\text{Ei}\left(\frac{3\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{e^2m^2}$$

```
[Out] 1/3*b*n*x^3/e/m-b*n*x^3*Ei(3*(d+e*ln(f*x^m))/e/m)*(d+e*ln(f*x^m))/e^2/exp(3
*d/e/m)/m^2/((f*x^m)^(3/m))+x^3*Ei(3*(d+e*ln(f*x^m))/e/m)*(a+b*ln(c*x^n))/e
/exp(3*d/e/m)/m/((f*x^m)^(3/m))
```

Rubi [A] time = 0.18, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2310, 2178, 2366, 12, 15, 6482}

$$\frac{x^3e^{-\frac{3d}{em}}\left(fx^m\right)^{-3/m}\left(a+b\log\left(cx^n\right)\right)\text{Ei}\left(\frac{3\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{em}-\frac{bnx^3e^{-\frac{3d}{em}}\left(fx^m\right)^{-3/m}\left(d+e\log\left(fx^m\right)\right)\text{Ei}\left(\frac{3\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{e^2m^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

```
[Out] (b*n*x^3)/(3*e*m) - (b*n*x^3*ExpIntegralEi[(3*(d + e*Log[f*x^m]))/(e*m)]*(d
 + e*Log[f*x^m]))/(e^2*E^((3*d)/(e*m))*m^2*(f*x^m)^(3/m)) + (x^3*ExpIntegra
lEi[(3*(d + e*Log[f*x^m]))/(e*m)]*(a + b*Log[c*x^n]))/(e*E^((3*d)/(e*m))*m*
(f*x^m)^(3/m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6482

```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*ExpInte
gralEi[a + b*x])/b, x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\log\left(cx^n\right)\right)}{d+e\log\left(fx^m\right)}\,dx = \frac{e^{-\frac{3d}{em}}x^3\left(fx^m\right)^{-3/m}\operatorname{Ei}\left(\frac{3\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log\left(cx^n\right)\right)}{em} - (bn)\int \frac{e^{-\frac{3d}{em}}x^2\left(fx^m\right)^{-3/m}\operatorname{Ei}\left(\right.}{em}$$

$$= \frac{e^{-\frac{3d}{em}}x^3\left(fx^m\right)^{-3/m}\operatorname{Ei}\left(\frac{3\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log\left(cx^n\right)\right)}{em} - \frac{\left(be^{-\frac{3d}{em}}n\right)\int x^2\left(fx^m\right)^{-3/m}\operatorname{Ei}\left(\right.}{em}$$

$$= \frac{e^{-\frac{3d}{em}}x^3\left(fx^m\right)^{-3/m}\operatorname{Ei}\left(\frac{3\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log\left(cx^n\right)\right)}{em} - \frac{\left(be^{-\frac{3d}{em}}nx^3\left(fx^m\right)^{-3/m}\right)\int \frac{\operatorname{Ei}\left(\frac{3}{-}\right.}{}}{em}$$

$$= \frac{e^{-\frac{3d}{em}}x^3\left(fx^m\right)^{-3/m}\operatorname{Ei}\left(\frac{3\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log\left(cx^n\right)\right)}{em} - \frac{\left(be^{-\frac{3d}{em}}nx^3\left(fx^m\right)^{-3/m}\right)\operatorname{Subst}}{}$$

$$= \frac{e^{-\frac{3d}{em}}x^3\left(fx^m\right)^{-3/m}\operatorname{Ei}\left(\frac{3\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log\left(cx^n\right)\right)}{em} - \frac{\left(be^{-\frac{3d}{em}}nx^3\left(fx^m\right)^{-3/m}\right)\operatorname{Subst}}{3}$$

$$= \frac{bnx^3}{3em} - \frac{be^{-\frac{3d}{em}}nx^3\left(fx^m\right)^{-3/m}\operatorname{Ei}\left(\frac{3d}{em}+\frac{3\log\left(fx^m\right)}{m}\right)\left(\frac{d}{em}+\frac{\log\left(fx^m\right)}{m}\right)}{em} + \frac{e^{-\frac{3d}{em}}x^3\left(fx^m\right)^{-3/m}}{}$$

Mathematica [A] time = 0.17, size = 93, normalized size = 0.66

$$\frac{x^3\left(3e^{-\frac{3d}{em}}\left(fx^m\right)^{-3/m}\operatorname{Ei}\left(\frac{3\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(aem+bem\log\left(cx^n\right)-bdn-ben\log\left(fx^m\right)\right)+bemn\right)}{3e^2m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

```
[Out] (x^3*(b*e*m*n + (3*ExpIntegralEi[(3*(d + e*Log[f*x^m]))/(e*m)]*(a*e*m - b*d
*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(E^((3*d)/(e*m))*(f*x^m)^(3/m)))
)/(3*e^2*m^2)
```

fricas [A] time = 0.79, size = 92, normalized size = 0.65

$$\frac{\left(bemnx^3e^{\left(\frac{3\left(e\log(f)+d\right)}{em}\right)}+3\left(bem\log(c)-ben\log(f)+aem-bdn\right)\operatorname{log\_integral}\left(x^3e^{\left(\frac{3\left(e\log(f)+d\right)}{em}\right)}\right)\right)e^{\left(-\frac{3\left(e\log(f)+d\right)}{em}\right)}}{3\,e^2m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*e*m*n*x^3*e^(3*(e*log(f) + d)/(e*m)) + 3*(b*e*m*log(c) - b*e*n*log(f
) + a*e*m - b*d*n)*log_integral(x^3*e^(3*(e*log(f) + d)/(e*m))))*e^(-3*(e*l
og(f) + d)/(e*m))/(e^2*m^2)
```

giac [A] time = 0.43, size = 206, normalized size = 1.46

$$\frac{bnx^3e^{(-1)}}{3\,m}-\frac{bdn\text{Ei}\left(\frac{3\,de^{(-1)}}{m}+\frac{3\,\log(f)}{m}+3\,\log(x)\right)e^{\left(-\frac{3\,de^{(-1)}}{m}-2\right)}}{f^{\frac{3}{m}}m^2}+\frac{b\text{Ei}\left(\frac{3\,de^{(-1)}}{m}+\frac{3\,\log(f)}{m}+3\,\log(x)\right)e^{\left(-\frac{3\,de^{(-1)}}{m}-1\right)}\log(}{f^{\frac{3}{m}}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="giac")
```

```
[Out] 1/3*b*n*x^3*e^(-1)/m - b*d*n*Ei(3*d*e^(-1)/m + 3*log(f)/m + 3*log(x))*e^(-3
*d*e^(-1)/m - 2)/(f^(3/m)*m^2) + b*Ei(3*d*e^(-1)/m + 3*log(f)/m + 3*log(x))
*e^(-3*d*e^(-1)/m - 1)*log(c)/(f^(3/m)*m) - b*n*Ei(3*d*e^(-1)/m + 3*log(f)/
m + 3*log(x))*e^(-3*d*e^(-1)/m - 1)*log(f)/(f^(3/m)*m^2) + a*Ei(3*d*e^(-1)/
m + 3*log(f)/m + 3*log(x))*e^(-3*d*e^(-1)/m - 1)/(f^(3/m)*m)
```

maple [F] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{(b\ln(c\,x^n)+a)\,x^2}{e\ln\left(f\,x^m\right)+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)/(d+e*ln(f*x^m)),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)/(d+e*ln(f*x^m)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log(cx^n)+a\right)x^2}{e\log\left(fx^m\right)+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*log(f*x^m) + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\ln(c\,x^n)\right)}{d+e\,\ln\left(f\,x^m\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*log(f*x^m)),x)
```

```
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*log(f*x^m)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\log(cx^n)\right)}{d+e\log\left(fx^m\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(d+e*ln(f*x**m)),x)
```

```
[Out] Integral(x**2*(a + b*log(c*x**n))/(d + e*log(f*x**m)), x)
```

3.171 $\int \frac{x\left(a+b\log(cx^n)\right)}{d+e\log\left(fx^m\right)}\,dx$

Optimal. Leaf size=141

$$\frac{x^2e^{-\frac{2d}{em}}\left(fx^m\right)^{-2/m}\left(a+b\log\left(cx^n\right)\right)\text{Ei}\left(\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{em}-\frac{bnx^2e^{-\frac{2d}{em}}\left(fx^m\right)^{-2/m}\left(d+e\log\left(fx^m\right)\right)\text{Ei}\left(\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{e^2m^2}+$$

```
[Out] 1/2*b*n*x^2/e/m-b*n*x^2*Ei(2*(d+e*ln(f*x^m))/e/m)*(d+e*ln(f*x^m))/e^2/exp(2
*d/e/m)/m^2/((f*x^m)^(2/m))+x^2*Ei(2*(d+e*ln(f*x^m))/e/m)*(a+b*ln(c*x^n))/e
/exp(2*d/e/m)/m/((f*x^m)^(2/m))
```

Rubi [A] time = 0.15, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2310, 2178, 2366, 12, 15, 6482}

$$\frac{x^2e^{-\frac{2d}{em}}\left(fx^m\right)^{-2/m}\left(a+b\log\left(cx^n\right)\right)\text{Ei}\left(\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{em}-\frac{bnx^2e^{-\frac{2d}{em}}\left(fx^m\right)^{-2/m}\left(d+e\log\left(fx^m\right)\right)\text{Ei}\left(\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{e^2m^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

```
[Out] (b*n*x^2)/(2*e*m) - (b*n*x^2*ExpIntegralEi[(2*(d + e*Log[f*x^m]))/(e*m)]*(d
 + e*Log[f*x^m]))/(e^2*E^((2*d)/(e*m))*m^2*(f*x^m)^(2/m)) + (x^2*ExpIntegra
lEi[(2*(d + e*Log[f*x^m]))/(e*m)]*(a + b*Log[c*x^n]))/(e*E^((2*d)/(e*m))*m*
(f*x^m)^(2/m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6482

```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*ExpInte
gralEi[a + b*x])/b, x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x\left(a+b\log\left(cx^n\right)\right)}{d+e\log\left(fx^m\right)}\,dx = \frac{e^{-\frac{2d}{em}}x^2\left(fx^m\right)^{-2/m}\operatorname{Ei}\left(\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log\left(cx^n\right)\right)}{em} - (bn)\int \frac{e^{-\frac{2d}{em}}x\left(fx^m\right)^{-2/m}\operatorname{Ei}}{em}$$

$$= \frac{e^{-\frac{2d}{em}}x^2\left(fx^m\right)^{-2/m}\operatorname{Ei}\left(\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log\left(cx^n\right)\right)}{em} - \frac{\left(be^{-\frac{2d}{em}}n\right)\int x\left(fx^m\right)^{-2/m}\operatorname{Ei}}{em}$$

$$= \frac{e^{-\frac{2d}{em}}x^2\left(fx^m\right)^{-2/m}\operatorname{Ei}\left(\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log\left(cx^n\right)\right)}{em} - \frac{\left(be^{-\frac{2d}{em}}nx^2\left(fx^m\right)^{-2/m}\right)\int \frac{\operatorname{Ei}}{}}{em}$$

$$= \frac{e^{-\frac{2d}{em}}x^2\left(fx^m\right)^{-2/m}\operatorname{Ei}\left(\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log\left(cx^n\right)\right)}{em} - \frac{\left(be^{-\frac{2d}{em}}nx^2\left(fx^m\right)^{-2/m}\right)\operatorname{Sub}}{}$$

$$= \frac{e^{-\frac{2d}{em}}x^2\left(fx^m\right)^{-2/m}\operatorname{Ei}\left(\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log\left(cx^n\right)\right)}{em} - \frac{\left(be^{-\frac{2d}{em}}nx^2\left(fx^m\right)^{-2/m}\right)\operatorname{Sub}}{}$$

$$= \frac{bnx^2}{2em} - \frac{be^{-\frac{2d}{em}}nx^2\left(fx^m\right)^{-2/m}\operatorname{Ei}\left(\frac{2d}{em}+\frac{2\log\left(fx^m\right)}{m}\right)\left(\frac{d}{em}+\frac{\log\left(fx^m\right)}{m}\right)}{em} + \frac{e^{-\frac{2d}{em}}x^2\left(fx^m\right)^{-2/}}{}$$

Mathematica [A] time = 0.15, size = 93, normalized size = 0.66

$$\frac{x^2\left(2e^{-\frac{2d}{em}}\left(fx^m\right)^{-2/m}\operatorname{Ei}\left(\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(aem+bem\log\left(cx^n\right)-bdn-ben\log\left(fx^m\right)\right)+bemn\right)}{2e^2m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

```
[Out] (x^2*(b*e*m*n + (2*ExpIntegralEi[(2*(d + e*Log[f*x^m]))/(e*m)]*(a*e*m - b*d
*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(E^((2*d)/(e*m))*(f*x^m)^(2/m)))
)/(2*e^2*m^2)
```

fricas [A] time = 0.78, size = 92, normalized size = 0.65

$$\frac{\left(bemnx^2e^{\left(\frac{2\left(e\log(f)+d\right)}{em}\right)}+2\left(bem\log(c)-ben\log(f)+aem-bdn\right)\operatorname{log\_integral}\left(x^2e^{\left(\frac{2\left(e\log(f)+d\right)}{em}\right)}\right)\right)e^{\left(-\frac{2\left(e\log(f)+d\right)}{em}\right)}}{2\,e^2m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*e*m*n*x^2*e^(2*(e*log(f) + d)/(e*m)) + 2*(b*e*m*log(c) - b*e*n*log(f
) + a*e*m - b*d*n)*log_integral(x^2*e^(2*(e*log(f) + d)/(e*m))))*e^(-2*(e*l
og(f) + d)/(e*m))/(e^2*m^2)
```

giac [A] time = 0.51, size = 206, normalized size = 1.46

$$\frac{bnx^2e^{(-1)}}{2m} - \frac{bdn\mathrm{Ei}\left(\frac{2de^{(-1)}}{m} + \frac{2\log(f)}{m} + 2\log(x)\right)e^{\left(-\frac{2de^{(-1)}}{m}-2\right)}}{f^{\frac{2}{m}}m^2} + \frac{b\mathrm{Ei}\left(\frac{2de^{(-1)}}{m} + \frac{2\log(f)}{m} + 2\log(x)\right)e^{\left(-\frac{2de^{(-1)}}{m}-1\right)}\log(c)}{f^{\frac{2}{m}}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="giac")
```

```
[Out] 1/2*b*n*x^2*e^(-1)/m - b*d*n*Ei(2*d*e^(-1)/m + 2*log(f)/m + 2*log(x))*e^(-2
*d*e^(-1)/m - 2)/(f^(2/m)*m^2) + b*Ei(2*d*e^(-1)/m + 2*log(f)/m + 2*log(x))
*e^(-2*d*e^(-1)/m - 1)*log(c)/(f^(2/m)*m) - b*n*Ei(2*d*e^(-1)/m + 2*log(f)/
m + 2*log(x))*e^(-2*d*e^(-1)/m - 1)*log(f)/(f^(2/m)*m^2) + a*Ei(2*d*e^(-1)/
m + 2*log(f)/m + 2*log(x))*e^(-2*d*e^(-1)/m - 1)/(f^(2/m)*m)
```

maple [F] time = 7.43, size = 0, normalized size = 0.00

$$\int \frac{(b\ln(cx^n) + a)\,x}{e\ln\left(fx^m\right) + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)/(e*ln(f*x^m)+d),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)/(e*ln(f*x^m)+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log(cx^n) + a\right)x}{e\log\left(fx^m\right) + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)*x/(e*log(f*x^m) + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a + b\,\ln(c\,x^n)\right)}{d + e\,\ln\left(f\,x^m\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*log(f*x^m)),x)
```

```
[Out] int((x*(a + b*log(c*x^n)))/(d + e*log(f*x^m)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a + b\log(cx^n)\right)}{d + e\log\left(fx^m\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))/(d+e*ln(f*x**m)),x)
```

```
[Out] Integral(x*(a + b*log(c*x**n))/(d + e*log(f*x**m)), x)
```

3.172 $\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx$

Optimal. Leaf size=130

$$\frac{xe^{-\frac{d}{em}}\left(fx^m\right)^{-1/m}\left(a+b\log\left(cx^n\right)\right)\operatorname{Ei}\left(\frac{d+e\log\left(fx^m\right)}{em}\right)}{em}-\frac{bnxe^{-\frac{d}{em}}\left(fx^m\right)^{-1/m}\left(d+e\log\left(fx^m\right)\right)\operatorname{Ei}\left(\frac{d+e\log\left(fx^m\right)}{em}\right)}{e^2m^2}+\frac{bnx}{em}$$

```
[Out] b*n*x/e/m-b*n*x*Ei((d+e*ln(f*x^m))/e/m)*(d+e*ln(f*x^m))/e^2/exp(d/e/m)/m^2/
((f*x^m)^(1/m))+x*Ei((d+e*ln(f*x^m))/e/m)*(a+b*ln(c*x^n))/e/exp(d/e/m)/m/((
f*x^m)^(1/m))
```

Rubi [A] time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2300, 2178, 2361, 12, 15, 6482}

$$\frac{xe^{-\frac{d}{em}}\left(fx^m\right)^{-1/m}\left(a+b\log\left(cx^n\right)\right)\operatorname{Ei}\left(\frac{d+e\log\left(fx^m\right)}{em}\right)}{em}-\frac{bnxe^{-\frac{d}{em}}\left(fx^m\right)^{-1/m}\left(d+e\log\left(fx^m\right)\right)\operatorname{Ei}\left(\frac{d+e\log\left(fx^m\right)}{em}\right)}{e^2m^2}+\frac{bnx}{em}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(d + e*Log[f*x^m]),x]
```

```
[Out] (b*n*x)/(e*m) - (b*n*x*ExpIntegralEi[(d + e*Log[f*x^m])/(e*m)]*(d + e*Log[f
*x^m]))/(e^2*E^(d/(e*m))*m^2*(f*x^m)^m^(-1)) + (x*ExpIntegralEi[(d + e*Log[
f*x^m])/(e*m)]*(a + b*Log[c*x^n]))/(e*E^(d/(e*m))*m*(f*x^m)^m^(-1))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Dist[
d + e*Log[f*x^r], u, x] - Dist[e*r, Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Rule 6482

```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*ExpInte
gralEi[a + b*x])/b, x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}dx = \frac{e^{-\frac{d}{em}}x\left(fx^m\right)^{-1/m}\text{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)\left(a+b\log(cx^n)\right)}{em} - (bn)\int \frac{e^{-\frac{d}{em}}\left(fx^m\right)^{-1/m}\text{Ei}\left(\frac{d+e\log(fx^m)}{em}\right.}{em}$$

$$= \frac{e^{-\frac{d}{em}}x\left(fx^m\right)^{-1/m}\text{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)\left(a+b\log(cx^n)\right)}{em} - \frac{\left(be^{-\frac{d}{em}}n\right)\int\left(fx^m\right)^{-1/m}\text{Ei}\left(\frac{d+e\log(fx^m)}{em}\right.}{em}$$

$$= \frac{e^{-\frac{d}{em}}x\left(fx^m\right)^{-1/m}\text{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)\left(a+b\log(cx^n)\right)}{em} - \frac{\left(be^{-\frac{d}{em}}nx\left(fx^m\right)^{-1/m}\right)\int\frac{\text{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)}{x}}{em}$$

$$= \frac{e^{-\frac{d}{em}}x\left(fx^m\right)^{-1/m}\text{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)\left(a+b\log(cx^n)\right)}{em} - \frac{\left(be^{-\frac{d}{em}}nx\left(fx^m\right)^{-1/m}\right)\text{Subst}\left(\int \text{Ei}\left(\right.\right.}{em^2}$$

$$= \frac{e^{-\frac{d}{em}}x\left(fx^m\right)^{-1/m}\text{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)\left(a+b\log(cx^n)\right)}{em} - \frac{\left(be^{-\frac{d}{em}}nx\left(fx^m\right)^{-1/m}\right)\text{Subst}\left(\int \text{Ei}(x\right.}{em}$$

$$= \frac{bnx}{em} - \frac{be^{-\frac{d}{em}}nx\left(fx^m\right)^{-1/m}\text{Ei}\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)}{em} + \frac{e^{-\frac{d}{em}}x\left(fx^m\right)^{-1/m}\text{Ei}\left(\frac{d+e\log}{}\right.}{em}$$

Mathematica [A] time = 0.13, size = 86, normalized size = 0.66

$$\frac{x\left(e^{-\frac{d}{em}}\left(fx^m\right)^{-1/m}\text{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)\left(aem+bem\log(cx^n)-bdn-ben\log\left(fx^m\right)\right)+bemn\right)}{e^2m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*Log[f*x^m]),x]
```

```
[Out] (x*(b*e*m*n + (ExpIntegralEi[(d + e*Log[f*x^m])/(e*m)]*(a*e*m - b*d*n - b*e
*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(E^(d/(e*m))*(f*x^m)^m^(-1))))/(e^2*m^2)
```

fricas [A] time = 0.77, size = 84, normalized size = 0.65

$$\frac{\left(bemnxe^{\left(\frac{e\log(f)+d}{em}\right)}+\left(bem\log(c)-ben\log(f)+aem-bdn\right)\text{log\_integral}\left(xe^{\left(\frac{e\log(f)+d}{em}\right)}\right)\right)e^{\left(-\frac{e\log(f)+d}{em}\right)}}{e^2m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="fricas")
```

```
[Out] (b*e*m*n*x*e^((e*log(f) + d)/(e*m)) + (b*e*m*log(c) - b*e*n*log(f) + a*e*m
- b*d*n)*log_integral(x*e^((e*log(f) + d)/(e*m))))*e^(-(e*log(f) + d)/(e*m)
)/(e^2*m^2)
```

giac [A] time = 0.43, size = 179, normalized size = 1.38

$$\frac{bnxe^{(-1)}}{m} - \frac{bdn\text{Ei}\left(\frac{de^{(-1)}}{m}+\frac{\log(f)}{m}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{m}-2\right)}}{f^{\left(\frac{1}{m}\right)}m^2} + \frac{b\text{Ei}\left(\frac{de^{(-1)}}{m}+\frac{\log(f)}{m}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{m}-1\right)}\log(c)}{f^{\left(\frac{1}{m}\right)}m} - \frac{bn\text{Ei}\left(\frac{de^{(-1)}}{m}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="giac")
```

```
[Out] b*n*x*e^(-1)/m - b*d*n*Ei(d*e^(-1)/m + log(f)/m + log(x))*e^(-d*e^(-1)/m -
2)/(f^(1/m)*m^2) + b*Ei(d*e^(-1)/m + log(f)/m + log(x))*e^(-d*e^(-1)/m - 1)
*log(c)/(f^(1/m)*m) - b*n*Ei(d*e^(-1)/m + log(f)/m + log(x))*e^(-d*e^(-1)/m
 - 1)*log(f)/(f^(1/m)*m^2) + a*Ei(d*e^(-1)/m + log(f)/m + log(x))*e^(-d*e^(
-1)/m - 1)/(f^(1/m)*m)
```

maple [C] time = 1.34, size = 2329, normalized size = 17.92

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)/(e*ln(f*x^m)+d),x)
```

```
[Out] -1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+2*b*ln(c)+
2*a)/e/m*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn
(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)
^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*csg
n(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn
(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)
/e/m)-b/e/m*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*c
sgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x
^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*
csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*c
sgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I
*d)/e/m)*ln(x^n)+b*n*x/e/m-1/2*I*b*n/e/m^2*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2
*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^
2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(
1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csg
n(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln
(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x
^m)+1/2*I*b*n/e/m^2*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(
I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*cs
gn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*cs
gn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(
I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln
(x))+2*I*d)/e/m)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I*b*n/e/m^2*x*(x^m)^(-1/m
)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(
I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m
)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)
-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(
I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*x^m)*c
sgn(I*f*x^m)^2-1/2*I*b*n/e/m^2*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn
(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csg
n(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2
*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2
-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(l
n(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f*x^m)^3+b*n/e/m^2*x*(x^m)^(-1/m)*f^(
-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*
csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e
+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi
*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x
^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*ln(f)+b*n/e/m^2*x*(x^
m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*
Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn
(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(
```

```
I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*
Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*ln(x^m)+
b*n/e^2/m^2*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*c
sgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x
^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*
csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*c
sgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I
*d)/e/m)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log\left(cx^{n}\right)+a}{e\log\left(fx^{m}\right)+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)/(e*log(f*x^m) + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,x^{n}\right)}{d+e\,\ln\left(f\,x^{m}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(d + e*log(f*x^m)),x)
[Out] int((a + b*log(c*x^n))/(d + e*log(f*x^m)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(cx^{n}\right)}{d+e\log\left(fx^{m}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/(d+e*ln(f*x**m)),x)
[Out] Integral((a + b*log(c*x**n))/(d + e*log(f*x**m)), x)
```

3.173 $\int \frac{a+b\log(cx^n)}{x\left(d+e\log\left(fx^m\right)\right)}\,dx$

Optimal. Leaf size=71

$$\frac{(a+b\log(cx^n))\log\left(d+e\log\left(fx^m\right)\right)}{em}-\frac{bn\left(d+e\log\left(fx^m\right)\right)\log\left(d+e\log\left(fx^m\right)\right)}{e^2m^2}+\frac{bn\log(x)}{em}$$

```
[Out] b*n*ln(x)/e/m-b*n*(d+e*ln(f*x^m))*ln(d+e*ln(f*x^m))/e^2/m^2+(a+b*ln(c*x^n))
*ln(d+e*ln(f*x^m))/e/m
```

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2302, 29, 2366, 12, 2389, 2295}

$$\frac{(a+b\log(cx^n))\log\left(d+e\log\left(fx^m\right)\right)}{em}-\frac{bn\left(d+e\log\left(fx^m\right)\right)\log\left(d+e\log\left(fx^m\right)\right)}{e^2m^2}+\frac{bn\log(x)}{em}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*Log[f*x^m])),x]
```

```
[Out] (b*n*Log[x])/(e*m) - (b*n*(d + e*Log[f*x^m])*Log[d + e*Log[f*x^m]])/(e^2*m^
2) + ((a + b*Log[c*x^n])*Log[d + e*Log[f*x^m]])/(e*m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \frac{a+b\log(cx^n)}{x\left(d+e\log\left(fx^m\right)\right)}\,dx = \frac{\left(a+b\log(cx^n)\right)\log\left(d+e\log\left(fx^m\right)\right)}{em} - (bn)\int \frac{\log\left(d+e\log\left(fx^m\right)\right)}{emx}\,dx$$
$$= \frac{\left(a+b\log(cx^n)\right)\log\left(d+e\log\left(fx^m\right)\right)}{em} - \frac{(bn)\int \frac{\log\left(d+e\log\left(fx^m\right)\right)}{x}\,dx}{em}$$
$$= \frac{\left(a+b\log(cx^n)\right)\log\left(d+e\log\left(fx^m\right)\right)}{em} - \frac{(bn)\,\text{Subst}\left(\int \log(d+ex)\,dx, x, \log\left(fx^m\right)\right)}{em^2}$$
$$= \frac{\left(a+b\log(cx^n)\right)\log\left(d+e\log\left(fx^m\right)\right)}{em} - \frac{(bn)\,\text{Subst}\left(\int \log(x)\,dx, x, d+e\log\left(fx^m\right)\right)}{e^2m^2}$$
$$= \frac{bn\log(x)}{em} - \frac{bn\left(d+e\log\left(fx^m\right)\right)\log\left(d+e\log\left(fx^m\right)\right)}{e^2m^2} + \frac{\left(a+b\log(cx^n)\right)\log\left(d+e\right.}{em}$$

Mathematica [A] time = 0.07, size = 58, normalized size = 0.82

$$\frac{\log\left(d+e\log\left(fx^m\right)\right)\left(aem+bem\log(cx^n)-bdn-ben\log\left(fx^m\right)\right)+bemn\log(x)}{e^2m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*Log[f*x^m])),x]
```

```
[Out] (b*e*m*n*Log[x] + (a*e*m - b*d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n])*Log
[d + e*Log[f*x^m]])/(e^2*m^2)
```

fricas [A] time = 0.75, size = 51, normalized size = 0.72

$$\frac{bemn\log(x)+\left(bem\log(c)-ben\log(f)+aem-bdn\right)\log\left(em\log(x)+e\log(f)+d\right)}{e^2m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*log(f*x^m)),x, algorithm="fricas")
```

```
[Out] (b*e*m*n*log(x) + (b*e*m*log(c) - b*e*n*log(f) + a*e*m - b*d*n)*log(e*m*log
(x) + e*log(f) + d))/(e^2*m^2)
```

giac [A] time = 0.39, size = 85, normalized size = 1.20

$$\frac{bne^{(-1)}\log(x)}{m} + \frac{\left(bme\log(c)-bne\log(f)-bdn+ame\right)e^{(-2)}\log\left(\frac{1}{4}\left(\pi m\left(\text{sgn}(x)-1\right)e+\pi\left(\text{sgn}(f)-1\right)e\right)^2+\left(me\,\mathrm{l}\right.\right.}{2\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*log(f*x^m)),x, algorithm="giac")
```

```
[Out] b*n*e^(-1)*log(x)/m + 1/2*(b*m*e*log(c) - b*n*e*log(f) - b*d*n + a*m*e)*e^(
-2)*log(1/4*(pi*m*(sgn(x) - 1)*e + pi*(sgn(f) - 1)*e)^2 + (m*e*log(abs(x))
+ e*log(abs(f)) + d)^2)/m^2
```

maple [C] time = 0.44, size = 1744, normalized size = 24.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)/x/(e*ln(f*x^m)+d),x)
```

```
[Out] 1/2*I/m*ln(Pi*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*f)*csgn(I*f
*x^m)^2-Pi*e*csgn(I*x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*I*e*ln(f)+2
*I*ln(x^m)*e+2*I*d)/e*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I/m*ln(Pi*e*csgn
(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*f)*csgn(I*f*x^m)^2-Pi*e*csgn(I*
x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*ln(x^m)*e+2*I*d)/
e*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-1/2*I/m*ln(Pi*e*csgn(I*f)*csgn(I
*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*f)*csgn(I*f*x^m)^2-Pi*e*csgn(I*x^m)*csgn(I*
f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*ln(x^m)*e+2*I*d)/e*b*Pi*csgn(
I*c*x^n)^3+1/2*I/m*ln(Pi*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*
f)*csgn(I*f*x^m)^2-Pi*e*csgn(I*x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*
I*e*ln(f)+2*I*ln(x^m)*e+2*I*d)/e*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+1/m*ln(Pi*e
*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*f)*csgn(I*f*x^m)^2-Pi*e*cs
gn(I*x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*ln(x^m)*e+2*
I*d)/e*b*ln(c)+1/m*ln(Pi*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*
f)*csgn(I*f*x^m)^2-Pi*e*csgn(I*x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*
I*e*ln(f)+2*I*ln(x^m)*e+2*I*d)/e*a+b*n*ln(x)/e/m+1/2*I*b/e/m^2*ln(Pi*e*csgn
(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*f)*csgn(I*f*x^m)^2-Pi*e*csgn(I*
x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*I*ln(x)*e*m+2*I*e*ln(f)+2*I*(-m
*ln(x)+ln(x^m))*e+2*I*d)*Pi*n*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2*I*b/e
/m^2*ln(Pi*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*f)*csgn(I*f*x^
m)^2-Pi*e*csgn(I*x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*I*ln(x)*e*m+2*
I*e*ln(f)+2*I*(-m*ln(x)+ln(x^m))*e+2*I*d)*Pi*n*csgn(I*f)*csgn(I*f*x^m)^2-1/
2*I*b/e/m^2*ln(Pi*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*f)*csgn
(I*f*x^m)^2-Pi*e*csgn(I*x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*I*ln(x)
*e*m+2*I*e*ln(f)+2*I*(-m*ln(x)+ln(x^m))*e+2*I*d)*Pi*n*csgn(I*x^m)*csgn(I*f*
x^m)^2+1/2*I*b/e/m^2*ln(Pi*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(
I*f)*csgn(I*f*x^m)^2-Pi*e*csgn(I*x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+
2*I*ln(x)*e*m+2*I*e*ln(f)+2*I*(-m*ln(x)+ln(x^m))*e+2*I*d)*Pi*n*csgn(I*f*x^m
)^3-b/e/m^2*ln(Pi*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*f)*csgn
(I*f*x^m)^2-Pi*e*csgn(I*x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*I*ln(x)
*e*m+2*I*e*ln(f)+2*I*(-m*ln(x)+ln(x^m))*e+2*I*d)*ln(f)*n+b/e/m*ln(Pi*e*csgn
(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csgn(I*f)*csgn(I*f*x^m)^2-Pi*e*csgn(I*
x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^3+2*I*ln(x)*e*m+2*I*e*ln(f)+2*I*(-m
*ln(x)+ln(x^m))*e+2*I*d)*ln(x^n)-b/e/m^2*ln(Pi*e*csgn(I*f)*csgn(I*x^m)*csgn
(I*f*x^m)-Pi*e*csgn(I*f)*csgn(I*f*x^m)^2-Pi*e*csgn(I*x^m)*csgn(I*f*x^m)^2+P
i*e*csgn(I*f*x^m)^3+2*I*ln(x)*e*m+2*I*e*ln(f)+2*I*(-m*ln(x)+ln(x^m))*e+2*I*
d)*n*ln(x^m)-b/e^2/m^2*ln(Pi*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-Pi*e*csg
n(I*f)*csgn(I*f*x^m)^2-Pi*e*csgn(I*x^m)*csgn(I*f*x^m)^2+Pi*e*csgn(I*f*x^m)^
3+2*I*ln(x)*e*m+2*I*e*ln(f)+2*I*(-m*ln(x)+ln(x^m))*e+2*I*d)*d*n
```

maxima [A] time = 0.67, size = 118, normalized size = 1.66

$$\frac{b\log(cx^n)\log\left(\frac{e\log(f)+e\log(x^m)+d}{e}\right)}{em} - \frac{bn\left(\frac{\left(e\log(f)+e\log(x^m)+d\right)\log\left(\frac{e\log(f)+e\log(x^m)+d}{e}\right)}{e} - \frac{e\log(f)+e\log(x^m)+d}{e}\right)}{em^2} + \frac{a\log\left(\frac{e\log(f)+e\log(x^m)+d}{e}\right)}{em}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*log(f*x^m)),x, algorithm="maxima")
```

```
[Out] b*log(c*x^n)*log((e*log(f) + e*log(x^m) + d)/e)/(e*m) - b*n*((e*log(f) + e*
log(x^m) + d)*log((e*log(f) + e*log(x^m) + d)/e)/e - (e*log(f) + e*log(x^m)
 + d)/e)/(e*m^2) + a*log((e*log(f) + e*log(x^m) + d)/e)/(e*m)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,x^n\right)}{x\,\left(d+e\,\ln\left(f\,x^m\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(x*(d + e*log(f*x^m))),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*log(f*x^m))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \log{(c x^{n})}}{x \left(d + e \log{\left(f x^{m} \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/x/(d+e*ln(f*x**m)),x)
[Out] Integral((a + b*log(c*x**n))/(x*(d + e*log(f*x**m))), x)
```

3.174 $\int \frac{a+b\log(cx^n)}{x^2\left(d+e\log\left(fx^m\right)\right)}\,dx$

Optimal. Leaf size=133

$$\frac{e^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\left(a+b\log\left(cx^n\right)\right)\mathrm{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)}{emx}-\frac{bne^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\left(d+e\log\left(fx^m\right)\right)\mathrm{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)}{e^2m^2x}-\frac{bn}{emx}$$

```
[Out] -b*n/e/m/x-b*exp(d/e/m)*n*(f*x^m)^(1/m)*Ei((-d-e*ln(f*x^m))/e/m)*(d+e*ln(f*
x^m))/e^2/m^2/x+exp(d/e/m)*(f*x^m)^(1/m)*Ei((-d-e*ln(f*x^m))/e/m)*(a+b*ln(c
*x^n))/e/m/x
```

Rubi [A] time = 0.17, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2310, 2178, 2366, 12, 15, 6482}

$$\frac{e^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\left(a+b\log\left(cx^n\right)\right)\mathrm{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)}{emx}-\frac{bne^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\left(d+e\log\left(fx^m\right)\right)\mathrm{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)}{e^2m^2x}-\frac{bn}{emx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*Log[f*x^m])),x]
```

```
[Out] -((b*n)/(e*m*x)) - (b*E^(d/(e*m))*n*(f*x^m)^m^(-1)*ExpIntegralEi[-((d + e*L
og[f*x^m])/(e*m))]*(d + e*Log[f*x^m]))/(e^2*m^2*x) + (E^(d/(e*m))*(f*x^m)^m
^(-1)*ExpIntegralEi[-((d + e*Log[f*x^m])/(e*m))]*(a + b*Log[c*x^n]))/(e*m*x
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6482

```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*ExpInte
gralEi[a + b*x])/b, x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\log(cx^n)}{x^2\left(d+e\log\left(fx^m\right)\right)}\,dx = \frac{e^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)\left(a+b\log(cx^n)\right)}{emx} - (bn)\int \frac{e^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log\left(fx\right.}{em}\right.}{emx^2}$$

$$= \frac{e^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)\left(a+b\log(cx^n)\right)}{emx} - \frac{\left(be^{\frac{d}{em}}n\right)\int \frac{\left(fx^m\right)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)}{x^2}\,dx}{em}$$

$$= \frac{e^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)\left(a+b\log(cx^n)\right)}{emx} - \frac{\left(be^{\frac{d}{em}}n\left(fx^m\right)^{\frac{1}{m}}\right)\int \frac{\operatorname{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)}{x}}{emx}$$

$$= \frac{e^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)\left(a+b\log(cx^n)\right)}{emx} - \frac{\left(be^{\frac{d}{em}}n\left(fx^m\right)^{\frac{1}{m}}\right)\operatorname{Subst}\left(\int \operatorname{Ei}\left(-\frac{d+}{e}\right.\right.}{em^2x}$$

$$= \frac{e^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)\left(a+b\log(cx^n)\right)}{emx} + \frac{\left(be^{\frac{d}{em}}n\left(fx^m\right)^{\frac{1}{m}}\right)\operatorname{Subst}\left(\int \operatorname{Ei}(x)\,d\right.}{emx}$$

$$= -\frac{bn}{emx} - \frac{be^{\frac{d}{em}}n\left(fx^m\right)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d}{em}-\frac{\log\left(fx^m\right)}{m}\right)\left(\frac{d}{em}+\frac{\log\left(fx^m\right)}{m}\right)}{emx} + \frac{e^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\,\mathrm{l}}{}\right.}{}$$

Mathematica [A] time = 0.13, size = 87, normalized size = 0.65

$$\frac{e^{\frac{d}{em}}\left(fx^m\right)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log\left(fx^m\right)}{em}\right)\left(aem+bem\log(cx^n)-bdn-ben\log\left(fx^m\right)\right)-bemn}{e^2m^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*Log[f*x^m])),x]
```

```
[Out] (-(b*e*m*n) + E^(d/(e*m))*(f*x^m)^m^(-1)*ExpIntegralEi[-((d + e*Log[f*x^m])
/(e*m))]*(a*e*m - b*d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(e^2*m^2*x)
```

fricas [A] time = 0.61, size = 81, normalized size = 0.61

$$-\frac{bemn-\left(bemx\log(c)-benx\log(f)+(aem-bdn)x\right)e^{\left(\frac{e\log(f)+d}{em}\right)}\operatorname{log\_integral}\left(\frac{e^{\left(-\frac{e\log(f)+d}{em}\right)}}{x}\right)}{e^2m^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*log(f*x^m)),x, algorithm="fricas")
```

```
[Out] -(b*e*m*n - (b*e*m*x*log(c) - b*e*n*x*log(f) + (a*e*m - b*d*n)*x)*e^((e*log
(f) + d)/(e*m))*log_integral(e^(-(e*log(f) + d)/(e*m))/x))/(e^2*m^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log(cx^n)+a}{\left(e\log\left(fx^m\right)+d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*log(f*x^m)),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^2), x)
```

maple [F] time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{b \ln \left(c\,x^{n}\right)+a}{\left(e \ln \left(f\,x^{m}\right)+d\right) x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)/x^2/(e*ln(f*x^m)+d),x)
[Out] int((b*ln(c*x^n)+a)/x^2/(e*ln(f*x^m)+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \log \left(cx^{n}\right)+a}{\left(e \log \left(fx^{m}\right)+d\right)x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*log(f*x^m)),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln \left(c\,x^n\right)}{x^2\,\left(d+e\,\ln \left(f\,x^m\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*log(f*x^m))),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*log(f*x^m))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \log {\left (cx^{n} \right)}}{x^{2} \left(d + e \log {\left (fx^{m} \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/x**2/(d+e*ln(f*x**m)),x)
[Out] Integral((a + b*log(c*x**n))/(x**2*(d + e*log(f*x**m))), x)
```

3.175 $\int \frac{a+b\log(cx^n)}{x^3\left(d+e\log\left(fx^m\right)\right)}\,dx$

Optimal. Leaf size=141

$$\frac{e^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\left(a+b\log\left(cx^n\right)\right)\text{Ei}\left(-\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{emx^2}-\frac{bne^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\left(d+e\log\left(fx^m\right)\right)\text{Ei}\left(-\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{e^2m^2x^2}-\frac{bn}{2emx^2}$$

```
[Out] -1/2*b*n/e/m/x^2-b*exp(2*d/e/m)*n*(f*x^m)^(2/m)*Ei(-2*(d+e*ln(f*x^m))/e/m)*
(d+e*ln(f*x^m))/e^2/m^2/x^2+exp(2*d/e/m)*(f*x^m)^(2/m)*Ei(-2*(d+e*ln(f*x^m)
)/e/m)*(a+b*ln(c*x^n))/e/m/x^2
```

Rubi [A] time = 0.17, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2310, 2178, 2366, 12, 15, 6482}

$$\frac{e^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\left(a+b\log\left(cx^n\right)\right)\text{Ei}\left(-\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{emx^2}-\frac{bne^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\left(d+e\log\left(fx^m\right)\right)\text{Ei}\left(-\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)}{e^2m^2x^2}-\frac{bn}{2emx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*Log[f*x^m])),x]
```

```
[Out] -(b*n)/(2*e*m*x^2) - (b*E^((2*d)/(e*m))*n*(f*x^m)^(2/m)*ExpIntegralEi[(-2*(
d + e*Log[f*x^m]))/(e*m)]*(d + e*Log[f*x^m]))/(e^2*m^2*x^2) + (E^((2*d)/(e*
m))*(f*x^m)^(2/m)*ExpIntegralEi[(-2*(d + e*Log[f*x^m]))/(e*m)]*(a + b*Log[c
*x^n]))/(e*m*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6482

```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*ExpInte
gralEi[a + b*x])/b, x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\log(cx^n)}{x^3\left(d+e\log\left(fx^m\right)\right)}\,dx = \frac{e^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\operatorname{Ei}\left(-\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log(cx^n)\right)}{emx^2} - (bn)\int \frac{e^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\operatorname{Ei}\left(-\frac{2}{\cdots}\right.}{emx^3}$$

$$= \frac{e^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\operatorname{Ei}\left(-\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log(cx^n)\right)}{emx^2} - \frac{\left(be^{\frac{2d}{em}}n\right)\int \frac{\left(fx^m\right)^{2/m}\operatorname{Ei}\left(-\frac{2(d+e\log}{em}\right.}{x^3}}{em}$$

$$= \frac{e^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\operatorname{Ei}\left(-\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log(cx^n)\right)}{emx^2} - \frac{\left(be^{\frac{2d}{em}}n\left(fx^m\right)^{2/m}\right)\int \frac{\operatorname{Ei}\left(-\frac{2(d+}{\cdots}\right.}{\cdots}}{emx^2}$$

$$= \frac{e^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\operatorname{Ei}\left(-\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log(cx^n)\right)}{emx^2} - \frac{\left(be^{\frac{2d}{em}}n\left(fx^m\right)^{2/m}\right)\operatorname{Subst}\left(\int \cdots\right.}{e\cdots}$$

$$= \frac{e^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\operatorname{Ei}\left(-\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(a+b\log(cx^n)\right)}{emx^2} + \frac{\left(be^{\frac{2d}{em}}n\left(fx^m\right)^{2/m}\right)\operatorname{Subst}\left(\int \cdots\right.}{2e\cdots}$$

$$= -\frac{bn}{2emx^2} - \frac{be^{\frac{2d}{em}}n\left(fx^m\right)^{2/m}\operatorname{Ei}\left(-\frac{2d}{em}-\frac{2\log\left(fx^m\right)}{m}\right)\left(\frac{d}{em}+\frac{\log\left(fx^m\right)}{m}\right)}{emx^2} + \frac{e^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\cdots}{\cdots}$$

Mathematica [A] time = 0.13, size = 94, normalized size = 0.67

$$\frac{2e^{\frac{2d}{em}}\left(fx^m\right)^{2/m}\operatorname{Ei}\left(-\frac{2\left(d+e\log\left(fx^m\right)\right)}{em}\right)\left(aem+bem\log(cx^n)-bdn-ben\log\left(fx^m\right)\right)-bemn}{2e^2m^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*Log[f*x^m])),x]
```

```
[Out] (-(b*e*m*n) + 2*E^((2*d)/(e*m))*(f*x^m)^(2/m)*ExpIntegralEi[(-2*(d + e*Log[
f*x^m]))/(e*m)]*(a*e*m - b*d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(2*e
^2*m^2*x^2)
```

fricas [A] time = 0.79, size = 88, normalized size = 0.62

$$-\frac{bemn-2\left(bemx^2\log(c)-benx^2\log(f)+(aem-bdn)x^2\right)e^{\left(\frac{2\left(e\log(f)+d\right)}{em}\right)}\operatorname{log\_integral}\left(\frac{e^{\left(-\frac{2\left(e\log(f)+d\right)}{em}\right)}}{x^2}\right)}{2\,e^2m^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*log(f*x^m)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*e*m*n - 2*(b*e*m*x^2*log(c) - b*e*n*x^2*log(f) + (a*e*m - b*d*n)*x^
2)*e^(2*(e*log(f) + d)/(e*m))*log_integral(e^(-2*(e*log(f) + d)/(e*m))/x^2)
)/(e^2*m^2*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log\left(cx^n\right)+a}{\left(e\log\left(fx^m\right)+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*log(f*x^m)),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^3), x)
```

maple [F] time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\,x^n\right)+a}{\left(e\ln\left(f\,x^m\right)+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)/x^3/(e*ln(f*x^m)+d),x)
[Out] int((b*ln(c*x^n)+a)/x^3/(e*ln(f*x^m)+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log\left(cx^n\right)+a}{\left(e\log\left(fx^m\right)+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*log(f*x^m)),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^3\,\left(d+e\,\ln\left(f\,x^m\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*log(f*x^m))),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*log(f*x^m))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+e\log\left(fx^m\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/x**3/(d+e*ln(f*x**m)),x)
[Out] Integral((a + b*log(c*x**n))/(x**3*(d + e*log(f*x**m))), x)
```

3.176 $\int \frac{a+b\log(cx^n)}{\left(d+e\log(cx^n)\right)^2}\,dx$

Optimal. Leaf size=89

$$\frac{x\left(cx^n\right)^{-1/n}e^{-\frac{d}{en}}(ae-bd+ben)\text{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^3n^2}+\frac{x(bd-ae)}{e^2n\left(e\log\left(cx^n\right)+d\right)}$$

```
[Out] (b*e*n+a*e-b*d)*x*Ei((d+e*ln(c*x^n))/e/n)/e^3/exp(d/e/n)/n^2/((c*x^n)^(1/n)
)+(-a*e+b*d)*x/e^2/n/(d+e*ln(c*x^n))
```

Rubi [A] time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.52, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2360, 2297, 2300, 2178}

$$-\frac{x\left(cx^n\right)^{-1/n}e^{-\frac{d}{en}}(bd-ae)\text{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^3n^2}+\frac{x(bd-ae)}{e^2n\left(e\log\left(cx^n\right)+d\right)}+\frac{bx\left(cx^n\right)^{-1/n}e^{-\frac{d}{en}}\text{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(d + e*Log[c*x^n])^2,x]
```

```
[Out] -(((b*d - a*e)*x*ExpIntegralEi[(d + e*Log[c*x^n])/(e*n)])/(e^3*E^(d/(e*n))*
n^2*(c*x^n)^n^(-1))) + (b*x*ExpIntegralEi[(d + e*Log[c*x^n])/(e*n)])/(e^2*E
^(d/(e*n))*n*(c*x^n)^n^(-1)) + ((b*d - a*e)*x)/(e^2*n*(d + e*Log[c*x^n]))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

Rubi steps

$$\int \frac{a+b\log(cx^n)}{\left(d+e\log(cx^n)\right)^2}dx = \int \left(\frac{-bd+ae}{e\left(d+e\log(cx^n)\right)^2}+\frac{b}{e\left(d+e\log(cx^n)\right)}\right)dx$$

$$= \frac{b\int \frac{1}{d+e\log(cx^n)}dx}{e}+\frac{(-bd+ae)\int \frac{1}{\left(d+e\log(cx^n)\right)^2}dx}{e}$$

$$= \frac{(bd-ae)x}{e^2n\left(d+e\log(cx^n)\right)}-\frac{(bd-ae)\int \frac{1}{d+e\log(cx^n)}dx}{e^2n}+\frac{\left(bx(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{d+ex}dx,x,\text{l}\right.}{en}$$

$$= \frac{be^{-\frac{d}{en}}x(cx^n)^{-1/n}\text{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^2n}+\frac{(bd-ae)x}{e^2n\left(d+e\log(cx^n)\right)}-\frac{\left((bd-ae)x(cx^n)^{-1/n}\right)\text{Subst}\left(\right.}{e^2n^2}$$

$$= -\frac{(bd-ae)e^{-\frac{d}{en}}x(cx^n)^{-1/n}\text{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^3n^2}+\frac{be^{-\frac{d}{en}}x(cx^n)^{-1/n}\text{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^2n}+\frac{(bd}{e^2n\left(d+\right.}$$

Mathematica [A] time = 0.15, size = 87, normalized size = 0.98

$$\frac{x(cx^n)^{-1/n}e^{-\frac{d}{en}}(ae-bd+ben)\text{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)-\frac{enx(ae-bd)}{e\log(cx^n)+d}}{e^3n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*Log[c*x^n])^2,x]
```

```
[Out] (((-(b*d) + a*e + b*e*n)*x*ExpIntegralEi[(d + e*Log[c*x^n])/(e*n)])/(E^(d/(
e*n))*(c*x^n)^n^(-1)) - (e*(-(b*d) + a*e)*n*x)/(d + e*Log[c*x^n]))/(e^3*n^2
)
```

fricas [A] time = 0.70, size = 154, normalized size = 1.73

$$\frac{\left(\left(bde-ae^2\right)nxe^{\left(\frac{e\log(c)+d}{en}\right)}+\left(bden-bd^2+ade+\left(be^2n-bde+ae^2\right)\log(c)+\left(be^2n^2-\left(bde-ae^2\right)n\right)\log(x)\right)\text{log\_in}\right.}{e^4n^3\log(x)+e^4n^2\log(c)+de^3n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] ((b*d*e - a*e^2)*n*x*e^((e*log(c) + d)/(e*n)) + (b*d*e*n - b*d^2 + a*d*e +
(b*e^2*n - b*d*e + a*e^2)*log(c) + (b*e^2*n^2 - (b*d*e - a*e^2)*n)*log(x))*
log_integral(x*e^((e*log(c) + d)/(e*n))))*e^(-(e*log(c) + d)/(e*n))/(e^4*n^
3*log(x) + e^4*n^2*log(c) + d*e^3*n^2)
```

giac [B] time = 0.52, size = 661, normalized size = 7.43

$$\frac{bdnxe}{n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c)}+\frac{bn^2\text{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{n}+2\right)}\log(x)}{\left(n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c)\right)c^{\left(\frac{1}{n}\right)}}-\frac{bdn\text{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(\right.}{\left(n^3e^4\log(x)+dn^2e^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] b*d*n*x*e/(n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c)) + b*n^2*Ei(d*e^(-1)
/n + log(c)/n + log(x))*e^(-d*e^(-1)/n + 2)*log(x)/((n^3*e^4*log(x) + d*n^2
*e^3 + n^2*e^4*log(c))*c^(1/n)) - b*d*n*Ei(d*e^(-1)/n + log(c)/n + log(x))*
e^(-d*e^(-1)/n + 1)*log(x)/((n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c))*c
^(1/n)) - a*n*x*e^2/(n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c)) + b*d*n*E
i(d*e^(-1)/n + log(c)/n + log(x))*e^(-d*e^(-1)/n + 1)/((n^3*e^4*log(x) + d*
n^2*e^3 + n^2*e^4*log(c))*c^(1/n)) - b*d^2*Ei(d*e^(-1)/n + log(c)/n + log(x
))*e^(-d*e^(-1)/n)/((n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c))*c^(1/n))
+ b*n*Ei(d*e^(-1)/n + log(c)/n + log(x))*e^(-d*e^(-1)/n + 2)*log(c)/((n^3*e
^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c))*c^(1/n)) - b*d*Ei(d*e^(-1)/n + log(
c)/n + log(x))*e^(-d*e^(-1)/n + 1)*log(c)/((n^3*e^4*log(x) + d*n^2*e^3 + n^
2*e^4*log(c))*c^(1/n)) + a*n*Ei(d*e^(-1)/n + log(c)/n + log(x))*e^(-d*e^(-1
)/n + 2)*log(x)/((n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c))*c^(1/n)) + a
*d*Ei(d*e^(-1)/n + log(c)/n + log(x))*e^(-d*e^(-1)/n + 1)/((n^3*e^4*log(x)
+ d*n^2*e^3 + n^2*e^4*log(c))*c^(1/n)) + a*Ei(d*e^(-1)/n + log(c)/n + log(x
))*e^(-d*e^(-1)/n + 2)*log(c)/((n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c)
)*c^(1/n))
```

maple [C] time = 0.65, size = 370, normalized size = 4.16

$$-\frac{(ben + ae - bd)\, x\, c^{-\frac{1}{n}} (x^n)^{-\frac{1}{n}} \operatorname{Ei}\left(1, -\ln(x) - \frac{-i\pi e \operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n) + i\pi e \operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2 + i\pi e \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}{2en}\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)/(d+e*ln(c*x^n))^2,x)
```

```
[Out] -2/e^2/n*x*(a*e-b*d)/(2*d+2*e*ln(c)+2*e*ln(x^n)+I*e*Pi*csgn(I*x^n)*csgn(I*c
*x^n)^2-I*e*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*e*Pi*csgn(I*c*x^n)^3+I
*e*Pi*csgn(I*c*x^n)^2*csgn(I*c))-(b*e*n+a*e-b*d)/e^3/n^2*x*(x^n)^(-1/n)*c^(
-1/n)*exp(-1/2*(I*e*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*e*Pi*csgn(I*x^n)*csgn(
I*c*x^n)*csgn(I*c)-I*e*Pi*csgn(I*c*x^n)^3+I*e*Pi*csgn(I*c*x^n)^2*csgn(I*c)+
2*d)/e/n)*Ei(1,-ln(x)-1/2*(I*e*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*e*Pi*csgn(I
*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*e*Pi*csgn(I*c*x^n)^3+I*e*Pi*csgn(I*c*x^n)^2
*csgn(I*c)+2*e*ln(c)+2*e*(-n*ln(x)+ln(x^n))+2*d)/e/n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$((en - d)b + ae) \int \frac{1}{e^3 n \log(c) + e^3 n \log(x^n) + de^2 n}\, dx + \frac{(bd - ae)x}{e^3 n \log(c) + e^3 n \log(x^n) + de^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] ((e*n - d)*b + a*e)*integrate(1/(e^3*n*log(c) + e^3*n*log(x^n) + d*e^2*n),
x) + (b*d - a*e)*x/(e^3*n*log(c) + e^3*n*log(x^n) + d*e^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\ln(c\,x^n)}{(d + e\,\ln(c\,x^n))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(d + e*log(c*x^n))^2,x)
```

```
[Out] int((a + b*log(c*x^n))/(d + e*log(c*x^n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log(cx^n)}{\left(d + e\log(cx^n)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/(d+e*ln(c*x**n))**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))/(d + e*log(c*x**n))**2, x)
```

3.177 $\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx$

Optimal. Leaf size=29

$$\log(\log(x))\left(a+b\log\left(cx^n\right)\right)+bn\log(x)-bn\log(\log(x))\log(x)$$

```
[Out] b*n*ln(x)-b*n*ln(x)*ln(ln(x))+(a+b*ln(c*x^n))*ln(ln(x))
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2302, 29, 2366, 2521}

$$\log(\log(x))\left(a+b\log\left(cx^n\right)\right)+bn\log(x)-bn\log(\log(x))\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(x*Log[x]),x]
```

```
[Out] b*n*Log[x] - b*n*Log[x]*Log[Log[x]] + (a + b*Log[c*x^n])*Log[Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2521

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n, x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(cx^n\right)}{x\log(x)}\,dx &= \left(a+b\log\left(cx^n\right)\right)\log(\log(x))-(bn)\int \frac{\log(\log(x))}{x}\,dx\\ &= bn\log(x)-bn\log(x)\log(\log(x))+\left(a+b\log\left(cx^n\right)\right)\log(\log(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.97

$$a\log(\log(x))+b\log(\log(x))\left(\log\left(cx^n\right)-n\log(x)\right)+bn\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(x*Log[x]),x]
```

```
[Out] b*n*Log[x] + a*Log[Log[x]] + b*(-(n*Log[x]) + Log[c*x^n])*Log[Log[x]]
```

fricas [A] time = 0.73, size = 16, normalized size = 0.55

$$bn\log(x) + \left(b\log(c) + a\right)\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/log(x),x, algorithm="fricas")
```

```
[Out] b*n*log(x) + (b*log(c) + a)*log(log(x))
```

giac [A] time = 0.32, size = 17, normalized size = 0.59

$$bn\log(x) + \left(b\log(c) + a\right)\log\left(\left|\log(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/log(x),x, algorithm="giac")
```

```
[Out] b*n*log(x) + (b*log(c) + a)*log(abs(log(x)))
```

maple [C] time = 0.19, size = 131, normalized size = 4.52

$$-\frac{i\pi b\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)\ln\left(\ln(x)\right)}{2}+\frac{i\pi b\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\ln\left(\ln(x)\right)}{2}+\frac{i\pi b\,\mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)/x/ln(x),x)
```

```
[Out] -b*n*ln(x)*ln(ln(x))+b*n*ln(x)+ln(ln(x))*ln(x^n)*b+1/2*I*ln(ln(x))*b*Pi*csg
n(I*x^n)*csgn(I*c*x^n)^2-1/2*I*ln(ln(x))*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csg
n(I*c)-1/2*I*ln(ln(x))*b*Pi*csgn(I*c*x^n)^3+1/2*I*ln(ln(x))*b*Pi*csgn(I*c*x
^n)^2*csgn(I*c)+ln(ln(x))*b*ln(c)+ln(ln(x))*a
```

maxima [A] time = 0.62, size = 32, normalized size = 1.10

$$-\left(\log(x)\log\left(\log(x)\right) - \log(x)\right)bn + b\log\left(cx^{n}\right)\log\left(\log(x)\right) + a\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/log(x),x, algorithm="maxima")
```

```
[Out] -(log(x)*log(log(x)) - log(x))*b*n + b*log(c*x^n)*log(log(x)) + a*log(log(x
))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b\,\ln\left(c\,x^{n}\right)}{x\,\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(x*log(x)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*log(x)), x)
```

sympy [A] time = 10.19, size = 32, normalized size = 1.10

$$a\log\left(\log\left(x\right)\right) - b\left(n\left(\log\left(x\right)\log\left(\log\left(x\right)\right) - \log\left(x\right)\right) - \log\left(cx^{n}\right)\log\left(\log\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/x/ln(x),x)
```

```
[Out] a*log(log(x)) - b*(n*(log(x)*log(log(x)) - log(x)) - log(c*x**n)*log(log(x)
))
```

3.178 $\int (gx)^m \left(a+b\log\left(cx^n\right)\right)^p \left(d+e\log\left(fx^r\right)\right)\,dx$

Optimal. Leaf size=347

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(m+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{g(m+1)}-$$

```
[Out] -e*r*x*(g*x)^m*GAMMA(2+p,-a*(1+m)/b/n-(1+m)*ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/
exp(a*(1+m)/b/n)/(1+m)^2/((c*x^n)^((1+m)/n))/((-(1+m)*(a+b*ln(c*x^n))/b/n)^
p)-e*r*x*(g*x)^m*GAMMA(1+p,-a*(1+m)/b/n-(1+m)*ln(c*x^n)/n)*(a+b*ln(c*x^n))^
(1+p)/b/exp(a*(1+m)/b/n)/(1+m)/n/((c*x^n)^((1+m)/n))/((-(1+m)*(a+b*ln(c*x^n
))/b/n)^p)+(g*x)^(1+m)*GAMMA(1+p,-(1+m)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n)
)^p*(d+e*ln(f*x^r))/exp(a*(1+m)/b/n)/g/(1+m)/((c*x^n)^((1+m)/n))/((-(1+m)*(
a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A] time = 0.36, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2310, 2181, 2366, 12, 15, 19, 6557}

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}\left(cx^n\right)^{-\frac{m+1}{n}}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(m+1)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{(m+1)\left(a+b\log\right.}{bn}\right.}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((e*r*x*(g*x)^m*Gamma[2 + p, -((a*(1 + m))/(b*n)) - ((1 + m)*Log[c*x^n])/n
]*(a + b*Log[c*x^n])^p)/(E^((a*(1 + m))/(b*n))*(1 + m)^2*(c*x^n)^((1 + m)/n
)*(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n)))^p)) - (e*r*x*(g*x)^m*Gamma[1 + p,
 -((a*(1 + m))/(b*n)) - ((1 + m)*Log[c*x^n])/n]*(a + b*Log[c*x^n])^(1 + p))
/(b*E^((a*(1 + m))/(b*n))*(1 + m)*n*(c*x^n)^((1 + m)/n)*(-(((1 + m)*(a + b*
Log[c*x^n]))/(b*n)))^p) + ((g*x)^(1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log
[c*x^n]))/(b*n))]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^((a*(1 + m))/
(b*n))*g*(1 + m)*(c*x^n)^((1 + m)/n)*(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n))
)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
```

```
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6557

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Gamma[n, a
+ b*x])/b, x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int (gx)^m \left(a+b\log\left(cx^n\right)\right)^p \left(d+e\log\left(fx^r\right)\right) dx = \frac{e^{-\frac{a(1+m)}{bn}} (gx)^{1+m} \left(cx^n\right)^{-\frac{1+m}{n}} \Gamma\left(1+p,-\frac{(1+m)\left(a+b\log\left(cx^n\right)\right)}{bn}\right) \left(a+b\right.}{g(1+m)}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}} (gx)^{1+m} \left(cx^n\right)^{-\frac{1+m}{n}} \Gamma\left(1+p,-\frac{(1+m)\left(a+b\log\left(cx^n\right)\right)}{bn}\right) \left(a+b\right.}{g(1+m)}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}} (gx)^{1+m} \left(cx^n\right)^{-\frac{1+m}{n}} \Gamma\left(1+p,-\frac{(1+m)\left(a+b\log\left(cx^n\right)\right)}{bn}\right) \left(a+b\right.}{g(1+m)}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}} (gx)^{1+m} \left(cx^n\right)^{-\frac{1+m}{n}} \Gamma\left(1+p,-\frac{(1+m)\left(a+b\log\left(cx^n\right)\right)}{bn}\right) \left(a+b\right.}{g(1+m)}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}} (gx)^{1+m} \left(cx^n\right)^{-\frac{1+m}{n}} \Gamma\left(1+p,-\frac{(1+m)\left(a+b\log\left(cx^n\right)\right)}{bn}\right) \left(a+b\right.}{g(1+m)}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}} (gx)^{1+m} \left(cx^n\right)^{-\frac{1+m}{n}} \Gamma\left(1+p,-\frac{(1+m)\left(a+b\log\left(cx^n\right)\right)}{bn}\right) \left(a+b\right.}{g(1+m)}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}} (gx)^{1+m} \left(cx^n\right)^{-\frac{1+m}{n}} \Gamma\left(1+p,-\frac{(1+m)\left(a+b\log\left(cx^n\right)\right)}{bn}\right) \left(a+b\right.}{g(1+m)}$$

$$= -\frac{ee^{-\frac{a(1+m)}{bn}} rx(gx)^m \left(cx^n\right)^{-\frac{1+m}{n}} \Gamma\left(2+p,-\frac{a(1+m)}{bn}-\frac{(1+m)\log\left(cx^n\right)}{n}\right.}{(1+m)^2}$$

Mathematica [A] time = 0.62, size = 179, normalized size = 0.52

$$-\frac{x^{-m}(gx)^m\left(a+b\log\left(cx^n\right)\right)^{p-1}\exp\left(-\frac{(m+1)\left(a+b\log(cx^n)-bn\log(x)\right)}{bn}\right)\left(-\frac{(m+1)\left(a+b\log(cx^n)\right)}{bn}\right)^{1-p}\left((m+1)\Gamma\left(p+1,-\frac{(m+1)(\cdots}{\cdots}\right.\right.}{(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((((g*x)^m*(a + b*Log[c*x^n])^(-1 + p)*(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n
)))^(1 - p)*(-(b*e*n*r*Gamma[2 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))])
 + (1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))]*(b*d*n - a*e
*r - b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/(E^(((1 + m)*(a - b*n*Log[x] +
b*Log[c*x^n]))/(b*n))*(1 + m)^3*x^m))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(gx\right)^{m}e\log\left(fx^{r}\right)+\left(gx\right)^{m}d\right)\left(b\log\left(cx^{n}\right)+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas"
)
```

```
[Out] integral(((g*x)^m*e*log(f*x^r) + (g*x)^m*d)*(b*log(c*x^n) + a)^p, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Simplification assuming t_nostep near 0Simplification ass
uming g near 0Simplification assuming t_nostep near 0Simplification assumin
g g near 0Simplification assuming t_nostep near 0Simplification assuming g
near 0Simplification assuming t_nostep near 0Simplification assuming g near
 0Simplification assuming t_nostep near 0Simplification assuming g near 0Si
mplification assuming t_nostep near 0Simplification assuming g near 0Simpli
fication assuming t_nostep near 0Simplification assuming g near 0Simplifica
tion assuming t_nostep near 0Simplification assuming g near 0Simplification
 assuming t_nostep near 0Simplification assuming g near 0Simplification ass
uming t_nostep near 0Simplification assuming g near 0Simplification assumin
g t_nostep near 0Simplification assuming g near 0Simplification assuming t_
nostep near 0Simplification assuming g near 0Simplification assuming t_nost
ep near 0Simplification assuming g near 0Simplification assuming t_nostep n
ear 0Simplification assuming g near 0Evaluation time: 0.64Unable to divide,
 perhaps due to rounding error%%%{1,[0,2,2,2,0,2,0,0]%%%}+%%%{2,[0,2,2,2,0,
1,0,0]%%%}+%%%{1,[0,2,2,2,0,0,0,0]%%%}+%%%{1,[0,2,2,1,1,1,1,0]%%%}+%%%{1,[0
,2,2,1,1,0,1,0]%%%}+%%%{2,[0,2,2,1,0,1,1,0]%%%}+%%%{2,[0,2,2,1,0,0,1,0]%%%}
+%%%{1,[0,2,1,1,1,1,0,1]%%%}+%%%{1,[0,2,1,1,1,0,0,1]%%%}+%%%{2,[0,2,1,1,0,1
,0,1]%%%}+%%%{2,[0,2,1,1,0,0,0,1]%%%} / %%%{1,[0,3,3,3,0,2,0,0]%%%}+%%%{2,[
0,3,3,3,0,1,0,0]%%%}+%%%{1,[0,3,3,3,0,0,0,0]%%%} Error: Bad Argument Value
```

maple [F] time = 2.06, size = 0, normalized size = 0.00

$$\int\left(e\ln\left(f\,x^{r}\right)+d\right)\left(gx\right)^{m}\left(b\ln\left(c\,x^{n}\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(b*ln(c*x^n)+a)^p*(d+e*ln(f*x^r)),x)
```

```
[Out] int((g*x)^m*(b*ln(c*x^n)+a)^p*(d+e*ln(f*x^r)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d+e\,\ln\left(f\,x^r\right)\right)\,{\left(g\,x\right)}^m\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*log(f*x^r))*(g*x)^m*(a + b*log(c*x^n))^p,x)
```

```
[Out] int((d + e*log(f*x^r))*(g*x)^m*(a + b*log(c*x^n))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
```

```
[Out] Timed out
```

3.179 $\int x^2 \left(a+b\log\left(cx^n\right)\right)^p \left(d+e\log\left(fx^r\right)\right) dx$

Optimal. Leaf size=298

$$3^{-p-1}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+e\left(-\right.$$

```
[Out] -3^(-2-p)*e*r*x^3*GAMMA(2+p,-3*a/b/n-3*ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/exp(3
*a/b/n)/((c*x^n)^(3/n))/(((-a-b*ln(c*x^n))/b/n)^p)-3^(-1-p)*e*r*x^3*GAMMA(1
+p,-3*a/b/n-3*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(1+p)/b/exp(3*a/b/n)/n/((c*x^n)^
(3/n))/(((-a-b*ln(c*x^n))/b/n)^p)+3^(-1-p)*x^3*GAMMA(1+p,-3*(a+b*ln(c*x^n))
/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/exp(3*a/b/n)/((c*x^n)^(3/n))/(((-a-
b*ln(c*x^n))/b/n)^p)
```

Rubi [A] time = 0.25, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2310, 2181, 2366, 12, 15, 19, 6557}

$$3^{-p-1}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(cx^n\right)\right.}{bn}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((3^(-2 - p)*e*r*x^3*Gamma[2 + p, (-3*a)/(b*n) - (3*Log[c*x^n])/n]*(a + b*
Log[c*x^n])^p)/(E^((3*a)/(b*n))*(c*x^n)^(3/n)*(-((a + b*Log[c*x^n])/(b*n)))
^p)) - (3^(-1 - p)*e*r*x^3*Gamma[1 + p, (-3*a)/(b*n) - (3*Log[c*x^n])/n]*(a
 + b*Log[c*x^n])^(1 + p))/(b*E^((3*a)/(b*n))*n*(c*x^n)^(3/n)*(-((a + b*Log[
c*x^n])/(b*n)))^p) + (3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*x^n]))/(
b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^((3*a)/(b*n))*(c*x^n)^(3/
n)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6557

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Gamma[n, a
+ b*x])/b, x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(cx^n\right)\right)^p \left(d+e\log\left(fx^r\right)\right)dx = 3^{-1-p}e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log(\right.$$

$$= 3^{-1-p}e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log(\right.$$

$$= 3^{-1-p}e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log(\right.$$

$$= 3^{-1-p}e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log(\right.$$

$$= 3^{-1-p}e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log(\right.$$

$$= 3^{-1-p}e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log(\right.$$

$$= -3^{-2-p}ee^{-\frac{3a}{bn}}rx^3\left(cx^n\right)^{-3/n}\Gamma\left(2+p,-\frac{3a}{bn}-\frac{3\log\left(cx^n\right)}{n}\right)\left(a+b\log\right.$$

Mathematica [A] time = 0.40, size = 156, normalized size = 0.52

$$-3^{-p-2}x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\left(a+b\log\left(cx^n\right)\right)^{p-1}\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)^{1-p}\left(3\Gamma\left(p+1,-\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(-aer-ber\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((3^(-2 - p)*x^3*(a + b*Log[c*x^n])^(-1 + p)*(-((a + b*Log[c*x^n])/(b*n)))
^(1 - p)*(-(b*e*n*r*Gamma[2 + p, (-3*(a + b*Log[c*x^n]))/(b*n)]) + 3*Gamma[
1 + p, (-3*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r - b*e*r*Log[c*x^n] + b
*e*n*Log[f*x^r])))/(E^((3*a)/(b*n))*(c*x^n)^(3/n)))
```

fricas **[F]** time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^2\log\left(fx^r\right)+dx^2\right)\left(b\log\left(cx^n\right)+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas")
[Out] integral((e*x^2*log(f*x^r) + d*x^2)*(b*log(c*x^n) + a)^p, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(e\log\left(fx^r\right)+d\right)\left(b\log\left(cx^n\right)+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p*x^2, x)
```

maple **[F]** time = 1.44, size = 0, normalized size = 0.00

$$\int\left(e\ln\left(f\,x^r\right)+d\right)x^2\left(b\ln\left(c\,x^n\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d),x)
[Out] int(x^2*(b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d),x)
```

maxima **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(d+e\,\ln\left(f\,x^r\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p,x)
[Out] int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
[Out] Timed out
```

3.180 $\int x\left(a+b\log\left(cx^n\right)\right)^p\left(d+e\log\left(fx^r\right)\right)\,dx$

Optimal. Leaf size=298

$$2^{-p-1}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\cdots$$

```
[Out] -2^(-2-p)*e*r*x^2*GAMMA(2+p,-2*a/b/n-2*ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/exp(2
*a/b/n)/((c*x^n)^(2/n))/(((-a-b*ln(c*x^n))/b/n)^p)-2^(-1-p)*e*r*x^2*GAMMA(1
+p,-2*a/b/n-2*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(1+p)/b/exp(2*a/b/n)/n/((c*x^n)^
(2/n))/(((-a-b*ln(c*x^n))/b/n)^p)+2^(-1-p)*x^2*GAMMA(1+p,-2*(a+b*ln(c*x^n))
/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/exp(2*a/b/n)/((c*x^n)^(2/n))/(((-a-
b*ln(c*x^n))/b/n)^p)
```

Rubi [A] time = 0.22, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2310, 2181, 2366, 12, 15, 19, 6557}

$$2^{-p-1}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\cdots\right.\right.}{bn}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((2^(-2 - p)*e*r*x^2*Gamma[2 + p, (-2*a)/(b*n) - (2*Log[c*x^n])/n]*(a + b*
Log[c*x^n])^p)/(E^((2*a)/(b*n))*(c*x^n)^(2/n)*(-((a + b*Log[c*x^n])/(b*n)))
^p)) - (2^(-1 - p)*e*r*x^2*Gamma[1 + p, (-2*a)/(b*n) - (2*Log[c*x^n])/n]*(a
 + b*Log[c*x^n])^(1 + p))/(b*E^((2*a)/(b*n))*n*(c*x^n)^(2/n)*(-((a + b*Log[
c*x^n])/(b*n)))^p) + (2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*x^n]))/(
b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^((2*a)/(b*n))*(c*x^n)^(2/
n)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6557

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Gamma[n, a
+ b*x])/b, x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)^p\left(d+e\log\left(fx^r\right)\right)dx = 2^{-1-p}e^{-\frac{2a}{bn}}x^2\left(cx^n\right)^{-2/n}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right.$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2\left(cx^n\right)^{-2/n}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right.$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2\left(cx^n\right)^{-2/n}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right.$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2\left(cx^n\right)^{-2/n}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right.$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2\left(cx^n\right)^{-2/n}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right.$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2\left(cx^n\right)^{-2/n}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right.$$

$$= -2^{-2-p}ee^{-\frac{2a}{bn}}rx^2\left(cx^n\right)^{-2/n}\Gamma\left(2+p,-\frac{2a}{bn}-\frac{2\log\left(cx^n\right)}{n}\right)\left(a+b\log\left(cx\right.\right.$$

Mathematica [A] time = 0.38, size = 156, normalized size = 0.52

$$-2^{-p-2}x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\left(a+b\log\left(cx^n\right)\right)^{p-1}\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)^{1-p}\left(2\Gamma\left(p+1,-\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(-aer-ber\log\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((2^(-2 - p)*x^2*(a + b*Log[c*x^n])^(-1 + p)*(-((a + b*Log[c*x^n])/(b*n)))
^(1 - p)*(-(b*e*n*r*Gamma[2 + p, (-2*(a + b*Log[c*x^n]))/(b*n)]) + 2*Gamma[
1 + p, (-2*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r - b*e*r*Log[c*x^n] + b
*e*n*Log[f*x^r])))/(E^((2*a)/(b*n))*(c*x^n)^(2/n)))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex\log\left(fx^{r}\right)+dx\right)\left(b\log\left(cx^{n}\right)+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas")
[Out] integral((e*x*log(f*x^r) + d*x)*(b*log(c*x^n) + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(e\log\left(fx^{r}\right)+d\right)\left(b\log\left(cx^{n}\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p*x, x)
```

maple [F] time = 1.63, size = 0, normalized size = 0.00

$$\int\left(e\ln\left(f\,x^{r}\right)+d\right)x\left(b\ln\left(c\,x^{n}\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d),x)
[Out] int(x*(b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(d+e\,\ln\left(f\,x^{r}\right)\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p,x)
[Out] int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
[Out] Timed out
```

3.181 $\int \left(a+b\log\left(cx^n\right)\right)^p \left(d+e\log\left(fx^r\right)\right)\,dx$

Optimal. Leaf size=271

$$xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(cx^n\right)}{bn}\right)-erxe^{-\frac{a}{bn}}\left(cx^n\right)^{-}$$

```
[Out] -e*r*x*GAMMA(2+p,-a/b/n-ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/exp(a/b/n)/((c*x^n)^
(1/n))/(((-a-b*ln(c*x^n))/b/n)^p)-e*r*x*GAMMA(1+p,-a/b/n-ln(c*x^n)/n)*(a+b*
ln(c*x^n))^(1+p)/b/exp(a/b/n)/n/((c*x^n)^(1/n))/(((-a-b*ln(c*x^n))/b/n)^p)+
x*GAMMA(1+p,(-a-b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/exp(a/b
/n)/((c*x^n)^(1/n))/(((-a-b*ln(c*x^n))/b/n)^p)
```

Rubi [A] time = 0.17, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2300, 2181, 2361, 12, 15, 19, 6557}

$$xe^{-\frac{a}{bn}}\left(cx^n\right)^{-1/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(cx^n\right)}{bn}\right)-erxe^{-\frac{a}{bn}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((e*r*x*Gamma[2 + p, -(a/(b*n)) - Log[c*x^n]/n]*(a + b*Log[c*x^n])^p)/(E^(
a/(b*n))*(c*x^n)^n^(-1)*(-((a + b*Log[c*x^n])/(b*n)))^p)) - (e*r*x*Gamma[1
+ p, -(a/(b*n)) - Log[c*x^n]/n]*(a + b*Log[c*x^n])^(1 + p))/(b*E^(a/(b*n))*
n*(c*x^n)^n^(-1)*(-((a + b*Log[c*x^n])/(b*n)))^p) + (x*Gamma[1 + p, -((a +
b*Log[c*x^n])/(b*n))]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^(a/(b*n))
*(c*x^n)^n^(-1)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Dist[
d + e*Log[f*x^r], u, x] - Dist[e*r, Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Rule 6557

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Gamma[n, a
+ b*x])/b, x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)^p\left(d+e\log\left(fx^r\right)\right)dx = e^{-\frac{a}{bn}}x\left(cx^n\right)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log\left(cx^n\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+}{}\right.$$

$$= e^{-\frac{a}{bn}}x\left(cx^n\right)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log\left(cx^n\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+}{}\right.$$

$$= e^{-\frac{a}{bn}}x\left(cx^n\right)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log\left(cx^n\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+}{}\right.$$

$$= e^{-\frac{a}{bn}}x\left(cx^n\right)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log\left(cx^n\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+}{}\right.$$

$$= e^{-\frac{a}{bn}}x\left(cx^n\right)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log\left(cx^n\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+}{}\right.$$

$$= e^{-\frac{a}{bn}}x\left(cx^n\right)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log\left(cx^n\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{a+}{}\right.$$

$$= -ee^{-\frac{a}{bn}}rx\left(cx^n\right)^{-1/n}\Gamma\left(2+p,-\frac{a}{bn}-\frac{\log\left(cx^n\right)}{n}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\right.$$

Mathematica [A] time = 0.31, size = 146, normalized size = 0.54

$$x\left(-e^{-\frac{a}{bn}}\right)\left(cx^n\right)^{-1/n}\left(a+b\log\left(cx^n\right)\right)^{p-1}\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)^{1-p}\left(\Gamma\left(p+1,-\frac{a+b\log\left(cx^n\right)}{bn}\right)\left(-aer-ber\log\left(cx^n\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((x*(a + b*Log[c*x^n])^(-1 + p)*(-((a + b*Log[c*x^n])/(b*n)))^(1 - p)*(-(b
*e*n*r*Gamma[2 + p, -((a + b*Log[c*x^n])/(b*n))]) + Gamma[1 + p, -((a + b*L
og[c*x^n])/(b*n))]*(b*d*n - a*e*r - b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/
(E^(a/(b*n))*(c*x^n)^n^(-1)))
```

fricas [A] time = 0.84, size = 131, normalized size = 0.48

$$-\frac{\left(ber\log(c)-ben\log(f)-bdn+\left(benp+ben+ae\right)r\right)e^{\left(-\frac{bnp\log\left(-\frac{1}{bn}\right)+b\log(c)+a}{bn}\right)}\Gamma\left(p+1,-\frac{bn\log(x)+b\log(c)+a}{bn}\right)-\left(ben\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] -((b*e*r*log(c) - b*e*n*log(f) - b*d*n + (b*e*n*p + b*e*n + a*e)*r)*e^(-(b*
n*p*log(-1/(b*n)) + b*log(c) + a)/(b*n))*gamma(p + 1, -(b*n*log(x) + b*log(
c) + a)/(b*n)) - (b*e*n*r*x*log(x) + b*e*r*x*log(c) + a*e*r*x)*(b*n*log(x)
+ b*log(c) + a)^p)/(b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e \log \left(f x^{r}\right) + d\right) \left(b \log \left(c x^{n}\right) + a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p, x)
```

maple [F] time = 1.42, size = 0, normalized size = 0.00

$$\int \left(e \ln \left(f \,x^{r}\right)+d \right) \left(b \ln \left(c \,x^{n}\right)+a \right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d),x)
```

```
[Out] int((b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d+e\,\ln\left(f\,x^r\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*log(f*x^r))*(a + b*log(c*x^n))^p,x)
```

```
[Out] int((d + e*log(f*x^r))*(a + b*log(c*x^n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log{\left(c x^{n} \right)}\right)^{p} \left(d + e \log{\left(f x^{r} \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
```

```
[Out] Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r)), x)
```

3.182 $\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x}\,dx$

Optimal. Leaf size=71

$$\frac{(d+e\log(fx^r))(a+b\log(cx^n))^{p+1}}{bn(p+1)}-\frac{er(a+b\log(cx^n))^{p+2}}{b^2n^2(p+1)(p+2)}$$

```
[Out] -e*r*(a+b*ln(c*x^n))^(2+p)/b^2/n^2/(1+p)/(2+p)+(a+b*ln(c*x^n))^(1+p)*(d+e*l
n(f*x^r))/b/n/(1+p)
```

Rubi [A] time = 0.16, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2302, 30, 2366, 12}

$$\frac{(d+e\log(fx^r))(a+b\log(cx^n))^{p+1}}{bn(p+1)}-\frac{er(a+b\log(cx^n))^{p+2}}{b^2n^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] -((e*r*(a + b*Log[c*x^n])^(2 + p))/(b^2*n^2*(1 + p)*(2 + p))) + ((a + b*Log
[c*x^n])^(1 + p)*(d + e*Log[f*x^r]))/(b*n*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx = \frac{(a+b\log(cx^n))^{1+p}(d+e\log(fx^r))}{bn(1+p)} - (er)\int \frac{(a+b\log(cx^n))^{1+p}}{bn(1+p)x}\,d$$

$$= \frac{(a+b\log(cx^n))^{1+p}(d+e\log(fx^r))}{bn(1+p)} - \frac{(er)\int \frac{(a+b\log(cx^n))^{1+p}}{x}\,dx}{bn(1+p)}$$

$$= \frac{(a+b\log(cx^n))^{1+p}(d+e\log(fx^r))}{bn(1+p)} - \frac{(er)\,\mathrm{Subst}\left(\int x^{1+p}\,dx, x, a+b\right.}{b^2n^2(1+p)}$$

$$= -\frac{er(a+b\log(cx^n))^{2+p}}{b^2n^2(1+p)(2+p)} + \frac{(a+b\log(cx^n))^{1+p}(d+e\log(fx^r))}{bn(1+p)}$$

Mathematica [A] time = 0.14, size = 71, normalized size = 1.00

$$\frac{(a+b\log(cx^n))^{p+1}\left(-aer-ber\log(cx^n)+bdnp+2bdn+ben(p+2)\log(fx^r)\right)}{b^2n^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^(1 + p)*(2*b*d*n + b*d*n*p - a*e*r - b*e*r*Log[c*x^n] +
 b*e*n*(2 + p)*Log[f*x^r]))/(b^2*n^2*(1 + p)*(2 + p))
```

fricas [B] time = 0.62, size = 222, normalized size = 3.13

$$-\frac{\left(b^2er\log(c)^2 - abdnp - 2\,abdn + a^2er - \left(b^2en^2p + b^2en^2\right)r\log(x)^2 - \left(b^2dnp + 2\,b^2dn - 2\,aber\right)\log(c) - \left(abenp\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x,x, algorithm="fricas")
```

```
[Out] -(b^2*e*r*log(c)^2 - a*b*d*n*p - 2*a*b*d*n + a^2*e*r - (b^2*e*n^2*p + b^2*e
*n^2)*r*log(x)^2 - (b^2*d*n*p + 2*b^2*d*n - 2*a*b*e*r)*log(c) - (a*b*e*n*p
+ 2*a*b*e*n + (b^2*e*n*p + 2*b^2*e*n)*log(c))*log(f) - (b^2*e*n*p*r*log(c)
+ b^2*d*n^2*p + a*b*e*n*p*r + 2*b^2*d*n^2 + (b^2*e*n^2*p + 2*b^2*e*n^2)*log
(f))*log(x))*(b*n*log(x) + b*log(c) + a)^p/(b^2*n^2*p^2 + 3*b^2*n^2*p + 2*b
^2*n^2)
```

giac [B] time = 0.40, size = 246, normalized size = 3.46

$$\frac{\frac{(bn\log(x)+b\log(c)+a)^{p+1}e\log(f)}{p+1} + \frac{(bn\log(x)+b\log(c)+a)^{p+1}d}{p+1} - \frac{\left((bn\log(x)+b\log(c)+a)(bn\log(x)+b\log(c)+a)^p bp\log(c) - (bn\log(x)+b\log(c)+a\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x,x, algorithm="giac")
```

```
[Out] ((b*n*log(x) + b*log(c) + a)^(p + 1)*e*log(f)/(p + 1) + (b*n*log(x) + b*log
(c) + a)^(p + 1)*d/(p + 1) - ((b*n*log(x) + b*log(c) + a)*(b*n*log(x) + b*l
og(c) + a)^p*b*p*log(c) - (b*n*log(x) + b*log(c) + a)^2*(b*n*log(x) + b*log
(c) + a)^p*p + (b*n*log(x) + b*log(c) + a)*(b*n*log(x) + b*log(c) + a)^p*a*
p + 2*(b*n*log(x) + b*log(c) + a)*(b*n*log(x) + b*log(c) + a)^p*b*log(c) -
(b*n*log(x) + b*log(c) + a)^2*(b*n*log(x) + b*log(c) + a)^p + 2*(b*n*log(x)
```

```
 + b*log(c) + a)*(b*n*log(x) + b*log(c) + a)^p*a)*r*e/((p^2 + 3*p + 2)*b*n)
)/(b*n)
```

maple [C] time = 1.19, size = 854, normalized size = 12.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d)/x,x)
```

```
[Out] -1/2*I*(b*n*ln(x)+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*
c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))+a)^(p+1)/b/n/(p+1)*e*Pi*csg
n(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+1/2*I*(b*n*ln(x)+b*(ln(c)+ln(x^n)-n*ln(x)-
1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^
n)))+a)^(p+1)/b/n/(p+1)*e*Pi*csgn(I*f)*csgn(I*f*x^r)^2+1/2*I*(b*n*ln(x)+b*(
ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-c
sgn(I*c*x^n)+csgn(I*x^n)))+a)^(p+1)/b/n/(p+1)*e*Pi*csgn(I*x^r)*csgn(I*f*x^r
)^2-1/2*I*(b*n*ln(x)+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn
(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))+a)^(p+1)/b/n/(p+1)*e*Pi*
csgn(I*f*x^r)^3+(b*n*ln(x)+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*
(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))+a)^(p+1)/b/n/(p+1)
*e*r*ln(x)+(b*n*ln(x)+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-csg
n(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))+a)^(p+1)/b/n/(p+1)*e*ln
(f)+(b*n*ln(x)+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))+a)^(p+1)/b/n/(p+1)*e*(ln(x^r)-
r*ln(x))+(b*n*ln(x)+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(
I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))+a)^(p+1)/b/n/(p+1)*d-1/b^
2/n^2/(p+1)*e*r*(b*n*ln(x)+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*
(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))+a)^(p+2)/(p+2)
```

maxima [A] time = 0.66, size = 95, normalized size = 1.34

$$\frac{\left(b\log\left(cx^{n}\right)+a\right)^{p+1}e\log\left(fx^{r}\right)}{bn\left(p+1\right)}+\frac{\left(b\log\left(cx^{n}\right)+a\right)^{p+1}d}{bn\left(p+1\right)}-\frac{\left(b\log\left(cx^{n}\right)+a\right)^{p+2}er}{b^{2}n^{2}\left(p+2\right)\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x,x, algorithm="maxima")
```

```
[Out] (b*log(c*x^n) + a)^(p + 1)*e*log(f*x^r)/(b*n*(p + 1)) + (b*log(c*x^n) + a)^
(p + 1)*d/(b*n*(p + 1)) - (b*log(c*x^n) + a)^(p + 2)*e*r/(b^2*n^2*(p + 2)*(
p + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(d+e\,\ln\left(f\,x^{r}\right)\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x,x)
```

```
[Out] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\log\left(cx^{n}\right)\right)^{p}\left(d+e\log\left(fx^{r}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r))/x, x)
```

3.183 $\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^2}\,dx$

Optimal. Leaf size=260

$$-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}-\frac{ere^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\left(a+b\log\left(cx^n\right)\right)}{}$$

```
[Out] -e*exp(a/b/n)*r*(c*x^n)^(1/n)*GAMMA(2+p,a/b/n+ln(c*x^n)/n)*(a+b*ln(c*x^n))^
p/x/(((a+b*ln(c*x^n))/b/n)^p)+e*exp(a/b/n)*r*(c*x^n)^(1/n)*GAMMA(1+p,a/b/n+
ln(c*x^n)/n)*(a+b*ln(c*x^n))^(1+p)/b/n/x/(((a+b*ln(c*x^n))/b/n)^p)-exp(a/b/
n)*(c*x^n)^(1/n)*GAMMA(1+p,(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f
*x^r))/x/(((a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A] time = 0.23, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2310, 2181, 2366, 12, 15, 19, 6557}

$$-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}-\frac{ere^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\left(a+b\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] -((e*E^(a/(b*n))*r*(c*x^n)^n^(-1)*Gamma[2 + p, a/(b*n) + Log[c*x^n]/n]*(a +
 b*Log[c*x^n])^p)/(x*((a + b*Log[c*x^n])/(b*n))^p)) + (e*E^(a/(b*n))*r*(c*x
^n)^n^(-1)*Gamma[1 + p, a/(b*n) + Log[c*x^n]/n]*(a + b*Log[c*x^n])^(1 + p))
/(b*n*x*((a + b*Log[c*x^n])/(b*n))^p) - (E^(a/(b*n))*(c*x^n)^n^(-1)*Gamma[1
 + p, (a + b*Log[c*x^n])/(b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(x
*((a + b*Log[c*x^n])/(b*n))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6557

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Gamma[n, a
+ b*x])/b, x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^p\left(d+e\log\left(fx^r\right)\right)}{x^2}\,dx = -\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\right.}{x}$$

$$= -\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\right.}{x}$$

$$= -\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\right.}{x}$$

$$= -\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\right.}{x}$$

$$= -\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\right.}{x}$$

$$= -\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\left(\right.}{x}$$

$$= -\frac{ee^{\frac{a}{bn}}r\left(cx^n\right)^{\frac{1}{n}}\Gamma\left(2+p,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right.}{x}$$

Mathematica [A] time = 0.32, size = 141, normalized size = 0.54

$$-\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\left(a+b\log\left(cx^n\right)\right)^{p-1}\left(\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)\left(-aer-ber\log\left(cx^n\right)+bdn+ben\log\left(f\right.\right.\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] -((E^(a/(b*n))*(c*x^n)^n^(-1)*(a + b*Log[c*x^n])^(-1 + p)*((a + b*Log[c*x^n
])/(b*n))^(1 - p)*(b*e*n*r*Gamma[2 + p, (a + b*Log[c*x^n])/(b*n)] + Gamma[1
 + p, (a + b*Log[c*x^n])/(b*n)]*(b*d*n - a*e*r - b*e*r*Log[c*x^n] + b*e*n*L
og[f*x^r])))/x)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e\log\left(fx^r\right)+d\right)\left(b\log\left(cx^n\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^2,x, algorithm="fricas")
[Out] integral((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\log\left(fx^r\right)+d\right)\left(b\log\left(cx^n\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^2,x, algorithm="giac")
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^2, x)
```

maple [F] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\left(e\ln\left(f\,x^r\right)+d\right)\left(b\ln\left(c\,x^n\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d)/x^2,x)
[Out] int((b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d)/x^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d+e\,\ln\left(f\,x^r\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^2,x)
[Out] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^p\left(d+e\log\left(fx^r\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x**2,x)
[Out] Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r))/x**2, x)
```

3.184 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^3}\,dx$

Optimal. Leaf size=295

$$-\frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}-\frac{e2^{-p-2}re^{\frac{2a}{bn}}(cx^n)^{2}\cdots}{\cdots}$$

```
[Out] -2^(-2-p)*e*exp(2*a/b/n)*r*(c*x^n)^(2/n)*GAMMA(2+p,2*a/b/n+2*ln(c*x^n)/n)*(
a+b*ln(c*x^n))^p/x^2/(((a+b*ln(c*x^n))/b/n)^p)+2^(-1-p)*e*exp(2*a/b/n)*r*(c
*x^n)^(2/n)*GAMMA(1+p,2*a/b/n+2*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(1+p)/b/n/x^2/
(((a+b*ln(c*x^n))/b/n)^p)-2^(-1-p)*exp(2*a/b/n)*(c*x^n)^(2/n)*GAMMA(1+p,2*(
a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^2/(((a+b*ln(c*x^n))
/b/n)^p)
```

Rubi [A] time = 0.23, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2310, 2181, 2366, 12, 15, 19, 6557}

$$-\frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}-\frac{e2^{-p-2}re^{\frac{2}{b\cdots}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -((2^(-2 - p)*e*E^((2*a)/(b*n))*r*(c*x^n)^(2/n)*Gamma[2 + p, (2*a)/(b*n) +
(2*Log[c*x^n])/n]*(a + b*Log[c*x^n])^p)/(x^2*((a + b*Log[c*x^n])/(b*n))^p))
 + (2^(-1 - p)*e*E^((2*a)/(b*n))*r*(c*x^n)^(2/n)*Gamma[1 + p, (2*a)/(b*n) +
 (2*Log[c*x^n])/n]*(a + b*Log[c*x^n])^(1 + p))/(b*n*x^2*((a + b*Log[c*x^n])
/(b*n))^p) - (2^(-1 - p)*E^((2*a)/(b*n))*(c*x^n)^(2/n)*Gamma[1 + p, (2*(a +
 b*Log[c*x^n]))/(b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(x^2*((a +
b*Log[c*x^n])/(b*n))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !I
ntegerQ[m] && !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6557

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Gamma[n, a
+ b*x])/b, x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^p\left(d+e\log\left(fx^r\right)\right)}{x^3}\,dx = -\frac{2^{-1-p}e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\Gamma\left(1+p,\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx}{bn}\right.}{x^2}$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\Gamma\left(1+p,\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx}{bn}\right.}{x^2}$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\Gamma\left(1+p,\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx}{bn}\right.}{x^2}$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\Gamma\left(1+p,\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx}{bn}\right.}{x^2}$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\Gamma\left(1+p,\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx}{bn}\right.}{x^2}$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\Gamma\left(1+p,\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx}{bn}\right.}{x^2}$$

$$= -\frac{2^{-2-p}ee^{\frac{2a}{bn}}r\left(cx^n\right)^{2/n}\Gamma\left(2+p,\frac{2a}{bn}+\frac{2\log\left(cx^n\right)}{n}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log}{bn}\right.}{x^2}$$

Mathematica [A] time = 0.38, size = 154, normalized size = 0.52

$$-\frac{2^{-p-2}e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\left(a+b\log\left(cx^n\right)\right)^{p-1}\left(\frac{a+b\log\left(cx^n\right)}{bn}\right)^{1-p}\left(2\Gamma\left(p+1,\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(-aer-ber\log\left(cx^n\right)+bdn+ben\right.}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -((2^(-2 - p)*E^((2*a)/(b*n))*(c*x^n)^(2/n)*(a + b*Log[c*x^n])^(-1 + p)*((a
 + b*Log[c*x^n])/(b*n))^(1 - p)*(b*e*n*r*Gamma[2 + p, (2*(a + b*Log[c*x^n])
```

```
)/(b*n)] + 2*Gamma[1 + p, (2*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r - b*
e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/x^2)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e\log\left(fx^{r}\right)+d\right)\left(b\log\left(cx^{n}\right)+a\right)^{p}}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^3,x, algorithm="fricas")
```

```
[Out] integral((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(e\log\left(fx^{r}\right)+d\right)\left(b\log\left(cx^{n}\right)+a\right)^{p}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^3, x)
```

maple [F] time = 0.94, size = 0, normalized size = 0.00

$$\int\frac{\left(e\ln\left(f\,x^{r}\right)+d\right)\left(b\ln\left(c\,x^{n}\right)+a\right)^{p}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d)/x^3,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(d+e\,\ln\left(f\,x^{r}\right)\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{p}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^3,x)
```

```
[Out] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x**3,x)
```

```
[Out] Timed out
```

3.185 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx$

Optimal. Leaf size=295

$$-\frac{3^{-p-1}e^{\frac{3a}{bn}}(cx^n)^{3/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}-\frac{e3^{-p-2}re^{\frac{3a}{bn}}(cx^n)^{3/n}\left(\right.}{}$$

```
[Out] -3^(-2-p)*e*exp(3*a/b/n)*r*(c*x^n)^(3/n)*GAMMA(2+p,3*a/b/n+3*ln(c*x^n)/n)*(
a+b*ln(c*x^n))^p/x^3/(((a+b*ln(c*x^n))/b/n)^p)+3^(-1-p)*e*exp(3*a/b/n)*r*(c
*x^n)^(3/n)*GAMMA(1+p,3*a/b/n+3*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(1+p)/b/n/x^3/
(((a+b*ln(c*x^n))/b/n)^p)-3^(-1-p)*exp(3*a/b/n)*(c*x^n)^(3/n)*GAMMA(1+p,3*(
a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^3/(((a+b*ln(c*x^n))
/b/n)^p)
```

Rubi [A] time = 0.24, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2310, 2181, 2366, 12, 15, 19, 6557}

$$-\frac{3^{-p-1}e^{\frac{3a}{bn}}(cx^n)^{3/n}\left(d+e\log\left(fx^r\right)\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}-\frac{e3^{-p-2}re^{\frac{3a}{bn}}(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] -((3^(-2 - p)*e*E^((3*a)/(b*n))*r*(c*x^n)^(3/n)*Gamma[2 + p, (3*a)/(b*n) +
(3*Log[c*x^n])/n]*(a + b*Log[c*x^n])^p)/(x^3*((a + b*Log[c*x^n])/(b*n))^p))
 + (3^(-1 - p)*e*E^((3*a)/(b*n))*r*(c*x^n)^(3/n)*Gamma[1 + p, (3*a)/(b*n) +
 (3*Log[c*x^n])/n]*(a + b*Log[c*x^n])^(1 + p))/(b*n*x^3*((a + b*Log[c*x^n])
/(b*n))^p) - (3^(-1 - p)*E^((3*a)/(b*n))*(c*x^n)^(3/n)*Gamma[1 + p, (3*(a +
 b*Log[c*x^n]))/(b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(x^3*((a +
b*Log[c*x^n])/(b*n))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2366

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6557

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Gamma[n, a
+ b*x])/b, x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^p\left(d+e\log\left(fx^r\right)\right)}{x^4}\,dx = -\frac{3^{-1-p}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\Gamma\left(1+p,\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log}{b}\right.}{x^3}$$

$$= -\frac{3^{-1-p}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\Gamma\left(1+p,\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log}{b}\right.}{x^3}$$

$$= -\frac{3^{-1-p}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\Gamma\left(1+p,\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log}{b}\right.}{x^3}$$

$$= -\frac{3^{-1-p}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\Gamma\left(1+p,\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log}{b}\right.}{x^3}$$

$$= -\frac{3^{-1-p}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\Gamma\left(1+p,\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log}{b}\right.}{x^3}$$

$$= -\frac{3^{-1-p}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\Gamma\left(1+p,\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+b\log}{b}\right.}{x^3}$$

$$= -\frac{3^{-2-p}ee^{\frac{3a}{bn}}r\left(cx^n\right)^{3/n}\Gamma\left(2+p,\frac{3a}{bn}+\frac{3\log\left(cx^n\right)}{n}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(\frac{a+}{}\right.}{x^3}$$

Mathematica [A] time = 0.38, size = 154, normalized size = 0.52

$$-\frac{3^{-p-2}e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\left(a+b\log\left(cx^n\right)\right)^{p-1}\left(\frac{a+b\log\left(cx^n\right)}{bn}\right)^{1-p}\left(3\Gamma\left(p+1,\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)\left(-aer-ber\log\left(cx^n\right)+bdn+\right.\right.}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] -((3^(-2 - p)*E^((3*a)/(b*n))*(c*x^n)^(3/n)*(a + b*Log[c*x^n])^(-1 + p)*((a
 + b*Log[c*x^n])/(b*n))^(1 - p)*(b*e*n*r*Gamma[2 + p, (3*(a + b*Log[c*x^n])
```

```
)/(b*n)] + 3*Gamma[1 + p, (3*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r - b*
e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/x^3)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e\log\left(fx^{r}\right)+d\right)\left(b\log\left(cx^{n}\right)+a\right)^{p}}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^4,x, algorithm="fricas")
[Out] integral((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(e\log\left(fx^{r}\right)+d\right)\left(b\log\left(cx^{n}\right)+a\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^4,x, algorithm="giac")
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^4, x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int\frac{\left(e\ln\left(f\,x^{r}\right)+d\right)\left(b\ln\left(c\,x^{n}\right)+a\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d)/x^4,x)
[Out] int((b*ln(c*x^n)+a)^p*(e*ln(f*x^r)+d)/x^4,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(d+e\,\ln\left(f\,x^{r}\right)\right)\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^4,x)
[Out] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x**4,x)
[Out] Timed out
```

3.186 $\int \left(d+ex^2\right) \sin^{-1}(ax) \log \left(cx^n\right) \, dx$

Optimal. Leaf size=246

$$-\frac{dn\sqrt{1-a^2x^2}}{a}+\frac{\sqrt{1-a^2x^2}\left(3a^2d+e\right)\log\left(cx^n\right)}{3a^3}-\frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3}-\frac{n\sqrt{1-a^2x^2}\left(3a^2d+e\right)}{3a^3}+\frac{n\left(3a^2d+\right.}{}$$

```
[Out] 2/27*e*n*(-a^2*x^2+1)^(3/2)/a^3-d*n*x*arcsin(a*x)-1/9*e*n*x^3*arcsin(a*x)-1
/9*e*n*arctanh((-a^2*x^2+1)^(1/2))/a^3+1/3*(3*a^2*d+e)*n*arctanh((-a^2*x^2+
1)^(1/2))/a^3-1/9*e*(-a^2*x^2+1)^(3/2)*ln(c*x^n)/a^3+d*x*arcsin(a*x)*ln(c*x
^n)+1/3*e*x^3*arcsin(a*x)*ln(c*x^n)-d*n*(-a^2*x^2+1)^(1/2)/a-1/3*(3*a^2*d+e
)*n*(-a^2*x^2+1)^(1/2)/a^3+1/3*(3*a^2*d+e)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.23, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {4665, 444, 43, 2387, 266, 50, 63, 208, 4619, 261, 4627}

$$\frac{\sqrt{1-a^2x^2}\left(3a^2d+e\right)\log\left(cx^n\right)}{3a^3}-\frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3}-\frac{n\sqrt{1-a^2x^2}\left(3a^2d+e\right)}{3a^3}+\frac{n\left(3a^2d+e\right)\tanh^{-1}\left(\sqrt{1-\right.}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcSin[a*x]*Log[c*x^n],x]
```

```
[Out] -((d*n*Sqrt[1 - a^2*x^2])/a) - ((3*a^2*d + e)*n*Sqrt[1 - a^2*x^2])/(3*a^3)
+ (2*e*n*(1 - a^2*x^2)^(3/2))/(27*a^3) - d*n*x*ArcSin[a*x] - (e*n*x^3*ArcSi
n[a*x])/9 - (e*n*ArcTanh[Sqrt[1 - a^2*x^2]])/(9*a^3) + ((3*a^2*d + e)*n*Arc
Tanh[Sqrt[1 - a^2*x^2]])/(3*a^3) + ((3*a^2*d + e)*Sqrt[1 - a^2*x^2]*Log[c*x
^n])/(3*a^3) - (e*(1 - a^2*x^2)^(3/2)*Log[c*x^n])/(9*a^3) + d*x*ArcSin[a*x]
*Log[c*x^n] + (e*x^3*ArcSin[a*x]*Log[c*x^n])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4665

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSin[c*x], u, x] -
 Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0]
)
```

Rubi steps

$$\int \left(d+ex^2\right)\sin^{-1}(ax)\log\left(cx^n\right)\,dx = \frac{\left(3a^2d+e\right)\sqrt{1-a^2x^2}\,\log\left(cx^n\right)}{3a^3} - \frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3} + dx\sin^{-1}(ax$$

$$= \frac{\left(3a^2d+e\right)\sqrt{1-a^2x^2}\,\log\left(cx^n\right)}{3a^3} - \frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3} + dx\sin^{-1}(ax$$

$$= -dnx\sin^{-1}(ax) - \frac{1}{9}enx^3\sin^{-1}(ax) + \frac{\left(3a^2d+e\right)\sqrt{1-a^2x^2}\,\log\left(cx^n\right)}{3a^3} - \frac{e}{}$$

$$= -\frac{dn\sqrt{1-a^2x^2}}{a} - \frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3} + \frac{en\left(1-a^2x^2\right)^{3/2}}{27a^3} - dnx\sin^{-1}$$

$$= -\frac{dn\sqrt{1-a^2x^2}}{a} + \frac{en\sqrt{1-a^2x^2}}{9a^3} - \frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3} + \frac{en\left(1-a^2x^2\right.}{27a^3}$$

$$= -\frac{dn\sqrt{1-a^2x^2}}{a} - \frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3} + \frac{2en\left(1-a^2x^2\right)^{3/2}}{27a^3} - dnx\sin^{-}$$

$$= -\frac{dn\sqrt{1-a^2x^2}}{a} - \frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3} + \frac{2en\left(1-a^2x^2\right)^{3/2}}{27a^3} - dnx\sin^{-}$$

Mathematica [A] time = 0.18, size = 248, normalized size = 1.01

$$-3a^3x\sin^{-1}(ax)\left(n\left(9d+ex^2\right)-3\left(3d+ex^2\right)\log\left(cx^n\right)\right)+27a^2d\sqrt{1-a^2x^2}\,\log\left(cx^n\right)+3a^2ex^2\sqrt{1-a^2x^2}\,\log(c$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcSin[a*x]*Log[c*x^n],x]
```

```
[Out] (-54*a^2*d*n*Sqrt[1 - a^2*x^2] - 7*e*n*Sqrt[1 - a^2*x^2] - 2*a^2*e*n*x^2*Sq
rt[1 - a^2*x^2] - 3*(9*a^2*d + 2*e)*n*Log[x] + 27*a^2*d*Sqrt[1 - a^2*x^2]*L
og[c*x^n] + 6*e*Sqrt[1 - a^2*x^2]*Log[c*x^n] + 3*a^2*e*x^2*Sqrt[1 - a^2*x^2
]*Log[c*x^n] - 3*a^3*x*ArcSin[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log[c
*x^n]) + 27*a^2*d*n*Log[1 + Sqrt[1 - a^2*x^2]] + 6*e*n*Log[1 + Sqrt[1 - a^2
*x^2]])/(27*a^3)
```

fricas [A] time = 1.27, size = 221, normalized size = 0.90

$$18\left(a^3ex^3+3a^3dx\right)\arcsin(ax)\log(c)+18\left(a^3enx^3+3a^3dnx\right)\arcsin(ax)\log(x)+3\left(9a^2d+2e\right)n\log\left(\sqrt{-a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] 1/54*(18*(a^3*e*x^3 + 3*a^3*d*x)*arcsin(a*x)*log(c) + 18*(a^3*e*n*x^3 + 3*a
^3*d*n*x)*arcsin(a*x)*log(x) + 3*(9*a^2*d + 2*e)*n*log(sqrt(-a^2*x^2 + 1) +
 1) - 3*(9*a^2*d + 2*e)*n*log(sqrt(-a^2*x^2 + 1) - 1) - 6*(a^3*e*n*x^3 + 9*
a^3*d*n*x)*arcsin(a*x) - 2*(2*a^2*e*n*x^2 + (54*a^2*d + 7*e)*n - 3*(a^2*e*x
^2 + 9*a^2*d + 2*e)*log(c) - 3*(a^2*e*n*x^2 + (9*a^2*d + 2*e)*n)*log(x))*sq
rt(-a^2*x^2 + 1))/a^3
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)*log(c*x^n),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 8.33, size = 6894, normalized size = 28.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arcsin(a*x)*ln(c*x^n),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)*log(c*x^n),x, algorithm="maxima")
[Out] -1/54*(-I*(27*a^2*d*n*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3) + a^2
*e*n*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5 + 3*log(a*x - 1)/a^5) - 16
2*a^2*e*n*integrate(1/9*x^4*log(x)/(a^2*x^2 - 1), x) - 486*a^2*d*n*integrat
e(1/9*x^2*log(x)/(a^2*x^2 - 1), x) - 27*a^2*d*(2*x/a^2 - log(a*x + 1)/a^3 +
 log(a*x - 1)/a^3)*log(c) - 3*a^2*e*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)
/a^5 + 3*log(a*x - 1)/a^5)*log(c))*a^3 + (4*I*a^3*e*n - 6*I*a^3*e*log(c))*x
^3 - 54*a^3*integrate(-1/9*((a*e*n - 3*a*e*log(c))*x^3 + 9*(a*d*n - a*d*log
(c))*x - 3*(a*e*x^3 + 3*a*d*x)*log(x^n))*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^2*
x^2 - 1), x) + (-27*I*a^2*d - 9*I*e)*n*dilog(a*x) + (27*I*a^2*d + 9*I*e)*n*
dilog(-a*x) + (-54*I*a^3*d*log(c) - 18*I*a*e*log(c) + (108*I*a^3*d + 24*I*a
*e)*n)*x + 6*((a^3*e*n - 3*a^3*e*log(c))*x^3 + 9*(a^3*d*n - a^3*d*log(c))*x
)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1)) + (27*I*a^2*d*log(c) + (-27*I*
a^2*d - 3*I*e)*n + 9*I*e*log(c))*log(a*x + 1) + (-27*I*a^2*d*log(c) + (27*I
*a^2*d + 3*I*e)*n - 9*I*e*log(c))*log(a*x - 1) + (-6*I*a^3*e*x^3 + (-54*I*a
^3*d - 18*I*a*e)*x - 18*(a^3*e*x^3 + 3*a^3*d*x)*arctan2(a*x, sqrt(a*x + 1)*
sqrt(-a*x + 1)) + (27*I*a^2*d + 9*I*e)*log(a*x + 1) + (-27*I*a^2*d - 9*I*e)
*log(-a*x + 1))*log(x^n))/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,\mathrm{asin}\left(a\,x\right)\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*asin(a*x)*(d + e*x^2),x)
[Out] int(log(c*x^n)*asin(a*x)*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + e x^{2}\right) \log{\left(c x^{n} \right)} \operatorname{asin}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*asin(a*x)*ln(c*x**n),x)
[Out] Integral((d + e*x**2)*log(c*x**n)*asin(a*x), x)
```

3.187 $\int \left(d+ex^2\right)\cos^{-1}(ax)\log\left(cx^n\right)\,dx$

Optimal. Leaf size=245

$$\frac{dn\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}\,\left(3a^2d+e\right)\log\left(cx^n\right)}{3a^3}+\frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3}+\frac{n\sqrt{1-a^2x^2}\,\left(3a^2d+e\right)}{3a^3}-\frac{n\left(3a^2d+e\right.}{}$$

```
[Out] -2/27*e*n*(-a^2*x^2+1)^(3/2)/a^3-d*n*x*arccos(a*x)-1/9*e*n*x^3*arccos(a*x)+
1/9*e*n*arctanh((-a^2*x^2+1)^(1/2))/a^3-1/3*(3*a^2*d+e)*n*arctanh((-a^2*x^2
+1)^(1/2))/a^3+1/9*e*(-a^2*x^2+1)^(3/2)*ln(c*x^n)/a^3+d*x*arccos(a*x)*ln(c*
x^n)+1/3*e*x^3*arccos(a*x)*ln(c*x^n)+d*n*(-a^2*x^2+1)^(1/2)/a+1/3*(3*a^2*d+
e)*n*(-a^2*x^2+1)^(1/2)/a^3-1/3*(3*a^2*d+e)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)/a^
3
```

Rubi [A] time = 0.23, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {4666, 444, 43, 2387, 266, 50, 63, 208, 4620, 261, 4628}

$$-\frac{\sqrt{1-a^2x^2}\,\left(3a^2d+e\right)\log\left(cx^n\right)}{3a^3}+\frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3}+\frac{n\sqrt{1-a^2x^2}\,\left(3a^2d+e\right)}{3a^3}-\frac{n\left(3a^2d+e\right)\tanh^{-1}\left(\sqrt{1\right.}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCos[a*x]*Log[c*x^n],x]
```

```
[Out] (d*n*Sqrt[1 - a^2*x^2])/a + ((3*a^2*d + e)*n*Sqrt[1 - a^2*x^2])/(3*a^3) - (
2*e*n*(1 - a^2*x^2)^(3/2))/(27*a^3) - d*n*x*ArcCos[a*x] - (e*n*x^3*ArcCos[a
*x])/9 + (e*n*ArcTanh[Sqrt[1 - a^2*x^2]])/(9*a^3) - ((3*a^2*d + e)*n*ArcTan
h[Sqrt[1 - a^2*x^2]])/(3*a^3) - ((3*a^2*d + e)*Sqrt[1 - a^2*x^2]*Log[c*x^n]
)/(3*a^3) + (e*(1 - a^2*x^2)^(3/2)*Log[c*x^n])/(9*a^3) + d*x*ArcCos[a*x]*Lo
g[c*x^n] + (e*x^3*ArcCos[a*x]*Log[c*x^n])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4666

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[c*x], u, x] +
 Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0]
)
```

Rubi steps

$$\int \left(d+ex^2\right)\cos^{-1}(ax)\log\left(cx^n\right)\,dx = -\frac{\left(3a^2d+e\right)\sqrt{1-a^2x^2}\,\log\left(cx^n\right)}{3a^3}+\frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3}+dx\cos^{-1}(\ldots$$

$$= -\frac{\left(3a^2d+e\right)\sqrt{1-a^2x^2}\,\log\left(cx^n\right)}{3a^3}+\frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3}+dx\cos^{-1}(\ldots$$

$$= -dnx\cos^{-1}(ax)-\frac{1}{9}enx^3\cos^{-1}(ax)-\frac{\left(3a^2d+e\right)\sqrt{1-a^2x^2}\,\log\left(cx^n\right)}{3a^3}+\ldots$$

$$= \frac{dn\sqrt{1-a^2x^2}}{a}+\frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3}-\frac{en\left(1-a^2x^2\right)^{3/2}}{27a^3}-dnx\cos^{-1}(\ldots$$

$$= \frac{dn\sqrt{1-a^2x^2}}{a}-\frac{en\sqrt{1-a^2x^2}}{9a^3}+\frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3}-\frac{en\left(1-a^2x^2\right)^{\ldots}}{27a^3}\ldots$$

$$= \frac{dn\sqrt{1-a^2x^2}}{a}+\frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3}-\frac{2en\left(1-a^2x^2\right)^{3/2}}{27a^3}-dnx\cos^{-1}\ldots$$

$$= \frac{dn\sqrt{1-a^2x^2}}{a}+\frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3}-\frac{2en\left(1-a^2x^2\right)^{3/2}}{27a^3}-dnx\cos^{-1}\ldots$$

Mathematica [A] time = 0.19, size = 248, normalized size = 1.01

$$-\frac{3a^3x\cos^{-1}(ax)\left(n\left(9d+ex^2\right)-3\left(3d+ex^2\right)\log\left(cx^n\right)\right)+27a^2d\sqrt{1-a^2x^2}\,\log\left(cx^n\right)+3a^2ex^2\sqrt{1-a^2x^2}\,\log(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcCos[a*x]*Log[c*x^n],x]
```

```
[Out] -1/27*(-54*a^2*d*n*Sqrt[1 - a^2*x^2] - 7*e*n*Sqrt[1 - a^2*x^2] - 2*a^2*e*n*
x^2*Sqrt[1 - a^2*x^2] - 3*(9*a^2*d + 2*e)*n*Log[x] + 27*a^2*d*Sqrt[1 - a^2*
x^2]*Log[c*x^n] + 6*e*Sqrt[1 - a^2*x^2]*Log[c*x^n] + 3*a^2*e*x^2*Sqrt[1 - a
^2*x^2]*Log[c*x^n] + 3*a^3*x*ArcCos[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)
*Log[c*x^n]) + 27*a^2*d*n*Log[1 + Sqrt[1 - a^2*x^2]] + 6*e*n*Log[1 + Sqrt[1
 - a^2*x^2]])/a^3
```

fricas [A] time = 1.34, size = 308, normalized size = 1.26

$$\frac{18\left(a^3ex^3+3\,a^3dx-3\,a^3d-a^3e\right)\arccos(ax)\log(c)+18\left(a^3enx^3+3\,a^3dnx\right)\arccos(ax)\log(x)-3\left(9\,a^2d+2\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] 1/54*(18*(a^3*e*x^3 + 3*a^3*d*x - 3*a^3*d - a^3*e)*arccos(a*x)*log(c) + 18*
(a^3*e*n*x^3 + 3*a^3*d*n*x)*arccos(a*x)*log(x) - 3*(9*a^2*d + 2*e)*n*log(sq
rt(-a^2*x^2 + 1) + 1) + 3*(9*a^2*d + 2*e)*n*log(sqrt(-a^2*x^2 + 1) - 1) - 6
*(a^3*e*n*x^3 + 9*a^3*d*n*x - (9*a^3*d + a^3*e)*n)*arccos(a*x) - 6*((9*a^3*
d + a^3*e)*n - 3*(3*a^3*d + a^3*e)*log(c))*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a
```

```
^2*x^2 - 1)) + 2*(2*a^2*e*n*x^2 + (54*a^2*d + 7*e)*n - 3*(a^2*e*x^2 + 9*a^2
*d + 2*e)*log(c) - 3*(a^2*e*n*x^2 + (9*a^2*d + 2*e)*n)*log(x))*sqrt(-a^2*x^
2 + 1))/a^3
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 11.06, size = 5618, normalized size = 22.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccos(a*x)*ln(c*x^n),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/54*(-I*(27*a^2*d*n*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3) + a^2
*e*n*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5 + 3*log(a*x - 1)/a^5) - 16
2*a^2*e*n*integrate(1/9*x^4*log(x)/(a^2*x^2 - 1), x) - 486*a^2*d*n*integrat
e(1/9*x^2*log(x)/(a^2*x^2 - 1), x) - 27*a^2*d*(2*x/a^2 - log(a*x + 1)/a^3 +
 log(a*x - 1)/a^3)*log(c) - 3*a^2*e*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)
/a^5 + 3*log(a*x - 1)/a^5)*log(c))*a^3 + (4*I*a^3*e*n - 6*I*a^3*e*log(c))*x
^3 + 54*a^3*integrate(-1/9*((a*e*n - 3*a*e*log(c))*x^3 + 9*(a*d*n - a*d*log
(c))*x - 3*(a*e*x^3 + 3*a*d*x)*log(x^n))*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^2*
x^2 - 1), x) + (-27*I*a^2*d - 9*I*e)*n*dilog(a*x) + (27*I*a^2*d + 9*I*e)*n*
dilog(-a*x) + (-54*I*a^3*d*log(c) - 18*I*a*e*log(c) + (108*I*a^3*d + 24*I*a
*e)*n)*x + 6*((a^3*e*n - 3*a^3*e*log(c))*x^3 + 9*(a^3*d*n - a^3*d*log(c))*x
)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x) + (27*I*a^2*d*log(c) + (-27*I*
a^2*d - 3*I*e)*n + 9*I*e*log(c))*log(a*x + 1) + (-27*I*a^2*d*log(c) + (27*I
*a^2*d + 3*I*e)*n - 9*I*e*log(c))*log(a*x - 1) + (-6*I*a^3*e*x^3 + (-54*I*a
^3*d - 18*I*a*e)*x - 18*(a^3*e*x^3 + 3*a^3*d*x)*arctan2(sqrt(a*x + 1)*sqrt(
-a*x + 1), a*x) + (27*I*a^2*d + 9*I*e)*log(a*x + 1) + (-27*I*a^2*d - 9*I*e)
*log(-a*x + 1))*log(x^n))/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,\mathrm{acos}\left(a\,x\right)\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acos(a*x)*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*acos(a*x)*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + e x^{2}\right) \log{\left(c x^{n} \right)} \operatorname{acos}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acos(a*x)*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*acos(a*x), x)
```

3.188 $\int \left(d+ex^2\right)\tan^{-1}(ax)\log\left(cx^n\right)\,dx$

Optimal. Leaf size=182

$$\frac{dn\log\left(a^2x^2+1\right)}{2a}-\frac{\left(3a^2d-e\right)\log\left(a^2x^2+1\right)\log\left(cx^n\right)}{6a^3}-\frac{n\left(3a^2d-e\right)\text{Li}_2\left(-a^2x^2\right)}{12a^3}-\frac{en\log\left(a^2x^2+1\right)}{18a^3}+dx\tan^{-1}(ax$$

```
[Out] 5/36*e*n*x^2/a-d*n*x*arctan(a*x)-1/9*e*n*x^3*arctan(a*x)-1/6*e*x^2*ln(c*x^n
)/a+d*x*arctan(a*x)*ln(c*x^n)+1/3*e*x^3*arctan(a*x)*ln(c*x^n)+1/2*d*n*ln(a^
2*x^2+1)/a-1/18*e*n*ln(a^2*x^2+1)/a^3-1/6*(3*a^2*d-e)*ln(c*x^n)*ln(a^2*x^2+
1)/a^3-1/12*(3*a^2*d-e)*n*polylog(2,-a^2*x^2)/a^3
```

Rubi [A] time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4912, 1593, 444, 43, 2388, 4846, 260, 4852, 266, 2391}

$$-\frac{n\left(3a^2d-e\right)\text{PolyLog}\left(2,-a^2x^2\right)}{12a^3}-\frac{\left(3a^2d-e\right)\log\left(a^2x^2+1\right)\log\left(cx^n\right)}{6a^3}+\frac{dn\log\left(a^2x^2+1\right)}{2a}-\frac{en\log\left(a^2x^2+1\right)}{18a^3}+dx$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcTan[a*x]*Log[c*x^n],x]
```

```
[Out] (5*e*n*x^2)/(36*a) - d*n*x*ArcTan[a*x] - (e*n*x^3*ArcTan[a*x])/9 - (e*x^2*L
og[c*x^n])/(6*a) + d*x*ArcTan[a*x]*Log[c*x^n] + (e*x^3*ArcTan[a*x]*Log[c*x^
n])/3 + (d*n*Log[1 + a^2*x^2])/(2*a) - (e*n*Log[1 + a^2*x^2])/(18*a^3) - ((
3*a^2*d - e)*Log[c*x^n]*Log[1 + a^2*x^2])/(6*a^3) - ((3*a^2*d - e)*n*PolyLo
g[2, -(a^2*x^2)])/(12*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2388

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Dist[a + b*L
og[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c
, d, e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh
, ArcCoth}, F]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4912

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\tan^{-1}(ax)\log\left(cx^n\right)\,dx = -\frac{ex^2\log\left(cx^n\right)}{6a}+dx\tan^{-1}(ax)\log\left(cx^n\right)+\frac{1}{3}ex^3\tan^{-1}(ax)\log\left(cx^n\right)-\frac{(3 \cdots}{\cdots}$$

$$=\frac{enx^2}{12a}-\frac{ex^2\log\left(cx^n\right)}{6a}+dx\tan^{-1}(ax)\log\left(cx^n\right)+\frac{1}{3}ex^3\tan^{-1}(ax)\log\left(cx^n\cdots\right.$$

$$=\frac{enx^2}{12a}-dnx\tan^{-1}(ax)-\frac{1}{9}enx^3\tan^{-1}(ax)-\frac{ex^2\log\left(cx^n\right)}{6a}+dx\tan^{-1}(ax)\cdots$$

$$=\frac{enx^2}{12a}-dnx\tan^{-1}(ax)-\frac{1}{9}enx^3\tan^{-1}(ax)-\frac{ex^2\log\left(cx^n\right)}{6a}+dx\tan^{-1}(ax)\cdots$$

$$=\frac{enx^2}{12a}-dnx\tan^{-1}(ax)-\frac{1}{9}enx^3\tan^{-1}(ax)-\frac{ex^2\log\left(cx^n\right)}{6a}+dx\tan^{-1}(ax)\cdots$$

$$=\frac{5enx^2}{36a}-dnx\tan^{-1}(ax)-\frac{1}{9}enx^3\tan^{-1}(ax)-\frac{ex^2\log\left(cx^n\right)}{6a}+dx\tan^{-1}(a\cdots$$

Mathematica [A] time = 0.12, size = 165, normalized size = 0.91

$$\frac{-4a^3x\tan^{-1}(ax)\left(n\left(9d+ex^2\right)-3\left(3d+ex^2\right)\log\left(cx^n\right)\right)-18a^2d\log\left(a^2x^2+1\right)\log\left(cx^n\right)-6a^2ex^2\log\left(cx^n\right)+\cdots}{36\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcTan[a*x]*Log[c*x^n],x]
```

```
[Out] (5*a^2*e*n*x^2 - 6*a^2*e*x^2*Log[c*x^n] - 4*a^3*x*ArcTan[a*x]*(n*(9*d + e*x
^2) - 3*(3*d + e*x^2)*Log[c*x^n]) + 18*a^2*d*n*Log[1 + a^2*x^2] - 2*e*n*Log
[1 + a^2*x^2] - 18*a^2*d*Log[c*x^n]*Log[1 + a^2*x^2] + 6*e*Log[c*x^n]*Log[1
 + a^2*x^2] + 3*(-3*a^2*d + e)*n*PolyLog[2, -(a^2*x^2)])/(36*a^3)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ex^2+d\right)\arctan\left(ax\right)\log\left(cx^n\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctan(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*arctan(a*x)*log(c*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex^2+d\right)\arctan\left(ax\right)\log\left(cx^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctan(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*arctan(a*x)*log(c*x^n), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(e\,x^2+d\right)\arctan\left(ax\right)\ln\left(c\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arctan(a*x)*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arctan(a*x)*ln(c*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2ex^2\log(c)-6\,a^3\int\left(ex^2+d\right)\arctan\left(ax\right)\log\left(x^n\right)\,dx-2\left(a^3ex^3\log(c)+3\,a^3dx\log(c)\right)\arctan\left(ax\right)+\left(3\,a^2d\log\right.}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctan(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/6*(a^2*e*x^2*log(c) - 3*a^3*integrate(2*(e*x^2 + d)*arctan(a*x)*log(x^n)
, x) - 2*(a^3*e*x^3*log(c) + 3*a^3*d*x*log(c))*arctan(a*x) + (3*a^2*d*log(c
) - e*log(c))*log(a^2*x^2 + 1))/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\ln\left(c\,x^n\right)\,\mathrm{atan}\left(a\,x\right)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*atan(a*x)*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*atan(a*x)*(d + e*x^2), x)
```

sympy [A] time = 91.78, size = 221, normalized size = 1.21

$$-dn\left(\left\{\begin{array}{cc}0 & \text{for } a=0\\ \left\{\begin{array}{cc}x\,\mathrm{atan}\,(ax)-\frac{\log(a^2x^2+1)}{2a} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{array}\right.+\frac{\mathrm{Li}_2(a^2x^2e^{i\pi})}{4a} & \text{otherwise}\end{array}\right.\right)+d\left(\left\{\begin{array}{cc}0 & \text{for } a\\ x\,\mathrm{atan}\,(ax)-\frac{\log(a^2x^2+1)}{2a} & \text{othe}\end{array}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*atan(a*x)*ln(c*x**n),x)
```

```
[Out] -d*n*Piecewise((0, Eq(a, 0)), (Piecewise((x*atan(a*x) - log(a**2*x**2 + 1)/
(2*a), Ne(a, 0)), (0, True)) + polylog(2, a**2*x**2*exp_polar(I*pi))/(4*a),
 True)) + d*Piecewise((0, Eq(a, 0)), (x*atan(a*x) - log(a**2*x**2 + 1)/(2*a
), True))*log(c*x**n) - e*n*x**3*atan(a*x)/9 + e*x**3*log(c*x**n)*atan(a*x)
/3 + 5*e*n*x**2/(36*a) - e*n*Piecewise((x**2/2, Eq(a, 0)), (-polylog(2, a**
2*x**2*exp_polar(I*pi))/(2*a**2), True))/(6*a) - e*n*Piecewise((x**2, Eq(a*
*2, 0)), (log(a**2*x**2 + 1)/a**2, True))/(18*a) - e*x**2*log(c*x**n)/(6*a)
 + e*Piecewise((x**2, Eq(a**2, 0)), (log(a**2*x**2 + 1)/a**2, True))*log(c*
x**n)/(6*a)
```

3.189 $\int \left(d+ex^2\right)\cot^{-1}(ax)\log\left(cx^n\right)\,dx$

Optimal. Leaf size=182

$$-\frac{dn\log\left(a^2x^2+1\right)}{2a}+\frac{\left(3a^2d-e\right)\log\left(a^2x^2+1\right)\log\left(cx^n\right)}{6a^3}+\frac{n\left(3a^2d-e\right)\text{Li}_2\left(-a^2x^2\right)}{12a^3}+\frac{en\log\left(a^2x^2+1\right)}{18a^3}+dx\cot^{-1}(a$$

```
[Out] -5/36*e*n*x^2/a-d*n*x*arccot(a*x)-1/9*e*n*x^3*arccot(a*x)+1/6*e*x^2*ln(c*x^
n)/a+d*x*arccot(a*x)*ln(c*x^n)+1/3*e*x^3*arccot(a*x)*ln(c*x^n)-1/2*d*n*ln(a
^2*x^2+1)/a+1/18*e*n*ln(a^2*x^2+1)/a^3+1/6*(3*a^2*d-e)*ln(c*x^n)*ln(a^2*x^2
+1)/a^3+1/12*(3*a^2*d-e)*n*polylog(2,-a^2*x^2)/a^3
```

Rubi [A] time = 0.15, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4913, 1593, 444, 43, 2388, 4847, 260, 4853, 266, 2391}

$$\frac{n\left(3a^2d-e\right)\text{PolyLog}\left(2,-a^2x^2\right)}{12a^3}+\frac{\left(3a^2d-e\right)\log\left(a^2x^2+1\right)\log\left(cx^n\right)}{6a^3}-\frac{dn\log\left(a^2x^2+1\right)}{2a}+\frac{en\log\left(a^2x^2+1\right)}{18a^3}+dx\,c$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCot[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*e*n*x^2)/(36*a) - d*n*x*ArcCot[a*x] - (e*n*x^3*ArcCot[a*x])/9 + (e*x^2*
Log[c*x^n])/(6*a) + d*x*ArcCot[a*x]*Log[c*x^n] + (e*x^3*ArcCot[a*x]*Log[c*x
^n])/3 - (d*n*Log[1 + a^2*x^2])/(2*a) + (e*n*Log[1 + a^2*x^2])/(18*a^3) + (
(3*a^2*d - e)*Log[c*x^n]*Log[1 + a^2*x^2])/(6*a^3) + ((3*a^2*d - e)*n*PolyL
og[2, -(a^2*x^2)])/(12*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2388

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Dist[a + b*L
og[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c
, d, e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh
, ArcCoth}, F]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4847

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cCot[c*x])^p, x] + Dist[b*c*p, Int[(x*(a + b*ArcCot[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4853

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCot[c*x])^p)/(d*(m + 1)), x] + Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCot[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4913

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\cot^{-1}(ax)\log\left(cx^n\right)\,dx = \frac{ex^2\log\left(cx^n\right)}{6a} + dx\cot^{-1}(ax)\log\left(cx^n\right) + \frac{1}{3}ex^3\cot^{-1}(ax)\log\left(cx^n\right) + \frac{\left(3a^2\right.}{}$$

$$= -\frac{enx^2}{12a} + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\cot^{-1}(ax)\log\left(cx^n\right) + \frac{1}{3}ex^3\cot^{-1}(ax)\log\left(cx\right.$$

$$= -\frac{enx^2}{12a} - dnx\cot^{-1}(ax) - \frac{1}{9}enx^3\cot^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\cot^{-1}(ax$$

$$= -\frac{enx^2}{12a} - dnx\cot^{-1}(ax) - \frac{1}{9}enx^3\cot^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\cot^{-1}(ax$$

$$= -\frac{enx^2}{12a} - dnx\cot^{-1}(ax) - \frac{1}{9}enx^3\cot^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\cot^{-1}(ax$$

$$= -\frac{5enx^2}{36a} - dnx\cot^{-1}(ax) - \frac{1}{9}enx^3\cot^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\cot^{-1}(a$$

Mathematica [A] time = 0.12, size = 178, normalized size = 0.98

$$\frac{-4a^3x\cot^{-1}(ax)\left(n\left(9d+ex^2\right)-3\left(3d+ex^2\right)\log\left(cx^n\right)\right)+18a^2d\log\left(a^2x^2+1\right)\log\left(cx^n\right)+6a^2ex^2\log\left(cx^n\right)-6e\,\mathrm{l}}{36a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcCot[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*a^2*e*n*x^2 + 36*a^2*d*n*Log[1/(a*Sqrt[1 + 1/(a^2*x^2)]*x)] + 6*a^2*e*x
^2*Log[c*x^n] - 4*a^3*x*ArcCot[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log[
c*x^n]) + 2*e*n*Log[1 + a^2*x^2] + 18*a^2*d*Log[c*x^n]*Log[1 + a^2*x^2] - 6
*e*Log[c*x^n]*Log[1 + a^2*x^2] + (9*a^2*d*n - 3*e*n)*PolyLog[2, -(a^2*x^2)]
)/(36*a^3)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ex^2+d\right)\mathrm{arccot}\left(ax\right)\log\left(cx^n\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccot(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*arccot(a*x)*log(c*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex^2+d\right)\mathrm{arccot}\left(ax\right)\log\left(cx^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccot(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*arccot(a*x)*log(c*x^n), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(e\,x^2+d\right)\mathrm{arccot}\left(ax\right)\ln\left(c\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccot(a*x)*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arccot(a*x)*ln(c*x^n),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccot(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\ln\left(c\,x^n\right)\,\mathrm{acot}\left(a\,x\right)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acot(a*x)*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*acot(a*x)*(d + e*x^2), x)
```

sympy [A] time = 95.10, size = 231, normalized size = 1.27

$$-dn\left(\begin{cases} \frac{\pi x}{2} & \text{for } a = 0 \\ \left(\begin{cases} x \operatorname{acot}(ax) + \frac{\log\left(a^2x^2+1\right)}{2a} & \text{for } a \neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}\right) - \frac{\operatorname{Li}_2\left(a^2x^2e^{i\pi}\right)}{4a} & \text{otherwise} \end{cases}\right) + d\left(\begin{cases} \frac{\pi x}{2} & \text{for } a \\ x \operatorname{acot}(ax) + \frac{\log\left(a^2x^2+1\right)}{2a} & \text{othe} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acot(a*x)*ln(c*x**n),x)
```

```
[Out] -d*n*Piecewise((pi*x/2, Eq(a, 0)), (Piecewise((x*acot(a*x) + log(a**2*x**2
+ 1)/(2*a), Ne(a, 0)), (pi*x/2, True)) - polylog(2, a**2*x**2*exp_polar(I*p
i))/(4*a), True)) + d*Piecewise((pi*x/2, Eq(a, 0)), (x*acot(a*x) + log(a**2
*x**2 + 1)/(2*a), True))*log(c*x**n) - e*n*x**3*acot(a*x)/9 + e*x**3*log(c*
x**n)*acot(a*x)/3 - 5*e*n*x**2/(36*a) + e*n*Piecewise((x**2/2, Eq(a, 0)), (
-polylog(2, a**2*x**2*exp_polar(I*pi))/(2*a**2), True))/(6*a) + e*n*Piecewi
se((x**2, Eq(a**2, 0)), (log(a**2*x**2 + 1)/a**2, True))/(18*a) + e*x**2*lo
g(c*x**n)/(6*a) - e*Piecewise((x**2, Eq(a**2, 0)), (log(a**2*x**2 + 1)/a**2
, True))*log(c*x**n)/(6*a)
```

3.190 $\int \left(d+ex^2\right)\sinh^{-1}(ax)\log\left(cx^n\right)\,dx$

Optimal. Leaf size=244

$$\frac{dn\sqrt{a^2x^2+1}}{a}-\frac{\sqrt{a^2x^2+1}\left(3a^2d-e\right)\log\left(cx^n\right)}{3a^3}-\frac{e\left(a^2x^2+1\right)^{3/2}\log\left(cx^n\right)}{9a^3}+\frac{n\sqrt{a^2x^2+1}\left(3a^2d-e\right)}{3a^3}-\frac{n\left(3a^2d-e\right)\text{ta}}{}$$

```
[Out] 2/27*e*n*(a^2*x^2+1)^(3/2)/a^3-d*n*x*arcsinh(a*x)-1/9*e*n*x^3*arcsinh(a*x)-
1/3*(3*a^2*d-e)*n*arctanh((a^2*x^2+1)^(1/2))/a^3-1/9*e*n*arctanh((a^2*x^2+1
)^(1/2))/a^3-1/9*e*(a^2*x^2+1)^(3/2)*ln(c*x^n)/a^3+d*x*arcsinh(a*x)*ln(c*x^
n)+1/3*e*x^3*arcsinh(a*x)*ln(c*x^n)+d*n*(a^2*x^2+1)^(1/2)/a+1/3*(3*a^2*d-e)
*n*(a^2*x^2+1)^(1/2)/a^3-1/3*(3*a^2*d-e)*ln(c*x^n)*(a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.22, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5704, 444, 43, 2387, 266, 50, 63, 208, 5653, 261, 5661}

$$-\frac{\sqrt{a^2x^2+1}\left(3a^2d-e\right)\log\left(cx^n\right)}{3a^3}-\frac{e\left(a^2x^2+1\right)^{3/2}\log\left(cx^n\right)}{9a^3}+\frac{n\sqrt{a^2x^2+1}\left(3a^2d-e\right)}{3a^3}-\frac{n\left(3a^2d-e\right)\tanh^{-1}\left(\sqrt{a^2x^2}\right.}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcSinh[a*x]*Log[c*x^n],x]
```

```
[Out] (d*n*Sqrt[1 + a^2*x^2])/a + ((3*a^2*d - e)*n*Sqrt[1 + a^2*x^2])/(3*a^3) + (
2*e*n*(1 + a^2*x^2)^(3/2))/(27*a^3) - d*n*x*ArcSinh[a*x] - (e*n*x^3*ArcSinh
[a*x])/9 - ((3*a^2*d - e)*n*ArcTanh[Sqrt[1 + a^2*x^2]])/(3*a^3) - (e*n*ArcT
anh[Sqrt[1 + a^2*x^2]])/(9*a^3) - ((3*a^2*d - e)*Sqrt[1 + a^2*x^2]*Log[c*x^
n])/(3*a^3) - (e*(1 + a^2*x^2)^(3/2)*Log[c*x^n])/(9*a^3) + d*x*ArcSinh[a*x]
*Log[c*x^n] + (e*x^3*ArcSinh[a*x]*Log[c*x^n])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\sinh^{-1}(ax)\log\left(cx^n\right)\,dx = -\frac{\left(3a^2d-e\right)\sqrt{1+a^2x^2}\,\log\left(cx^n\right)}{3a^3} - \frac{e\left(1+a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3} + dx\sinh^{-1}(a$$

$$= -\frac{\left(3a^2d-e\right)\sqrt{1+a^2x^2}\,\log\left(cx^n\right)}{3a^3} - \frac{e\left(1+a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3} + dx\sinh^{-1}(a$$

$$= -dnx\sinh^{-1}(ax) - \frac{1}{9}enx^3\sinh^{-1}(ax) - \frac{\left(3a^2d-e\right)\sqrt{1+a^2x^2}\,\log\left(cx^n\right)}{3a^3} -$$

$$= \frac{dn\sqrt{1+a^2x^2}}{a} + \frac{\left(3a^2d-e\right)n\sqrt{1+a^2x^2}}{3a^3} + \frac{en\left(1+a^2x^2\right)^{3/2}}{27a^3} - dnx\sinh^{-1}($$

$$= \frac{dn\sqrt{1+a^2x^2}}{a} + \frac{\left(3a^2d-e\right)n\sqrt{1+a^2x^2}}{3a^3} + \frac{en\sqrt{1+a^2x^2}}{9a^3} + \frac{en\left(1+a^2x^2\right)^{3/}}{27a^3}$$

$$= \frac{dn\sqrt{1+a^2x^2}}{a} + \frac{\left(3a^2d-e\right)n\sqrt{1+a^2x^2}}{3a^3} + \frac{2en\left(1+a^2x^2\right)^{3/2}}{27a^3} - dnx\sinh^{-1}$$

$$= \frac{dn\sqrt{1+a^2x^2}}{a} + \frac{\left(3a^2d-e\right)n\sqrt{1+a^2x^2}}{3a^3} + \frac{2en\left(1+a^2x^2\right)^{3/2}}{27a^3} - dnx\sinh^{-1}$$

Mathematica [A] time = 0.16, size = 240, normalized size = 0.98

$$\frac{-3a^3x\sinh^{-1}(ax)\left(n\left(9d+ex^2\right)-3\left(3d+ex^2\right)\log\left(cx^n\right)\right)-27a^2d\sqrt{a^2x^2+1}\,\log\left(cx^n\right)-3a^2ex^2\sqrt{a^2x^2+1}\,\log\left(cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcSinh[a*x]*Log[c*x^n],x]
```

```
[Out] (54*a^2*d*n*Sqrt[1 + a^2*x^2] - 7*e*n*Sqrt[1 + a^2*x^2] + 2*a^2*e*n*x^2*Sqr
t[1 + a^2*x^2] + 3*(9*a^2*d - 2*e)*n*Log[x] - 27*a^2*d*Sqrt[1 + a^2*x^2]*Lo
g[c*x^n] + 6*e*Sqrt[1 + a^2*x^2]*Log[c*x^n] - 3*a^2*e*x^2*Sqrt[1 + a^2*x^2]
*Log[c*x^n] - 3*a^3*x*ArcSinh[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log[c
*x^n]) - 27*a^2*d*n*Log[1 + Sqrt[1 + a^2*x^2]] + 6*e*n*Log[1 + Sqrt[1 + a^2
*x^2]])/(27*a^3)
```

fricas [A] time = 1.10, size = 307, normalized size = 1.26

$$-\frac{3\left(9\,a^2d-2\,e\right)n\log\left(-ax+\sqrt{a^2x^2+1}\,+1\right)-3\left(9\,a^2d-2\,e\right)n\log\left(-ax+\sqrt{a^2x^2+1}\,-1\right)+3\left(a^3enx^3+9\,a^3dnx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] -1/27*(3*(9*a^2*d - 2*e)*n*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*(9*a^2*d -
 2*e)*n*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + 3*(a^3*e*n*x^3 + 9*a^3*d*n*x -
(9*a^3*d + a^3*e)*n - 3*(a^3*e*x^3 + 3*a^3*d*x - 3*a^3*d - a^3*e)*log(c) -
3*(a^3*e*n*x^3 + 3*a^3*d*n*x)*log(x))*log(a*x + sqrt(a^2*x^2 + 1)) - 3*((9*
a^3*d + a^3*e)*n - 3*(3*a^3*d + a^3*e)*log(c))*log(-a*x + sqrt(a^2*x^2 + 1)
```

```
) - (2*a^2*e*n*x^2 + (54*a^2*d - 7*e)*n - 3*(a^2*e*x^2 + 9*a^2*d - 2*e)*log
(c) - 3*(a^2*e*n*x^2 + (9*a^2*d - 2*e)*n)*log(x))*sqrt(a^2*x^2 + 1))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right) \operatorname{arcsinh}\left(a x\right) \ln \left(c\,x^{n}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arcsinh(a*x)*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arcsinh(a*x)*ln(c*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a^{2}dn\left(\frac{2\,x}{a^{2}}+\frac{i\left(\log\left(i\,ax+1\right)-\log\left(-i\,ax+1\right)\right)}{a^{3}}\right)+\frac{1}{54}\,a^{2}en\left(\frac{2\left(a^{2}x^{3}-3\,x\right)}{a^{4}}-\frac{3i\left(\log\left(i\,ax+1\right)-\log\left(-i\,ax+1\right)\right)}{a^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*n*(2*x/a^2 + I*(log(I*a*x + 1) - log(-I*a*x + 1))/a^3) + 1/54*a^2
*e*n*(2*(a^2*x^3 - 3*x)/a^4 - 3*I*(log(I*a*x + 1) - log(-I*a*x + 1))/a^5) -
 3*a^2*e*n*integrate(1/9*x^4*log(x)/(a^2*x^2 + 1), x) - 9*a^2*d*n*integrate
(1/9*x^2*log(x)/(a^2*x^2 + 1), x) - 1/2*a^2*d*(2*x/a^2 + I*(log(I*a*x + 1)
- log(-I*a*x + 1))/a^3)*log(c) - 1/18*a^2*e*(2*(a^2*x^3 - 3*x)/a^4 - 3*I*(l
og(I*a*x + 1) - log(-I*a*x + 1))/a^5)*log(c) - 1/9*((e*n - 3*e*log(c))*x^3
+ 9*(d*n - d*log(c))*x - 3*(e*x^3 + 3*d*x)*log(x^n))*log(a*x + sqrt(a^2*x^2
 + 1)) - integrate(-1/9*((e*n - 3*e*log(c))*a*x^3 + 9*(d*n - d*log(c))*a*x
- 3*(a*e*x^3 + 3*a*d*x)*log(x^n))/(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,\mathrm{asinh}\left(a\,x\right)\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*asinh(a*x)*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*asinh(a*x)*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + e x^{2}\right) \log{\left(c x^{n} \right)} \operatorname{asinh}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*asinh(a*x)*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*asinh(a*x), x)
```

3.191 $\int (d+ex^2)\cosh^{-1}(ax)\log(cx^n)\,dx$

Optimal. Leaf size=312

$$\frac{en(ax-1)^{3/2}(ax+1)^{3/2}}{27a^3}+\frac{2en\sqrt{ax-1}\sqrt{ax+1}}{27a^3}-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(9a^2d+2e\right)\log(cx^n)}{9a^3}+\frac{n\sqrt{ax-1}\sqrt{ax+1}\left(9a^2\right.}{9a^3}$$

```
[Out] 1/27*e*n*(a*x-1)^(3/2)*(a*x+1)^(3/2)/a^3-d*n*x*arccosh(a*x)-1/9*e*n*x^3*arc
cosh(a*x)-1/9*(9*a^2*d+2*e)*n*arctan((a*x-1)^(1/2)*(a*x+1)^(1/2))/a^3+d*x*a
rccosh(a*x)*ln(c*x^n)+1/3*e*x^3*arccosh(a*x)*ln(c*x^n)+d*n*(a*x-1)^(1/2)*(a
*x+1)^(1/2)/a+2/27*e*n*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3+1/9*(9*a^2*d+2*e)*n*
(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3+1/27*e*n*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-
1/9*(9*a^2*d+2*e)*ln(c*x^n)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3-1/9*e*x^2*ln(c*
x^n)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A] time = 0.21, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5705, 460, 74, 2387, 101, 92, 205, 5654, 5662, 100, 12}

$$-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(9a^2d+2e\right)\log(cx^n)}{9a^3}+\frac{n\sqrt{ax-1}\sqrt{ax+1}\left(9a^2d+2e\right)}{9a^3}-\frac{n\left(9a^2d+2e\right)\tan^{-1}\left(\sqrt{ax-1}\sqrt{ax+1}\right.}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCosh[a*x]*Log[c*x^n],x]
```

```
[Out] (d*n*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a + (2*e*n*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
/(27*a^3) + ((9*a^2*d + 2*e)*n*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(9*a^3) + (e*n
*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(27*a) + (e*n*(-1 + a*x)^(3/2)*(1 + a*x)
^(3/2))/(27*a^3) - d*n*x*ArcCosh[a*x] - (e*n*x^3*ArcCosh[a*x])/9 - ((9*a^2*
d + 2*e)*n*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*x]])/(9*a^3) - ((9*a^2*d + 2*e)
*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[c*x^n])/(9*a^3) - (e*x^2*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*Log[c*x^n])/(9*a) + d*x*ArcCosh[a*x]*Log[c*x^n] + (e*x^3*ArcCo
sh[a*x]*Log[c*x^n])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\cosh^{-1}(ax)\log(cx^n)\,dx = -\frac{\left(9a^2d+2e\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\log(cx^n)}{9a^3} - \frac{ex^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\log(c}{9a}$$

$$= -\frac{\left(9a^2d+2e\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\log(cx^n)}{9a^3} - \frac{ex^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\log(c}{9a}$$

$$= \frac{\left(9a^2d+2e\right)n\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3} + \frac{en(-1+ax)^{3/2}(1+ax)^{3/2}}{27a^3} - dnx\cosh$$

$$= \frac{dn\sqrt{-1+ax}\,\sqrt{1+ax}}{a} + \frac{\left(9a^2d+2e\right)n\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3} + \frac{enx^2\sqrt{-1+a}}{27}$$

$$= \frac{dn\sqrt{-1+ax}\,\sqrt{1+ax}}{a} + \frac{\left(9a^2d+2e\right)n\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3} + \frac{enx^2\sqrt{-1+a}}{27}$$

$$= \frac{dn\sqrt{-1+ax}\,\sqrt{1+ax}}{a} + \frac{2en\sqrt{-1+ax}\,\sqrt{1+ax}}{27a^3} + \frac{\left(9a^2d+2e\right)n\sqrt{-1+ax}}{9a^3}$$

Mathematica [A] time = 0.24, size = 145, normalized size = 0.46

$$\frac{-3a^3x\cosh^{-1}(ax)\left(n\left(9d+ex^2\right)-3\left(3d+ex^2\right)\log(cx^n)\right)+\sqrt{ax-1}\,\sqrt{ax+1}\,\left(n\left(2a^2\left(27d+ex^2\right)+7e\right)-3\left(a^2\left(9\right.\right.\right.}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcCosh[a*x]*Log[c*x^n],x]
```

```
[Out] (3*(9*a^2*d + 2*e)*n*ArcTan[1/(Sqrt[-1 + a*x]*Sqrt[1 + a*x])] - 3*a^3*x*Arc
Cosh[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log[c*x^n]) + Sqrt[-1 + a*x]*S
qrt[1 + a*x]*(n*(7*e + 2*a^2*(27*d + e*x^2)) - 3*(2*e + a^2*(9*d + e*x^2))*
Log[c*x^n]))/(27*a^3)
```

fricas [A] time = 1.05, size = 275, normalized size = 0.88

$$-\frac{6\left(9\,a^2d+2\,e\right)n\arctan\left(-ax+\sqrt{a^2x^2-1}\right)+3\left(a^3enx^3+9\,a^3dnx-\left(9\,a^3d+a^3e\right)n-3\left(a^3ex^3+3\,a^3dx-3\,a^3d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] -1/27*(6*(9*a^2*d + 2*e)*n*arctan(-a*x + sqrt(a^2*x^2 - 1)) + 3*(a^3*e*n*x^
3 + 9*a^3*d*n*x - (9*a^3*d + a^3*e)*n - 3*(a^3*e*x^3 + 3*a^3*d*x - 3*a^3*d
- a^3*e)*log(c) - 3*(a^3*e*n*x^3 + 3*a^3*d*n*x)*log(x))*log(a*x + sqrt(a^2*
x^2 - 1)) - 3*((9*a^3*d + a^3*e)*n - 3*(3*a^3*d + a^3*e)*log(c))*log(-a*x +
 sqrt(a^2*x^2 - 1)) - (2*a^2*e*n*x^2 + (54*a^2*d + 7*e)*n - 3*(a^2*e*x^2 +
9*a^2*d + 2*e)*log(c) - 3*(a^2*e*n*x^2 + (9*a^2*d + 2*e)*n)*log(x))*sqrt(a^
2*x^2 - 1))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right) \operatorname{arccosh}\left(a x\right) \ln \left(c\,x^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccosh(a*x)*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arccosh(a*x)*ln(c*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\,a^{2} d n+e n\right)\left(\log\left(a x+1\right)\log(x)+\mathrm{Li}_2\left(-a x\right)\right)}{6\,a^{3}}-\frac{\left(3\,a^{2} d n+e n\right)\left(\log\left(-a x+1\right)\log(x)+\mathrm{Li}_2\left(a x\right)\right)}{6\,a^{3}}-\frac{\left(9\left(d n-d\log\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*d*n + e*n)*(log(a*x + 1)*log(x) + dilog(-a*x))/a^3 - 1/6*(3*a^2*
d*n + e*n)*(log(-a*x + 1)*log(x) + dilog(a*x))/a^3 - 1/18*(9*(d*n - d*log(c
))*a^2 + e*n - 3*e*log(c))*log(a*x + 1)/a^3 + 1/18*(9*(d*n - d*log(c))*a^2
+ e*n - 3*e*log(c))*log(a*x - 1)/a^3 + 1/54*(2*(2*e*n - 3*e*log(c))*a^3*x^3
 - 9*(3*a^2*d*n + e*n)*log(a*x + 1)*log(x) + 9*(3*a^2*d*n + e*n)*log(a*x -
1)*log(x) + 6*(9*(2*d*n - d*log(c))*a^3 + (4*e*n - 3*e*log(c))*a)*x - 6*((e
*n - 3*e*log(c))*a^3*x^3 + 9*(d*n - d*log(c))*a^3*x - 3*(a^3*e*x^3 + 3*a^3*
d*x)*log(x^n))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)) - 3*(2*a^3*e*x^3 + 6*
(3*a^3*d + a*e)*x - 3*(3*a^2*d + e)*log(a*x + 1) + 3*(3*a^2*d + e)*log(a*x
- 1))*log(x^n))/a^3 + integrate(-1/9*((e*n - 3*e*log(c))*a*x^3 + 9*(d*n - d
*log(c))*a*x - 3*(a*e*x^3 + 3*a*d*x)*log(x^n))/(a^3*x^3 + (a^2*x^2 - 1)*sqr
t(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,\mathrm{acosh}\left(a\,x\right)\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acosh(a*x)*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*acosh(a*x)*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + e x^{2}\right) \log{\left(c x^{n} \right)} \operatorname{acosh}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acosh(a*x)*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*acosh(a*x), x)
```

3.192 $\int (d+ex^2)\tanh^{-1}(ax)\log(cx^n)\,dx$

Optimal. Leaf size=180

$$-\frac{dn\log(1-a^2x^2)}{2a}+\frac{(3a^2d+e)\log(1-a^2x^2)\log(cx^n)}{6a^3}+\frac{n(3a^2d+e)\mathrm{Li}_2(a^2x^2)}{12a^3}-\frac{en\log(1-a^2x^2)}{18a^3}+dx\tanh^{-1}(a$$

```
[Out] -5/36*e*n*x^2/a-d*n*x*arctanh(a*x)-1/9*e*n*x^3*arctanh(a*x)+1/6*e*x^2*ln(c*
x^n)/a+d*x*arctanh(a*x)*ln(c*x^n)+1/3*e*x^3*arctanh(a*x)*ln(c*x^n)-1/2*d*n*
ln(-a^2*x^2+1)/a-1/18*e*n*ln(-a^2*x^2+1)/a^3+1/6*(3*a^2*d+e)*ln(c*x^n)*ln(-
a^2*x^2+1)/a^3+1/12*(3*a^2*d+e)*n*polylog(2,a^2*x^2)/a^3
```

Rubi [A] time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5976, 1593, 444, 43, 2388, 5910, 260, 5916, 266, 2391}

$$\frac{n(3a^2d+e)\,\mathrm{PolyLog}(2,a^2x^2)}{12a^3}+\frac{(3a^2d+e)\log(1-a^2x^2)\log(cx^n)}{6a^3}-\frac{dn\log(1-a^2x^2)}{2a}-\frac{en\log(1-a^2x^2)}{18a^3}+dx\tan$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcTanh[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*e*n*x^2)/(36*a) - d*n*x*ArcTanh[a*x] - (e*n*x^3*ArcTanh[a*x])/9 + (e*x^
2*Log[c*x^n])/(6*a) + d*x*ArcTanh[a*x]*Log[c*x^n] + (e*x^3*ArcTanh[a*x]*Log
[c*x^n])/3 - (d*n*Log[1 - a^2*x^2])/(2*a) - (e*n*Log[1 - a^2*x^2])/(18*a^3)
 + ((3*a^2*d + e)*Log[c*x^n]*Log[1 - a^2*x^2])/(6*a^3) + ((3*a^2*d + e)*n*P
olyLog[2, a^2*x^2])/(12*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2388

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Dist[a + b*L
og[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c
, d, e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh
, ArcCoth}, F]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\tanh^{-1}(ax)\log\left(cx^n\right)\,dx = \frac{ex^2\log\left(cx^n\right)}{6a} + dx\tanh^{-1}(ax)\log\left(cx^n\right) + \frac{1}{3}ex^3\tanh^{-1}(ax)\log\left(cx^n\right) +$$

$$= -\frac{enx^2}{12a} + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\tanh^{-1}(ax)\log\left(cx^n\right) + \frac{1}{3}ex^3\tanh^{-1}(ax)\text{lo}$$

$$= -\frac{enx^2}{12a} - dnx\tanh^{-1}(ax) - \frac{1}{9}enx^3\tanh^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\,\text{tan}$$

$$= -\frac{enx^2}{12a} - dnx\tanh^{-1}(ax) - \frac{1}{9}enx^3\tanh^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\,\text{tan}$$

$$= -\frac{enx^2}{12a} - dnx\tanh^{-1}(ax) - \frac{1}{9}enx^3\tanh^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\,\text{tan}$$

$$= -\frac{5enx^2}{36a} - dnx\tanh^{-1}(ax) - \frac{1}{9}enx^3\tanh^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\,\text{ta}$$

Mathematica [A] time = 0.14, size = 167, normalized size = 0.93

$$\frac{-4a^3x\tanh^{-1}(ax)\left(n\left(9d+ex^2\right)-3\left(3d+ex^2\right)\log\left(cx^n\right)\right)+18a^2d\log\left(1-a^2x^2\right)\log\left(cx^n\right)+6a^2ex^2\log\left(cx^n\right)+}{36}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcTanh[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*a^2*e*n*x^2 + 6*a^2*e*x^2*Log[c*x^n] - 4*a^3*x*ArcTanh[a*x]*(n*(9*d + e
*x^2) - 3*(3*d + e*x^2)*Log[c*x^n]) - 18*a^2*d*n*Log[1 - a^2*x^2] + 18*a^2*
d*Log[c*x^n]*Log[1 - a^2*x^2] + 6*e*Log[c*x^n]*Log[1 - a^2*x^2] - 2*e*n*Log
[-1 + a^2*x^2] + 3*(3*a^2*d + e)*n*PolyLog[2, a^2*x^2])/(36*a^3)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^2+d\right)\text{artanh}\left(ax\right)\log\left(cx^n\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctanh(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*arctanh(a*x)*log(c*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)\text{artanh}\left(ax\right)\log\left(cx^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctanh(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*arctanh(a*x)*log(c*x^n), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)\text{arctanh}\left(ax\right)\ln\left(c\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arctanh(a*x)*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arctanh(a*x)*ln(c*x^n),x)
```

maxima [C] time = 1.08, size = 354, normalized size = 1.97

$$-\frac{1}{36}n\left(\frac{18\,(i\,\pi d-2\,d)\log(x)}{a}+\frac{6\left(3\,a^2d+e\right)\left(\log\left(ax-1\right)\log\left(ax\right)+\text{Li}_2\left(-ax+1\right)\right)}{a^3}+\frac{6\left(3\,a^2d+e\right)\left(\log\left(ax+1\right)\text{lo}\right.}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctanh(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/36*n*(18*(I*pi*d - 2*d)*log(x)/a + 6*(3*a^2*d + e)*(log(a*x - 1)*log(a*x
) + dilog(-a*x + 1))/a^3 + 6*(3*a^2*d + e)*(log(a*x + 1)*log(-a*x) + dilog(
a*x + 1))/a^3 + 2*(9*a^2*d + e)*log(a*x + 1)/a^3 + (-2*I*pi*a^3*e*x^3 - 18*
I*pi*a^3*d*x + 5*a^2*e*x^2 + 2*(a^3*e*x^3 + 9*a^3*d*x)*log(a*x + 1) - 2*(a^
3*e*x^3 + 9*a^3*d*x - 9*a^2*d - e)*log(a*x - 1))/a^3) + 1/36*((6*x^3*log(a*
x + 1) - a*((2*a^2*x^3 - 3*a*x^2 + 6*x)/a^3 - 6*log(a*x + 1)/a^4))*e - (6*x
^3*log(-a*x + 1) - a*((2*a^2*x^3 + 3*a*x^2 + 6*x)/a^3 + 6*log(a*x - 1)/a^4)
)*e - 18*(a*x - (a*x + 1)*log(a*x + 1) + 1)*d/a + 18*(a*x - (a*x - 1)*log(-
a*x + 1) - 1)*d/a)*log(c*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,x^n\right)\,\mathrm{atanh}\left(a\,x\right)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*atanh(a*x)*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*atanh(a*x)*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex^2\right) \log \left(cx^n\right) \operatorname{atanh}\left(ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*atanh(a*x)*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*atanh(a*x), x)
```

3.193 $\int \left(d+ex^2\right)\coth^{-1}(ax)\log\left(cx^n\right)\,dx$

Optimal. Leaf size=180

$$-\frac{dn\log\left(1-a^2x^2\right)}{2a}+\frac{\left(3a^2d+e\right)\log\left(1-a^2x^2\right)\log\left(cx^n\right)}{6a^3}+\frac{n\left(3a^2d+e\right)\text{Li}_2\left(a^2x^2\right)}{12a^3}-\frac{en\log\left(1-a^2x^2\right)}{18a^3}+dx\coth^{-1}(a$$

```
[Out] -5/36*e*n*x^2/a-d*n*x*arccoth(a*x)-1/9*e*n*x^3*arccoth(a*x)+1/6*e*x^2*ln(c*
x^n)/a+d*x*arccoth(a*x)*ln(c*x^n)+1/3*e*x^3*arccoth(a*x)*ln(c*x^n)-1/2*d*n*
ln(-a^2*x^2+1)/a-1/18*e*n*ln(-a^2*x^2+1)/a^3+1/6*(3*a^2*d+e)*ln(c*x^n)*ln(-
a^2*x^2+1)/a^3+1/12*(3*a^2*d+e)*n*polylog(2,a^2*x^2)/a^3
```

Rubi [A] time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5977, 1593, 444, 43, 2388, 5911, 260, 5917, 266, 2391}

$$\frac{n\left(3a^2d+e\right)\text{PolyLog}\left(2,a^2x^2\right)}{12a^3}+\frac{\left(3a^2d+e\right)\log\left(1-a^2x^2\right)\log\left(cx^n\right)}{6a^3}-\frac{dn\log\left(1-a^2x^2\right)}{2a}-\frac{en\log\left(1-a^2x^2\right)}{18a^3}+dx\text{ co}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCoth[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*e*n*x^2)/(36*a) - d*n*x*ArcCoth[a*x] - (e*n*x^3*ArcCoth[a*x])/9 + (e*x^
2*Log[c*x^n])/(6*a) + d*x*ArcCoth[a*x]*Log[c*x^n] + (e*x^3*ArcCoth[a*x]*Log
[c*x^n])/3 - (d*n*Log[1 - a^2*x^2])/(2*a) - (e*n*Log[1 - a^2*x^2])/(18*a^3)
 + ((3*a^2*d + e)*Log[c*x^n]*Log[1 - a^2*x^2])/(6*a^3) + ((3*a^2*d + e)*n*P
olyLog[2, a^2*x^2])/(12*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2388

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Dist[a + b*L
og[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c
, d, e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh
, ArcCoth}, F]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\coth^{-1}(ax)\log\left(cx^n\right)\,dx=\frac{ex^2\log\left(cx^n\right)}{6a}+dx\coth^{-1}(ax)\log\left(cx^n\right)+\frac{1}{3}ex^3\coth^{-1}(ax)\log\left(cx^n\right)+$$

$$=-\frac{enx^2}{12a}+\frac{ex^2\log\left(cx^n\right)}{6a}+dx\coth^{-1}(ax)\log\left(cx^n\right)+\frac{1}{3}ex^3\coth^{-1}(ax)\log$$

$$=-\frac{enx^2}{12a}-dnx\coth^{-1}(ax)-\frac{1}{9}enx^3\coth^{-1}(ax)+\frac{ex^2\log\left(cx^n\right)}{6a}+dx\coth$$

$$=-\frac{enx^2}{12a}-dnx\coth^{-1}(ax)-\frac{1}{9}enx^3\coth^{-1}(ax)+\frac{ex^2\log\left(cx^n\right)}{6a}+dx\coth$$

$$=-\frac{enx^2}{12a}-dnx\coth^{-1}(ax)-\frac{1}{9}enx^3\coth^{-1}(ax)+\frac{ex^2\log\left(cx^n\right)}{6a}+dx\coth$$

$$=-\frac{5enx^2}{36a}-dnx\coth^{-1}(ax)-\frac{1}{9}enx^3\coth^{-1}(ax)+\frac{ex^2\log\left(cx^n\right)}{6a}+dx\coth$$

Mathematica [A] time = 0.13, size = 178, normalized size = 0.99

$$\frac{-4a^3x\coth^{-1}(ax)\left(n\left(9d+ex^2\right)-3\left(3d+ex^2\right)\log\left(cx^n\right)\right)+18a^2d\log\left(1-a^2x^2\right)\log\left(cx^n\right)+6a^2ex^2\log\left(cx^n\right)+6e}{36a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcCoth[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*a^2*e*n*x^2 + 36*a^2*d*n*Log[1/(a*Sqrt[1 - 1/(a^2*x^2)]*x)] + 6*a^2*e*x
^2*Log[c*x^n] - 4*a^3*x*ArcCoth[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log
[c*x^n]) + 18*a^2*d*Log[c*x^n]*Log[1 - a^2*x^2] + 6*e*Log[c*x^n]*Log[1 - a^
2*x^2] - 2*e*n*Log[-1 + a^2*x^2] + 3*(3*a^2*d + e)*n*PolyLog[2, a^2*x^2])/(
36*a^3)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ex^2+d\right)\mathrm{arcoth}\left(ax\right)\log\left(cx^n\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccoth(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*arccoth(a*x)*log(c*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex^2+d\right)\mathrm{arcoth}\left(ax\right)\log\left(cx^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccoth(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*arccoth(a*x)*log(c*x^n), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(e\,x^2+d\right)\mathrm{arccoth}\left(ax\right)\ln\left(c\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccoth(a*x)*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arccoth(a*x)*ln(c*x^n),x)
```

maxima [A] time = 0.95, size = 319, normalized size = 1.77

$$-\frac{1}{36}n\left(\frac{6\left(3\,a^2d+e\right)\left(\log\left(ax-1\right)\log\left(ax\right)+\mathrm{Li}_2\left(-ax+1\right)\right)}{a^3}+\frac{6\left(3\,a^2d+e\right)\left(\log\left(ax+1\right)\log\left(-ax\right)+\mathrm{Li}_2\left(ax+1\right)\right)}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccoth(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/36*n*(6*(3*a^2*d + e)*(log(a*x - 1)*log(a*x) + dilog(-a*x + 1))/a^3 + 6*
(3*a^2*d + e)*(log(a*x + 1)*log(-a*x) + dilog(a*x + 1))/a^3 + 2*(9*a^2*d +
e)*log(a*x + 1)/a^3 + (5*a^2*e*x^2 + 2*(a^3*e*x^3 + 9*a^3*d*x)*log(a*x + 1)
 - 2*(a^3*e*x^3 + 9*a^3*d*x - 9*a^2*d - e)*log(a*x - 1))/a^3) + 1/12*(6*(x*
log(1/(a*x) + 1) + log(a*x + 1)/a)*d - 6*(x*log(-1/(a*x) + 1) - log(a*x - 1
```

```
)/a)*d + (2*x^3*log(1/(a*x) + 1) + ((a*x^2 - 2*x)/a + 2*log(a*x + 1)/a^2)/a
)*e - (2*x^3*log(-1/(a*x) + 1) - ((a*x^2 + 2*x)/a + 2*log(a*x - 1)/a^2)/a)*
e)*log(c*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,x^n\right)\,\mathrm{acoth}\left(a\,x\right)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acoth(a*x)*(d + e*x^2),x)
[Out] int(log(c*x^n)*acoth(a*x)*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)\log\left(cx^n\right)\mathrm{acoth}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acoth(a*x)*ln(c*x**n),x)
[Out] Integral((d + e*x**2)*log(c*x**n)*acoth(a*x), x)
```

3.194 $\int \left(d+ex^2\right) \sin^{-1}(ax)^2 \log\left(cx^n\right) \, dx$

Optimal. Leaf size=482

$$\frac{2d\sqrt{1-a^2x^2}\,\sin^{-1}(ax)\log\left(cx^n\right)}{a}-\frac{4ex\log\left(cx^n\right)}{9a^2}+\frac{2ex^2\sqrt{1-a^2x^2}\,\sin^{-1}(ax)\log\left(cx^n\right)}{9a}+\frac{4}{9}nx\left(\frac{2e}{a^2}+9d\right)-\frac{2dn\sqrt{1-a}}{}$$

```
[Out] 2*d*n*x+2/27*e*n*x/a^2+4/9*(9*d+2*e/a^2)*n*x+2/27*e*n*x^3+2/27*e*n*(-a^2*x^
2+1)^(3/2)*arcsin(a*x)/a^3-d*n*x*arcsin(a*x)^2-1/9*e*n*x^3*arcsin(a*x)^2+4/
9*(9*a^2*d+2*e)*n*arcsin(a*x)*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))/a^3-2*d*x*l
n(c*x^n)-4/9*e*x*ln(c*x^n)/a^2-2/27*e*x^3*ln(c*x^n)+d*x*arcsin(a*x)^2*ln(c*
x^n)+1/3*e*x^3*arcsin(a*x)^2*ln(c*x^n)+2/9*I*(9*a^2*d+2*e)*n*polylog(2,I*a*
x+(-a^2*x^2+1)^(1/2))/a^3-2/9*I*(9*a^2*d+2*e)*n*polylog(2,-I*a*x-(-a^2*x^2+
1)^(1/2))/a^3-2*d*n*arcsin(a*x)*(-a^2*x^2+1)^(1/2)/a-4/27*e*n*arcsin(a*x)*(
-a^2*x^2+1)^(1/2)/a^3-2/9*(9*a^2*d+2*e)*n*arcsin(a*x)*(-a^2*x^2+1)^(1/2)/a^
3-2/27*e*n*x^2*arcsin(a*x)*(-a^2*x^2+1)^(1/2)/a+2*d*arcsin(a*x)*ln(c*x^n)*(
-a^2*x^2+1)^(1/2)/a+4/9*e*arcsin(a*x)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)/a^3+2/9*
e*x^2*arcsin(a*x)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.73, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4667, 4619, 4677, 8, 4627, 4707, 30, 2387, 6, 4697, 4709, 4183, 2279, 2391}

$$-\frac{2in\left(9a^2d+2e\right)\text{PolyLog}\left(2,-e^{i\sin^{-1}(ax)}\right)}{9a^3}+\frac{2in\left(9a^2d+2e\right)\text{PolyLog}\left(2,e^{i\sin^{-1}(ax)}\right)}{9a^3}+\frac{2d\sqrt{1-a^2x^2}\,\sin^{-1}(ax)\log(}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcSin[a*x]^2*Log[c*x^n],x]
```

```
[Out] 2*d*n*x + (2*e*n*x)/(27*a^2) + (4*(9*d + (2*e)/a^2)*n*x)/9 + (2*e*n*x^3)/27
 - (2*d*n*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/a - (4*e*n*Sqrt[1 - a^2*x^2]*ArcSi
n[a*x])/(27*a^3) - (2*(9*a^2*d + 2*e)*n*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(9*a
^3) - (2*e*n*x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(27*a) + (2*e*n*(1 - a^2*x^
2)^(3/2)*ArcSin[a*x])/(27*a^3) - d*n*x*ArcSin[a*x]^2 - (e*n*x^3*ArcSin[a*x]
^2)/9 + (4*(9*a^2*d + 2*e)*n*ArcSin[a*x]*ArcTanh[E^(I*ArcSin[a*x])])/(9*a^3
) - 2*d*x*Log[c*x^n] - (4*e*x*Log[c*x^n])/(9*a^2) - (2*e*x^3*Log[c*x^n])/27
 + (2*d*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*Log[c*x^n])/a + (4*e*Sqrt[1 - a^2*x^2
]*ArcSin[a*x]*Log[c*x^n])/(9*a^3) + (2*e*x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*
Log[c*x^n])/(9*a) + d*x*ArcSin[a*x]^2*Log[c*x^n] + (e*x^3*ArcSin[a*x]^2*Log
[c*x^n])/3 - (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[2, -E^(I*ArcSin[a*x])])/a
^3 + (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[2, E^(I*ArcSin[a*x])])/a^3
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4697

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcS
in[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
```

```
^2]), Int[((f*x)^m*(a + b*ArcSin[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] - Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4709

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Sin
[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d +
e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)\sin^{-1}(ax)^2\log\left(cx^n\right)\,dx = -2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{2}{27}ex^3\log\left(cx^n\right)+\frac{2d\sqrt{1-a^2x^2}\,\sin^{-1}(ax)}{a}$$

$$= -2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{2}{27}ex^3\log\left(cx^n\right)+\frac{2d\sqrt{1-a^2x^2}\,\sin^{-1}(ax)}{a}$$

$$= \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{81}enx^3-2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{2}{27}ex^3\log\left(cx\right.$$

$$= \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{81}enx^3-\frac{2\left(9a^2d+2e\right)n\sqrt{1-a^2x^2}\,\sin^{-1}(ax)}{9a^3}+\frac{2en\left(1\right.}{}$$

$$= -\frac{2enx}{27a^2}+\frac{2\left(9a^2d+2e\right)nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{4}{81}enx^3-\frac{2dn\sqrt{1-a^2x^2}\,\mathrm{si}}{a}$$

$$= 2dnx-\frac{2enx}{27a^2}+\frac{2\left(9a^2d+2e\right)nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3-\frac{2dn\sqrt{1-}}{}$$

$$= 2dnx+\frac{2enx}{27a^2}+\frac{2\left(9a^2d+2e\right)nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3-\frac{2dn\sqrt{1-}}{}$$

$$= 2dnx+\frac{2enx}{27a^2}+\frac{2\left(9a^2d+2e\right)nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3-\frac{2dn\sqrt{1-}}{}$$

Mathematica [A] time = 0.85, size = 456, normalized size = 0.95

$$\frac{-54a^3dx\log\left(cx^n\right)+27a^3dx\sin^{-1}(ax)^2\log\left(cx^n\right)-2a^3ex^3\log\left(cx^n\right)+9a^3ex^3\sin^{-1}(ax)^2\log\left(cx^n\right)+162a^3dnx-2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*ArcSin[a*x]^2*Log[c*x^n],x]
```

```
[Out] (162*a^3*d*n*x + 26*a*e*n*x + 2*a^3*e*n*x^3 - 108*a^2*d*n*Sqrt[1 - a^2*x^2]
*ArcSin[a*x] - 14*e*n*Sqrt[1 - a^2*x^2]*ArcSin[a*x] - 4*a^2*e*n*x^2*Sqrt[1
- a^2*x^2]*ArcSin[a*x] - 27*a^3*d*n*x*ArcSin[a*x]^2 - 3*a^3*e*n*x^3*ArcSin[
a*x]^2 - 54*a^2*d*n*ArcSin[a*x]*Log[1 - E^(I*ArcSin[a*x])] - 12*e*n*ArcSin[
a*x]*Log[1 - E^(I*ArcSin[a*x])] + 54*a^2*d*n*ArcSin[a*x]*Log[1 + E^(I*ArcSi
n[a*x])] + 12*e*n*ArcSin[a*x]*Log[1 + E^(I*ArcSin[a*x])] - 54*a^3*d*x*Log[c
*x^n] - 12*a*e*x*Log[c*x^n] - 2*a^3*e*x^3*Log[c*x^n] + 54*a^2*d*Sqrt[1 - a^
2*x^2]*ArcSin[a*x]*Log[c*x^n] + 12*e*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*Log[c*x^
n] + 6*a^2*e*x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*Log[c*x^n] + 27*a^3*d*x*ArcS
in[a*x]^2*Log[c*x^n] + 9*a^3*e*x^3*ArcSin[a*x]^2*Log[c*x^n] - (6*I)*(9*a^2*
d + 2*e)*n*PolyLog[2, -E^(I*ArcSin[a*x])] + (6*I)*(9*a^2*d + 2*e)*n*PolyLog
[2, E^(I*ArcSin[a*x])])/(27*a^3)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ex^2+d\right)\arcsin\left(ax\right)^2\log\left(cx^n\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*arcsin(a*x)^2*log(c*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)\arcsin\left(ax\right)^2\log\left(cx^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)^2*log(c*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*arcsin(a*x)^2*log(c*x^n), x)
```

maple [F] time = 10.29, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)\arcsin\left(ax\right)^2\ln\left(c\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arcsin(a*x)^2*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arcsin(a*x)^2*ln(c*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(ex^3+3\,dx\right)\arctan\left(ax,\sqrt{ax+1}\,\sqrt{-ax+1}\right)^2\log\left(x^n\right)-\frac{1}{9}\left(\left(en-3\,e\log(c)\right)x^3+9\left(dn-d\log(c)\right)x\right)\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

```
[Out] 1/3*(e*x^3 + 3*d*x)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))^2*log(x^n) -
 1/9*((e*n - 3*e*log(c))*x^3 + 9*(d*n - d*log(c))*x)*arctan2(a*x, sqrt(a*x
+ 1)*sqrt(-a*x + 1))^2 + integrate(2/9*(3*(a*e*x^3 + 3*a*d*x)*arctan2(a*x,
sqrt(a*x + 1)*sqrt(-a*x + 1))*log(x^n) - ((a*e*n - 3*a*e*log(c))*x^3 + 9*(a
*d*n - a*d*log(c))*x)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1)))*sqrt(a*x
+ 1)*sqrt(-a*x + 1)/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(c\,x^n\right)\,\mathrm{asin}\left(a\,x\right)^2\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*asin(a*x)^2*(d + e*x^2),x)
[Out] int(log(c*x^n)*asin(a*x)^2*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex^{2}\right) \log \left(cx^{n}\right) \operatorname{asin}^{2}\left(ax\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*asin(a*x)**2*ln(c*x**n),x)
[Out] Integral((d + e*x**2)*log(c*x**n)*asin(a*x)**2, x)
```

3.195 $\int \left(d+ex^2\right) \cos^{-1}(ax)^2 \log\left(cx^n\right) \,dx$

Optimal. Leaf size=490

$$-\frac{2d\sqrt{1-a^2x^2}\,\cos^{-1}(ax)\log\left(cx^n\right)}{a}-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{2ex^2\sqrt{1-a^2x^2}\,\cos^{-1}(ax)\log\left(cx^n\right)}{9a}+\frac{4}{9}nx\left(\frac{2e}{a^2}+9d\right)+\frac{2dn\sqrt{}}{}$$

```
[Out] 2*d*n*x+2/27*e*n*x/a^2+4/9*(9*d+2*e/a^2)*n*x+2/27*e*n*x^3-2/27*e*n*(-a^2*x^
2+1)^(3/2)*arccos(a*x)/a^3-d*n*x*arccos(a*x)^2-1/9*e*n*x^3*arccos(a*x)^2-2/
9*I*(9*a^2*d+2*e)*n*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a^3-2*d*x*ln(c
*x^n)-4/9*e*x*ln(c*x^n)/a^2-2/27*e*x^3*ln(c*x^n)+d*x*arccos(a*x)^2*ln(c*x^n
)+1/3*e*x^3*arccos(a*x)^2*ln(c*x^n)+4/9*I*(9*a^2*d+2*e)*n*arccos(a*x)*arcta
n(a*x+I*(-a^2*x^2+1)^(1/2))/a^3+2/9*I*(9*a^2*d+2*e)*n*polylog(2,I*(a*x+I*(-
a^2*x^2+1)^(1/2)))/a^3+2*d*n*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a+4/27*e*n*arcc
os(a*x)*(-a^2*x^2+1)^(1/2)/a^3+2/9*(9*a^2*d+2*e)*n*arccos(a*x)*(-a^2*x^2+1)
^(1/2)/a^3+2/27*e*n*x^2*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a-2*d*arccos(a*x)*ln
(c*x^n)*(-a^2*x^2+1)^(1/2)/a-4/9*e*arccos(a*x)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)
/a^3-2/9*e*x^2*arccos(a*x)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.70, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4668, 4620, 4678, 8, 4628, 4708, 30, 2387, 6, 4698, 4710, 4181, 2279, 2391}

$$-\frac{2in\left(9a^2d+2e\right)\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)}{9a^3}+\frac{2in\left(9a^2d+2e\right)\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)}{9a^3}-\frac{2d\sqrt{1-a^2x^2}\,\cos^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCos[a*x]^2*Log[c*x^n],x]
```

```
[Out] 2*d*n*x + (2*e*n*x)/(27*a^2) + (4*(9*d + (2*e)/a^2)*n*x)/9 + (2*e*n*x^3)/27
 + (2*d*n*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a + (4*e*n*Sqrt[1 - a^2*x^2]*ArcCo
s[a*x])/(27*a^3) + (2*(9*a^2*d + 2*e)*n*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(9*a
^3) + (2*e*n*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(27*a) - (2*e*n*(1 - a^2*x^
2)^(3/2)*ArcCos[a*x])/(27*a^3) - d*n*x*ArcCos[a*x]^2 - (e*n*x^3*ArcCos[a*x]
^2)/9 + (((4*I)/9)*(9*a^2*d + 2*e)*n*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])])
/a^3 - 2*d*x*Log[c*x^n] - (4*e*x*Log[c*x^n])/(9*a^2) - (2*e*x^3*Log[c*x^n])
/27 - (2*d*Sqrt[1 - a^2*x^2]*ArcCos[a*x]*Log[c*x^n])/a - (4*e*Sqrt[1 - a^2*
x^2]*ArcCos[a*x]*Log[c*x^n])/(9*a^3) - (2*e*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*
x]*Log[c*x^n])/(9*a) + d*x*ArcCos[a*x]^2*Log[c*x^n] + (e*x^3*ArcCos[a*x]^2*
Log[c*x^n])/3 - (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[2, (-I)*E^(I*ArcCos[a*
x])])/a^3 + (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[2, I*E^(I*ArcCos[a*x])])/a
^3
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4668

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
```

```
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)\cos^{-1}(ax)^2\log\left(cx^n\right)\,dx = -2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{2}{27}ex^3\log\left(cx^n\right)-\frac{2d\sqrt{1-a^2x^2}\,\cos^{-1}}{a}$$

$$= -2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{2}{27}ex^3\log\left(cx^n\right)-\frac{2d\sqrt{1-a^2x^2}\,\cos^{-1}}{a}$$

$$= \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{81}enx^3-2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{2}{27}ex^3\log($$

$$= \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{81}enx^3+\frac{2\left(9a^2d+2e\right)n\sqrt{1-a^2x^2}\,\cos^{-1}(ax)}{9a^3}-\frac{2en}{}$$

$$= -\frac{2enx}{27a^2}+\frac{2\left(9a^2d+2e\right)nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{4}{81}enx^3+\frac{2dn\sqrt{1-a^2x}}{a}$$

$$= 2dnx-\frac{2enx}{27a^2}+\frac{2\left(9a^2d+2e\right)nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3+\frac{2dn\sqrt{1}}{}$$

$$= 2dnx+\frac{2enx}{27a^2}+\frac{2\left(9a^2d+2e\right)nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3+\frac{2dn\sqrt{1}}{}$$

$$= 2dnx+\frac{2enx}{27a^2}+\frac{2\left(9a^2d+2e\right)nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3+\frac{2dn\sqrt{1}}{}$$

Mathematica [A] time = 0.82, size = 564, normalized size = 1.15

$$\frac{d\left(ax\left(\cos^{-1}(ax)^2-2\right)-2\sqrt{1-a^2x^2}\,\cos^{-1}(ax)\right)\left(\log\left(cx^n\right)+n(-\log(x))-n\right)}{a}+\frac{2dn\left(\sqrt{1-a^2x^2}\,\cos^{-1}(ax)-i\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*ArcCos[a*x]^2*Log[c*x^n],x]
```

```
[Out] 2*d*n*x + (4*e*n*x)/(9*a^2) + (2*e*n*x^3)/81 + (e*n*(-9*a*x - 12*(1 - a^2*x
^2)^(3/2)*ArcCos[a*x] + Cos[3*ArcCos[a*x]]))/(162*a^3) + (d*n*(-2*a*x - 2*S
qrt[1 - a^2*x^2]*ArcCos[a*x] + a*x*ArcCos[a*x]^2)*Log[x])/a + (e*n*(-12*a*x
 - 2*a^3*x^3 - 12*Sqrt[1 - a^2*x^2]*ArcCos[a*x] - 6*a^2*x^2*Sqrt[1 - a^2*x^
2]*ArcCos[a*x] + 9*a^3*x^3*ArcCos[a*x]^2)*Log[x])/(27*a^3) + (d*(-2*Sqrt[1
- a^2*x^2]*ArcCos[a*x] + a*x*(-2 + ArcCos[a*x]^2))*(-n - n*Log[x] + Log[c*x
^n]))/a + (2*d*n*(a*x + Sqrt[1 - a^2*x^2]*ArcCos[a*x] - ArcCos[a*x]*Log[1 -
 I*E^(I*ArcCos[a*x])] + ArcCos[a*x]*Log[1 + I*E^(I*ArcCos[a*x])] - I*PolyLo
g[2, (-I)*E^(I*ArcCos[a*x])] + I*PolyLog[2, I*E^(I*ArcCos[a*x])]))/a + (4*e
*n*(a*x + Sqrt[1 - a^2*x^2]*ArcCos[a*x] - ArcCos[a*x]*Log[1 - I*E^(I*ArcCos
[a*x])] + ArcCos[a*x]*Log[1 + I*E^(I*ArcCos[a*x])] - I*PolyLog[2, (-I)*E^(I
*ArcCos[a*x])] + I*PolyLog[2, I*E^(I*ArcCos[a*x])]))/(9*a^3) + (e*(-n + 3*(
-(n*Log[x]) + Log[c*x^n]))*(27*a*x*(-2 + ArcCos[a*x]^2) - (2 - 9*ArcCos[a*x
]^2)*Cos[3*ArcCos[a*x]] - 6*ArcCos[a*x]*(9*Sqrt[1 - a^2*x^2] + Sin[3*ArcCos
[a*x]])))/(324*a^3)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ex^2+d\right)\arccos\left(ax\right)^2\log\left(cx^n\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*arccos(a*x)^2*log(c*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex^2+d\right)\arccos\left(ax\right)^2\log\left(cx^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)^2*log(c*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*arccos(a*x)^2*log(c*x^n), x)
```

maple [F] time = 11.59, size = 0, normalized size = 0.00

$$\int\left(e\,x^2+d\right)\arccos\left(ax\right)^2\ln\left(c\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccos(a*x)^2*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arccos(a*x)^2*ln(c*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(ex^3+3\,dx\right)\arctan\left(\sqrt{ax+1}\,\sqrt{-ax+1},ax\right)^2\log\left(x^n\right)-\frac{1}{9}\left(\left(en-3\,e\log(c)\right)x^3+9\left(dn-d\log(c)\right)x\right)\arctan\left(\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

```
[Out] 1/3*(e*x^3 + 3*d*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*log(x^n) -
 1/9*((e*n - 3*e*log(c))*x^3 + 9*(d*n - d*log(c))*x)*arctan2(sqrt(a*x + 1)*
sqrt(-a*x + 1), a*x)^2 - integrate(2/9*(3*(a*e*x^3 + 3*a*d*x)*arctan2(sqrt(
a*x + 1)*sqrt(-a*x + 1), a*x)*log(x^n) - ((a*e*n - 3*a*e*log(c))*x^3 + 9*(a
```

```
*d*n - a*d*log(c))*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))*sqrt(a*x
+ 1)*sqrt(-a*x + 1)/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,\mathrm{acos}\left(a\,x\right)^2\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acos(a*x)^2*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*acos(a*x)^2*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)\log\left(cx^n\right)\mathrm{acos}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acos(a*x)**2*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*acos(a*x)**2, x)
```

3.196 $\int (d+ex^2) \sinh^{-1}(ax)^2 \log(cx^n)\, dx$

Optimal. Leaf size=458

$$-\frac{2d\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\log(cx^n)}{a}-\frac{4ex\log(cx^n)}{9a^2}-\frac{2ex^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\log(cx^n)}{9a}-\frac{4}{9}nx\left(9d-\frac{2e}{a^2}\right)+\frac{2dn\sqrt{\ldots}}{\ldots}$$

```
[Out] -2*d*n*x+2/27*e*n*x/a^2-4/9*(9*d-2*e/a^2)*n*x-2/27*e*n*x^3+2/27*e*n*(a^2*x^
2+1)^(3/2)*arcsinh(a*x)/a^3-d*n*x*arcsinh(a*x)^2-1/9*e*n*x^3*arcsinh(a*x)^2
-4/9*(9*a^2*d-2*e)*n*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))/a^3+2*d*x*
ln(c*x^n)-4/9*e*x*ln(c*x^n)/a^2+2/27*e*x^3*ln(c*x^n)+d*x*arcsinh(a*x)^2*ln(
c*x^n)+1/3*e*x^3*arcsinh(a*x)^2*ln(c*x^n)-2/9*(9*a^2*d-2*e)*n*polylog(2,-a*
x-(a^2*x^2+1)^(1/2))/a^3+2/9*(9*a^2*d-2*e)*n*polylog(2,a*x+(a^2*x^2+1)^(1/2
))/a^3+2*d*n*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a+2/9*(9*a^2*d-2*e)*n*arcsinh(a
*x)*(a^2*x^2+1)^(1/2)/a^3-4/27*e*n*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3+2/27*
e*n*x^2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a-2*d*arcsinh(a*x)*ln(c*x^n)*(a^2*x^
2+1)^(1/2)/a+4/9*e*arcsinh(a*x)*ln(c*x^n)*(a^2*x^2+1)^(1/2)/a^3-2/9*e*x^2*a
rcsinh(a*x)*ln(c*x^n)*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.70, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5706, 5653, 5717, 8, 5661, 5758, 30, 2387, 6, 5742, 5760, 4182, 2279, 2391}

$$-\frac{2n(9a^2d-2e)\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)}{9a^3}+\frac{2n(9a^2d-2e)\,\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)}{9a^3}-\frac{2d\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\log(\ldots}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcSinh[a*x]^2*Log[c*x^n],x]
```

```
[Out] -2*d*n*x + (2*e*n*x)/(27*a^2) - (4*(9*d - (2*e)/a^2)*n*x)/9 - (2*e*n*x^3)/2
7 + (2*d*n*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + (2*(9*a^2*d - 2*e)*n*Sqrt[1
+ a^2*x^2]*ArcSinh[a*x])/(9*a^3) - (4*e*n*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(
27*a^3) + (2*e*n*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a) + (2*e*n*(1 + a
^2*x^2)^(3/2)*ArcSinh[a*x])/(27*a^3) - d*n*x*ArcSinh[a*x]^2 - (e*n*x^3*ArcS
inh[a*x]^2)/9 - (4*(9*a^2*d - 2*e)*n*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]])/
(9*a^3) + 2*d*x*Log[c*x^n] - (4*e*x*Log[c*x^n])/(9*a^2) + (2*e*x^3*Log[c*x^
n])/27 - (2*d*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*Log[c*x^n])/a + (4*e*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]*Log[c*x^n])/(9*a^3) - (2*e*x^2*Sqrt[1 + a^2*x^2]*ArcS
inh[a*x]*Log[c*x^n])/(9*a) + d*x*ArcSinh[a*x]^2*Log[c*x^n] + (e*x^3*ArcSinh
[a*x]^2*Log[c*x^n])/3 - (2*(9*a^2*d - 2*e)*n*PolyLog[2, -E^ArcSinh[a*x]])/(
9*a^3) + (2*(9*a^2*d - 2*e)*n*PolyLog[2, E^ArcSinh[a*x]])/(9*a^3)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
```

```
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)\sinh^{-1}(ax)^2\log\left(cx^n\right)\,dx = 2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}+\frac{2}{27}ex^3\log\left(cx^n\right)-\frac{2d\sqrt{1+a^2x^2}\,\sinh^{-1}(a}{a}$$

$$= 2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}+\frac{2}{27}ex^3\log\left(cx^n\right)-\frac{2d\sqrt{1+a^2x^2}\,\sinh^{-1}(a}{a}$$

$$= -\frac{2}{9}\left(9d-\frac{2e}{a^2}\right)nx-\frac{2}{81}enx^3+2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}+\frac{2}{27}ex^3\log($$

$$= -\frac{2}{9}\left(9d-\frac{2e}{a^2}\right)nx-\frac{2}{81}enx^3+\frac{2\left(9d-\frac{2e}{a^2}\right)n\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a}+\frac{2en\,(}{}$$

$$= -\frac{2enx}{27a^2}-\frac{4}{9}\left(9d-\frac{2e}{a^2}\right)nx-\frac{4}{81}enx^3+\frac{2dn\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a}+\frac{2\left(9d\right.}{}$$

$$= -2dnx-\frac{2enx}{27a^2}-\frac{4}{9}\left(9d-\frac{2e}{a^2}\right)nx-\frac{2}{27}enx^3+\frac{2dn\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a}$$

$$= -2dnx+\frac{2enx}{27a^2}-\frac{4}{9}\left(9d-\frac{2e}{a^2}\right)nx-\frac{2}{27}enx^3+\frac{2dn\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a}$$

$$= -2dnx+\frac{2enx}{27a^2}-\frac{4}{9}\left(9d-\frac{2e}{a^2}\right)nx-\frac{2}{27}enx^3+\frac{2dn\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a}$$

Mathematica [A] time = 0.73, size = 516, normalized size = 1.13

$$\frac{d\left(ax\left(\sinh^{-1}(ax)^2+2\right)-2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\right)\left(\log\left(cx^n\right)+n(-\log(x))-n\right)}{a}+\frac{2dn\left(\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*ArcSinh[a*x]^2*Log[c*x^n],x]
```

```
[Out] -2*d*n*x + (4*e*n*x)/(9*a^2) - (2*e*n*x^3)/81 + (2*e*n*(-1/3*(a*x) - (a^3*x
^3)/9 + ((1 + a^2*x^2)^(3/2)*ArcSinh[a*x])/3))/(9*a^3) + (d*n*(2*a*x - 2*Sq
rt[1 + a^2*x^2]*ArcSinh[a*x] + a*x*ArcSinh[a*x]^2)*Log[x])/a + (e*n*(-12*a*
x + 2*a^3*x^3 + 12*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] - 6*a^2*x^2*Sqrt[1 + a^2*
x^2]*ArcSinh[a*x] + 9*a^3*x^3*ArcSinh[a*x]^2)*Log[x])/(27*a^3) + (d*(-2*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x] + a*x*(2 + ArcSinh[a*x]^2))*(-n - n*Log[x] + Lo
g[c*x^n]))/a + (e*(27*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + a*x*(-26 - 9*ArcSinh
[a*x]^2 + (2 + 9*ArcSinh[a*x]^2)*Cosh[2*ArcSinh[a*x]]) - 3*ArcSinh[a*x]*Cos
h[3*ArcSinh[a*x]])*(-n + 3*(-(n*Log[x]) + Log[c*x^n])))/(162*a^3) + (2*d*n*
(-(a*x) + Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]*Log[1 - E^(-ArcSinh
[a*x])] - ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x])] + PolyLog[2, -E^(-ArcSinh
[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]))/a - (4*e*n*(-(a*x) + Sqrt[1 + a^2
*x^2]*ArcSinh[a*x] + ArcSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] - ArcSinh[a*x]
*Log[1 + E^(-ArcSinh[a*x])] + PolyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E
^(-ArcSinh[a*x])]))/(9*a^3)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^2+d\right)\text{arsinh}\left(ax\right)^2\log\left(cx^n\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*arcsinh(a*x)^2*log(c*x^n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)^2*log(c*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)\text{arcsinh}\left(ax\right)^2\ln\left(c\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arcsinh(a*x)^2*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arcsinh(a*x)^2*ln(c*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{9}\left(\left(en-3\,e\log(c)\right)x^3+9\left(dn-d\log(c)\right)x-3\left(ex^3+3\,dx\right)\log\left(x^n\right)\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2-\int -\frac{2\left(\left(en-3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/9*((e*n - 3*e*log(c))*x^3 + 9*(d*n - d*log(c))*x - 3*(e*x^3 + 3*d*x)*log
(x^n))*log(a*x + sqrt(a^2*x^2 + 1))^2 - integrate(-2/9*((e*n - 3*e*log(c))*
```

```
a^3*x^5 + (9*(d*n - d*log(c))*a^3 + (e*n - 3*e*log(c))*a)*x^3 + 9*(d*n - d*
log(c))*a*x - 3*(a^3*e*x^5 + (3*a^3*d + a*e)*x^3 + 3*a*d*x)*log(x^n) + ((e*
n - 3*e*log(c))*a^2*x^4 + 9*(d*n - d*log(c))*a^2*x^2 - 3*(a^2*e*x^4 + 3*a^2
*d*x^2)*log(x^n))*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*x^3
+ a*x + (a^2*x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,\mathrm{asinh}\left(a\,x\right)^2\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*asinh(a*x)^2*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*asinh(a*x)^2*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex^{2}\right) \log{\left(cx^{n} \right)} \operatorname{asinh}^{2}{\left(ax \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*asinh(a*x)**2*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*asinh(a*x)**2, x)
```

3.197 $\int \left(d+ex^2\right)\cosh^{-1}(ax)^2\log\left(cx^n\right)\,dx$

Optimal. Leaf size=508

$$-\frac{4e\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)\log\left(cx^n\right)}{9a^3}+\frac{2en(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)}{27a^3}+\frac{4en\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}}{27a^3}$$

```
[Out] -2*d*n*x-2/27*e*n*x/a^2-4/9*(9*d+2*e/a^2)*n*x-2/27*e*n*x^3+2/27*e*n*(a*x-1)
^(3/2)*(a*x+1)^(3/2)*arccosh(a*x)/a^3-d*n*x*arccosh(a*x)^2-1/9*e*n*x^3*arcc
osh(a*x)^2-4/9*(9*a^2*d+2*e)*n*arccosh(a*x)*arctan(a*x+(a*x-1)^(1/2)*(a*x+1
)^(1/2))/a^3+2*d*x*ln(c*x^n)+4/9*e*x*ln(c*x^n)/a^2+2/27*e*x^3*ln(c*x^n)+d*x
*arccosh(a*x)^2*ln(c*x^n)+1/3*e*x^3*arccosh(a*x)^2*ln(c*x^n)+2/9*I*(9*a^2*d
+2*e)*n*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/a^3-2/9*I*(9*a^2*d+
2*e)*n*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/a^3+2*d*n*arccosh(a*x
)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+4/27*e*n*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)
^(1/2)/a^3+2/9*(9*a^2*d+2*e)*n*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3
+2/27*e*n*x^2*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-2*d*arccosh(a*x)*l
n(c*x^n)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-4/9*e*arccosh(a*x)*ln(c*x^n)*(a*x-1)
^(1/2)*(a*x+1)^(1/2)/a^3-2/9*e*x^2*arccosh(a*x)*ln(c*x^n)*(a*x-1)^(1/2)*(a*
x+1)^(1/2)/a
```

Rubi [A] time = 1.55, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5707, 5654, 5718, 8, 5662, 5759, 30, 2387, 6, 5743, 5761, 4180, 2279, 2391}

$$\frac{2in\left(9a^2d+2e\right)\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{9a^3}-\frac{2in\left(9a^2d+2e\right)\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{9a^3}+\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{4e\sqrt{ax-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCosh[a*x]^2*Log[c*x^n],x]
```

```
[Out] -2*d*n*x - (2*e*n*x)/(27*a^2) - (4*(9*d + (2*e)/a^2)*n*x)/9 - (2*e*n*x^3)/2
7 + (2*d*n*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a + (4*e*n*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(27*a^3) + (2*(9*a^2*d + 2*e)*n*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(9*a^3) + (2*e*n*x^2*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*ArcCosh[a*x])/(27*a) + (2*e*n*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcC
osh[a*x])/(27*a^3) - d*n*x*ArcCosh[a*x]^2 - (e*n*x^3*ArcCosh[a*x]^2)/9 - (4
*(9*a^2*d + 2*e)*n*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/(9*a^3) + 2*d*x*Log
[c*x^n] + (4*e*x*Log[c*x^n])/(9*a^2) + (2*e*x^3*Log[c*x^n])/27 - (2*d*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*Log[c*x^n])/a - (4*e*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]*ArcCosh[a*x]*Log[c*x^n])/(9*a^3) - (2*e*x^2*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*ArcCosh[a*x]*Log[c*x^n])/(9*a) + d*x*ArcCosh[a*x]^2*Log[c*x^n] +
(e*x^3*ArcCosh[a*x]^2*Log[c*x^n])/3 + (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[
2, (-I)*E^ArcCosh[a*x]])/a^3 - (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[2, I*E^
ArcCosh[a*x]])/a^3
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
```

```
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)\cosh^{-1}(ax)^2\log\left(cx^n\right)\,dx = 2dx\log\left(cx^n\right)+\frac{4ex\log\left(cx^n\right)}{9a^2}+\frac{2}{27}ex^3\log\left(cx^n\right)-\frac{2d\sqrt{-1+ax}\,\sqrt{1+ax}}{a}$$

$$= 2dx\log\left(cx^n\right)+\frac{4ex\log\left(cx^n\right)}{9a^2}+\frac{2}{27}ex^3\log\left(cx^n\right)-\frac{2d\sqrt{-1+ax}\,\sqrt{1+ax}}{a}$$

$$= -\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx-\frac{2}{81}enx^3+2dx\log\left(cx^n\right)+\frac{4ex\log\left(cx^n\right)}{9a^2}+\frac{2}{27}ex^3\log$$

$$= -\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx-\frac{2}{81}enx^3+\frac{2\left(9a^2d+2e\right)n\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(}{9a^3}$$

$$= \frac{2enx}{27a^2}-\frac{2\left(9a^2d+2e\right)nx}{9a^2}-\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx-\frac{4}{81}enx^3+\frac{2dn\sqrt{-1+ax}\,\sqrt{}}{}$$

$$= -2dnx+\frac{2enx}{27a^2}-\frac{2\left(9a^2d+2e\right)nx}{9a^2}-\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx-\frac{2}{27}enx^3+\frac{2dn\sqrt{-}}{}$$

$$= -2dnx-\frac{2enx}{27a^2}-\frac{2\left(9a^2d+2e\right)nx}{9a^2}-\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx-\frac{2}{27}enx^3+\frac{2dn\sqrt{-}}{}$$

$$= -2dnx-\frac{2enx}{27a^2}-\frac{2\left(9a^2d+2e\right)nx}{9a^2}-\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx-\frac{2}{27}enx^3+\frac{2dn\sqrt{-}}{}$$

Mathematica [A] time = 3.88, size = 619, normalized size = 1.22

$$\frac{-648a^3dnx-8a^3enx^3+324a^2d\left(2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\cosh^{-1}(ax)-ax\left(\cosh^{-1}(ax)^2+2\right)\right)\left(-\log\left(cx^n\right)+n\log(x)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*ArcCosh[a*x]^2*Log[c*x^n],x]
```

```
[Out] (-648*a^3*d*n*x - 144*a*e*n*x - 8*a^3*e*n*x^3 + 2*e*n*(9*a*x + 12*((-1 + a*
x)/(1 + a*x))^(3/2)*(1 + a*x)^3*ArcCosh[a*x] - Cosh[3*ArcCosh[a*x]]) + 324*
a^2*d*n*(2*a*x - 2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + a*x*ArcCosh[
a*x]^2)*Log[x] + 12*e*n*(2*a*x*(6 + a^2*x^2) - 6*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]*(2 + a^2*x^2)*ArcCosh[a*x] + 9*a^3*x^3*ArcCosh[a*x]^2)*Log[x] + 324*a^2*
d*(2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x] - a*x*(2 + ArcCosh[a
*x]^2))*(n + n*Log[x] - Log[c*x^n]) + 648*a^2*d*n*(-(a*x) + Sqrt[(-1 + a*x)
/(1 + a*x)]*ArcCosh[a*x] + a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + I*
ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]] - I*ArcCosh[a*x]*Log[1 + I/E^ArcCosh
[a*x]] + I*PolyLog[2, (-I)/E^ArcCosh[a*x]] - I*PolyLog[2, I/E^ArcCosh[a*x]]
) + 144*e*n*(-(a*x) + Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + a*x*Sqrt[(-
1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + I*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]]
 - I*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] + I*PolyLog[2, (-I)/E^ArcCosh[a
*x]] - I*PolyLog[2, I/E^ArcCosh[a*x]]) - e*(n + 3*n*Log[x] - 3*Log[c*x^n])*
(27*a*x*(2 + ArcCosh[a*x]^2) + (2 + 9*ArcCosh[a*x]^2)*Cosh[3*ArcCosh[a*x]]
- 6*ArcCosh[a*x]*(9*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + Sinh[3*ArcCosh[a
*x]])))/(324*a^3)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^2+d\right)\text{arcosh}\left(ax\right)^2\log\left(cx^n\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*arccosh(a*x)^2*log(c*x^n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)^2*log(c*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.64sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right) \operatorname{arccosh}\left(a x\right)^{2} \ln\left(c\,x^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccosh(a*x)^2*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arccosh(a*x)^2*ln(c*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{9}\left(\left(en-3\,e\log(c)\right)x^{3}+9\left(dn-d\log(c)\right)x-3\left(ex^{3}+3\,dx\right)\log\left(x^{n}\right)\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)^{2}-\int -\frac{2\left(\left(e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/9*((e*n - 3*e*log(c))*x^3 + 9*(d*n - d*log(c))*x - 3*(e*x^3 + 3*d*x)*log
(x^n))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2 - integrate(-2/9*((e*n - 3*
e*log(c))*a^3*x^5 + (9*(d*n - d*log(c))*a^3 - (e*n - 3*e*log(c))*a)*x^3 - 9
*(d*n - d*log(c))*a*x + ((e*n - 3*e*log(c))*a^2*x^4 + 9*(d*n - d*log(c))*a^
2*x^2 - 3*(a^2*e*x^4 + 3*a^2*d*x^2)*log(x^n))*sqrt(a*x + 1)*sqrt(a*x - 1) -
 3*(a^3*e*x^5 + (3*a^3*d - a*e)*x^3 - 3*a*d*x)*log(x^n))*log(a*x + sqrt(a*x
 + 1)*sqrt(a*x - 1))/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) -
 a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^{n}\right)\,\mathrm{acosh}\left(a\,x\right)^{2}\,\left(e\,x^{2}+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acosh(a*x)^2*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*acosh(a*x)^2*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^{2}\right)\log\left(cx^{n}\right)\operatorname{acosh}^{2}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acosh(a*x)**2*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*acosh(a*x)**2, x)
```

3.198 $\int \frac{(a+b\log(cx^n))^p \mathbf{Li}_k(ex^q)}{x} dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\text{Li}_k(ex^q)\left(a+b\log(cx^n)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))^p*polylog(k,e*x^q)/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log(cx^n)\right)^p \text{PolyLog}(k,ex^q)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x,x]
[Out] Defer[Int][((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x, x]
```

Rubi steps

$$\int \frac{\left(a+b\log(cx^n)\right)^p \text{Li}_k(ex^q)}{x} dx = \int \frac{\left(a+b\log(cx^n)\right)^p \text{Li}_k(ex^q)}{x} dx$$

Mathematica [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log(cx^n)\right)^p \text{Li}_k(ex^q)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x,x]
[Out] Integrate[((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x, x]
```

fricas [A] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log(cx^n)+a\right)^p \text{polylog}(k,ex^q)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*polylog(k,e*x^q)/x,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)^p*polylog(k, e*x^q)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log(cx^n)+a\right)^p \text{Li}_k(ex^q)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*polylog(k,e*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^p*polylog(k, e*x^q)/x, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(b \ln (c x^n) + a)^p \operatorname{polylog}(k, e x^q)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^p*polylog(k,e*x^q)/x,x)
[Out] int((b*ln(c*x^n)+a)^p*polylog(k,e*x^q)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log (cx^n) + a\right)^p \operatorname{Li}_k(ex^q)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*polylog(k,e*x^q)/x,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)^p*polylog(k, e*x^q)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{polylog}(k, e x^q) \ (a + b \ \ln (c x^n))^p}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^p)/x,x)
[Out] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^p)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log (cx^n)\right)^p \operatorname{Li}_k(ex^q)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*polylog(k,e*x**q)/x,x)
[Out] Integral((a + b*log(c*x**n))**p*polylog(k, e*x**q)/x, x)
```

3.199 $\int \frac{(a+b\log(cx^n))^3\mathbf{Li}_k(ex^q)}{x}\,dx$

Optimal. Leaf size=104

$$\frac{6b^2n^2\text{Li}_{k+3}(ex^q)\left(a+b\log(cx^n)\right)}{q^3}-\frac{3bn\text{Li}_{k+2}(ex^q)\left(a+b\log(cx^n)\right)^2}{q^2}+\frac{\text{Li}_{k+1}(ex^q)\left(a+b\log(cx^n)\right)^3}{q}-\frac{6b^3n^3\text{Li}_{k+4}(}{q^4}$$

```
[Out] (a+b*ln(c*x^n))^3*polylog(1+k,e*x^q)/q-3*b*n*(a+b*ln(c*x^n))^2*polylog(2+k,
e*x^q)/q^2+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(3+k,e*x^q)/q^3-6*b^3*n^3*polyl
og(4+k,e*x^q)/q^4
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2383, 6589}

$$\frac{6b^2n^2\text{PolyLog}(k+3,ex^q)\left(a+b\log(cx^n)\right)}{q^3}-\frac{3bn\text{PolyLog}(k+2,ex^q)\left(a+b\log(cx^n)\right)^2}{q^2}+\frac{\text{PolyLog}(k+1,ex^q)\left(a\right.}{q}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*PolyLog[k, e*x^q])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^3*PolyLog[1 + k, e*x^q])/q - (3*b*n*(a + b*Log[c*x^n])^
2*PolyLog[2 + k, e*x^q])/q^2 + (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3 + k,
 e*x^q])/q^3 - (6*b^3*n^3*PolyLog[4 + k, e*x^q])/q^4
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\log(cx^n)\right)^3\text{Li}_k(ex^q)}{x}\,dx &= \frac{\left(a+b\log(cx^n)\right)^3\text{Li}_{1+k}(ex^q)}{q}-\frac{(3bn)\int \frac{\left(a+b\log(cx^n)\right)^2\text{Li}_{1+k}(ex^q)}{x}\,dx}{q}\\
&= \frac{\left(a+b\log(cx^n)\right)^3\text{Li}_{1+k}(ex^q)}{q}-\frac{3bn\left(a+b\log(cx^n)\right)^2\text{Li}_{2+k}(ex^q)}{q^2}+\frac{\left(6b^2n^2\right)\int}{}\\
&= \frac{\left(a+b\log(cx^n)\right)^3\text{Li}_{1+k}(ex^q)}{q}-\frac{3bn\left(a+b\log(cx^n)\right)^2\text{Li}_{2+k}(ex^q)}{q^2}+\frac{6b^2n^2\left(a\right.}{}\\
&= \frac{\left(a+b\log(cx^n)\right)^3\text{Li}_{1+k}(ex^q)}{q}-\frac{3bn\left(a+b\log(cx^n)\right)^2\text{Li}_{2+k}(ex^q)}{q^2}+\frac{6b^2n^2\left(a\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 99, normalized size = 0.95

$$\frac{q^3\text{Li}_{k+1}(ex^q)\left(a+b\log(cx^n)\right)^3-3bn\left(q^2\text{Li}_{k+2}(ex^q)\left(a+b\log(cx^n)\right)^2+2bn\left(bn\text{Li}_{k+4}(ex^q)-q\text{Li}_{k+3}(ex^q)\left(a+b\right.\right.\right.}{q^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*PolyLog[k, e*x^q])/x,x]
```

```
[Out] (q^3*(a + b*Log[c*x^n])^3*PolyLog[1 + k, e*x^q] - 3*b*n*(q^2*(a + b*Log[c*x
^n])^2*PolyLog[2 + k, e*x^q] + 2*b*n*(-(q*(a + b*Log[c*x^n])*PolyLog[3 + k,
 e*x^q]) + b*n*PolyLog[4 + k, e*x^q])))/q^4
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3 \log\left(cx^n\right)^3 + 3\,ab^2 \log\left(cx^n\right)^2 + 3\,a^2 b \log\left(cx^n\right) + a^3\right)\text{polylog}\left(k, ex^q\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*polylog(k,e*x^q)/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*polylog(k, e*x^q)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^n\right) + a\right)^3 \text{Li}_k(ex^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*polylog(k,e*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*polylog(k, e*x^q)/x, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^n\right) + a\right)^3 \text{polylog}\left(k, e\,x^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^3*polylog(k,e*x^q)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^3*polylog(k,e*x^q)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^n\right) + a\right)^3 \text{Li}_k(ex^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*polylog(k,e*x^q)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*polylog(k, e*x^q)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{polylog}\left(k, e\,x^q\right)\,\left(a + b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(cx^{n}\right)\right)^{3} \operatorname{Li}_{k}\left(ex^{q}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*polylog(k,e*x**q)/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))**3*polylog(k, e*x**q)/x, x)
```

3.200 $\int \frac{\left(a+b\log(cx^n)\right)^2\mathbf{Li}_k(ex^q)}{x}\,dx$

Optimal. Leaf size=72

$$-\frac{2bn\text{Li}_{k+2}\left(ex^q\right)\left(a+b\log\left(cx^n\right)\right)}{q^2}+\frac{\text{Li}_{k+1}\left(ex^q\right)\left(a+b\log\left(cx^n\right)\right)^2}{q}+\frac{2b^2n^2\text{Li}_{k+3}\left(ex^q\right)}{q^3}$$

```
[Out] (a+b*ln(c*x^n))^2*polylog(1+k,e*x^q)/q-2*b*n*(a+b*ln(c*x^n))*polylog(2+k,e*
x^q)/q^2+2*b^2*n^2*polylog(3+k,e*x^q)/q^3
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2383, 6589}

$$-\frac{2bn\text{PolyLog}\left(k+2,ex^q\right)\left(a+b\log\left(cx^n\right)\right)}{q^2}+\frac{\text{PolyLog}\left(k+1,ex^q\right)\left(a+b\log\left(cx^n\right)\right)^2}{q}+\frac{2b^2n^2\text{PolyLog}\left(k+3,ex\right.}{q^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*PolyLog[k, e*x^q])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^2*PolyLog[1 + k, e*x^q])/q - (2*b*n*(a + b*Log[c*x^n])*
PolyLog[2 + k, e*x^q])/q^2 + (2*b^2*n^2*PolyLog[3 + k, e*x^q])/q^3
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_k\left(ex^q\right)}{x}\,dx &= \frac{\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_{1+k}\left(ex^q\right)}{q}-\frac{(2bn)\int\frac{\left(a+b\log(cx^n)\right)\text{Li}_{1+k}(ex^q)}{x}\,dx}{q}\\ &= \frac{\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_{1+k}\left(ex^q\right)}{q}-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\text{Li}_{2+k}\left(ex^q\right)}{q^2}+\frac{\left(2b^2n^2\right)}{}\\ &= \frac{\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_{1+k}\left(ex^q\right)}{q}-\frac{2bn\left(a+b\log\left(cx^n\right)\right)\text{Li}_{2+k}\left(ex^q\right)}{q^2}+\frac{2b^2n^2\text{L}}{}\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.96

$$\frac{q^2\text{Li}_{k+1}\left(ex^q\right)\left(a+b\log\left(cx^n\right)\right)^2+2bn\left(bn\text{Li}_{k+3}\left(ex^q\right)-q\text{Li}_{k+2}\left(ex^q\right)\left(a+b\log\left(cx^n\right)\right)\right)}{q^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*PolyLog[k, e*x^q])/x,x]
```

```
[Out] (q^2*(a + b*Log[c*x^n])^2*PolyLog[1 + k, e*x^q] + 2*b*n*(-(q*(a + b*Log[c*x
^n])*PolyLog[2 + k, e*x^q]) + b*n*PolyLog[3 + k, e*x^q]))/q^3
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2 \log\left(cx^n\right)^2 + 2\,ab\log\left(cx^n\right) + a^2\right)\text{polylog}\left(k, ex^q\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*polylog(k,e*x^q)/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*polylog(k, e*x^q)/x, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right) + a\right)^2 \text{Li}_k(ex^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*polylog(k,e*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*polylog(k, e*x^q)/x, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right) + a\right)^2 \text{polylog}\left(k, e\,x^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)^2*polylog(k,e*x^q)/x,x)
```

```
[Out] int((b*ln(c*x^n)+a)^2*polylog(k,e*x^q)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right) + a\right)^2 \text{Li}_k(ex^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*polylog(k,e*x^q)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*polylog(k, e*x^q)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{polylog}\left(k, e\,x^q\right)\,\left(a + b\,\ln\left(c\,x^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(cx^n\right)\right)^2 \text{Li}_k\left(ex^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*polylog(k,e*x**q)/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))**2*polylog(k, e*x**q)/x, x)
```

3.201 $\int \frac{(a+b\log(cx^n))\mathbf{Li}_k(ex^q)}{x}\,dx$

Optimal. Leaf size=40

$$\frac{\operatorname{Li}_{k+1}(ex^q)\left(a+b\log(cx^n)\right)}{q}-\frac{bn\operatorname{Li}_{k+2}(ex^q)}{q^2}$$

```
[Out] (a+b*ln(c*x^n))*polylog(1+k,e*x^q)/q-b*n*polylog(2+k,e*x^q)/q^2
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2383, 6589}

$$\frac{\operatorname{PolyLog}(k+1,ex^q)\left(a+b\log(cx^n)\right)}{q}-\frac{bn\operatorname{PolyLog}(k+2,ex^q)}{q^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[k, e*x^q])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])*PolyLog[1 + k, e*x^q])/q - (b*n*PolyLog[2 + k, e*x^q])/
q^2
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log(cx^n)\right)\operatorname{Li}_k(ex^q)}{x}\,dx &= \frac{\left(a+b\log(cx^n)\right)\operatorname{Li}_{1+k}(ex^q)}{q}-\frac{(bn)\int \frac{\operatorname{Li}_{1+k}(ex^q)}{x}\,dx}{q}\\ &= \frac{\left(a+b\log(cx^n)\right)\operatorname{Li}_{1+k}(ex^q)}{q}-\frac{bn\operatorname{Li}_{2+k}(ex^q)}{q^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 51, normalized size = 1.28

$$\frac{a\operatorname{Li}_{k+1}(ex^q)}{q}+\frac{b\log(cx^n)\operatorname{Li}_{k+1}(ex^q)}{q}-\frac{bn\operatorname{Li}_{k+2}(ex^q)}{q^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[k, e*x^q])/x,x]
```

```
[Out] (a*PolyLog[1 + k, e*x^q])/q + (b*Log[c*x^n]*PolyLog[1 + k, e*x^q])/q - (b*n
*PolyLog[2 + k, e*x^q])/q^2
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cx^n\right)+a\right)\text{polylog}\left(k,ex^q\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(k,e*x^q)/x,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*polylog(k, e*x^q)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)\text{Li}_k(ex^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(k,e*x^q)/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*polylog(k, e*x^q)/x, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\,x^n\right)+a\right)\text{polylog}\left(k,e\,x^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*polylog(k,e*x^q)/x,x)
[Out] int((b*ln(c*x^n)+a)*polylog(k,e*x^q)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(cx^n\right)+a\right)\text{Li}_k(ex^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(k,e*x^q)/x,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*polylog(k, e*x^q)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\text{polylog}\left(k,e\,x^q\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(k, e*x^q)*(a + b*log(c*x^n)))/x,x)
[Out] int((polylog(k, e*x^q)*(a + b*log(c*x^n)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_k\left(ex^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(k,e*x**q)/x,x)
[Out] Integral((a + b*log(c*x**n))*polylog(k, e*x**q)/x, x)
```

3.202 $\int \frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)},x\right)$$

```
[Out] Unintegrable(polylog(k,e*x^q)/x/(a+b*ln(c*x^n)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{PolyLog}\left(k,ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])),x]
[Out] Defer[Int][PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\int \frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])),x]
[Out] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])), x]
```

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{polylog}\left(k,ex^q\right)}{bx\log\left(cx^n\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(polylog(k, e*x^q)/(b*x*log(c*x^n) + a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_k(ex^q)}{\left(b\log\left(cx^n\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)*x), x)
```

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{polylog}\left(k, e\,x^{q}\right)}{\left(b \ln\left(c\,x^{n}\right)+a\right) x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k,e*x^q)/x/(b*ln(c*x^n)+a),x)
[Out] int(polylog(k,e*x^q)/x/(b*ln(c*x^n)+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{k}(ex^{q})}{\left(b \log\left(cx^{n}\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{polylog}\left(k, e\,x^{q}\right)}{x\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))),x)
[Out] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{k}\left(ex^{q}\right)}{x\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x**q)/x/(a+b*ln(c*x**n)),x)
[Out] Integral(polylog(k, e*x**q)/(x*(a + b*log(c*x**n))), x)
```

3.203 $\int \frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))^2}\,dx$

Optimal. Leaf size=64

$$\frac{q\text{Int}\left(\frac{\text{Li}_{k-1}(ex^q)}{x(a+b\log(cx^n))},x\right)}{bn}-\frac{\text{Li}_k\left(ex^q\right)}{bn\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] -polylog(k,e*x^q)/b/n/(a+b*ln(c*x^n))+q*Unintegrable(polylog(-1+k,e*x^q)/x/
(a+b*ln(c*x^n)),x)/b/n
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{PolyLog}\left(k,ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^2),x]
```

```
[Out] -(PolyLog[k, e*x^q]/(b*n*(a + b*Log[c*x^n]))) + (q*Defer[Int][PolyLog[-1 +
k, e*x^q]/(x*(a + b*Log[c*x^n])), x])/(b*n)
```

Rubi steps

$$\int \frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx=-\frac{\text{Li}_k\left(ex^q\right)}{bn\left(a+b\log\left(cx^n\right)\right)}+\frac{q\int \frac{\text{Li}_{-1+k}(ex^q)}{x(a+b\log(cx^n))}\,dx}{bn}$$

Mathematica [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^2),x]
```

```
[Out] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^2), x]
```

fricas [A] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{polylog}\left(k,ex^q\right)}{b^2x\log\left(cx^n\right)^2+2\,abx\log\left(cx^n\right)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(polylog(k, e*x^q)/(b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x
), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_k(ex^q)}{\left(b\log\left(cx^n\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)^2*x), x)
```

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{polylog}\left(k, e\,x^{q}\right)}{\left(b \ln \left(c\,x^{n}\right)+a\right)^{2} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k,e*x^q)/x/(b*ln(c*x^n)+a)^2,x)
[Out] int(polylog(k,e*x^q)/x/(b*ln(c*x^n)+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_{k}(ex^{q})}{\left(b \log \left(cx^{n}\right)+a\right)^{2} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)^2*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{polylog}\left(k, e\,x^{q}\right)}{x\,\left(a+b\,\ln \left(c\,x^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^2),x)
[Out] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_{k}\left(ex^{q}\right)}{x \left(a+b \log \left(cx^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x**q)/x/(a+b*ln(c*x**n))**2,x)
[Out] Integral(polylog(k, e*x**q)/(x*(a + b*log(c*x**n))**2), x)
```

3.204 $\int \frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))^3} \, dx$

Optimal. Leaf size=103

$$\frac{q^2\text{Int}\left(\frac{\text{Li}_{k-2}(ex^q)}{x(a+b\log(cx^n))},x\right)}{2b^2n^2}-\frac{q\text{Li}_{k-1}\left(ex^q\right)}{2b^2n^2\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{Li}_k\left(ex^q\right)}{2bn\left(a+b\log\left(cx^n\right)\right)^2}$$

```
[Out] -1/2*q*polylog(-1+k,e*x^q)/b^2/n^2/(a+b*ln(c*x^n))-1/2*polylog(k,e*x^q)/b/n
/(a+b*ln(c*x^n))^2+1/2*q^2*Unintegrable(polylog(-2+k,e*x^q)/x/(a+b*ln(c*x^n
)),x)/b^2/n^2
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{PolyLog}\left(k,ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^3),x]
```

```
[Out] -(q*PolyLog[-1 + k, e*x^q])/(2*b^2*n^2*(a + b*Log[c*x^n])) - PolyLog[k, e*x
^q]/(2*b*n*(a + b*Log[c*x^n])^2) + (q^2*Defer[Int][PolyLog[-2 + k, e*x^q]/(
x*(a + b*Log[c*x^n])), x])/(2*b^2*n^2)
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^3} \, dx &= -\frac{\text{Li}_k\left(ex^q\right)}{2bn\left(a+b\log\left(cx^n\right)\right)^2}+\frac{q\int \frac{\text{Li}_{-1+k}(ex^q)}{x(a+b\log(cx^n))^2} \, dx}{2bn}\\ &= -\frac{q\text{Li}_{-1+k}\left(ex^q\right)}{2b^2n^2\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{Li}_k\left(ex^q\right)}{2bn\left(a+b\log\left(cx^n\right)\right)^2}+\frac{q^2\int \frac{\text{Li}_{-2+k}(ex^q)}{x(a+b\log(cx^n))} \, dx}{2b^2n^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^3),x]
```

```
[Out] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^3), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{polylog}\left(k,ex^q\right)}{b^3x\log\left(cx^n\right)^3+3\,ab^2x\log\left(cx^n\right)^2+3\,a^2bx\log\left(cx^n\right)+a^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(polylog(k, e*x^q)/(b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3
*a^2*b*x*log(c*x^n) + a^3*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_k(ex^q)}{\left(b\log\left(cx^n\right)+a\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^3,x, algorithm="giac")
[Out] integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)^3*x), x)
```

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{polylog}\left(k, e\,x^q\right)}{\left(b\ln\left(c\,x^n\right)+a\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k,e*x^q)/x/(b*ln(c*x^n)+a)^3,x)
[Out] int(polylog(k,e*x^q)/x/(b*ln(c*x^n)+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_k(ex^q)}{\left(b\log\left(cx^n\right)+a\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^3,x, algorithm="maxima")
[Out] integrate(polylog(k, e*x^q)/((b*log(c*x^n) + a)^3*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}\left(k, e\,x^q\right)}{x\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^3),x)
[Out] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_{k}\left(ex^{q}\right)}{x\left(a+b\log\left(cx^{n}\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x**q)/x/(a+b*ln(c*x**n))**3,x)
[Out] Integral(polylog(k, e*x**q)/(x*(a + b*log(c*x**n))**3), x)
```

3.205 $\int \frac{\log(x)\mathbf{Li}_n(ax)}{x}\,dx$

Optimal. Leaf size=20

$$\log(x)\mathrm{Li}_{n+1}(ax) - \mathrm{Li}_{n+2}(ax)$$

```
[Out] ln(x)*polylog(1+n,a*x)-polylog(2+n,a*x)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2383, 6589}

$$\log(x)\mathrm{PolyLog}(n+1, ax) - \mathrm{PolyLog}(n+2, ax)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]*PolyLog[n, a*x])/x,x]
[Out] Log[x]*PolyLog[1 + n, a*x] - PolyLog[2 + n, a*x]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)\mathrm{Li}_n(ax)}{x}\,dx &= \log(x)\mathrm{Li}_{1+n}(ax) - \int \frac{\mathrm{Li}_{1+n}(ax)}{x}\,dx\\ &= \log(x)\mathrm{Li}_{1+n}(ax) - \mathrm{Li}_{2+n}(ax)\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\log(x)\mathrm{Li}_{n+1}(ax) - \mathrm{Li}_{n+2}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]*PolyLog[n, a*x])/x,x]
[Out] Log[x]*PolyLog[1 + n, a*x] - PolyLog[2 + n, a*x]
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log(x)\mathrm{polylog}(n, ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*polylog(n,a*x)/x,x, algorithm="fricas")
[Out] integral(log(x)*polylog(n, a*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)\text{Li}_n(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*polylog(n,a*x)/x,x, algorithm="giac")
[Out] integrate(log(x)*polylog(n, a*x)/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\text{polylog}\,(n,ax)\ln(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*polylog(n,a*x)/x,x)
[Out] int(ln(x)*polylog(n,a*x)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)\text{Li}_n(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*polylog(n,a*x)/x,x, algorithm="maxima")
[Out] integrate(log(x)*polylog(n, a*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\ln(x)\,\text{polylog}\,(n,a\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)*polylog(n, a*x))/x,x)
[Out] int((log(x)*polylog(n, a*x))/x, x)
```

sympy [A] time = 2.33, size = 15, normalized size = 0.75

$$\log(x)\,\text{Li}_{n+1}(ax) - \text{Li}_{n+2}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*polylog(n,a*x)/x,x)
[Out] log(x)*polylog(n + 1, a*x) - polylog(n + 2, a*x)
```

3.206 $\int \frac{\log^2(x)\mathbf{Li}_n(ax)}{x}\,dx$

Optimal. Leaf size=33

$$2\mathrm{Li}_{n+3}(ax)+\log^2(x)\mathrm{Li}_{n+1}(ax)-2\log(x)\mathrm{Li}_{n+2}(ax)$$

```
[Out] ln(x)^2*polylog(1+n,a*x)-2*ln(x)*polylog(2+n,a*x)+2*polylog(3+n,a*x)
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2383, 6589}

$$2\mathrm{PolyLog}(n+3,ax)+\log^2(x)\mathrm{PolyLog}(n+1,ax)-2\log(x)\mathrm{PolyLog}(n+2,ax)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]^2*PolyLog[n, a*x])/x,x]
```

```
[Out] Log[x]^2*PolyLog[1 + n, a*x] - 2*Log[x]*PolyLog[2 + n, a*x] + 2*PolyLog[3 +
 n, a*x]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2(x)\mathrm{Li}_n(ax)}{x}\,dx &= \log^2(x)\mathrm{Li}_{1+n}(ax)-2\int \frac{\log(x)\mathrm{Li}_{1+n}(ax)}{x}\,dx\\ &= \log^2(x)\mathrm{Li}_{1+n}(ax)-2\log(x)\mathrm{Li}_{2+n}(ax)+2\int \frac{\mathrm{Li}_{2+n}(ax)}{x}\,dx\\ &= \log^2(x)\mathrm{Li}_{1+n}(ax)-2\log(x)\mathrm{Li}_{2+n}(ax)+2\mathrm{Li}_{3+n}(ax)\end{aligned}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$2\mathrm{Li}_{n+3}(ax)+\log^2(x)\mathrm{Li}_{n+1}(ax)-2\log(x)\mathrm{Li}_{n+2}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]^2*PolyLog[n, a*x])/x,x]
```

```
[Out] Log[x]^2*PolyLog[1 + n, a*x] - 2*Log[x]*PolyLog[2 + n, a*x] + 2*PolyLog[3 +
 n, a*x]
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log(x)^2\mathrm{polylog}(n,ax)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*polylog(n,a*x)/x,x, algorithm="fricas")
[Out] integral(log(x)^2*polylog(n, a*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)^2 \mathrm{Li}_n(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*polylog(n,a*x)/x,x, algorithm="giac")
[Out] integrate(log(x)^2*polylog(n, a*x)/x, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{polylog}\left(n, ax\right)\ln(x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2*polylog(n,a*x)/x,x)
[Out] int(ln(x)^2*polylog(n,a*x)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)^2 \mathrm{Li}_n(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*polylog(n,a*x)/x,x, algorithm="maxima")
[Out] integrate(log(x)^2*polylog(n, a*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\ln(x)}^2\,\mathrm{polylog}\left(n,a\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)^2*polylog(n, a*x))/x,x)
[Out] int((log(x)^2*polylog(n, a*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{\left(x \right)}^{2} \operatorname{Li}_{n}\left(a x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2*polylog(n,a*x)/x,x)
[Out] Integral(log(x)**2*polylog(n, a*x)/x, x)
```

3.207 $\int \left(\frac{q\mathbf{Li}_{-1+k}(ex^q)}{bnx\left(a+b\log(cx^n)\right)}-\frac{\mathbf{Li}_k(ex^q)}{x\left(a+b\log(cx^n)\right)^2}\right)dx$

Optimal. Leaf size=26

$$\frac{\text{Li}_k\left(ex^q\right)}{bn\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] polylog(k,e*x^q)/b/n/(a+b*ln(c*x^n))
```

Rubi [A] time = 0.11, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2384}

$$\frac{\text{PolyLog}\left(k,ex^q\right)}{bn\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(q*PolyLog[-1 + k, e*x^q])/(b*n*x*(a + b*Log[c*x^n])) - PolyLog[k, e*x^
q]/(x*(a + b*Log[c*x^n])^2),x]
```

```
[Out] PolyLog[k, e*x^q]/(b*n*(a + b*Log[c*x^n]))
```

Rule 2384

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k, e*x^q]*(a + b*Log[c*x^n])^(p + 1))
/(b*n*(p + 1)), x] - Dist[q/(b*n*(p + 1)), Int[(PolyLog[k - 1, e*x^q]*(a +
b*Log[c*x^n])^(p + 1))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && LtQ[
p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{q\text{Li}_{-1+k}\left(ex^q\right)}{bnx\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\right)dx &= \frac{q\int \frac{\text{Li}_{-1+k}(ex^q)}{x\left(a+b\log(cx^n)\right)}\,dx}{bn}-\int \frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx\\ &= \frac{\text{Li}_k\left(ex^q\right)}{bn\left(a+b\log\left(cx^n\right)\right)}\end{aligned}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(\frac{q\text{Li}_{-1+k}\left(ex^q\right)}{bnx\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{Li}_k\left(ex^q\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(q*PolyLog[-1 + k, e*x^q])/(b*n*x*(a + b*Log[c*x^n])) - PolyLog[k
, e*x^q]/(x*(a + b*Log[c*x^n])^2),x]
```

```
[Out] Integrate[(q*PolyLog[-1 + k, e*x^q])/(b*n*x*(a + b*Log[c*x^n])) - PolyLog[k
, e*x^q]/(x*(a + b*Log[c*x^n])^2), x]
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bn\text{polylog}\left(k,ex^q\right)-\left(bq\log\left(cx^n\right)+aq\right)\text{polylog}\left(k-1,ex^q\right)}{b^3nx\log\left(cx^n\right)^2+2\,ab^2nx\log\left(cx^n\right)+a^2bnx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*log(c*x^n))-polylog(k,e*x^q)/x/(
a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b*n*polylog(k, e*x^q) - (b*q*log(c*x^n) + a*q)*polylog(k - 1, e*
x^q))/(b^3*n*x*log(c*x^n)^2 + 2*a*b^2*n*x*log(c*x^n) + a^2*b*n*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{q\text{Li}_{k-1}(ex^q)}{\left(b\log\left(cx^n\right)+a\right)bnx}-\frac{\text{Li}_k(ex^q)}{\left(b\log\left(cx^n\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*log(c*x^n))-polylog(k,e*x^q)/x/(
a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(q*polylog(k - 1, e*x^q)/((b*log(c*x^n) + a)*b*n*x) - polylog(k, e
*x^q)/((b*log(c*x^n) + a)^2*x), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int -\frac{\mathrm{polylog}\left(k,e\,x^q\right)}{\left(b\ln\left(c\,x^n\right)+a\right)^2x}+\frac{q\,\mathrm{polylog}\left(k-1,e\,x^q\right)}{\left(b\ln\left(c\,x^n\right)+a\right)bnx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(q*polylog(-1+k,e*x^q)/b/n/x/(b*ln(c*x^n)+a)-polylog(k,e*x^q)/x/(b*ln(c*
x^n)+a)^2,x)
```

```
[Out] int(q*polylog(-1+k,e*x^q)/b/n/x/(b*ln(c*x^n)+a)-polylog(k,e*x^q)/x/(b*ln(c*
x^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{q\text{Li}_{k-1}(ex^q)}{\left(b\log\left(cx^n\right)+a\right)bnx}-\frac{\text{Li}_k(ex^q)}{\left(b\log\left(cx^n\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*log(c*x^n))-polylog(k,e*x^q)/x/(
a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] integrate(q*polylog(k - 1, e*x^q)/((b*log(c*x^n) + a)*b*n*x) - polylog(k, e
*x^q)/((b*log(c*x^n) + a)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{q\,\mathrm{polylog}\left(k-1,e\,x^q\right)}{b\,n\,x\,\left(a+b\,\ln\left(c\,x^n\right)\right)}-\frac{\mathrm{polylog}\left(k,e\,x^q\right)}{x\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((q*polylog(k - 1, e*x^q))/(b*n*x*(a + b*log(c*x^n))) - polylog(k, e*x^q
)/(x*(a + b*log(c*x^n))^2),x)
```

```
[Out] int((q*polylog(k - 1, e*x^q))/(b*n*x*(a + b*log(c*x^n))) - polylog(k, e*x^q
)/(x*(a + b*log(c*x^n))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{aq\operatorname{Li}_{k-1}(ex^q)}{a^2x+2abx\log\left(cx^n\right)+b^2x\log\left(cx^n\right)^2}\,dx+\int\left(-\frac{bn\operatorname{Li}_{k}(ex^q)}{a^2x+2abx\log\left(cx^n\right)+b^2x\log\left(cx^n\right)^2}\right)dx+\int \frac{bq\log\left(cx^n\right)\operatorname{Li}_{k-1}(ex^q)}{a^2x+2abx\log\left(cx^n\right)+b^2x\log\left(cx^n\right)^2}\,dx}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(q*polylog(-1+k,e*x**q)/b/n/x/(a+b*ln(c*x**n))-polylog(k,e*x**q)/x
/(a+b*ln(c*x**n))**2,x)
```

```
[Out] (Integral(a*q*polylog(k - 1, e*x**q)/(a**2*x + 2*a*b*x*log(c*x**n) + b**2*x
*log(c*x**n)**2), x) + Integral(-b*n*polylog(k, e*x**q)/(a**2*x + 2*a*b*x*l
og(c*x**n) + b**2*x*log(c*x**n)**2), x) + Integral(b*q*log(c*x**n)*polylog(
k - 1, e*x**q)/(a**2*x + 2*a*b*x*log(c*x**n) + b**2*x*log(c*x**n)**2), x))/
(b*n)
```

3.208 $\int x^2 (a + b \log (cx^n)) \mathrm{Li}_2(ex) \, dx$

Optimal. Leaf size=217

$$-\frac{\log(1-ex)(a+b\log(cx^n))}{9e^3}-\frac{x(a+b\log(cx^n))}{9e^2}+\frac{1}{3}x^3\mathrm{Li}_2(ex)(a+b\log(cx^n))+\frac{1}{9}x^3\log(1-ex)(a+b\log(cx$$

```
[Out] 5/27*b*n*x/e^2+7/108*b*n*x^2/e+1/27*b*n*x^3-1/9*x*(a+b*ln(c*x^n))/e^2-1/18*
x^2*(a+b*ln(c*x^n))/e-1/27*x^3*(a+b*ln(c*x^n))+2/27*b*n*ln(-e*x+1)/e^3-2/27
*b*n*x^3*ln(-e*x+1)-1/9*(a+b*ln(c*x^n))*ln(-e*x+1)/e^3+1/9*x^3*(a+b*ln(c*x^
n))*ln(-e*x+1)-1/9*b*n*polylog(2,e*x)/e^3-1/9*b*n*x^3*polylog(2,e*x)+1/3*x^
3*(a+b*ln(c*x^n))*polylog(2,e*x)
```

Rubi [A] time = 0.18, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2385, 2395, 43, 2376, 2391}

$$\frac{1}{3}x^3\mathrm{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{bn\mathrm{PolyLog}(2,ex)}{9e^3}-\frac{1}{9}bnx^3\mathrm{PolyLog}(2,ex)-\frac{x(a+b\log(cx^n))}{9e^2}-\frac{\log(1-e}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] (5*b*n*x)/(27*e^2) + (7*b*n*x^2)/(108*e) + (b*n*x^3)/27 - (x*(a + b*Log[c*x
^n]))/(9*e^2) - (x^2*(a + b*Log[c*x^n]))/(18*e) - (x^3*(a + b*Log[c*x^n]))/
27 + (2*b*n*Log[1 - e*x])/(27*e^3) - (2*b*n*x^3*Log[1 - e*x])/27 - ((a + b*
Log[c*x^n])*Log[1 - e*x])/(9*e^3) + (x^3*(a + b*Log[c*x^n])*Log[1 - e*x])/9
 - (b*n*PolyLog[2, e*x])/(9*e^3) - (b*n*x^3*PolyLog[2, e*x])/9 + (x^3*(a +
b*Log[c*x^n])*PolyLog[2, e*x])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> -Simp[(b*n*(d*x)^(m + 1)*PolyLog[k, e*x^q])/(
d*(m + 1)^2), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a +
 b*Log[c*x^n]), x], x] + Dist[(b*n*q)/(m + 1)^2, Int[(d*x)^m*PolyLog[k - 1,
 e*x^q], x], x] + Simp[((d*x)^(m + 1)*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]))
/(d*(m + 1)), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x^2\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)\,dx = -\frac{1}{9}bnx^3\text{Li}_2(ex)+\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)+\frac{1}{3}\int x^2\left(a+b\log\left(cx^n\right)\right)\log$$

$$= -\frac{x\left(a+b\log\left(cx^n\right)\right)}{9e^2}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{18e}-\frac{1}{27}x^3\left(a+b\log\left(cx^n\right)\right)-\frac{1}{27}bnx^3$$

$$= \frac{bnx}{9e^2}+\frac{bnx^2}{36e}+\frac{1}{81}bnx^3-\frac{x\left(a+b\log\left(cx^n\right)\right)}{9e^2}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{18e}-\frac{1}{27}x^3\left(a\right.$$

$$= \frac{4bnx}{27e^2}+\frac{5bnx^2}{108e}+\frac{2}{81}bnx^3-\frac{x\left(a+b\log\left(cx^n\right)\right)}{9e^2}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{18e}-\frac{1}{27}x^3$$

$$= \frac{4bnx}{27e^2}+\frac{5bnx^2}{108e}+\frac{2}{81}bnx^3-\frac{x\left(a+b\log\left(cx^n\right)\right)}{9e^2}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{18e}-\frac{1}{27}x^3$$

$$= \frac{5bnx}{27e^2}+\frac{7bnx^2}{108e}+\frac{1}{27}bnx^3-\frac{x\left(a+b\log\left(cx^n\right)\right)}{9e^2}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{18e}-\frac{1}{27}x^3$$

Mathematica [A] time = 0.55, size = 196, normalized size = 0.90

$$\frac{\left(18e^3x^3\text{Li}_2(ex)+6\left(e^3x^3-1\right)\log(1-ex)-ex\left(2e^2x^2+3ex+6\right)\right)\left(a+b\log\left(cx^n\right)-bn\log(x)\right)}{54e^3}+\frac{bn\left(12\text{Li}_2(ex)\left(-e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] ((a - b*n*Log[x] + b*Log[c*x^n])*(-(e*x*(6 + 3*e*x + 2*e^2*x^2)) + 6*(-1 +
e^3*x^3)*Log[1 - e*x] + 18*e^3*x^3*PolyLog[2, e*x]))/(54*e^3) + (b*n*(20*e*
x + 7*e^2*x^2 + 4*e^3*x^3 + 8*Log[1 - e*x] - 8*e^3*x^3*Log[1 - e*x] + 2*Log
[x]*(-(e*x*(6 + 3*e*x + 2*e^2*x^2)) + 6*(-1 + e^3*x^3)*Log[1 - e*x]) + 12*(
-1 - e^3*x^3 + 3*e^3*x^3*Log[x])*PolyLog[2, e*x]))/(108*e^3)
```

fricas [A] time = 0.69, size = 247, normalized size = 1.14

$$\frac{4\left(be^3n-ae^3\right)x^3+\left(7\,be^2n-6\,ae^2\right)x^2+4\,(5\,ben-3\,ae)x-12\left(\left(be^3n-3\,ae^3\right)x^3+bn\right)\text{Li}_2\,(ex)-4\left(\left(2\,be^3n-3\,ae^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="fricas")
```

```
[Out] 1/108*(4*(b*e^3*n - a*e^3)*x^3 + (7*b*e^2*n - 6*a*e^2)*x^2 + 4*(5*b*e*n - 3
*a*e)*x - 12*((b*e^3*n - 3*a*e^3)*x^3 + b*n)*dilog(e*x) - 4*((2*b*e^3*n - 3
*a*e^3)*x^3 - 2*b*n + 3*a)*log(-e*x + 1) + 2*(18*b*e^3*x^3*dilog(e*x) - 2*b
*e^3*x^3 - 3*b*e^2*x^2 - 6*b*e*x + 6*(b*e^3*x^3 - b)*log(-e*x + 1))*log(c)
+ 2*(18*b*e^3*n*x^3*dilog(e*x) - 2*b*e^3*n*x^3 - 3*b*e^2*n*x^2 - 6*b*e*n*x
+ 6*(b*e^3*n*x^3 - b*n)*log(-e*x + 1))*log(x))/e^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x^2\text{Li}_2\,(ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*dilog(e*x), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^{n}\right)+a\right)x^{2}\operatorname{polylog}\left(2,ex\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)*polylog(2,e*x),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)*polylog(2,e*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{54}b\left(\frac{6\left(3\,e^{3}x^{3}\log\left(x^{n}\right)-\left(e^{3}n-3\,e^{3}\log(c)\right)x^{3}\right)\mathrm{Li}_2\left(ex\right)-2\left(\left(2\,e^{3}n-3\,e^{3}\log(c)\right)x^{3}-3\,n\log(x)\right)\log\left(-ex+1\right)}{e^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="maxima")
```

```
[Out] 1/54*b*((6*(3*e^3*x^3*log(x^n) - (e^3*n - 3*e^3*log(c))*x^3)*dilog(e*x) - 2
*((2*e^3*n - 3*e^3*log(c))*x^3 - 3*n*log(x))*log(-e*x + 1) - (2*e^3*x^3 + 3
*e^2*x^2 + 6*e*x - 6*(e^3*x^3 - 1)*log(-e*x + 1))*log(x^n))/e^3 - 54*integr
ate(-1/54*(e^2*n*x^2 + 6*(e^3*n - e^3*log(c))*x^3 + 3*e*n*x - 6*n*log(x) -
6*n)/(e^3*x - e^2), x)) + 1/54*(18*e^3*x^3*dilog(e*x) - 2*e^3*x^3 - 3*e^2*x
^2 - 6*e*x + 6*(e^3*x^3 - 1)*log(-e*x + 1))*a/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{polylog}\left(2,e\,x\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2, e*x)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*polylog(2, e*x)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*polylog(2,e*x),x)
```

```
[Out] Timed out
```

3.209 $\int x\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)\,dx$

Optimal. Leaf size=185

$$-\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{4e^2}+\frac{1}{2}x^2\text{Li}_2(ex)\left(a+b\log\left(cx^n\right)\right)-\frac{x\left(a+b\log\left(cx^n\right)\right)}{4e}+\frac{1}{4}x^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] 1/2*b*n*x/e+3/16*b*n*x^2-1/4*x*(a+b*ln(c*x^n))/e-1/8*x^2*(a+b*ln(c*x^n))+1/
4*b*n*ln(-e*x+1)/e^2-1/4*b*n*x^2*ln(-e*x+1)-1/4*(a+b*ln(c*x^n))*ln(-e*x+1)/
e^2+1/4*x^2*(a+b*ln(c*x^n))*ln(-e*x+1)-1/4*b*n*polylog(2,e*x)/e^2-1/4*b*n*x
^2*polylog(2,e*x)+1/2*x^2*(a+b*ln(c*x^n))*polylog(2,e*x)
```

Rubi [A] time = 0.13, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2385, 2395, 43, 2376, 2391}

$$\frac{1}{2}x^2\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)-\frac{bn\text{PolyLog}(2,ex)}{4e^2}-\frac{1}{4}bnx^2\text{PolyLog}(2,ex)-\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{4e^2}+\frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] (b*n*x)/(2*e) + (3*b*n*x^2)/16 - (x*(a + b*Log[c*x^n]))/(4*e) - (x^2*(a + b
*Log[c*x^n]))/8 + (b*n*Log[1 - e*x])/(4*e^2) - (b*n*x^2*Log[1 - e*x])/4 - (
(a + b*Log[c*x^n])*Log[1 - e*x])/(4*e^2) + (x^2*(a + b*Log[c*x^n])*Log[1 -
e*x])/4 - (b*n*PolyLog[2, e*x])/(4*e^2) - (b*n*x^2*PolyLog[2, e*x])/4 + (x^
2*(a + b*Log[c*x^n])*PolyLog[2, e*x])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> -Simp[(b*n*(d*x)^(m + 1)*PolyLog[k, e*x^q])/(
d*(m + 1)^2), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a +
 b*Log[c*x^n]), x], x] + Dist[(b*n*q)/(m + 1)^2, Int[(d*x)^m*PolyLog[k - 1,
 e*x^q], x], x] + Simp[((d*x)^(m + 1)*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]))
/(d*(m + 1)), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2(ex)\,dx = -\frac{1}{4}bnx^2\mathrm{Li}_2(ex)+\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2(ex)+\frac{1}{2}\int x\left(a+b\log\left(cx^n\right)\right)\mathrm{lo}$$
$$= -\frac{x\left(a+b\log\left(cx^n\right)\right)}{4e}-\frac{1}{8}x^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{8}bnx^2\log(1-ex)-\frac{\left(a+b\,\mathrm{lo}\right.}{}$$
$$= \frac{bnx}{4e}+\frac{1}{16}bnx^2-\frac{x\left(a+b\log\left(cx^n\right)\right)}{4e}-\frac{1}{8}x^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{8}bnx^2\log(1$$
$$= \frac{3bnx}{8e}+\frac{1}{8}bnx^2-\frac{x\left(a+b\log\left(cx^n\right)\right)}{4e}-\frac{1}{8}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{bn\log(1-ex}{8e^2}$$
$$= \frac{3bnx}{8e}+\frac{1}{8}bnx^2-\frac{x\left(a+b\log\left(cx^n\right)\right)}{4e}-\frac{1}{8}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{bn\log(1-ex}{8e^2}$$
$$= \frac{bnx}{2e}+\frac{3}{16}bnx^2-\frac{x\left(a+b\log\left(cx^n\right)\right)}{4e}-\frac{1}{8}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{bn\log(1-ex}{4e^2}$$

Mathematica [A] time = 0.34, size = 168, normalized size = 0.91

$$\frac{\left(4e^2x^2\mathrm{Li}_2(ex)+2\left(e^2x^2-1\right)\log(1-ex)-ex(ex+2)\right)\left(a+b\log\left(cx^n\right)-bn\log(x)\right)}{8e^2}+\frac{bn\left(\mathrm{Li}_2(ex)\left(-4e^2x^2+8e^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] ((a - b*n*Log[x] + b*Log[c*x^n])*(-(e*x*(2 + e*x)) + 2*(-1 + e^2*x^2)*Log[1
 - e*x] + 4*e^2*x^2*PolyLog[2, e*x]))/(8*e^2) + (b*n*(8*e*x + 3*e^2*x^2 + 4
*Log[1 - e*x] - 4*e^2*x^2*Log[1 - e*x] + Log[x]*(-2*e*x*(2 + e*x) + 4*(-1 +
 e^2*x^2)*Log[1 - e*x]) + (-4 - 4*e^2*x^2 + 8*e^2*x^2*Log[x])*PolyLog[2, e*
x]))/(16*e^2)
```

fricas [A] time = 0.56, size = 207, normalized size = 1.12

$$\frac{\left(3\,be^2n-2\,ae^2\right)x^2+4\,(2\,ben-ae)x-4\left(\left(be^2n-2\,ae^2\right)x^2+bn\right)\mathrm{Li}_2\,(ex)-4\left(\left(be^2n-ae^2\right)x^2-bn+a\right)\log\,(-ex}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="fricas")
```

```
[Out] 1/16*((3*b*e^2*n - 2*a*e^2)*x^2 + 4*(2*b*e*n - a*e)*x - 4*((b*e^2*n - 2*a*e
^2)*x^2 + b*n)*dilog(e*x) - 4*((b*e^2*n - a*e^2)*x^2 - b*n + a)*log(-e*x +
1) + 2*(4*b*e^2*x^2*dilog(e*x) - b*e^2*x^2 - 2*b*e*x + 2*(b*e^2*x^2 - b)*lo
g(-e*x + 1))*log(c) + 2*(4*b*e^2*n*x^2*dilog(e*x) - b*e^2*n*x^2 - 2*b*e*n*x
 + 2*(b*e^2*n*x^2 - b*n)*log(-e*x + 1))*log(x))/e^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(cx^n\right)+a\right)x\mathrm{Li}_2\,(ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*dilog(e*x), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int (b\ln(c\,x^n)+a)\,x\,\mathrm{polylog}(2,ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)*polylog(2,e*x),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)*polylog(2,e*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}b\left(\frac{2\left(2e^2x^2\log(x^n)-\left(e^2n-2e^2\log(c)\right)x^2\right)\mathrm{Li}_2(ex)-2\left(\left(e^2n-e^2\log(c)\right)x^2-n\log(x)\right)\log(-ex+1)-\left(e^2x^2+\right.\cdots}{e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="maxima")
```

```
[Out] 1/8*b*((2*(2*e^2*x^2*log(x^n) - (e^2*n - 2*e^2*log(c))*x^2)*dilog(e*x) - 2*
((e^2*n - e^2*log(c))*x^2 - n*log(x))*log(-e*x + 1) - (e^2*x^2 + 2*e*x - 2*
(e^2*x^2 - 1)*log(-e*x + 1))*log(x^n))/e^2 - 8*integrate(-1/8*(e*n*x + (3*e
^2*n - 2*e^2*log(c))*x^2 - 2*n*log(x) - 2*n)/(e^2*x - e), x)) + 1/8*(4*e^2*
x^2*dilog(e*x) - e^2*x^2 - 2*e*x + 2*(e^2*x^2 - 1)*log(-e*x + 1))*a/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{polylog}(2,e\,x)\,\left(a+b\,\ln(c\,x^n)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2, e*x)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*polylog(2, e*x)*(a + b*log(c*x^n)), x)
```

sympy [A] time = 83.20, size = 264, normalized size = 1.43

$$\begin{cases} -\frac{ax^2\,\mathrm{Li}_1(ex)}{4}+\frac{ax^2\,\mathrm{Li}_2(ex)}{2}-\frac{ax^2}{8}-\frac{ax}{4e}+\frac{a\,\mathrm{Li}_1(ex)}{4e^2}-\frac{bnx^2\log(x)\,\mathrm{Li}_1(ex)}{4}+\frac{bnx^2\log(x)\,\mathrm{Li}_2(ex)}{2}-\frac{bnx^2\log(x)}{8}+\frac{bnx^2\,\mathrm{Li}_1(ex)}{4}-\frac{bnx^2\,\mathrm{Li}_2(\cdots}{4} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*polylog(2,e*x),x)
```

```
[Out] Piecewise((-a*x**2*polylog(1, e*x)/4 + a*x**2*polylog(2, e*x)/2 - a*x**2/8
- a*x/(4*e) + a*polylog(1, e*x)/(4*e**2) - b*n*x**2*log(x)*polylog(1, e*x)/
4 + b*n*x**2*log(x)*polylog(2, e*x)/2 - b*n*x**2*log(x)/8 + b*n*x**2*polylo
g(1, e*x)/4 - b*n*x**2*polylog(2, e*x)/4 + 3*b*n*x**2/16 - b*x**2*log(c)*po
lylog(1, e*x)/4 + b*x**2*log(c)*polylog(2, e*x)/2 - b*x**2*log(c)/8 - b*n*x
*log(x)/(4*e) + b*n*x/(2*e) - b*x*log(c)/(4*e) + b*n*log(x)*polylog(1, e*x)
/(4*e**2) - b*n*polylog(1, e*x)/(4*e**2) - b*n*polylog(2, e*x)/(4*e**2) + b
*log(c)*polylog(1, e*x)/(4*e**2), Ne(e, 0)), (0, True))
```

3.210 $\int \left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2(ex)\,dx$

Optimal. Leaf size=106

$$x\mathrm{Li}_2(ex)\left(a+b\log\left(cx^n\right)\right)-\frac{(1-ex)\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{e}-x\left(a+b\log\left(cx^n\right)\right)-bnx\mathrm{Li}_2(ex)-\frac{bn\mathrm{Li}_2(ex)}{e}+\frac{2}{\ }$$

```
[Out] 3*b*n*x-x*(a+b*ln(c*x^n))+2*b*n*(-e*x+1)*ln(-e*x+1)/e-(-e*x+1)*(a+b*ln(c*x^
n))*ln(-e*x+1)/e-b*n*polylog(2,e*x)/e-b*n*x*polylog(2,e*x)+x*(a+b*ln(c*x^n)
)*polylog(2,e*x)
```

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2381, 2389, 2295, 2370, 2411, 43, 2351, 2315}

$$x\mathrm{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)-bnx\mathrm{PolyLog}(2,ex)-\frac{bn\mathrm{PolyLog}(2,ex)}{e}-\frac{(1-ex)\log(1-ex)\left(a+b\log\left(cx^n\right)\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] 3*b*n*x - x*(a + b*Log[c*x^n]) + (2*b*n*(1 - e*x)*Log[1 - e*x])/e - ((1 - e
*x)*(a + b*Log[c*x^n])*Log[1 - e*x])/e - (b*n*PolyLog[2, e*x])/e - b*n*x*Po
lyLog[2, e*x] + x*(a + b*Log[c*x^n])*PolyLog[2, e*x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*PolyLog[k_, (e_.)*(x_)^(q_.)], x_
Symbol] :> -Simp[b*n*x*PolyLog[k, e*x^q], x] + (-Dist[q, Int[PolyLog[k - 1,
 e*x^q]*(a + b*Log[c*x^n]), x], x] + Dist[b*n*q, Int[PolyLog[k - 1, e*x^q],
 x], x] + Simp[x*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]), x]) /; FreeQ[{a, b,
c, e, n, q}, x] && IGtQ[k, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2(ex)\,dx = -bnx\mathrm{Li}_2(ex)+x\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2(ex)-(bn)\int\log(1-ex)\,dx+\int\left(a+b\log\right.$$

$$= -x\left(a+b\log\left(cx^n\right)\right)-\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)\log(1-ex)}{e}-bnx\mathrm{Li}_2(ex)+x\left(a\right.$$

$$= 2bnx-x\left(a+b\log\left(cx^n\right)\right)+\frac{bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)\mathrm{lo}}{e}$$

$$= 2bnx-x\left(a+b\log\left(cx^n\right)\right)+\frac{bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)\mathrm{lo}}{e}$$

$$= 2bnx-x\left(a+b\log\left(cx^n\right)\right)+\frac{bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)\mathrm{lo}}{e}$$

$$= 2bnx-x\left(a+b\log\left(cx^n\right)\right)+\frac{bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)\mathrm{lo}}{e}$$

$$= 3bnx-x\left(a+b\log\left(cx^n\right)\right)+\frac{2bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)}{e}$$

Mathematica [A] time = 0.08, size = 113, normalized size = 1.07

$$\left(x\mathrm{Li}_2(ex)+\left(x-\frac{1}{e}\right)\log(1-ex)-x\right)\left(a+b\left(\log\left(cx^n\right)-n\log(x)\right)\right)+\frac{bn(\mathrm{Li}_2(ex)(-ex+ex\log(x)-1)+3ex-2ex\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] (a + b*(-(n*Log[x]) + Log[c*x^n]))*(-x + (-e^(-1) + x)*Log[1 - e*x] + x*Pol
yLog[2, e*x]) + (b*n*(3*e*x + 2*Log[1 - e*x] - 2*e*x*Log[1 - e*x] + Log[x]*
(-(e*x) + (-1 + e*x)*Log[1 - e*x]) + (-1 - e*x + e*x*Log[x])*PolyLog[2, e*x
]))/e
```

fricas [A] time = 1.04, size = 137, normalized size = 1.29

$$\frac{(3\,ben-ae)x-(bn+(ben-ae)x)\mathrm{Li}_2\,(ex)+(2\,bn-(2\,ben-ae)x-a)\log{(-ex+1)}+\left(bex\mathrm{Li}_2\,(ex)-bex+(be\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="fricas")
```

```
[Out] ((3*b*e*n - a*e)*x - (b*n + (b*e*n - a*e)*x)*dilog(e*x) + (2*b*n - (2*b*e*n
 - a*e)*x - a)*log(-e*x + 1) + (b*e*x*dilog(e*x) - b*e*x + (b*e*x - b)*log(
-e*x + 1))*log(c) + (b*e*n*x*dilog(e*x) - b*e*n*x + (b*e*n*x - b*n)*log(-e*
x + 1))*log(x))/e
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log{(cx^n)}+a\right)\mathrm{Li}_2\,(ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*dilog(e*x), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (b\ln{(c\,x^n)}+a)\operatorname{polylog}{(2,ex)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*polylog(2,e*x),x)
```

```
[Out] int((b*ln(c*x^n)+a)*polylog(2,e*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{\left(ex\log{(x^n)}-\left(en-e\log(c)\right)x\right)\mathrm{Li}_2\,(ex)-\left(\left(2\,en-e\log(c)\right)x-n\log(x)\right)\log{(-ex+1)}-\left(ex-(ex-1)\log{(-\right.}{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="maxima")
```

```
[Out] b*(((e*x*log(x^n) - (e*n - e*log(c))*x)*dilog(e*x) - ((2*e*n - e*log(c))*x
- n*log(x))*log(-e*x + 1) - (e*x - (e*x - 1)*log(-e*x + 1))*log(x^n))/e - i
ntegrate(-((3*e*n - e*log(c))*x - n*log(x) - n)/(e*x - 1), x)) + (e*x*dilog
(e*x) - e*x + (e*x - 1)*log(-e*x + 1))*a/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{polylog}{(2,e\,x)}\;\left(a+b\;\ln{(c\,x^n)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, e*x)*(a + b*log(c*x^n)),x)
```

```
[Out] int(polylog(2, e*x)*(a + b*log(c*x^n)), x)
```

sympy [A] time = 23.64, size = 172, normalized size = 1.62

$$\begin{cases} -ax\,\mathrm{Li}_1\,(ex)+ax\,\mathrm{Li}_2\,(ex)-ax+\frac{a\,\mathrm{Li}_1(ex)}{e}-bnx\log{(x)}\,\mathrm{Li}_1\,(ex)+bnx\log{(x)}\,\mathrm{Li}_2\,(ex)-bnx\log{(x)}+2bnx\,\mathrm{Li}_1 \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(2,e*x),x)
```

```
[Out] Piecewise((-a*x*polylog(1, e*x) + a*x*polylog(2, e*x) - a*x + a*polylog(1,
e*x)/e - b*n*x*log(x)*polylog(1, e*x) + b*n*x*log(x)*polylog(2, e*x) - b*n*
x*log(x) + 2*b*n*x*polylog(1, e*x) - b*n*x*polylog(2, e*x) + 3*b*n*x - b*x*
log(c)*polylog(1, e*x) + b*x*log(c)*polylog(2, e*x) - b*x*log(c) + b*n*log(
x)*polylog(1, e*x)/e - 2*b*n*polylog(1, e*x)/e - b*n*polylog(2, e*x)/e + b*
log(c)*polylog(1, e*x)/e, Ne(e, 0)), (0, True))
```

3.211 $\int \frac{(a+b\log(cx^n))\text{Li}_2(ex)}{x}\,dx$

Optimal. Leaf size=26

$$\text{Li}_3(ex)\left(a+b\log\left(cx^n\right)\right)-bn\text{Li}_4(ex)$$

```
[Out] (a+b*ln(c*x^n))*polylog(3,e*x)-b*n*polylog(4,e*x)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2383, 6589}

$$\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)-bn\text{PolyLog}(4,ex)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x,x]
```

```
[Out] (a + b*Log[c*x^n])*PolyLog[3, e*x] - b*n*PolyLog[4, e*x]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)}{x}\,dx&=\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)-(bn)\int \frac{\text{Li}_3(ex)}{x}\,dx\\&=\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)-bn\text{Li}_4(ex)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.15

$$a\text{Li}_3(ex)+b\text{Li}_3(ex)\log\left(cx^n\right)-bn\text{Li}_4(ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x,x]
```

```
[Out] a*PolyLog[3, e*x] + b*Log[c*x^n]*PolyLog[3, e*x] - b*n*PolyLog[4, e*x]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\text{Li}_2\left(ex\right)\log\left(cx^n\right)+a\text{Li}_2\left(ex\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x,x, algorithm="fricas")
```

```
[Out] integral((b*dilog(e*x)*log(c*x^n) + a*dilog(e*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right)\mathrm{Li}_{2}\left(ex\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*dilog(e*x)/x, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^{n}\right) + a\right) \mathrm{polylog}\left(2, ex\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*polylog(2,e*x)/x,x)
[Out] int((b*ln(c*x^n)+a)*polylog(2,e*x)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(bn\log(x)^{2} - 2\,b\log(x)\log\left(x^{n}\right) - 2\left(b\log(c) + a\right)\log(x)\right)\mathrm{Li}_{2}\left(ex\right)+\frac{1}{2}\int \frac{2\,b\log\left(-ex+1\right)\log(x)\log\left(x^{n}\right) - \left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x,x, algorithm="maxima")
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*dilog(e
*x) + 1/2*integrate((2*b*log(-e*x + 1)*log(x)*log(x^n) - (b*n*log(x)^2 - 2*
(b*log(c) + a)*log(x))*log(-e*x + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{polylog}\left(2, e\,x\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x,x)
[Out] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x, x)
```

sympy [A] time = 15.63, size = 26, normalized size = 1.00

$$a\,\mathrm{Li}_{3}\left(ex\right) + b\left(-n\,\mathrm{Li}_{4}\left(ex\right) + \log\left(cx^{n}\right)\mathrm{Li}_{3}\left(ex\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(2,e*x)/x,x)
[Out] a*polylog(3, e*x) + b*(-n*polylog(4, e*x) + log(c*x**n)*polylog(3, e*x))
```

3.212 $\int \frac{(a+b\log(cx^n))\mathbf{Li}_2(ex)}{x^2}\,dx$

Optimal. Leaf size=142

$$-\frac{\text{Li}_2(ex)\left(a+b\log\left(cx^n\right)\right)}{x}+e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{\log(1-ex)\left(a+b\log\left(cx^n\right.\right.}{x}$$

```
[Out] 2*b*e*n*ln(x)-1/2*b*e*n*ln(x)^2+e*ln(x)*(a+b*ln(c*x^n))-2*b*e*n*ln(-e*x+1)+
2*b*n*ln(-e*x+1)/x-e*(a+b*ln(c*x^n))*ln(-e*x+1)+(a+b*ln(c*x^n))*ln(-e*x+1)/
x-b*e*n*polylog(2,e*x)-b*n*polylog(2,e*x)/x-(a+b*ln(c*x^n))*polylog(2,e*x)/
x
```

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2385, 2395, 36, 29, 31, 2376, 2301, 2391}

$$-\frac{\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)}{x}-ben\text{PolyLog}(2,ex)-\frac{bn\text{PolyLog}(2,ex)}{x}+e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\log(1-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^2,x]
```

```
[Out] 2*b*e*n*Log[x] - (b*e*n*Log[x]^2)/2 + e*Log[x]*(a + b*Log[c*x^n]) - 2*b*e*n
*Log[1 - e*x] + (2*b*n*Log[1 - e*x])/x - e*(a + b*Log[c*x^n])*Log[1 - e*x]
+ ((a + b*Log[c*x^n])*Log[1 - e*x])/x - b*e*n*PolyLog[2, e*x] - (b*n*PolyLo
g[2, e*x])/x - ((a + b*Log[c*x^n])*PolyLog[2, e*x])/x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> -Simp[(b*n*(d*x)^(m + 1)*PolyLog[k, e*x^q])/(
```

```
d*(m + 1)^2), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a +
 b*Log[c*x^n]), x], x] + Dist[(b*n*q)/(m + 1)^2, Int[(d*x)^m*PolyLog[k - 1,
 e*x^q], x], x] + Simp[((d*x)^(m + 1)*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]))
/(d*(m + 1)), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)}{x^2}\,dx = -\frac{bn\text{Li}_2(ex)}{x} - \frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)}{x} - (bn)\int \frac{\log(1-ex)}{x^2}\,dx - \int \frac{\left(a+b\log\right.}{}$$

$$= e\log(x)\left(a+b\log\left(cx^n\right)\right) + \frac{bn\log(1-ex)}{x} - e\left(a+b\log\left(cx^n\right)\right)\log(1-ex) + \frac{(a}{}$$

$$= e\log(x)\left(a+b\log\left(cx^n\right)\right) + \frac{bn\log(1-ex)}{x} - e\left(a+b\log\left(cx^n\right)\right)\log(1-ex) + \frac{(a}{}$$

$$= ben\log(x) - \frac{1}{2}ben\log^2(x) + e\log(x)\left(a+b\log\left(cx^n\right)\right) - ben\log(1-ex) + \frac{2bn\,\mathrm{l}}{}$$

$$= ben\log(x) - \frac{1}{2}ben\log^2(x) + e\log(x)\left(a+b\log\left(cx^n\right)\right) - ben\log(1-ex) + \frac{2bn\,\mathrm{l}}{}$$

$$= 2ben\log(x) - \frac{1}{2}ben\log^2(x) + e\log(x)\left(a+b\log\left(cx^n\right)\right) - 2ben\log(1-ex) + \frac{2b}{}$$

Mathematica [A] time = 0.16, size = 115, normalized size = 0.81

$$\frac{(-\text{Li}_2(ex)+ex\log(x)+(1-ex)\log(1-ex))\left(a+b\log\left(cx^n\right)-bn\log(x)\right)}{x}+\frac{bn\left(-2\text{Li}_2(ex)(ex+\log(x)+1)+ex\,\mathrm{l}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^2,x]
```

```
[Out] ((a - b*n*Log[x] + b*Log[c*x^n])*(e*x*Log[x] + (1 - e*x)*Log[1 - e*x] - Pol
yLog[2, e*x]))/x + (b*n*(e*x*Log[x]^2 - 4*(-1 + e*x)*Log[1 - e*x] + Log[x]*
(4*e*x + (2 - 2*e*x)*Log[1 - e*x]) - 2*(1 + e*x + Log[x])*PolyLog[2, e*x]))
/(2*x)
```

fricas [A] time = 0.69, size = 134, normalized size = 0.94

$$\frac{benx\log(x)^2 - 2\,(benx+bn+a)\text{Li}_2\,(ex) + 2\,(2\,bn-(2\,ben+ae)x+a)\log\,(-ex+1) - 2\left(b\text{Li}_2\,(ex)+(bex-b)\log\right.}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*e*n*x*log(x)^2 - 2*(b*e*n*x + b*n + a)*dilog(e*x) + 2*(2*b*n - (2*b*
e*n + a*e)*x + a)*log(-e*x + 1) - 2*(b*dilog(e*x) + (b*e*x - b)*log(-e*x +
1))*log(c) + 2*(b*e*x*log(c) - b*n*dilog(e*x) + (2*b*e*n + a*e)*x - (b*e*n*
x - b*n)*log(-e*x + 1))*log(x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right)\mathrm{Li}_2\left(ex\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*dilog(e*x)/x^2, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^{n}\right) + a\right)\mathrm{polylog}\left(2, ex\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*polylog(2,e*x)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)*polylog(2,e*x)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(e\log(x) - \frac{(ex-1)\log\left(-ex+1\right) + \mathrm{Li}_2\left(ex\right)}{x}\right)a - b\left(\frac{\left(n + \log(c) + \log\left(x^{n}\right)\right)\mathrm{Li}_2\left(ex\right) - \left(enx\log(x) + 2\,n + \log(c)\right)}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^2,x, algorithm="maxima")
```

```
[Out] (e*log(x) - ((e*x - 1)*log(-e*x + 1) + dilog(e*x))/x)*a - b*(((n + log(c) +
 log(x^n))*dilog(e*x) - (e*n*x*log(x) + 2*n + log(c))*log(-e*x + 1) - (e*x*
log(x) - (e*x - 1)*log(-e*x + 1))*log(x^n))/x + integrate((2*e*n + e*log(c)
 + (2*e^2*n*x - e*n)*log(x))/(e*x^2 - x), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}\left(2, e\,x\right)\,\left(a + b\,\ln\left(c\,x^{n}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(cx^{n}\right)\right)\mathrm{Li}_2\left(ex\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(2,e*x)/x**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))*polylog(2, e*x)/x**2, x)
```

3.213 $\int \frac{(a+b\log(cx^n))\textbf{Li}_2(ex)}{x^3}\,dx$

Optimal. Leaf size=202

$$\frac{1}{4}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}e^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)-\frac{\text{Li}_2(ex)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{e\left(a+b\log\left(cx^n\right)\right)}{4x}+\frac{\log(1$$

[Out] -1/2*b*e*n/x+1/4*b*e^2*n*ln(x)-1/8*b*e^2*n*ln(x)^2-1/4*e*(a+b*ln(c*x^n))/x+1/4*e^2*ln(x)*(a+b*ln(c*x^n))-1/4*b*e^2*n*ln(-e*x+1)+1/4*b*n*ln(-e*x+1)/x^2-1/4*e^2*(a+b*ln(c*x^n))*ln(-e*x+1)+1/4*(a+b*ln(c*x^n))*ln(-e*x+1)/x^2-1/4*b*e^2*n*polylog(2,e*x)-1/4*b*n*polylog(2,e*x)/x^2-1/2*(a+b*ln(c*x^n))*polylog(2,e*x)/x^2

Rubi [A] time = 0.16, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2385, 2395, 44, 2376, 2301, 2391}

$$-\frac{\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{1}{4}be^2n\text{PolyLog}(2,ex)-\frac{bn\text{PolyLog}(2,ex)}{4x^2}+\frac{1}{4}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}e^2\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^3,x]
```

```
[Out] -(b*e*n)/(2*x) + (b*e^2*n*Log[x])/4 - (b*e^2*n*Log[x]^2)/8 - (e*(a + b*Log[c*x^n]))/(4*x) + (e^2*Log[x]*(a + b*Log[c*x^n]))/4 - (b*e^2*n*Log[1 - e*x])/4 + (b*n*Log[1 - e*x])/(4*x^2) - (e^2*(a + b*Log[c*x^n])*Log[1 - e*x])/4 + ((a + b*Log[c*x^n])*Log[1 - e*x])/(4*x^2) - (b*e^2*n*PolyLog[2, e*x])/4 - (b*n*PolyLog[2, e*x])/(4*x^2) - ((a + b*Log[c*x^n])*PolyLog[2, e*x])/(2*x^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e_.)*(x_)^(q_.)], x_Symbol] :> -Simp[(b*n*(d*x)^(m + 1)*PolyLog[k, e*x^q])/(d*(m + 1)^2), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a + b*Log[c*x^n]), x], x] + Dist[(b*n*q)/(m + 1)^2, Int[(d*x)^m*PolyLog[k - 1, e*x^q], x], x] + Simp[((d*x)^(m + 1)*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]))/(d*(m + 1)), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)}{x^3}\,dx = -\frac{bn\text{Li}_2(ex)}{4x^2} - \frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)}{2x^2} - \frac{1}{2}\int \frac{\left(a+b\log\left(cx^n\right)\right)\log(1-ex)}{x^3}\,d$$

$$= -\frac{e\left(a+b\log\left(cx^n\right)\right)}{4x} + \frac{1}{4}e^2\log(x)\left(a+b\log\left(cx^n\right)\right) + \frac{bn\log(1-ex)}{8x^2} - \frac{1}{4}e^2\left(a\right.$$

$$= -\frac{ben}{4x} - \frac{e\left(a+b\log\left(cx^n\right)\right)}{4x} + \frac{1}{4}e^2\log(x)\left(a+b\log\left(cx^n\right)\right) + \frac{bn\log(1-ex)}{8x^2} -$$

$$= -\frac{3ben}{8x} + \frac{1}{8}be^2n\log(x) - \frac{1}{8}be^2n\log^2(x) - \frac{e\left(a+b\log\left(cx^n\right)\right)}{4x} + \frac{1}{4}e^2\log(x)\left(a\right.$$

$$= -\frac{3ben}{8x} + \frac{1}{8}be^2n\log(x) - \frac{1}{8}be^2n\log^2(x) - \frac{e\left(a+b\log\left(cx^n\right)\right)}{4x} + \frac{1}{4}e^2\log(x)\left(a\right.$$

$$= -\frac{ben}{2x} + \frac{1}{4}be^2n\log(x) - \frac{1}{8}be^2n\log^2(x) - \frac{e\left(a+b\log\left(cx^n\right)\right)}{4x} + \frac{1}{4}e^2\log(x)\left(a+\right.$$

Mathematica [A] time = 0.20, size = 163, normalized size = 0.81

$$\frac{\left(e^2x^2\log(x)-e^2x^2\log(1-ex)-2\text{Li}_2(ex)-ex+\log(1-ex)\right)\left(a+b\log\left(cx^n\right)-bn\log(x)\right)}{4x^2}+\frac{bn\left(-2\text{Li}_2(ex)\left(e^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^3,x]
```

```
[Out] ((a - b*n*Log[x] + b*Log[c*x^n])*(-(e*x) + e^2*x^2*Log[x] + Log[1 - e*x] -
e^2*x^2*Log[1 - e*x] - 2*PolyLog[2, e*x]))/(4*x^2) + (b*n*(-4*e*x + e^2*x^2
*Log[x]^2 + 2*Log[1 - e*x] - 2*e^2*x^2*Log[1 - e*x] - 2*(-1 + e*x)*Log[x]*(
-(e*x) + (1 + e*x)*Log[1 - e*x]) - 2*(1 + e^2*x^2 + 2*Log[x])*PolyLog[2, e*
x]))/(8*x^2)
```

fricas [A] time = 0.94, size = 190, normalized size = 0.94

$$\frac{be^2nx^2\log(x)^2-2\,(2\,ben+ae)x-2\left(be^2nx^2+bn+2\,a\right)\text{Li}_2\left(ex\right)-2\left(\left(be^2n+ae^2\right)x^2-bn-a\right)\log\left(-ex+1\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(b*e^2*n*x^2*log(x)^2 - 2*(2*b*e*n + a*e)*x - 2*(b*e^2*n*x^2 + b*n + 2*
a)*dilog(e*x) - 2*((b*e^2*n + a*e^2)*x^2 - b*n - a)*log(-e*x + 1) - 2*(b*e*
x + 2*b*dilog(e*x) + (b*e^2*x^2 - b)*log(-e*x + 1))*log(c) + 2*(b*e^2*x^2*l
```

```
og(c) - b*e*n*x + (b*e^2*n + a*e^2)*x^2 - 2*b*n*dilog(e*x) - (b*e^2*n*x^2 -
 b*n)*log(-e*x + 1))*log(x))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right)+a\right)\mathrm{Li}_2\left(ex\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*dilog(e*x)/x^3, x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right)+a\right)\mathrm{polylog}\left(2,ex\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*polylog(2,e*x)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)*polylog(2,e*x)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(e^2\log(x)-\frac{ex+\left(e^2x^2-1\right)\log\left(-ex+1\right)+2\,\mathrm{Li}_2\left(ex\right)}{x^2}\right)a-\frac{1}{4}\,b\left(\frac{\left(n+2\,\log(c)+2\,\log\left(x^n\right)\right)\mathrm{Li}_2\left(ex\right)-\left(e^2nx^2\log(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(e^2*log(x) - (e*x + (e^2*x^2 - 1)*log(-e*x + 1) + 2*dilog(e*x))/x^2)*a
 - 1/4*b*(((n + 2*log(c) + 2*log(x^n))*dilog(e*x) - (e^2*n*x^2*log(x) + n +
 log(c))*log(-e*x + 1) - (e^2*x^2*log(x) - e*x - (e^2*x^2 - 1)*log(-e*x + 1
))*log(x^n))/x^2 + 4*integrate(-1/4*(e^2*n*x - 2*e*n - e*log(c) - (2*e^3*n*
x^2 - e^2*n*x)*log(x))/(e*x^3 - x^2), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}\left(2,e\,x\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2\left(ex\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(2,e*x)/x**3,x)
```

```
[Out] Integral((a + b*log(c*x**n))*polylog(2, e*x)/x**3, x)
```

3.214 $\int x^2 \left(a + b\log\left(cx^n\right)\right) \text{Li}_3(ex)\,dx$

Optimal. Leaf size=253

$$\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{27e^3}+\frac{x\left(a+b\log\left(cx^n\right)\right)}{27e^2}-\frac{1}{9}x^3\text{Li}_2(ex)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{3}x^3\text{Li}_3(ex)\left(a+b\log\left(cx^n\right)\right)-$$

```
[Out] -2/27*b*n*x/e^2-1/36*b*n*x^2/e-4/243*b*n*x^3+1/27*x*(a+b*ln(c*x^n))/e^2+1/5
4*x^2*(a+b*ln(c*x^n))/e+1/81*x^3*(a+b*ln(c*x^n))-1/27*b*n*ln(-e*x+1)/e^3+1/
27*b*n*x^3*ln(-e*x+1)+1/27*(a+b*ln(c*x^n))*ln(-e*x+1)/e^3-1/27*x^3*(a+b*ln(
c*x^n))*ln(-e*x+1)+1/27*b*n*polylog(2,e*x)/e^3+2/27*b*n*x^3*polylog(2,e*x)-
1/9*x^3*(a+b*ln(c*x^n))*polylog(2,e*x)-1/9*b*n*x^3*polylog(3,e*x)+1/3*x^3*(
a+b*ln(c*x^n))*polylog(3,e*x)
```

Rubi [A] time = 0.25, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2385, 2395, 43, 2376, 2391, 6591}

$$-\frac{1}{9}x^3\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{3}x^3\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)+\frac{bn\text{PolyLog}(2,ex)}{27e^3}+\frac{2}{27}bnx^3\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

```
[Out] (-2*b*n*x)/(27*e^2) - (b*n*x^2)/(36*e) - (4*b*n*x^3)/243 + (x*(a + b*Log[c*
x^n]))/(27*e^2) + (x^2*(a + b*Log[c*x^n]))/(54*e) + (x^3*(a + b*Log[c*x^n])
)/81 - (b*n*Log[1 - e*x])/(27*e^3) + (b*n*x^3*Log[1 - e*x])/27 + ((a + b*Lo
g[c*x^n])*Log[1 - e*x])/(27*e^3) - (x^3*(a + b*Log[c*x^n])*Log[1 - e*x])/27
 + (b*n*PolyLog[2, e*x])/(27*e^3) + (2*b*n*x^3*PolyLog[2, e*x])/27 - (x^3*(
a + b*Log[c*x^n])*PolyLog[2, e*x])/9 - (b*n*x^3*PolyLog[3, e*x])/9 + (x^3*(
a + b*Log[c*x^n])*PolyLog[3, e*x])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> -Simp[(b*n*(d*x)^(m + 1)*PolyLog[k, e*x^q])/(
d*(m + 1)^2), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a +
 b*Log[c*x^n]), x], x] + Dist[(b*n*q)/(m + 1)^2, Int[(d*x)^m*PolyLog[k - 1,
 e*x^q], x], x] + Simp[((d*x)^(m + 1)*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]))
/(d*(m + 1)), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int x^2\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)\,dx=-\frac{1}{9}bnx^3\text{Li}_3(ex)+\frac{1}{3}x^3\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)-\frac{1}{3}\int x^2\left(a+b\log\left(cx^n\right)\right)\text{Li}_2($$

$$=\frac{2}{27}bnx^3\text{Li}_2(ex)-\frac{1}{9}x^3\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)-\frac{1}{9}bnx^3\text{Li}_3(ex)+\frac{1}{3}x^3\left(a+b\text{lo}\right.$$

$$=\frac{x\left(a+b\log\left(cx^n\right)\right)}{27e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{54e}+\frac{1}{81}x^3\left(a+b\log\left(cx^n\right)\right)+\frac{\left(a+b\text{lo}\right.}{}$$

$$=-\frac{bnx}{27e^2}-\frac{bnx^2}{108e}-\frac{1}{243}bnx^3+\frac{x\left(a+b\log\left(cx^n\right)\right)}{27e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{54e}+\frac{1}{81}x^3$$

$$=-\frac{bnx}{27e^2}-\frac{bnx^2}{108e}-\frac{1}{243}bnx^3+\frac{x\left(a+b\log\left(cx^n\right)\right)}{27e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{54e}+\frac{1}{81}x^3$$

$$=-\frac{bnx}{27e^2}-\frac{bnx^2}{108e}-\frac{1}{243}bnx^3+\frac{x\left(a+b\log\left(cx^n\right)\right)}{27e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{54e}+\frac{1}{81}x^3$$

$$=-\frac{4bnx}{81e^2}-\frac{5bnx^2}{324e}-\frac{2}{243}bnx^3+\frac{x\left(a+b\log\left(cx^n\right)\right)}{27e^2}+\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{54e}+\frac{1}{81}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

```
[Out] Integrate[x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x], x]
```

fricas [C] time = 0.79, size = 296, normalized size = 1.17

$$-\frac{4\left(4\,be^3n-3\,ae^3\right)x^3+9\left(3\,be^2n-2\,ae^2\right)x^2+36\,(2\,ben-ae)x-36\left(\left(2\,be^3n-3\,ae^3\right)x^3+bn\right)\text{Li}_2\,(ex)-36\left(\left(be^3n\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="fricas")
```

```
[Out] -1/972*(4*(4*b*e^3*n - 3*a*e^3)*x^3 + 9*(3*b*e^2*n - 2*a*e^2)*x^2 + 36*(2*b
*e*n - a*e)*x - 36*((2*b*e^3*n - 3*a*e^3)*x^3 + b*n)*dilog(e*x) - 36*((b*e^
3*n - a*e^3)*x^3 - b*n + a)*log(-e*x + 1) + 6*(18*b*e^3*x^3*dilog(e*x) - 2*
b*e^3*x^3 - 3*b*e^2*x^2 - 6*b*e*x + 6*(b*e^3*x^3 - b)*log(-e*x + 1))*log(c)
 + 6*(18*b*e^3*n*x^3*dilog(e*x) - 2*b*e^3*n*x^3 - 3*b*e^2*n*x^2 - 6*b*e*n*x
 + 6*(b*e^3*n*x^3 - b*n)*log(-e*x + 1))*log(x) - 108*(3*b*e^3*n*x^3*log(x)
+ 3*b*e^3*x^3*log(c) - (b*e^3*n - 3*a*e^3)*x^3)*polylog(3, e*x))/e^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right)+a\right)x^2\mathrm{Li}_3(ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*polylog(3, e*x), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right)+a\right)x^2\,\mathrm{polylog}\left(3,ex\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*x^n)+a)*polylog(3,e*x),x)
```

```
[Out] int(x^2*(b*ln(c*x^n)+a)*polylog(3,e*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{162}\,b\left(\frac{6\left(3\,e^3x^3\log\left(x^n\right)-\left(2\,e^3n-3\,e^3\log(c)\right)x^3\right)\mathrm{Li}_2\left(ex\right)-6\left(\left(e^3n-e^3\log(c)\right)x^3-n\log(x)\right)\log\left(-ex+1\right)-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="maxima")
```

```
[Out] -1/162*b*((6*(3*e^3*x^3*log(x^n) - (2*e^3*n - 3*e^3*log(c))*x^3)*dilog(e*x)
 - 6*((e^3*n - e^3*log(c))*x^3 - n*log(x))*log(-e*x + 1) - (2*e^3*x^3 + 3*e
^2*x^2 + 6*e*x - 6*(e^3*x^3 - 1)*log(-e*x + 1))*log(x^n) - 18*(3*e^3*x^3*lo
g(x^n) - (e^3*n - 3*e^3*log(c))*x^3)*polylog(3, e*x))/e^3 - 162*integrate(-
1/162*(e^2*n*x^2 + 2*(4*e^3*n - 3*e^3*log(c))*x^3 + 3*e*n*x - 6*n*log(x) -
6*n)/(e^3*x - e^2), x)) - 1/162*(18*e^3*x^3*dilog(e*x) - 54*e^3*x^3*polylog
(3, e*x) - 2*e^3*x^3 - 3*e^2*x^2 - 6*e*x + 6*(e^3*x^3 - 1)*log(-e*x + 1))*a
/e^3
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3, e*x)*(a + b*log(c*x^n)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3\left(ex\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*polylog(3,e*x),x)
```

```
[Out] Integral(x**2*(a + b*log(c*x**n))*polylog(3, e*x), x)
```

3.215 $\int x\left(a+b\log\left(cx^n\right)\right)\mathbf{Li}_3(ex)\,dx$

Optimal. Leaf size=221

$$\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{8e^2}-\frac{1}{4}x^2\text{Li}_2(ex)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}x^2\text{Li}_3(ex)\left(a+b\log\left(cx^n\right)\right)+\frac{x\left(a+b\log\left(cx^n\right)\right)}{8e}-\frac{1}{8}x^2$$

```
[Out] -5/16*b*n*x/e-1/8*b*n*x^2+1/8*x*(a+b*ln(c*x^n))/e+1/16*x^2*(a+b*ln(c*x^n))-
3/16*b*n*ln(-e*x+1)/e^2+3/16*b*n*x^2*ln(-e*x+1)+1/8*(a+b*ln(c*x^n))*ln(-e*x
+1)/e^2-1/8*x^2*(a+b*ln(c*x^n))*ln(-e*x+1)+1/8*b*n*polylog(2,e*x)/e^2+1/4*b
*n*x^2*polylog(2,e*x)-1/4*x^2*(a+b*ln(c*x^n))*polylog(2,e*x)-1/4*b*n*x^2*po
lylog(3,e*x)+1/2*x^2*(a+b*ln(c*x^n))*polylog(3,e*x)
```

Rubi [A] time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2385, 2395, 43, 2376, 2391, 6591}

$$-\frac{1}{4}x^2\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}x^2\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)+\frac{bn\text{PolyLog}(2,ex)}{8e^2}+\frac{1}{4}bnx^2\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

```
[Out] (-5*b*n*x)/(16*e) - (b*n*x^2)/8 + (x*(a + b*Log[c*x^n]))/(8*e) + (x^2*(a +
b*Log[c*x^n]))/16 - (3*b*n*Log[1 - e*x])/(16*e^2) + (3*b*n*x^2*Log[1 - e*x]
)/16 + ((a + b*Log[c*x^n])*Log[1 - e*x])/(8*e^2) - (x^2*(a + b*Log[c*x^n])*
Log[1 - e*x])/8 + (b*n*PolyLog[2, e*x])/(8*e^2) + (b*n*x^2*PolyLog[2, e*x])
/4 - (x^2*(a + b*Log[c*x^n])*PolyLog[2, e*x])/4 - (b*n*x^2*PolyLog[3, e*x])
/4 + (x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> -Simp[(b*n*(d*x)^(m + 1)*PolyLog[k, e*x^q])/(
d*(m + 1)^2), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a +
 b*Log[c*x^n]), x], x] + Dist[(b*n*q)/(m + 1)^2, Int[(d*x)^m*PolyLog[k - 1,
 e*x^q], x], x] + Simp[((d*x)^(m + 1)*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]))
/(d*(m + 1)), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)\,dx &= -\frac{1}{4}bnx^2\text{Li}_3(ex)+\frac{1}{2}x^2\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)-\frac{1}{2}\int x\left(a+b\log\left(cx^n\right)\right)\text{Li}_2 \\
&= \frac{1}{4}bnx^2\text{Li}_2(ex)-\frac{1}{4}x^2\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)-\frac{1}{4}bnx^2\text{Li}_3(ex)+\frac{1}{2}x^2\left(a+b\,\text{l}\right. \\
&= \frac{x\left(a+b\log\left(cx^n\right)\right)}{8e}+\frac{1}{16}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(a+b\log\left(cx^n\right)\right)\log(1-ex)}{8e^2} \\
&= -\frac{bnx}{8e}-\frac{1}{32}bnx^2+\frac{x\left(a+b\log\left(cx^n\right)\right)}{8e}+\frac{1}{16}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{\left(a+b\log\right.}{} \\
&= -\frac{bnx}{8e}-\frac{1}{32}bnx^2+\frac{x\left(a+b\log\left(cx^n\right)\right)}{8e}+\frac{1}{16}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{1}{16}bnx^2\,\text{lo} \\
&= -\frac{bnx}{8e}-\frac{1}{32}bnx^2+\frac{x\left(a+b\log\left(cx^n\right)\right)}{8e}+\frac{1}{16}x^2\left(a+b\log\left(cx^n\right)\right)+\frac{1}{16}bnx^2\,\text{lo} \\
&= -\frac{3bnx}{16e}-\frac{1}{16}bnx^2+\frac{x\left(a+b\log\left(cx^n\right)\right)}{8e}+\frac{1}{16}x^2\left(a+b\log\left(cx^n\right)\right)-\frac{bn\log(1}{16e^2}
\end{aligned}$$

Mathematica [F] time = 0.12, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

```
[Out] Integrate[x*(a + b*Log[c*x^n])*PolyLog[3, e*x], x]
```

fricas [C] time = 0.88, size = 257, normalized size = 1.16

$$-\frac{\left(2\,be^2n-ae^2\right)x^2+(5\,ben-2\,ae)x-2\left(2\left(be^2n-ae^2\right)x^2+bn\right)\text{Li}_2\,(ex)-\left(\left(3\,be^2n-2\,ae^2\right)x^2-3\,bn+2\,a\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="fricas")
```

```
[Out] -1/16*((2*b*e^2*n - a*e^2)*x^2 + (5*b*e*n - 2*a*e)*x - 2*(2*(b*e^2*n - a*e^
2)*x^2 + b*n)*dilog(e*x) - ((3*b*e^2*n - 2*a*e^2)*x^2 - 3*b*n + 2*a)*log(-e
*x + 1) + (4*b*e^2*x^2*dilog(e*x) - b*e^2*x^2 - 2*b*e*x + 2*(b*e^2*x^2 - b)
```

```
*log(-e*x + 1))*log(c) + (4*b*e^2*n*x^2*dilog(e*x) - b*e^2*n*x^2 - 2*b*e*n*
x + 2*(b*e^2*n*x^2 - b*n)*log(-e*x + 1))*log(x) - 4*(2*b*e^2*n*x^2*log(x) +
 2*b*e^2*x^2*log(c) - (b*e^2*n - 2*a*e^2)*x^2)*polylog(3, e*x))/e^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\log(cx^n) + a)x\mathrm{Li}_3(ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*polylog(3, e*x), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int (b\ln(c\,x^n) + a)\,x\,\mathrm{polylog}(3, ex)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*x^n)+a)*polylog(3,e*x),x)
```

```
[Out] int(x*(b*ln(c*x^n)+a)*polylog(3,e*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{16}b\left(\frac{4\left(e^2x^2\log(x^n) - \left(e^2n - e^2\log(c)\right)x^2\right)\mathrm{Li}_2(ex) - \left(\left(3\,e^2n - 2\,e^2\log(c)\right)x^2 - 2\,n\log(x)\right)\log(-ex+1) - \left(e^2x^2\right.}{e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="maxima")
```

```
[Out] -1/16*b*((4*(e^2*x^2*log(x^n) - (e^2*n - e^2*log(c))*x^2)*dilog(e*x) - ((3*
e^2*n - 2*e^2*log(c))*x^2 - 2*n*log(x))*log(-e*x + 1) - (e^2*x^2 + 2*e*x -
2*(e^2*x^2 - 1)*log(-e*x + 1))*log(x^n) - 4*(2*e^2*x^2*log(x^n) - (e^2*n -
2*e^2*log(c))*x^2)*polylog(3, e*x))/e^2 - 16*integrate(-1/16*(e*n*x + 2*(2*
e^2*n - e^2*log(c))*x^2 - 2*n*log(x) - 2*n)/(e^2*x - e), x)) - 1/16*(4*e^2*
x^2*dilog(e*x) - 8*e^2*x^2*polylog(3, e*x) - e^2*x^2 - 2*e*x + 2*(e^2*x^2 -
 1)*log(-e*x + 1))*a/e^2
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3, e*x)*(a + b*log(c*x^n)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + b\log(cx^n)\right)\mathrm{Li}_3(ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*polylog(3,e*x),x)
```

```
[Out] Integral(x*(a + b*log(c*x**n))*polylog(3, e*x), x)
```

3.216 $\int \left(a+b\log\left(cx^n\right)\right) \mathbf{Li}_3(ex)\,dx$

Optimal. Leaf size=131

$$-x\text{Li}_2(ex)\left(a+b\log\left(cx^n\right)\right)+x\text{Li}_3(ex)\left(a+b\log\left(cx^n\right)\right)+\frac{(1-ex)\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{e}+x\left(a+b\log\left(cx^n\right.\right.$$

```
[Out] -4*b*n*x+x*(a+b*ln(c*x^n))-3*b*n*(-e*x+1)*ln(-e*x+1)/e+(-e*x+1)*(a+b*ln(c*x
^n))*ln(-e*x+1)/e+b*n*polylog(2,e*x)/e+2*b*n*x*polylog(2,e*x)-x*(a+b*ln(c*x
^n))*polylog(2,e*x)-b*n*x*polylog(3,e*x)+x*(a+b*ln(c*x^n))*polylog(3,e*x)
```

Rubi [A] time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2381, 2389, 2295, 2370, 2411, 43, 2351, 2315, 6586}

$$-x\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)+x\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)+2bnx\text{PolyLog}(2,ex)-bnx\text{PolyLog}(3,e$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

```
[Out] -4*b*n*x + x*(a + b*Log[c*x^n]) - (3*b*n*(1 - e*x)*Log[1 - e*x])/e + ((1 -
e*x)*(a + b*Log[c*x^n])*Log[1 - e*x])/e + (b*n*PolyLog[2, e*x])/e + 2*b*n*x
*PolyLog[2, e*x] - x*(a + b*Log[c*x^n])*PolyLog[2, e*x] - b*n*x*PolyLog[3,
e*x] + x*(a + b*Log[c*x^n])*PolyLog[3, e*x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2370

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*PolyLog[k_, (e_.)*(x_)^(q_.)], x_
Symbol] :> -Simp[b*n*x*PolyLog[k, e*x^q], x] + (-Dist[q, Int[PolyLog[k - 1,
 e*x^q]*(a + b*Log[c*x^n]), x], x] + Dist[b*n*q, Int[PolyLog[k - 1, e*x^q],
 x], x] + Simp[x*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]), x]) /; FreeQ[{a, b,
c, e, n, q}, x] && IGtQ[k, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rubi steps

$$\int \left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3(ex)\,dx = -bnx\mathrm{Li}_3(ex)+x\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3(ex)+(bn)\int \mathrm{Li}_2(ex)\,dx-\int \left(a+b\log\left(cx\right.\right.$$

$$= 2bnx\mathrm{Li}_2(ex)-x\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2(ex)-bnx\mathrm{Li}_3(ex)+x\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3(e$$

$$= x\left(a+b\log\left(cx^n\right)\right)+\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)\log(1-ex)}{e}+2bnx\mathrm{Li}_2(ex)-x\left(a\right.$$

$$= -bnx+x\left(a+b\log\left(cx^n\right)\right)+\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)\log(1-ex)}{e}-2\left(bnx+\frac{bn}{}\right.$$

$$= -bnx+x\left(a+b\log\left(cx^n\right)\right)+\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)\log(1-ex)}{e}-2\left(bnx+\frac{bn}{}\right.$$

$$= -bnx+x\left(a+b\log\left(cx^n\right)\right)+\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)\log(1-ex)}{e}-2\left(bnx+\frac{bn}{}\right.$$

$$= -bnx+x\left(a+b\log\left(cx^n\right)\right)+\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)\log(1-ex)}{e}-2\left(bnx+\frac{bn}{}\right.$$

$$= -2bnx+x\left(a+b\log\left(cx^n\right)\right)-\frac{bn(1-ex)\log(1-ex)}{e}+\frac{(1-ex)\left(a+b\log\left(cx^n\right)\right)}{e}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3(ex)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
[Out] Integrate[(a + b*Log[c*x^n])*PolyLog[3, e*x], x]
```

fricas [C] time = 0.66, size = 172, normalized size = 1.31

$$-\frac{(4\,ben - ae)x - (bn + (2\,ben - ae)x)\mathrm{Li}_2\,(ex) + (3\,bn - (3\,ben - ae)x - a)\log\,(-ex + 1) + \left(bex\mathrm{Li}_2\,(ex) - bex + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="fricas")
[Out] -((4*b*e*n - a*e)*x - (b*n + (2*b*e*n - a*e)*x)*dilog(e*x) + (3*b*n - (3*b*
e*n - a*e)*x - a)*log(-e*x + 1) + (b*e*x*dilog(e*x) - b*e*x + (b*e*x - b)*l
og(-e*x + 1))*log(c) + (b*e*n*x*dilog(e*x) - b*e*n*x + (b*e*n*x - b*n)*log(
-e*x + 1))*log(x) - (b*e*n*x*log(x) + b*e*x*log(c) - (b*e*n - a*e)*x)*polyl
og(3, e*x))/e
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right) + a\right)\mathrm{Li}_3(ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*polylog(3, e*x), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,x^n\right) + a\right)\mathrm{polylog}\left(3, ex\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*polylog(3,e*x),x)
[Out] int((b*ln(c*x^n)+a)*polylog(3,e*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\left(\frac{\left(ex\log\left(x^n\right) - \left(2\,en - e\log(c)\right)x\right)\mathrm{Li}_2\,(ex) - \left(\left(3\,en - e\log(c)\right)x - n\log(x)\right)\log\,(-ex + 1) - \left(ex - (ex - 1)\log\right.}{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="maxima")
[Out] -b*(((e*x*log(x^n) - (2*e*n - e*log(c))*x)*dilog(e*x) - ((3*e*n - e*log(c))
*x - n*log(x))*log(-e*x + 1) - (e*x - (e*x - 1)*log(-e*x + 1))*log(x^n) - (
e*x*log(x^n) - (e*n - e*log(c))*x)*polylog(3, e*x))/e - integrate(-((4*e*n
- e*log(c))*x - n*log(x) - n)/(e*x - 1), x)) - (e*x*dilog(e*x) - e*x*polylo
g(3, e*x) - e*x + (e*x - 1)*log(-e*x + 1))*a/e
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, e*x)*(a + b*log(c*x^n)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(cx^{n}\right)\right)\mathrm{Li}_{3}\left(ex\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(3,e*x),x)
```

```
[Out] Integral((a + b*log(c*x**n))*polylog(3, e*x), x)
```

3.217 $\int \frac{(a+b\log(cx^n))\mathbf{Li}_3(ex)}{x}\,dx$

Optimal. Leaf size=26

$$\text{Li}_4(ex)\left(a+b\log\left(cx^n\right)\right)-bn\text{Li}_5(ex)$$

```
[Out] (a+b*ln(c*x^n))*polylog(4,e*x)-b*n*polylog(5,e*x)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2383, 6589}

$$\text{PolyLog}(4,ex)\left(a+b\log\left(cx^n\right)\right)-bn\text{PolyLog}(5,ex)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x,x]
```

```
[Out] (a + b*Log[c*x^n])*PolyLog[4, e*x] - b*n*PolyLog[5, e*x]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)}{x}\,dx&=\left(a+b\log\left(cx^n\right)\right)\text{Li}_4(ex)-(bn)\int \frac{\text{Li}_4(ex)}{x}\,dx\\&=\left(a+b\log\left(cx^n\right)\right)\text{Li}_4(ex)-bn\text{Li}_5(ex)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.15

$$a\text{Li}_4(ex)+b\text{Li}_4(ex)\log\left(cx^n\right)-bn\text{Li}_5(ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x,x]
```

```
[Out] a*PolyLog[4, e*x] + b*Log[c*x^n]*PolyLog[4, e*x] - b*n*PolyLog[5, e*x]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(cx^n\right)\text{polylog}\left(3,ex\right)+a\text{polylog}\left(3,ex\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n)*polylog(3, e*x) + a*polylog(3, e*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^{n}\right)+a\right)\mathrm{Li}_3(ex)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*polylog(3, e*x)/x, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(b\ln\left(c\,x^{n}\right)+a)\,\mathrm{polylog}\left(3,ex\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*polylog(3,e*x)/x,x)
[Out] int((b*ln(c*x^n)+a)*polylog(3,e*x)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(2\,bn\log(x)^{3}-3\,b\log(x)^{2}\log\left(x^{n}\right)-3\left(b\log(c)+a\right)\log(x)^{2}\right)\mathrm{Li}_2\left(ex\right)-\frac{1}{2}\left(bn\log(x)^{2}-2\,b\log(x)\log\left(x^{n}\right)-2\left(b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x,x, algorithm="maxima")
[Out] 1/6*(2*b*n*log(x)^3 - 3*b*log(x)^2*log(x^n) - 3*(b*log(c) + a)*log(x)^2)*di
log(e*x) - 1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x
))*polylog(3, e*x) - 1/6*integrate((3*b*log(-e*x + 1)*log(x)^2*log(x^n) - (
2*b*n*log(x)^3 - 3*(b*log(c) + a)*log(x)^2)*log(-e*x + 1))/x, x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.04

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(3, e*x)*(a + b*log(c*x^n)))/x,x)
[Out] \text{Hanged}
```

sympy [A] time = 9.05, size = 26, normalized size = 1.00

$$a\,\mathrm{Li}_{4}\left(ex\right)+b\left(-n\,\mathrm{Li}_{5}\left(ex\right)+\log\left(cx^{n}\right)\mathrm{Li}_{4}\left(ex\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(3,e*x)/x,x)
[Out] a*polylog(4, e*x) + b*(-n*polylog(5, e*x) + log(c*x**n)*polylog(4, e*x))
```

3.218 $\int \frac{(a+b\log(cx^n))\text{Li}_3(ex)}{x^2}\,dx$

Optimal. Leaf size=174

$$-\frac{\text{Li}_2(ex)\left(a+b\log\left(cx^n\right)\right)}{x}-\frac{\text{Li}_3(ex)\left(a+b\log\left(cx^n\right)\right)}{x}+e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+$$

```
[Out] 3*b*e*n*ln(x)-1/2*b*e*n*ln(x)^2+e*ln(x)*(a+b*ln(c*x^n))-3*b*e*n*ln(-e*x+1)+
3*b*n*ln(-e*x+1)/x-e*(a+b*ln(c*x^n))*ln(-e*x+1)+(a+b*ln(c*x^n))*ln(-e*x+1)/
x-b*e*n*polylog(2,e*x)-2*b*n*polylog(2,e*x)/x-(a+b*ln(c*x^n))*polylog(2,e*x
)/x-b*n*polylog(3,e*x)/x-(a+b*ln(c*x^n))*polylog(3,e*x)/x
```

Rubi [A] time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {2385, 2395, 36, 29, 31, 2376, 2301, 2391, 6591}

$$-\frac{\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)}{x}-\frac{\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)}{x}-ben\text{PolyLog}(2,ex)-\frac{2bn\text{PolyLog}(2,ex)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^2,x]
```

```
[Out] 3*b*e*n*Log[x] - (b*e*n*Log[x]^2)/2 + e*Log[x]*(a + b*Log[c*x^n]) - 3*b*e*n
*Log[1 - e*x] + (3*b*n*Log[1 - e*x])/x - e*(a + b*Log[c*x^n])*Log[1 - e*x]
+ ((a + b*Log[c*x^n])*Log[1 - e*x])/x - b*e*n*PolyLog[2, e*x] - (2*b*n*Poly
Log[2, e*x])/x - ((a + b*Log[c*x^n])*PolyLog[2, e*x])/x - (b*n*PolyLog[3, e
*x])/x - ((a + b*Log[c*x^n])*PolyLog[3, e*x])/x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> -Simp[(b*n*(d*x)^(m + 1)*PolyLog[k, e*x^q])/(
d*(m + 1)^2), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a +
 b*Log[c*x^n]), x], x] + Dist[(b*n*q)/(m + 1)^2, Int[(d*x)^m*PolyLog[k - 1,
 e*x^q], x], x] + Simp[((d*x)^(m + 1)*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]))
/(d*(m + 1)), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)}{x^2}\,dx=-\frac{bn\text{Li}_3(ex)}{x}-\frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)}{x}+(bn)\int\frac{\text{Li}_2(ex)}{x^2}\,dx+\int\frac{\left(a+b\log\left(cx^{\prime}\right.\right.}{x^2}$$

$$=-\frac{2bn\text{Li}_2(ex)}{x}-\frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_2(ex)}{x}-\frac{bn\text{Li}_3(ex)}{x}-\frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)}{x}$$

$$=e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\left(a+b\log\left(cx^n\right)\right)\log(1-ex)+\frac{\left(a+b\log\left(cx^n\right)\right)\log}{x}$$

$$=e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\left(a+b\log\left(cx^n\right)\right)\log(1-ex)+\frac{\left(a+b\log\left(cx^n\right)\right)\log}{x}$$

$$=-\frac{1}{2}ben\log^2(x)+e\log(x)\left(a+b\log\left(cx^n\right)\right)+\frac{bn\log(1-ex)}{x}-e\left(a+b\log\left(cx^n\right)\right)$$

$$=-\frac{1}{2}ben\log^2(x)+e\log(x)\left(a+b\log\left(cx^n\right)\right)+\frac{bn\log(1-ex)}{x}-e\left(a+b\log\left(cx^n\right)\right)$$

$$=ben\log(x)-\frac{1}{2}ben\log^2(x)+e\log(x)\left(a+b\log\left(cx^n\right)\right)-ben\log(1-ex)+\frac{bn\log}{}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\text{Li}_3(ex)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^2,x]
```

```
[Out] Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^2, x]
```

fricas [C] time = 0.62, size = 156, normalized size = 0.90

$$\frac{benx\log(x)^2 - 2\,(benx + 2\,bn + a)\mathrm{Li}_2\,(ex) + 2\,(3\,bn - (3\,ben + ae)x + a)\log\,(-ex + 1) - 2\,\Big(b\mathrm{Li}_2\,(ex) + (bex - b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*e*n*x*log(x)^2 - 2*(b*e*n*x + 2*b*n + a)*dilog(e*x) + 2*(3*b*n - (3*
b*e*n + a*e)*x + a)*log(-e*x + 1) - 2*(b*dilog(e*x) + (b*e*x - b)*log(-e*x
+ 1))*log(c) + 2*(b*e*x*log(c) - b*n*dilog(e*x) + (3*b*e*n + a*e)*x - (b*e*
n*x - b*n)*log(-e*x + 1))*log(x) - 2*(b*n*log(x) + b*n + b*log(c) + a)*poly
log(3, e*x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(cx^n\right) + a\right)\mathrm{Li}_3(ex)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*polylog(3, e*x)/x^2, x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,x^n\right) + a\right)\mathrm{polylog}\left(3, ex\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*polylog(3,e*x)/x^2,x)
```

```
[Out] int((b*ln(c*x^n)+a)*polylog(3,e*x)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(e\log(x) - \frac{(ex-1)\log\,(-ex+1) + \mathrm{Li}_2\,(ex) + \mathrm{Li}_3(ex)}{x}\right)a - b\left(\frac{\left(2\,n + \log(c) + \log\left(x^n\right)\right)\mathrm{Li}_2\,(ex) - \left(enx\log(x) + 3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^2,x, algorithm="maxima")
```

```
[Out] (e*log(x) - ((e*x - 1)*log(-e*x + 1) + dilog(e*x) + polylog(3, e*x))/x)*a -
 b*(((2*n + log(c) + log(x^n))*dilog(e*x) - (e*n*x*log(x) + 3*n + log(c))*l
og(-e*x + 1) - (e*x*log(x) - (e*x - 1)*log(-e*x + 1))*log(x^n) + (n + log(c
) + log(x^n))*polylog(3, e*x))/x + integrate((3*e*n + e*log(c) + (2*e^2*n*x
 - e*n)*log(x))/(e*x^2 - x), x))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(3, e*x)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] \text{Hanged}
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(cx^{n}\right)\right)\mathrm{Li}_{3}\left(ex\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(3,e*x)/x**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))*polylog(3, e*x)/x**2, x)
```

3.219 $\int \frac{(a+b\log(cx^n))\text{Li}_3(ex)}{x^3}\,dx$

Optimal. Leaf size=238

$$\frac{1}{8}e^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{8}e^2\log(1-ex)\left(a+b\log\left(cx^n\right)\right)-\frac{\text{Li}_2(ex)\left(a+b\log\left(cx^n\right)\right)}{4x^2}-\frac{\text{Li}_3(ex)\left(a+b\log\left(cx^n\right.\right.}{2x^2}$$

```
[Out] -5/16*b*e*n/x+3/16*b*e^2*n*ln(x)-1/16*b*e^2*n*ln(x)^2-1/8*e*(a+b*ln(c*x^n))
/x+1/8*e^2*ln(x)*(a+b*ln(c*x^n))-3/16*b*e^2*n*ln(-e*x+1)+3/16*b*n*ln(-e*x+1
)/x^2-1/8*e^2*(a+b*ln(c*x^n))*ln(-e*x+1)+1/8*(a+b*ln(c*x^n))*ln(-e*x+1)/x^2
-1/8*b*e^2*n*polylog(2,e*x)-1/4*b*n*polylog(2,e*x)/x^2-1/4*(a+b*ln(c*x^n))*
polylog(2,e*x)/x^2-1/4*b*n*polylog(3,e*x)/x^2-1/2*(a+b*ln(c*x^n))*polylog(3
,e*x)/x^2
```

Rubi [A] time = 0.22, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2385, 2395, 44, 2376, 2301, 2391, 6591}

$$-\frac{\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)}{4x^2}-\frac{\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{1}{8}be^2n\text{PolyLog}(2,ex)-\frac{bn\text{PolyLog}(2,ex}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^3,x]
```

```
[Out] (-5*b*e*n)/(16*x) + (3*b*e^2*n*Log[x])/16 - (b*e^2*n*Log[x]^2)/16 - (e*(a +
 b*Log[c*x^n]))/(8*x) + (e^2*Log[x]*(a + b*Log[c*x^n]))/8 - (3*b*e^2*n*Log[
1 - e*x])/16 + (3*b*n*Log[1 - e*x])/(16*x^2) - (e^2*(a + b*Log[c*x^n])*Log[
1 - e*x])/8 + ((a + b*Log[c*x^n])*Log[1 - e*x])/(8*x^2) - (b*e^2*n*PolyLog[
2, e*x])/8 - (b*n*PolyLog[2, e*x])/(4*x^2) - ((a + b*Log[c*x^n])*PolyLog[2,
 e*x])/(4*x^2) - (b*n*PolyLog[3, e*x])/(4*x^2) - ((a + b*Log[c*x^n])*PolyLo
g[3, e*x])/(2*x^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2376

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> -Simp[(b*n*(d*x)^(m + 1)*PolyLog[k, e*x^q])/(
d*(m + 1)^2), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a +
 b*Log[c*x^n]), x], x] + Dist[(b*n*q)/(m + 1)^2, Int[(d*x)^m*PolyLog[k - 1,
 e*x^q], x], x] + Simp[((d*x)^(m + 1)*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]))
```

```
/(d*(m + 1)), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3(ex)}{x^3}\,dx = -\frac{bn\mathrm{Li}_3(ex)}{4x^2} - \frac{\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3(ex)}{2x^2} + \frac{1}{2}\int \frac{\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2(ex)}{x^3}\,dx + \frac{1}{4}($$

$$= -\frac{bn\mathrm{Li}_2(ex)}{4x^2} - \frac{\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2(ex)}{4x^2} - \frac{bn\mathrm{Li}_3(ex)}{4x^2} - \frac{\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3(ex)}{2x^2}$$

$$= -\frac{e\left(a+b\log\left(cx^n\right)\right)}{8x} + \frac{1}{8}e^2\log(x)\left(a+b\log\left(cx^n\right)\right) - \frac{1}{8}e^2\left(a+b\log\left(cx^n\right)\right)\log(1$$

$$= -\frac{ben}{8x} - \frac{e\left(a+b\log\left(cx^n\right)\right)}{8x} + \frac{1}{8}e^2\log(x)\left(a+b\log\left(cx^n\right)\right) - \frac{1}{8}e^2\left(a+b\log\left(cx^n\right)\right)$$

$$= -\frac{ben}{8x} - \frac{1}{16}be^2n\log^2(x) - \frac{e\left(a+b\log\left(cx^n\right)\right)}{8x} + \frac{1}{8}e^2\log(x)\left(a+b\log\left(cx^n\right)\right) + \frac{bn}{}$$

$$= -\frac{ben}{8x} - \frac{1}{16}be^2n\log^2(x) - \frac{e\left(a+b\log\left(cx^n\right)\right)}{8x} + \frac{1}{8}e^2\log(x)\left(a+b\log\left(cx^n\right)\right) + \frac{bn}{}$$

$$= -\frac{3ben}{16x} + \frac{1}{16}be^2n\log(x) - \frac{1}{16}be^2n\log^2(x) - \frac{e\left(a+b\log\left(cx^n\right)\right)}{8x} + \frac{1}{8}e^2\log(x)\left(a\right.$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_3(ex)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^3,x]
```

```
[Out] Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^3, x]
```

fricas [C] time = 0.55, size = 221, normalized size = 0.93

$$be^2nx^2\log(x)^2 - (5\,ben + 2\,ae)x - 2\left(be^2nx^2 + 2\,bn + 2\,a\right)\mathrm{Li}_2(ex) - \left(\left(3\,be^2n + 2\,ae^2\right)x^2 - 3\,bn - 2\,a\right)\log(-ex+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(b*e^2*n*x^2*log(x)^2 - (5*b*e*n + 2*a*e)*x - 2*(b*e^2*n*x^2 + 2*b*n +
 2*a)*dilog(e*x) - ((3*b*e^2*n + 2*a*e^2)*x^2 - 3*b*n - 2*a)*log(-e*x + 1)
- 2*(b*e*x + 2*b*dilog(e*x) + (b*e^2*x^2 - b)*log(-e*x + 1))*log(c) + (2*b*
e^2*x^2*log(c) - 2*b*e*n*x + (3*b*e^2*n + 2*a*e^2)*x^2 - 4*b*n*dilog(e*x) -
 2*(b*e^2*n*x^2 - b*n)*log(-e*x + 1))*log(x) - 4*(2*b*n*log(x) + b*n + 2*b*
log(c) + 2*a)*polylog(3, e*x))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(cx^{n}\right) + a\right)\mathrm{Li}_{3}(ex)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*polylog(3, e*x)/x^3, x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,x^{n}\right) + a\right)\mathrm{polylog}\left(3, ex\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*x^n)+a)*polylog(3,e*x)/x^3,x)
```

```
[Out] int((b*ln(c*x^n)+a)*polylog(3,e*x)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(e^{2}\log(x) - \frac{ex + \left(e^{2}x^{2} - 1\right)\log\left(-ex + 1\right) + 2\,\mathrm{Li}_{2}\left(ex\right) + 4\,\mathrm{Li}_{3}(ex)}{x^{2}}\right)a - \frac{1}{16}\,b\left(\frac{4\left(n + \log(c) + \log\left(x^{n}\right)\right)\mathrm{Li}_{2}\left(ex\right)\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(e^2*log(x) - (e*x + (e^2*x^2 - 1)*log(-e*x + 1) + 2*dilog(e*x) + 4*pol
ylog(3, e*x))/x^2)*a - 1/16*b*((4*(n + log(c) + log(x^n))*dilog(e*x) - (2*e
^2*n*x^2*log(x) + 3*n + 2*log(c))*log(-e*x + 1) - 2*(e^2*x^2*log(x) - e*x -
 (e^2*x^2 - 1)*log(-e*x + 1))*log(x^n) + 4*(n + 2*log(c) + 2*log(x^n))*poly
log(3, e*x))/x^2 + 16*integrate(-1/16*(2*e^2*n*x - 5*e*n - 2*e*log(c) - 2*(
2*e^3*n*x^2 - e^2*n*x)*log(x))/(e*x^3 - x^2), x))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(3, e*x)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log\left(cx^{n}\right)\right)\mathrm{Li}_{3}\left(ex\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(3,e*x)/x**3,x)
```

```
[Out] Integral((a + b*log(c*x**n))*polylog(3, e*x)/x**3, x)
```

3.220 $\int -(dx)^m \left(a+b\log\left(cx^n\right)\right)\log\left(1-ex^q\right)\,dx$

Optimal. Leaf size=30

$$-\text{Int}\left((dx)^m\log\left(1-ex^q\right)\left(a+b\log\left(cx^n\right)\right),x\right)$$

```
[Out] -Unintegrable((d*x)^m*(a+b*ln(c*x^n))*ln(1-e*x^q),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int -(dx)^m\left(a+b\log\left(cx^n\right)\right)\log\left(1-ex^q\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[-((d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q]),x]
```

```
[Out] -Defer[Int][(d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q], x]
```

Rubi steps

$$\int -(dx)^m\left(a+b\log\left(cx^n\right)\right)\log\left(1-ex^q\right)\,dx=-\int (dx)^m\left(a+b\log\left(cx^n\right)\right)\log\left(1-ex^q\right)\,dx$$

Mathematica [A] time = 0.24, size = 266, normalized size = 8.87

$$-\frac{x(dx)^m\left(-bnq\,{}_3F_2\left(1,\frac{m}{q}+\frac{1}{q},\frac{m}{q}+\frac{1}{q};\frac{m}{q}+\frac{1}{q}+1,\frac{m}{q}+\frac{1}{q}+1;ex^q\right)+q\,{}_2F_1\left(1,\frac{m+1}{q};\frac{m+q+1}{q};ex^q\right)\left(am+a+b(m+1\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[-((d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q]),x]
```

```
[Out] -((x*(d*x)^m*(-(a*q) - a*m*q + 2*b*n*q - b*n*q*HypergeometricPFQ[{1, q^(-1)
 + m/q, q^(-1) + m/q}, {1 + q^(-1) + m/q, 1 + q^(-1) + m/q}, e*x^q] - b*q*L
og[c*x^n] - b*m*q*Log[c*x^n] + q*Hypergeometric2F1[1, (1 + m)/q, (1 + m + q
)/q, e*x^q]*(a + a*m - b*n + b*(1 + m)*Log[c*x^n]) + a*Log[1 - e*x^q] + 2*a
*m*Log[1 - e*x^q] + a*m^2*Log[1 - e*x^q] - b*n*Log[1 - e*x^q] - b*m*n*Log[1
 - e*x^q] + b*Log[c*x^n]*Log[1 - e*x^q] + 2*b*m*Log[c*x^n]*Log[1 - e*x^q] +
 b*m^2*Log[c*x^n]*Log[1 - e*x^q]))/(1 + m)^3)
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(dx\right)^m b\log\left(cx^n\right)\log\left(-ex^q+1\right)-\left(dx\right)^m a\log\left(-ex^q+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x)^m*(a+b*log(c*x^n))*log(1-e*x^q),x, algorithm="fricas")
```

```
[Out] integral(-(d*x)^m*b*log(c*x^n)*log(-e*x^q + 1) - (d*x)^m*a*log(-e*x^q + 1),
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(b\log\left(cx^n\right)+a\right)(dx)^m\log\left(-ex^q+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x)^m*(a+b*log(c*x^n))*log(1-e*x^q),x, algorithm="giac")
```

```
[Out] integrate(-(b*log(c*x^n) + a)*(d*x)^m*log(-e*x^q + 1), x)
```

maple [A] time = 0.87, size = 844, normalized size = 28.13

$$-\frac{\left(-\frac{\left(-m-q-1\right)eq\,x^{m+q+1}(-e)^{\frac{m}{q}+\frac{1}{q}}\Phi\left(e\,x^{q},1,\frac{m+q+1}{q}\right)}{\left(m+q+1\right)\left(m+1\right)}+\frac{q\,x^{m+1}(-e)^{\frac{m}{q}+\frac{1}{q}}\ln(-e\,x^{q}+1)}{m+1}\right)b\,x^{-m}\,(dx)^{m}\,(-e)^{-\frac{m}{q}-\frac{1}{q}}\ln(c)}{q}-\frac{\left(-\frac{\left(-m-q-1\right)eq\,x^{m+q+1}(}{\left(m+q\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d*x)^m*(b*ln(c*x^n)+a)*ln(1-e*x^q),x)
```

```
[Out] -(d*x)^m*x^(-m)*(-e)^(-1/q*m-1/q)*b*ln(c)/q*(q*x^(m+1)*(-e)^(1/q*m+1/q)/(m+
1)*ln(1-e*x^q)-q/(1+m+q)*x^(1+m+q)*e*(-e)^(1/q*m+1/q)*(-q-m-1)/(m+1)*LerchP
hi(e*x^q,1,(1+m+q)/q))+((-e)^(-1/q*m-1/q)*ln(-e)/q^2*(d*x)^m*x^(-m)*b*n*(q*
x^m*(-e)^(1/q*m+1/q)/(m+1)*ln(1-e*x^q)-q/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)
*(-q-m-1)/(m+1)*LerchPhi(e*x^q,1,(1+m+q)/q))-(-e)^(-1/q*m-1/q)*(d*x)^m*x^(-
m)*b*n/q*(q*x^m*(-e)^(1/q*m+1/q)*ln(x)/(m+1)*ln(1-e*x^q)+x^m*(-e)^(1/q*m+1/
q)*ln(-e)/(m+1)*ln(1-e*x^q)-q*x^m*(-e)^(1/q*m+1/q)/(m+1)^2*ln(1-e*x^q)+q/(1
+m+q)^2*x^(q+m)*e*(-e)^(1/q*m+1/q)*(-q-m-1)/(m+1)*LerchPhi(e*x^q,1,(1+m+q)/
q)-q/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)*ln(x)*(-q-m-1)/(m+1)*LerchPhi(e*x^q
,1,(1+m+q)/q)-1/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)*ln(-e)*(-q-m-1)/(m+1)*Le
rchPhi(e*x^q,1,(1+m+q)/q)+q/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)/(m+1)*LerchP
hi(e*x^q,1,(1+m+q)/q)+q/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)*(-q-m-1)/(m+1)^2
*LerchPhi(e*x^q,1,(1+m+q)/q)+1/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)*(-q-m-1)/
(m+1)*LerchPhi(e*x^q,2,(1+m+q)/q)))*x-(d*x)^m*x^(-m)*(-e)^(-1/q*m-1/q)*a/q*
(q*x^(m+1)*(-e)^(1/q*m+1/q)/(m+1)*ln(1-e*x^q)-q/(1+m+q)*x^(1+m+q)*e*(-e)^(1
/q*m+1/q)*(-q-m-1)/(m+1)*LerchPhi(e*x^q,1,(1+m+q)/q))
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(bd^{m}(m+1)xx^{m}\log\left(x^{n}\right)+\left(ad^{m}(m+1)+\left(d^{m}(m+1)\log(c)-d^{m}n\right)b\right)xx^{m}\right)\log\left(-ex^{q}+1\right)}{m^{2}+2m+1}+\int\frac{\left(mq+q\right)bd^{m}ee^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x)^m*(a+b*log(c*x^n))*log(1-e*x^q),x, algorithm="maxima")
```

```
[Out] -(b*d^m*(m + 1)*x*x^m*log(x^n) + (a*d^m*(m + 1) + (d^m*(m + 1)*log(c) - d^m
*n)*b)*x*x^m)*log(-e*x^q + 1)/(m^2 + 2*m + 1) + integrate(((m*q + q)*b*d^m*
e*e^(m*log(x) + q*log(x))*log(x^n) + ((m*q + q)*a*d^m*e - (d^m*e*n*q - (m*q
 + q)*d^m*e*log(c))*b)*e^(m*log(x) + q*log(x)))/((m^2 + 2*m + 1)*e*x^q - m^
2 - 2*m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int -\ln\left(1-e\,x^q\right)\,{\left(d\,x\right)}^m\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(1 - e*x^q)*(d*x)^m*(a + b*log(c*x^n)),x)
```

```
[Out] int(-log(1 - e*x^q)*(d*x)^m*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x)**m*(a+b*ln(c*x**n))*ln(1-e*x**q),x)
```

```
[Out] Timed out
```

3.221 $\int (dx)^m \left(a+b\log\left(cx^n\right)\right) \mathrm{Li}_2\left(ex^q\right)\, dx$

Optimal. Leaf size=178

$$\frac{q\mathrm{Int}\left((dx)^m\log\left(1-ex^q\right)\left(a+b\log\left(cx^n\right)\right),x\right)}{m+1}+\frac{(dx)^{m+1}\mathrm{Li}_2\left(ex^q\right)\left(a+b\log\left(cx^n\right)\right)}{d(m+1)}-\frac{benq^2x^{q+1}(dx)^m\,{}_2F_1\left(1,\frac{m+q+1}{q};\right.}{(m+1)^3(m+q+1}$$

```
[Out] -b*e*n*q^2*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],e*x^q)/(1
+m)^3/(1+m+q)-b*n*q*(d*x)^(1+m)*ln(1-e*x^q)/d/(1+m)^3-b*n*(d*x)^(1+m)*polyl
og(2,e*x^q)/d/(1+m)^2+(d*x)^(1+m)*(a+b*ln(c*x^n))*polylog(2,e*x^q)/d/(1+m)+
q*Unintegrable((d*x)^m*(a+b*ln(c*x^n))*ln(1-e*x^q),x)/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\log\left(cx^n\right)\right) \mathrm{PolyLog}\left(2,ex^q\right)\, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[2, e*x^q],x]
```

```
[Out] -((b*e*n*q^2*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m +
 2*q)/q, e*x^q])/((1 + m)^3*(1 + m + q))) - (b*n*q*(d*x)^(1 + m)*Log[1 - e*
x^q])/(d*(1 + m)^3) - (b*n*(d*x)^(1 + m)*PolyLog[2, e*x^q])/(d*(1 + m)^2) +
 ((d*x)^(1 + m)*(a + b*Log[c*x^n])*PolyLog[2, e*x^q])/(d*(1 + m)) + (q*Defe
r[Int][(d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q], x])/(1 + m)
```

Rubi steps

$$\int (dx)^m \left(a+b\log\left(cx^n\right)\right) \mathrm{Li}_2\left(ex^q\right)\, dx=-\frac{bn(dx)^{1+m}\mathrm{Li}_2\left(ex^q\right)}{d(1+m)^2}+\frac{(dx)^{1+m}\left(a+b\log\left(cx^n\right)\right)\mathrm{Li}_2\left(ex^q\right)}{d(1+m)}+\frac{q\int (dx)^m \left(a\right.}{}$$

$$=-\frac{bnq(dx)^{1+m}\log\left(1-ex^q\right)}{d(1+m)^3}-\frac{bn(dx)^{1+m}\mathrm{Li}_2\left(ex^q\right)}{d(1+m)^2}+\frac{(dx)^{1+m}\left(a+b\log\left(c\right.\right.}{d(1+m)}$$

$$=-\frac{bnq(dx)^{1+m}\log\left(1-ex^q\right)}{d(1+m)^3}-\frac{bn(dx)^{1+m}\mathrm{Li}_2\left(ex^q\right)}{d(1+m)^2}+\frac{(dx)^{1+m}\left(a+b\log\left(c\right.\right.}{d(1+m)}$$

$$=-\frac{benq^2x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ex^q\right)}{(1+m)^3(1+m+q)}-\frac{bnq(dx)^{1+m}\log\left(1-ex^q\right)}{d(1+m)^3}$$

Mathematica [A] time = 0.11, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\log\left(cx^n\right)\right) \mathrm{Li}_2\left(ex^q\right)\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[2, e*x^q],x]
```

```
[Out] Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[2, e*x^q], x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((dx)^m\, b\mathrm{Li}_2\left(ex^q\right)\log\left(cx^n\right)+(dx)^m\, a\mathrm{Li}_2\left(ex^q\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(2,e*x^q),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*b*dilog(e*x^q)*log(c*x^n) + (d*x)^m*a*dilog(e*x^q), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(cx^{n}\right) + a\right) (dx)^{m} \operatorname{Li}_2\left(ex^{q}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(2,e*x^q),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(d*x)^m*dilog(e*x^q), x)
```

maple [A] time = 0.35, size = 867, normalized size = 4.87

$$-\frac{\left(-\frac{e q^{2} x^{m+q+1} (-e)^{\frac{m}{q}+\frac{1}{q}} \Phi\left(e x^{q},1,\frac{m+q+1}{q}\right)}{(m+1)^{2}} - \frac{q^{2} x^{m+1} (-e)^{\frac{m}{q}+\frac{1}{q}} \ln(-e x^{q}+1)}{(m+1)^{2}} - \frac{q x^{m+1} (-e)^{\frac{m}{q}+\frac{1}{q}} \operatorname{polylog}(2,e x^{q})}{m+1}\right) b x^{-m} (dx)^{m} (-e)^{-\frac{m}{q}-\frac{1}{q}} \ln\cdots}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*ln(c*x^n)+a)*polylog(2,e*x^q),x)
```

```
[Out] -(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*b*ln(c)/q*(-q^2*x^(m+1)*(-e)^(m/q+1/q)/(m+1
)^2*ln(-e*x^q+1)-q*x^(m+1)*(-e)^(m/q+1/q)/(m+1)*polylog(2,e*x^q)-q^2*x^(m+q
+1)*e*(-e)^(m/q+1/q)/(m+1)^2*LerchPhi(e*x^q,1,(m+q+1)/q))+((-e)^(-m/q-1/q)*
ln(-e)/q^2*(d*x)^m*x^(-m)*b*n*(-q^2*x^m*(-e)^(m/q+1/q)/(m+1)^2*ln(-e*x^q+1)
-q*x^m*(-e)^(m/q+1/q)/(m+1)*polylog(2,e*x^q)-q^2*x^(m+q)*e*(-e)^(m/q+1/q)/(
m+1)^2*LerchPhi(e*x^q,1,(m+q+1)/q))-(-e)^(-m/q-1/q)*(d*x)^m*x^(-m)*b*n/q*(-
q^2*x^m*(-e)^(m/q+1/q)*ln(x)/(m+1)^2*ln(-e*x^q+1)-q*x^m*(-e)^(m/q+1/q)*ln(-
e)/(m+1)^2*ln(-e*x^q+1)+2*q^2*x^m*(-e)^(m/q+1/q)/(m+1)^3*ln(-e*x^q+1)-q*x^m
*(-e)^(m/q+1/q)*ln(x)/(m+1)*polylog(2,e*x^q)-x^m*(-e)^(m/q+1/q)*ln(-e)/(m+1
)*polylog(2,e*x^q)+q*x^m*(-e)^(m/q+1/q)/(m+1)^2*polylog(2,e*x^q)-q^2*x^(m+q
)*e*(-e)^(m/q+1/q)*ln(x)/(m+1)^2*LerchPhi(e*x^q,1,(m+q+1)/q)-q*x^(m+q)*e*(-
e)^(m/q+1/q)*ln(-e)/(m+1)^2*LerchPhi(e*x^q,1,(m+q+1)/q)+2*q^2*x^(m+q)*e*(-e
)^(m/q+1/q)/(m+1)^3*LerchPhi(e*x^q,1,(m+q+1)/q)+q*x^(m+q)*e*(-e)^(m/q+1/q)/
(m+1)^2*LerchPhi(e*x^q,2,(m+q+1)/q)))*x-(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*a/q*
(-q^2*x^(m+1)*(-e)^(m/q+1/q)/(m+1)^2*ln(-e*x^q+1)-q*x^(m+1)*(-e)^(m/q+1/q)/
(m+1)*polylog(2,e*x^q)-q^2*x^(m+q+1)*e*(-e)^(m/q+1/q)/(m+1)^2*LerchPhi(e*x^
q,1,(m+q+1)/q))
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(bd^{m}m^{2} + 2\,bd^{m}m + bd^{m}\right)xx^{m}\log\left(x^{n}\right) + \left(\left(b\log(c) + a\right)d^{m}m^{2} + 2\left(b\log(c) + a\right)d^{m}m + \left(b\log(c) + a\right)d^{m} - (b\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(2,e*x^q),x, algorithm="maxima")
```

```
[Out] (((b*d^m*m^2 + 2*b*d^m*m + b*d^m)*x*x^m*log(x^n) + ((b*log(c) + a)*d^m*m^2
+ 2*(b*log(c) + a)*d^m*m + (b*log(c) + a)*d^m - (b*d^m*m + b*d^m)*n)*x*x^m)
*dilog(e*x^q) + ((b*d^m*m + b*d^m)*q*x*x^m*log(x^n) + ((b*log(c) + a)*d^m*m
 - 2*b*d^m*n + (b*log(c) + a)*d^m)*q*x*x^m)*log(-e*x^q + 1))/(m^3 + 3*m^2 +
 3*m + 1) - integrate(-((b*d^m*e*m + b*d^m*e)*q^2*e^(m*log(x) + q*log(x))*l
og(x^n) + ((b*log(c) + a)*d^m*e*m - 2*b*d^m*e*n + (b*log(c) + a)*d^m*e)*q^2
```

```
*e^(m*log(x) + q*log(x)))/(m^3 + 3*m^2 - (e*m^3 + 3*e*m^2 + 3*e*m + e)*x^q
+ 3*m + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \operatorname{polylog}(2, e\,x^q)\ (a + b\,\ln(c\,x^n))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2, e*x^q)*(a + b*log(c*x^n)),x)
[Out] int((d*x)^m*polylog(2, e*x^q)*(a + b*log(c*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*ln(c*x**n))*polylog(2,e*x**q),x)
[Out] Timed out
```

3.222 $\int (dx)^m \left(a+b\log\left(cx^n\right)\right) \mathbf{Li}_3\left(ex^q\right)\, dx$

Optimal. Leaf size=245

$$-\frac{q^2\text{Int}\left((dx)^m\log\left(1-ex^q\right)\left(a+b\log\left(cx^n\right)\right),x\right)}{(m+1)^2}-\frac{q(dx)^{m+1}\text{Li}_2\left(ex^q\right)\left(a+b\log\left(cx^n\right)\right)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{Li}_3\left(ex^q\right)\left(a+b\log\left(cx^n\right)\right)}{d(m+1)}$$

```
[Out] 2*b*e*n*q^3*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],e*x^q)/(
1+m)^4/(1+m+q)+2*b*n*q^2*(d*x)^(1+m)*ln(1-e*x^q)/d/(1+m)^4+2*b*n*q*(d*x)^(1
+m)*polylog(2,e*x^q)/d/(1+m)^3-q*(d*x)^(1+m)*(a+b*ln(c*x^n))*polylog(2,e*x^
q)/d/(1+m)^2-b*n*(d*x)^(1+m)*polylog(3,e*x^q)/d/(1+m)^2+(d*x)^(1+m)*(a+b*ln
(c*x^n))*polylog(3,e*x^q)/d/(1+m)-q^2*Unintegrable((d*x)^m*(a+b*ln(c*x^n))*
ln(1-e*x^q),x)/(1+m)^2
```

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\log\left(cx^n\right)\right) \text{PolyLog}\left(3,ex^q\right)\, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[3, e*x^q],x]
```

```
[Out] (2*b*e*n*q^3*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m +
 2*q)/q, e*x^q])/((1 + m)^4*(1 + m + q)) + (2*b*n*q^2*(d*x)^(1 + m)*Log[1 -
 e*x^q])/(d*(1 + m)^4) + (2*b*n*q*(d*x)^(1 + m)*PolyLog[2, e*x^q])/(d*(1 +
m)^3) - (q*(d*x)^(1 + m)*(a + b*Log[c*x^n])*PolyLog[2, e*x^q])/(d*(1 + m)^2
) - (b*n*(d*x)^(1 + m)*PolyLog[3, e*x^q])/(d*(1 + m)^2) + ((d*x)^(1 + m)*(a
 + b*Log[c*x^n])*PolyLog[3, e*x^q])/(d*(1 + m)) - (q^2*Defer[Int][(d*x)^m*(
a + b*Log[c*x^n])*Log[1 - e*x^q], x])/(1 + m)^2
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(a+b\log\left(cx^n\right)\right) \text{Li}_3\left(ex^q\right)\, dx &= -\frac{bn(dx)^{1+m}\text{Li}_3\left(ex^q\right)}{d(1+m)^2}+\frac{(dx)^{1+m}\left(a+b\log\left(cx^n\right)\right)\text{Li}_3\left(ex^q\right)}{d(1+m)}-\frac{q\int (dx)^m
\\
&= \frac{2bnq(dx)^{1+m}\text{Li}_2\left(ex^q\right)}{d(1+m)^3}-\frac{q(dx)^{1+m}\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(ex^q\right)}{d(1+m)^2}-\frac{bn(dx)
\\
&= \frac{2bnq(dx)^{1+m}\text{Li}_2\left(ex^q\right)}{d(1+m)^3}-\frac{q(dx)^{1+m}\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(ex^q\right)}{d(1+m)^2}-\frac{bn(dx)
\\
&= \frac{2bnq(dx)^{1+m}\text{Li}_2\left(ex^q\right)}{d(1+m)^3}-\frac{q(dx)^{1+m}\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(ex^q\right)}{d(1+m)^2}-\frac{bn(dx)
\\
&= 2\left(\frac{benq^3x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ex^q\right)}{(1+m)^4(1+m+q)}+\frac{bnq^2(dx)^{1+m}\log\left(1-\right.}{d(1+m)^4}\right.
\end{aligned}$$

Mathematica [A] time = 0.07, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\log\left(cx^n\right)\right) \text{Li}_3\left(ex^q\right)\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[3, e*x^q],x]
```

```
[Out] Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[3, e*x^q], x]
```

fricas [A] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left((dx)^m b\log\left(cx^n\right) + (dx)^m a\right)\text{polylog}\left(3, ex^q\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(3,e*x^q),x, algorithm="fricas")
```

```
[Out] integral(((d*x)^m*b*log(c*x^n) + (d*x)^m*a)*polylog(3, e*x^q), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(cx^n\right) + a\right)(dx)^m \operatorname{Li}_3(ex^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(3,e*x^q),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(d*x)^m*polylog(3, e*x^q), x)
```

maple [A] time = 2.28, size = 1065, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*ln(c*x^n)+a)*polylog(3,e*x^q),x)
```

```
[Out] -(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*b*ln(c)/q*(q^3*x^(m+1)*(-e)^(m/q+1/q)/(m+1)
^3*ln(-e*x^q+1)+q^2*x^(m+1)*(-e)^(m/q+1/q)/(m+1)^2*polylog(2,e*x^q)-q*x^(m+
1)*(-e)^(m/q+1/q)/(m+1)*polylog(3,e*x^q)+q^3*x^(m+q+1)*e*(-e)^(m/q+1/q)/(m+
1)^3*LerchPhi(e*x^q,1,(m+q+1)/q))+((-e)^(-m/q-1/q)/q^2*ln(-e)*(d*x)^m*x^(-m
)*b*n*(q^3*x^m*(-e)^(m/q+1/q)/(m+1)^3*ln(-e*x^q+1)+q^2*x^m*(-e)^(m/q+1/q)/(
m+1)^2*polylog(2,e*x^q)-q*x^m*(-e)^(m/q+1/q)/(m+1)*polylog(3,e*x^q)+q^3*x^(
m+q)*e*(-e)^(m/q+1/q)/(m+1)^3*LerchPhi(e*x^q,1,(m+q+1)/q))-(-e)^(-m/q-1/q)*
(d*x)^m*x^(-m)*b*n/q*(q^3*x^m*(-e)^(m/q+1/q)*ln(x)/(m+1)^3*ln(-e*x^q+1)+q^2
*x^m*(-e)^(m/q+1/q)*ln(-e)/(m+1)^3*ln(-e*x^q+1)-3*q^3*x^m*(-e)^(m/q+1/q)/(m
+1)^4*ln(-e*x^q+1)+q^2*x^m*(-e)^(m/q+1/q)*ln(x)/(m+1)^2*polylog(2,e*x^q)+q*
x^m*(-e)^(m/q+1/q)*ln(-e)/(m+1)^2*polylog(2,e*x^q)-2*q^2*x^m*(-e)^(m/q+1/q)
/(m+1)^3*polylog(2,e*x^q)-q*x^m*(-e)^(m/q+1/q)*ln(x)/(m+1)*polylog(3,e*x^q)
-x^m*(-e)^(m/q+1/q)*ln(-e)/(m+1)*polylog(3,e*x^q)+q*x^m*(-e)^(m/q+1/q)/(m+1
)^2*polylog(3,e*x^q)+q^3*x^(m+q)*e*(-e)^(m/q+1/q)*ln(x)/(m+1)^3*LerchPhi(e*
x^q,1,(m+q+1)/q)+q^2*x^(m+q)*e*(-e)^(m/q+1/q)*ln(-e)/(m+1)^3*LerchPhi(e*x^q
,1,(m+q+1)/q)-3*q^3*x^(m+q)*e*(-e)^(m/q+1/q)/(m+1)^4*LerchPhi(e*x^q,1,(m+q+
1)/q)-q^2*x^(m+q)*e*(-e)^(m/q+1/q)/(m+1)^3*LerchPhi(e*x^q,2,(m+q+1)/q)))*x-
(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*a/q*(q^3*x^(m+1)*(-e)^(m/q+1/q)/(m+1)^3*ln(-
e*x^q+1)+q^2*x^(m+1)*(-e)^(m/q+1/q)/(m+1)^2*polylog(2,e*x^q)-q*x^(m+1)*(-e)
^(m/q+1/q)/(m+1)*polylog(3,e*x^q)+q^3*x^(m+q+1)*e*(-e)^(m/q+1/q)/(m+1)^3*Le
rchPhi(e*x^q,1,(m+q+1)/q))
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(m^2q + 2\,mq + q\right)bd^m x x^m \log\left(x^n\right) + \left(\left(m^2q + 2\,mq + q\right)ad^m + \left(\left(m^2q + 2\,mq + q\right)d^m \log(c) - 2\left(mnq + nq\right)d^m\right)b \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(3,e*x^q),x, algorithm="maxima")
```

```
[Out] -(((m^2*q + 2*m*q + q)*b*d^m*x*x^m*log(x^n) + ((m^2*q + 2*m*q + q)*a*d^m +
((m^2*q + 2*m*q + q)*d^m*log(c) - 2*(m*n*q + n*q)*d^m)*b)*x*x^m)*dilog(e*x^
q) + ((m*q^2 + q^2)*b*d^m*x*x^m*log(x^n) + ((m*q^2 + q^2)*a*d^m - (3*d^m*n*
q^2 - (m*q^2 + q^2)*d^m*log(c))*b)*x*x^m)*log(-e*x^q + 1) - ((m^3 + 3*m^2 +
 3*m + 1)*b*d^m*x*x^m*log(x^n) + ((m^3 + 3*m^2 + 3*m + 1)*a*d^m + ((m^3 + 3
*m^2 + 3*m + 1)*d^m*log(c) - (m^2*n + 2*m*n + n)*d^m)*b)*x*x^m)*polylog(3,
e*x^q))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + integrate(-((m*q^3 + q^3)*b*d^m*e
*e^(m*log(x) + q*log(x))*log(x^n) + ((m*q^3 + q^3)*a*d^m*e - (3*d^m*e*n*q^3
 - (m*q^3 + q^3)*d^m*e*log(c))*b)*e^(m*log(x) + q*log(x)))/(m^4 + 4*m^3 - (
m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*e*x^q + 6*m^2 + 4*m + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\mathrm{polylog}\left(3,e\,x^q\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3, e*x^q)*(a + b*log(c*x^n)),x)
```

```
[Out] int((d*x)^m*polylog(3, e*x^q)*(a + b*log(c*x^n)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b\log\left(cx^n\right)\right) \mathrm{Li}_3\left(ex^q\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*ln(c*x**n))*polylog(3,e*x**q),x)
```

```
[Out] Integral((d*x)**m*(a + b*log(c*x**n))*polylog(3, e*x**q), x)
```

3.223 $\int x^2 \log\left(c\left(bx^n\right)^p\right)\,dx$

Optimal. Leaf size=27

$$\frac{1}{3}x^3\log\left(c\left(bx^n\right)^p\right)-\frac{1}{9}npx^3$$

```
[Out] -1/9*n*p*x^3+1/3*x^3*ln(c*(b*x^n)^p)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2304, 2445}

$$\frac{1}{3}x^3\log\left(c\left(bx^n\right)^p\right)-\frac{1}{9}npx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(b*x^n)^p],x]
```

```
[Out] -(n*p*x^3)/9 + (x^3*Log[c*(b*x^n)^p])/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(c\left(bx^n\right)^p\right)\,dx &= \text{Subst}\left(\int x^2\log\left(b^pcx^{np}\right)\,dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\ &= -\frac{1}{9}npx^3+\frac{1}{3}x^3\log\left(c\left(bx^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{1}{3}x^3\log\left(c\left(bx^n\right)^p\right)-\frac{1}{9}npx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(b*x^n)^p],x]
```

```
[Out] -1/9*(n*p*x^3) + (x^3*Log[c*(b*x^n)^p])/3
```

fricas [A] time = 0.73, size = 32, normalized size = 1.19

$$\frac{1}{3}npx^3\log(x)-\frac{1}{9}npx^3+\frac{1}{3}px^3\log(b)+\frac{1}{3}x^3\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p),x, algorithm="fricas")
[Out] 1/3*n*p*x^3*log(x) - 1/9*n*p*x^3 + 1/3*p*x^3*log(b) + 1/3*x^3*log(c)
```

giac [A] time = 0.35, size = 32, normalized size = 1.19

$$\frac{1}{3}\,npx^3\log(x) - \frac{1}{9}\,npx^3 + \frac{1}{3}\,px^3\log(b) + \frac{1}{3}\,x^3\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p),x, algorithm="giac")
[Out] 1/3*n*p*x^3*log(x) - 1/9*n*p*x^3 + 1/3*p*x^3*log(b) + 1/3*x^3*log(c)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int x^2\ln\left(c\,(b\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^n)^p),x)
[Out] int(x^2*ln(c*(b*x^n)^p),x)
```

maxima [A] time = 1.10, size = 23, normalized size = 0.85

$$-\frac{1}{9}\,npx^3 + \frac{1}{3}\,x^3\log\left((bx^n)^p\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p),x, algorithm="maxima")
[Out] -1/9*n*p*x^3 + 1/3*x^3*log((b*x^n)^p*c)
```

mupad [B] time = 3.84, size = 23, normalized size = 0.85

$$\frac{x^3\,\ln\left(c\,(b\,x^n)^p\right)}{3} - \frac{n\,p\,x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(b*x^n)^p),x)
[Out] (x^3*log(c*(b*x^n)^p))/3 - (n*p*x^3)/9
```

sympy [A] time = 2.21, size = 37, normalized size = 1.37

$$\frac{npx^3\log(x)}{3} - \frac{npx^3}{9} + \frac{px^3\log(b)}{3} + \frac{x^3\log(c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**n)**p),x)
[Out] n*p*x**3*log(x)/3 - n*p*x**3/9 + p*x**3*log(b)/3 + x**3*log(c)/3
```

3.224 $\int x \log\left(c (bx^n)^p\right) dx$

Optimal. Leaf size=27

$$\frac{1}{2}x^2 \log\left(c (bx^n)^p\right) - \frac{1}{4}npx^2$$

```
[Out] -1/4*n*p*x^2+1/2*x^2*ln(c*(b*x^n)^p)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304, 2445}

$$\frac{1}{2}x^2 \log\left(c (bx^n)^p\right) - \frac{1}{4}npx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(b*x^n)^p],x]
```

```
[Out] -(n*p*x^2)/4 + (x^2*Log[c*(b*x^n)^p])/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int x \log\left(c (bx^n)^p\right) dx &= \text{Subst}\left(\int x \log\left(b^p cx^{np}\right) dx, b^p cx^{np}, c (bx^n)^p\right)\\ &= -\frac{1}{4}npx^2 + \frac{1}{2}x^2 \log\left(c (bx^n)^p\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{1}{2}x^2 \log\left(c (bx^n)^p\right) - \frac{1}{4}npx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(b*x^n)^p],x]
```

```
[Out] -1/4*(n*p*x^2) + (x^2*Log[c*(b*x^n)^p])/2
```

fricas [A] time = 0.65, size = 32, normalized size = 1.19

$$\frac{1}{2}npx^2 \log(x) - \frac{1}{4}npx^2 + \frac{1}{2}px^2 \log(b) + \frac{1}{2}x^2 \log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p),x, algorithm="fricas")
```

```
[Out] 1/2*n*p*x^2*log(x) - 1/4*n*p*x^2 + 1/2*p*x^2*log(b) + 1/2*x^2*log(c)
```

giac [A] time = 0.31, size = 32, normalized size = 1.19

$$\frac{1}{2}\,npx^2\log(x) - \frac{1}{4}\,npx^2 + \frac{1}{2}\,px^2\log(b) + \frac{1}{2}\,x^2\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p),x, algorithm="giac")
```

```
[Out] 1/2*n*p*x^2*log(x) - 1/4*n*p*x^2 + 1/2*p*x^2*log(b) + 1/2*x^2*log(c)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\ln\left(c\,(b\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^n)^p),x)
```

```
[Out] int(x*ln(c*(b*x^n)^p),x)
```

maxima [A] time = 1.16, size = 23, normalized size = 0.85

$$-\frac{1}{4}\,npx^2 + \frac{1}{2}\,x^2\log\left((bx^n)^p\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/4*n*p*x^2 + 1/2*x^2*log((b*x^n)^p*c)
```

mupad [B] time = 3.80, size = 23, normalized size = 0.85

$$\frac{x^2\,\ln\left(c\,(b\,x^n)^p\right)}{2} - \frac{n\,p\,x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(b*x^n)^p),x)
```

```
[Out] (x^2*log(c*(b*x^n)^p))/2 - (n*p*x^2)/4
```

sympy [A] time = 0.95, size = 37, normalized size = 1.37

$$\frac{npx^2\log\left(x\right)}{2} - \frac{npx^2}{4} + \frac{px^2\log\left(b\right)}{2} + \frac{x^2\log\left(c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**n)**p),x)
```

```
[Out] n*p*x**2*log(x)/2 - n*p*x**2/4 + p*x**2*log(b)/2 + x**2*log(c)/2
```

3.225 $\int \log \left(c \left(b x^n\right)^p\right) dx$

Optimal. Leaf size=18

$$x \log \left(c \left(b x^n\right)^p\right)-n p x$$

```
[Out] -n*p*x+x*ln(c*(b*x^n)^p)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2295, 2445}

$$x \log \left(c \left(b x^n\right)^p\right)-n p x$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p],x]
```

```
[Out] -(n*p*x) + x*Log[c*(b*x^n)^p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned} \int \log \left(c \left(b x^n\right)^p\right) dx &= \text{Subst}\left(\int \log \left(b^p c x^{n p}\right) dx, b^p c x^{n p}, c \left(b x^n\right)^p\right)\\ &= -n p x+x \log \left(c \left(b x^n\right)^p\right) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$x \log \left(c \left(b x^n\right)^p\right)-n p x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p],x]
```

```
[Out] -(n*p*x) + x*Log[c*(b*x^n)^p]
```

fricas [A] time = 0.48, size = 21, normalized size = 1.17

$$n p x \log (x)-n p x+p x \log (b)+x \log (c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p),x, algorithm="fricas")
```

```
[Out] n*p*x*log(x) - n*p*x + p*x*log(b) + x*log(c)
```

giac [**A**] time = 0.36, size = 21, normalized size = 1.17

$$npx\log(x) - npx + px\log(b) + x\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p),x, algorithm="giac")
[Out] n*p*x*log(x) - n*p*x + p*x*log(b) + x*log(c)
```

maple [**A**] time = 0.04, size = 19, normalized size = 1.06

$$-npx + x\ln\left(c\,(b\,x^{n})^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p),x)
[Out] -n*p*x+x*ln(c*(b*x^n)^p)
```

maxima [**A**] time = 1.39, size = 18, normalized size = 1.00

$$-npx + x\log\left((bx^{n})^{p}\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p),x, algorithm="maxima")
[Out] -n*p*x + x*log((b*x^n)^p*c)
```

mupad [**B**] time = 0.03, size = 17, normalized size = 0.94

$$x\,\left(\ln\left(c\,(b\,x^{n})^{p}\right) - n\,p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p),x)
[Out] x*(log(c*(b*x^n)^p) - n*p)
```

sympy [**A**] time = 0.41, size = 24, normalized size = 1.33

$$npx\log\left(x\right) - npx + px\log\left(b\right) + x\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p),x)
[Out] n*p*x*log(x) - n*p*x + p*x*log(b) + x*log(c)
```

3.226 $\int \frac{\log\left(c(bx^n)^p\right)}{x}\,dx$

Optimal. Leaf size=22

$$\frac{\log^2\left(c\left(bx^n\right)^p\right)}{2np}$$

```
[Out] 1/2*ln(c*(b*x^n)^p)^2/n/p
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2301, 2445\}$

$$\frac{\log^2\left(c\left(bx^n\right)^p\right)}{2np}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]/x,x]
[Out] Log[c*(b*x^n)^p]^2/(2*n*p)
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(bx^n\right)^p\right)}{x}\,dx &= \text{Subst}\left(\int \frac{\log\left(b^p c x^{np}\right)}{x}\,dx, b^p c x^{np}, c\left(bx^n\right)^p\right)\\ &= \frac{\log^2\left(c\left(bx^n\right)^p\right)}{2np}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log^2\left(c\left(bx^n\right)^p\right)}{2np}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]/x,x]
[Out] Log[c*(b*x^n)^p]^2/(2*n*p)
```

fricas [A] time = 0.67, size = 19, normalized size = 0.86

$$\frac{1}{2}\,np\log(x)^2 + \left(p\log(b) + \log(c)\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] 1/2*n*p*log(x)^2 + (p*log(b) + log(c))*log(x)
```

giac [A] time = 0.30, size = 20, normalized size = 0.91

$$\frac{1}{2}\,np\log(x)^2 + p\log(b)\log(x) + \log(c)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] 1/2*n*p*log(x)^2 + p*log(b)*log(x) + log(c)*log(x)
```

maple [A] time = 0.04, size = 21, normalized size = 0.95

$$\frac{\ln\left(c\,(b\,x^n)^p\right)^2}{2np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)/x,x)
```

```
[Out] 1/2*ln(c*(b*x^n)^p)^2/p/n
```

maxima [A] time = 1.20, size = 20, normalized size = 0.91

$$\frac{\log\left((bx^n)^p\,c\right)^2}{2\,np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*log((b*x^n)^p*c)^2/(n*p)
```

mupad [B] time = 3.80, size = 20, normalized size = 0.91

$$\frac{\ln\left(c\,(b\,x^n)^p\right)^2}{2\,n\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)/x,x)
```

```
[Out] log(c*(b*x^n)^p)^2/(2*n*p)
```

sympy [A] time = 2.02, size = 37, normalized size = 1.68

$$-\begin{cases} -\log{(x)}\log{(b^p c)} & \text{for}\: n = 0 \\ -\log{(c)}\log{(x)} & \text{for}\: p = 0 \\ -\frac{\log{\left(c(bx^n)^p\right)}^2}{2np} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)/x,x)
```

```
[Out] -Piecewise((-log(x)*log(b**p*c), Eq(n, 0)), (-log(c)*log(x), Eq(p, 0)), (-l
og(c*(b*x**n)**p)**2/(2*n*p), True))
```

3.227 $\int \frac{\log\left(c(bx^n)^p\right)}{x^2}\,dx$

Optimal. Leaf size=23

$$-\frac{\log\left(c\left(bx^n\right)^p\right)}{x}-\frac{np}{x}$$

```
[Out] -n*p/x-ln(c*(b*x^n)^p)/x
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2304, 2445}

$$-\frac{\log\left(c\left(bx^n\right)^p\right)}{x}-\frac{np}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]/x^2,x]
```

```
[Out] -((n*p)/x) - Log[c*(b*x^n)^p]/x
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(bx^n\right)^p\right)}{x^2}\,dx &= \text{Subst}\left(\int \frac{\log\left(b^p c x^{np}\right)}{x^2}\,dx, b^p c x^{np}, c\left(bx^n\right)^p\right)\\ &= -\frac{np}{x}-\frac{\log\left(c\left(bx^n\right)^p\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{\log\left(c\left(bx^n\right)^p\right)}{x}-\frac{np}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]/x^2,x]
```

```
[Out] -((n*p)/x) - Log[c*(b*x^n)^p]/x
```

fricas [A] time = 0.66, size = 20, normalized size = 0.87

$$-\frac{np\log(x)+np+p\log(b)+\log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^2,x, algorithm="fricas")
```

```
[Out] -(n*p*log(x) + n*p + p*log(b) + log(c))/x
```

giac [A] time = 0.24, size = 25, normalized size = 1.09

$$-\frac{np\log(x)}{x}-\frac{np+p\log(b)+\log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^2,x, algorithm="giac")
```

```
[Out] -n*p*log(x)/x - (n*p + p*log(b) + log(c))/x
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,(b\,x^n)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)/x^2,x)
```

```
[Out] int(ln(c*(b*x^n)^p)/x^2,x)
```

maxima [A] time = 1.05, size = 23, normalized size = 1.00

$$-\frac{np}{x}-\frac{\log\left((bx^n)^p\,c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^2,x, algorithm="maxima")
```

```
[Out] -n*p/x - log((b*x^n)^p*c)/x
```

mupad [B] time = 3.88, size = 19, normalized size = 0.83

$$-\frac{\ln\left(c\,(b\,x^n)^p\right)+n\,p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)/x^2,x)
```

```
[Out] -(log(c*(b*x^n)^p) + n*p)/x
```

sympy [A] time = 1.00, size = 26, normalized size = 1.13

$$-\frac{np\log\left(x\right)}{x}-\frac{np}{x}-\frac{p\log\left(b\right)}{x}-\frac{\log\left(c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)/x**2,x)
```

```
[Out] -n*p*log(x)/x - n*p/x - p*log(b)/x - log(c)/x
```

3.228 $\int \frac{\log(c(bx^n)^p)}{x^3}\,dx$

Optimal. Leaf size=27

$$-\frac{\log\left(c\left(bx^n\right)^p\right)}{2x^2}-\frac{np}{4x^2}$$

```
[Out] -1/4*n*p/x^2-1/2*ln(c*(b*x^n)^p)/x^2
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2304, 2445}

$$-\frac{\log\left(c\left(bx^n\right)^p\right)}{2x^2}-\frac{np}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]/x^3,x]
```

```
[Out] -(n*p)/(4*x^2) - Log[c*(b*x^n)^p]/(2*x^2)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(bx^n\right)^p\right)}{x^3}\,dx &= \text{Subst}\left(\int \frac{\log\left(b^pcx^{np}\right)}{x^3}\,dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\ &= -\frac{np}{4x^2}-\frac{\log\left(c\left(bx^n\right)^p\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{\log\left(c\left(bx^n\right)^p\right)}{2x^2}-\frac{np}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]/x^3,x]
```

```
[Out] -1/4*(n*p)/x^2 - Log[c*(b*x^n)^p]/(2*x^2)
```

fricas [A] time = 0.59, size = 24, normalized size = 0.89

$$-\frac{2\,np\log(x)+np+2\,p\log(b)+2\,\log(c)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^3,x, algorithm="fricas")
[Out] -1/4*(2*n*p*log(x) + n*p + 2*p*log(b) + 2*log(c))/x^2
```

giac [A] time = 0.29, size = 28, normalized size = 1.04

$$-\frac{np\log(x)}{2\,x^2} - \frac{np + 2\,p\log(b) + 2\,\log(c)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^3,x, algorithm="giac")
[Out] -1/2*n*p*log(x)/x^2 - 1/4*(n*p + 2*p*log(b) + 2*log(c))/x^2
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,(b\,x^n)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)/x^3,x)
[Out] int(ln(c*(b*x^n)^p)/x^3,x)
```

maxima [A] time = 1.11, size = 23, normalized size = 0.85

$$-\frac{np}{4\,x^2} - \frac{\log\left((bx^n)^p\,c\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^3,x, algorithm="maxima")
[Out] -1/4*n*p/x^2 - 1/2*log((b*x^n)^p*c)/x^2
```

mupad [B] time = 3.84, size = 23, normalized size = 0.85

$$-\frac{\ln\left(c\,(b\,x^n)^p\right)}{2\,x^2} - \frac{n\,p}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)/x^3,x)
[Out] - log(c*(b*x^n)^p)/(2*x^2) - (n*p)/(4*x^2)
```

sympy [A] time = 2.32, size = 39, normalized size = 1.44

$$-\frac{np\log(x)}{2x^2} - \frac{np}{4x^2} - \frac{p\log(b)}{2x^2} - \frac{\log(c)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)/x**3,x)
[Out] -n*p*log(x)/(2*x**2) - n*p/(4*x**2) - p*log(b)/(2*x**2) - log(c)/(2*x**2)
```

3.229 $\int \frac{\log\left(c(bx^n)^p\right)}{x^4}\,dx$

Optimal. Leaf size=27

$$-\frac{\log\left(c\left(bx^n\right)^p\right)}{3x^3}-\frac{np}{9x^3}$$

```
[Out] -1/9*n*p/x^3-1/3*ln(c*(b*x^n)^p)/x^3
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2304, 2445}

$$-\frac{\log\left(c\left(bx^n\right)^p\right)}{3x^3}-\frac{np}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]/x^4,x]
[Out] -(n*p)/(9*x^3) - Log[c*(b*x^n)^p]/(3*x^3)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(bx^n\right)^p\right)}{x^4}\,dx &= \text{Subst}\left(\int \frac{\log\left(b^pcx^{np}\right)}{x^4}\,dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\ &= -\frac{np}{9x^3}-\frac{\log\left(c\left(bx^n\right)^p\right)}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{\log\left(c\left(bx^n\right)^p\right)}{3x^3}-\frac{np}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]/x^4,x]
[Out] -1/9*(n*p)/x^3 - Log[c*(b*x^n)^p]/(3*x^3)
```

fricas [A] time = 0.69, size = 24, normalized size = 0.89

$$-\frac{3\,np\log(x)+np+3\,p\log(b)+3\,\log(c)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^4,x, algorithm="fricas")
[Out] -1/9*(3*n*p*log(x) + n*p + 3*p*log(b) + 3*log(c))/x^3
```

giac [A] time = 0.28, size = 28, normalized size = 1.04

$$-\frac{np\log(x)}{3\,x^3} - \frac{np + 3\,p\log(b) + 3\,\log(c)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^4,x, algorithm="giac")
[Out] -1/3*n*p*log(x)/x^3 - 1/9*(n*p + 3*p*log(b) + 3*log(c))/x^3
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,(b\,x^n)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)/x^4,x)
[Out] int(ln(c*(b*x^n)^p)/x^4,x)
```

maxima [A] time = 1.25, size = 23, normalized size = 0.85

$$-\frac{np}{9\,x^3} - \frac{\log\left((bx^n)^p\,c\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^4,x, algorithm="maxima")
[Out] -1/9*n*p/x^3 - 1/3*log((b*x^n)^p*c)/x^3
```

mupad [B] time = 3.87, size = 23, normalized size = 0.85

$$-\frac{\ln\left(c\,(b\,x^n)^p\right)}{3\,x^3} - \frac{n\,p}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)/x^4,x)
[Out] - log(c*(b*x^n)^p)/(3*x^3) - (n*p)/(9*x^3)
```

sympy [A] time = 5.09, size = 39, normalized size = 1.44

$$-\frac{np\log(x)}{3x^3} - \frac{np}{9x^3} - \frac{p\log(b)}{3x^3} - \frac{\log(c)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)/x**4,x)
[Out] -n*p*log(x)/(3*x**3) - n*p/(9*x**3) - p*log(b)/(3*x**3) - log(c)/(3*x**3)
```

3.230 $\int x^2 \log^2 \left(c \left(b x^n\right)^p\right) \, dx$

Optimal. Leaf size=52

$$\frac{1}{3}x^3\log^2\left(c\left(bx^n\right)^p\right)-\frac{2}{9}npx^3\log\left(c\left(bx^n\right)^p\right)+\frac{2}{27}n^2p^2x^3$$

```
[Out] 2/27*n^2*p^2*x^3-2/9*n*p*x^3*ln(c*(b*x^n)^p)+1/3*x^3*ln(c*(b*x^n)^p)^2
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2305, 2304, 2445}

$$\frac{1}{3}x^3\log^2\left(c\left(bx^n\right)^p\right)-\frac{2}{9}npx^3\log\left(c\left(bx^n\right)^p\right)+\frac{2}{27}n^2p^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(b*x^n)^p]^2,x]
```

```
[Out] (2*n^2*p^2*x^3)/27 - (2*n*p*x^3*Log[c*(b*x^n)^p])/9 + (x^3*Log[c*(b*x^n)^p]
^2)/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int x^2\log^2\left(c\left(bx^n\right)^p\right)\,dx&=\text{Subst}\left(\int x^2\log^2\left(b^pcx^{np}\right)\,dx,b^pcx^{np},c\left(bx^n\right)^p\right)\\&=\frac{1}{3}x^3\log^2\left(c\left(bx^n\right)^p\right)-\text{Subst}\left(\frac{1}{3}(2np)\int x^2\log\left(b^pcx^{np}\right)\,dx,b^pcx^{np},c\left(bx^n\right)^p\right)\\&=\frac{2}{27}n^2p^2x^3-\frac{2}{9}npx^3\log\left(c\left(bx^n\right)^p\right)+\frac{1}{3}x^3\log^2\left(c\left(bx^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 52, normalized size = 1.00

$$\frac{1}{3}x^3\log^2\left(c\left(bx^n\right)^p\right)-\frac{2}{9}npx^3\log\left(c\left(bx^n\right)^p\right)+\frac{2}{27}n^2p^2x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(b*x^n)^p]^2,x]
```

```
[Out] (2*n^2*p^2*x^3)/27 - (2*n*p*x^3*Log[c*(b*x^n)^p])/9 + (x^3*Log[c*(b*x^n)^p]
^2)/3
```

fricas [B] time = 0.71, size = 113, normalized size = 2.17

$$\frac{1}{3}n^2p^2x^3\log(x)^2+\frac{2}{27}n^2p^2x^3-\frac{2}{9}np^2x^3\log(b)+\frac{1}{3}p^2x^3\log(b)^2+\frac{1}{3}x^3\log(c)^2-\frac{2}{9}\left(npx^3-3px^3\log(b)\right)\log(c)-\frac{2}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/3*n^2*p^2*x^3*log(x)^2 + 2/27*n^2*p^2*x^3 - 2/9*n*p^2*x^3*log(b) + 1/3*p^
2*x^3*log(b)^2 + 1/3*x^3*log(c)^2 - 2/9*(n*p*x^3 - 3*p*x^3*log(b))*log(c) -
 2/9*(n^2*p^2*x^3 - 3*n*p^2*x^3*log(b) - 3*n*p*x^3*log(c))*log(x)
```

giac [B] time = 0.28, size = 115, normalized size = 2.21

$$\frac{1}{3}n^2p^2x^3\log(x)^2-\frac{2}{9}n^2p^2x^3\log(x)+\frac{2}{3}np^2x^3\log(b)\log(x)+\frac{2}{27}n^2p^2x^3-\frac{2}{9}np^2x^3\log(b)+\frac{1}{3}p^2x^3\log(b)^2+\frac{2}{3}npx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] 1/3*n^2*p^2*x^3*log(x)^2 - 2/9*n^2*p^2*x^3*log(x) + 2/3*n*p^2*x^3*log(b)*lo
g(x) + 2/27*n^2*p^2*x^3 - 2/9*n*p^2*x^3*log(b) + 1/3*p^2*x^3*log(b)^2 + 2/3
*n*p*x^3*log(c)*log(x) - 2/9*n*p*x^3*log(c) + 2/3*p*x^3*log(b)*log(c) + 1/3
*x^3*log(c)^2
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^2\ln\left(c\,(b\,x^n)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^n)^p)^2,x)
```

```
[Out] int(x^2*ln(c*(b*x^n)^p)^2,x)
```

maxima [A] time = 1.08, size = 46, normalized size = 0.88

$$\frac{2}{27}n^2p^2x^3-\frac{2}{9}npx^3\log\left((bx^n)^p\,c\right)+\frac{1}{3}x^3\log\left((bx^n)^p\,c\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] 2/27*n^2*p^2*x^3 - 2/9*n*p*x^3*log((b*x^n)^p*c) + 1/3*x^3*log((b*x^n)^p*c)^
2
```

mupad [B] time = 3.80, size = 46, normalized size = 0.88

$$\frac{2\,n^2\,p^2\,x^3}{27}-\frac{2\,n\,p\,x^3\,\ln\left(c\,(b\,x^n)^p\right)}{9}+\frac{x^3\,\ln\left(c\,(b\,x^n)^p\right)^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(b*x^n)^p)^2,x)
```

```
[Out] (x^3*log(c*(b*x^n)^p)^2)/3 + (2*n^2*p^2*x^3)/27 - (2*n*p*x^3*log(c*(b*x^n)^
p))/9
```

sympy [B] time = 5.00, size = 150, normalized size = 2.88

$$\frac{n^2p^2x^3\log(x)^2}{3}-\frac{2n^2p^2x^3\log(x)}{9}+\frac{2n^2p^2x^3}{27}+\frac{2np^2x^3\log(b)\log(x)}{3}-\frac{2np^2x^3\log(b)}{9}+\frac{2npx^3\log(c)\log(x)}{3}-\frac{2npx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**n)**p)**2,x)
```

```
[Out] n**2*p**2*x**3*log(x)**2/3 - 2*n**2*p**2*x**3*log(x)/9 + 2*n**2*p**2*x**3/2
7 + 2*n*p**2*x**3*log(b)*log(x)/3 - 2*n*p**2*x**3*log(b)/9 + 2*n*p*x**3*log
(c)*log(x)/3 - 2*n*p*x**3*log(c)/9 + p**2*x**3*log(b)**2/3 + 2*p*x**3*log(b
)*log(c)/3 + x**3*log(c)**2/3
```

3.231 $\int x \log^2\left(c\left(bx^n\right)^p\right) dx$

Optimal. Leaf size=52

$$\frac{1}{2}x^2\log^2\left(c\left(bx^n\right)^p\right)-\frac{1}{2}npx^2\log\left(c\left(bx^n\right)^p\right)+\frac{1}{4}n^2p^2x^2$$

```
[Out] 1/4*n^2*p^2*x^2-1/2*n*p*x^2*ln(c*(b*x^n)^p)+1/2*x^2*ln(c*(b*x^n)^p)^2
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2305, 2304, 2445}

$$\frac{1}{2}x^2\log^2\left(c\left(bx^n\right)^p\right)-\frac{1}{2}npx^2\log\left(c\left(bx^n\right)^p\right)+\frac{1}{4}n^2p^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(b*x^n)^p]^2,x]
```

```
[Out] (n^2*p^2*x^2)/4 - (n*p*x^2*Log[c*(b*x^n)^p])/2 + (x^2*Log[c*(b*x^n)^p]^2)/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int x\log^2\left(c\left(bx^n\right)^p\right)dx &= \text{Subst}\left(\int x\log^2\left(b^pcx^{np}\right)dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\ &= \frac{1}{2}x^2\log^2\left(c\left(bx^n\right)^p\right)-\text{Subst}\left((np)\int x\log\left(b^pcx^{np}\right)dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\ &= \frac{1}{4}n^2p^2x^2-\frac{1}{2}npx^2\log\left(c\left(bx^n\right)^p\right)+\frac{1}{2}x^2\log^2\left(c\left(bx^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.83

$$\frac{1}{4}x^2\left(2\log^2\left(c\left(bx^n\right)^p\right)-2np\log\left(c\left(bx^n\right)^p\right)+n^2p^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(b*x^n)^p]^2,x]
```

```
[Out] (x^2*(n^2*p^2 - 2*n*p*Log[c*(b*x^n)^p] + 2*Log[c*(b*x^n)^p]^2))/4
```

fricas [B] time = 0.76, size = 113, normalized size = 2.17

$$\frac{1}{2}n^2p^2x^2\log(x)^2+\frac{1}{4}n^2p^2x^2-\frac{1}{2}np^2x^2\log(b)+\frac{1}{2}p^2x^2\log(b)^2+\frac{1}{2}x^2\log(c)^2-\frac{1}{2}\left(npx^2-2px^2\log(b)\right)\log(c)-\frac{1}{2}\left(n^2p\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/2*n^2*p^2*x^2*log(x)^2 + 1/4*n^2*p^2*x^2 - 1/2*n*p^2*x^2*log(b) + 1/2*p^2
*x^2*log(b)^2 + 1/2*x^2*log(c)^2 - 1/2*(n*p*x^2 - 2*p*x^2*log(b))*log(c) -
1/2*(n^2*p^2*x^2 - 2*n*p^2*x^2*log(b) - 2*n*p*x^2*log(c))*log(x)
```

giac [B] time = 0.38, size = 112, normalized size = 2.15

$$\frac{1}{2}n^2p^2x^2\log(x)^2-\frac{1}{2}n^2p^2x^2\log(x)+np^2x^2\log(b)\log(x)+\frac{1}{4}n^2p^2x^2-\frac{1}{2}np^2x^2\log(b)+\frac{1}{2}p^2x^2\log(b)^2+npx^2\log(c)\,l$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] 1/2*n^2*p^2*x^2*log(x)^2 - 1/2*n^2*p^2*x^2*log(x) + n*p^2*x^2*log(b)*log(x)
 + 1/4*n^2*p^2*x^2 - 1/2*n*p^2*x^2*log(b) + 1/2*p^2*x^2*log(b)^2 + n*p*x^2*
log(c)*log(x) - 1/2*n*p*x^2*log(c) + p*x^2*log(b)*log(c) + 1/2*x^2*log(c)^2
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\ln\left(c\,(b\,x^n)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^n)^p)^2,x)
```

```
[Out] int(x*ln(c*(b*x^n)^p)^2,x)
```

maxima [A] time = 1.14, size = 46, normalized size = 0.88

$$\frac{1}{4}n^2p^2x^2-\frac{1}{2}npx^2\log\left((bx^n)^p c\right)+\frac{1}{2}x^2\log\left((bx^n)^p c\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/4*n^2*p^2*x^2 - 1/2*n*p*x^2*log((b*x^n)^p*c) + 1/2*x^2*log((b*x^n)^p*c)^2
```

mupad [B] time = 3.85, size = 46, normalized size = 0.88

$$\frac{n^2\,p^2\,x^2}{4}-\frac{n\,p\,x^2\,\ln\left(c\,(b\,x^n)^p\right)}{2}+\frac{x^2\,\ln\left(c\,(b\,x^n)^p\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(b*x^n)^p)^2,x)
```

```
[Out] (x^2*log(c*(b*x^n)^p)^2)/2 + (n^2*p^2*x^2)/4 - (n*p*x^2*log(c*(b*x^n)^p))/2
```

sympy [B] time = 2.31, size = 133, normalized size = 2.56

$$\frac{n^2p^2x^2\log(x)^2}{2}-\frac{n^2p^2x^2\log(x)}{2}+\frac{n^2p^2x^2}{4}+np^2x^2\log(b)\log(x)-\frac{np^2x^2\log(b)}{2}+npx^2\log(c)\log(x)-\frac{npx^2\log(c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**n)**p)**2,x)
```

```
[Out] n**2*p**2*x**2*log(x)**2/2 - n**2*p**2*x**2*log(x)/2 + n**2*p**2*x**2/4 + n
*p**2*x**2*log(b)*log(x) - n*p**2*x**2*log(b)/2 + n*p*x**2*log(c)*log(x) -
n*p*x**2*log(c)/2 + p**2*x**2*log(b)**2/2 + p*x**2*log(b)*log(c) + x**2*log
(c)**2/2
```

3.232 $\int \log^2\left(c\left(bx^n\right)^p\right)\,dx$

Optimal. Leaf size=39

$$x\log^2\left(c\left(bx^n\right)^p\right)-2npx\log\left(c\left(bx^n\right)^p\right)+2n^2p^2x$$

```
[Out] 2*n^2*p^2*x-2*n*p*x*ln(c*(b*x^n)^p)+x*ln(c*(b*x^n)^p)^2
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2296, 2295, 2445}

$$x\log^2\left(c\left(bx^n\right)^p\right)-2npx\log\left(c\left(bx^n\right)^p\right)+2n^2p^2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]^2,x]
```

```
[Out] 2*n^2*p^2*x - 2*n*p*x*Log[c*(b*x^n)^p] + x*Log[c*(b*x^n)^p]^2
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int \log^2\left(c\left(bx^n\right)^p\right)\,dx &= \text{Subst}\left(\int \log^2\left(b^pcx^{np}\right)\,dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\ &= x\log^2\left(c\left(bx^n\right)^p\right)-\text{Subst}\left((2np)\int \log\left(b^pcx^{np}\right)\,dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\ &= 2n^2p^2x-2npx\log\left(c\left(bx^n\right)^p\right)+x\log^2\left(c\left(bx^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 0.95

$$x\log^2\left(c\left(bx^n\right)^p\right)-2np\left(x\log\left(c\left(bx^n\right)^p\right)-npx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]^2,x]
```

```
[Out] x*Log[c*(b*x^n)^p]^2 - 2*n*p*(-(n*p*x) + x*Log[c*(b*x^n)^p])
```

fricas [B] time = 0.57, size = 90, normalized size = 2.31

$$n^2p^2x\log(x)^2+2\,n^2p^2x-2\,np^2x\log(b)+p^2x\log(b)^2+x\log(c)^2-2\left(npx-px\log(b)\right)\log(c)-2\left(n^2p^2x-np^2x\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] n^2*p^2*x*log(x)^2 + 2*n^2*p^2*x - 2*n*p^2*x*log(b) + p^2*x*log(b)^2 + x*lo
g(c)^2 - 2*(n*p*x - p*x*log(b))*log(c) - 2*(n^2*p^2*x - n*p^2*x*log(b) - n*
p*x*log(c))*log(x)
```

giac [B] time = 0.29, size = 92, normalized size = 2.36

$$n^2p^2x\log(x)^2-2\,n^2p^2x\log(x)+2\,np^2x\log(b)\log(x)+2\,n^2p^2x-2\,np^2x\log(b)+p^2x\log(b)^2+2\,npx\log(c)\log(x)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] n^2*p^2*x*log(x)^2 - 2*n^2*p^2*x*log(x) + 2*n*p^2*x*log(b)*log(x) + 2*n^2*p
^2*x - 2*n*p^2*x*log(b) + p^2*x*log(b)^2 + 2*n*p*x*log(c)*log(x) - 2*n*p*x*
log(c) + 2*p*x*log(b)*log(c) + x*log(c)^2
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \ln\left(c\,(b\,x^n)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)^2,x)
```

```
[Out] int(ln(c*(b*x^n)^p)^2,x)
```

maxima [A] time = 1.17, size = 39, normalized size = 1.00

$$2\,n^2p^2x-2\,npx\log\left((bx^n)^p\,c\right)+x\log\left((bx^n)^p\,c\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] 2*n^2*p^2*x - 2*n*p*x*log((b*x^n)^p*c) + x*log((b*x^n)^p*c)^2
```

mupad [B] time = 3.84, size = 39, normalized size = 1.00

$$2\,x\,n^2\,p^2-2\,x\,n\,p\,\ln\left(c\,(b\,x^n)^p\right)+x\ln\left(c\,(b\,x^n)^p\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)^2,x)
```

```
[Out] x*log(c*(b*x^n)^p)^2 + 2*n^2*p^2*x - 2*n*p*x*log(c*(b*x^n)^p)
```

sympy [B] time = 1.03, size = 116, normalized size = 2.97

$$n^2p^2x\log{(x)}^2-2n^2p^2x\log{(x)}+2n^2p^2x+2np^2x\log{(b)}\log{(x)}-2np^2x\log{(b)}+2npx\log{(c)}\log{(x)}-2npx\log{(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)**2,x)
```

```
[Out] n**2*p**2*x*log(x)**2 - 2*n**2*p**2*x*log(x) + 2*n**2*p**2*x + 2*n*p**2*x*l
og(b)*log(x) - 2*n*p**2*x*log(b) + 2*n*p*x*log(c)*log(x) - 2*n*p*x*log(c) +
 p**2*x*log(b)**2 + 2*p*x*log(b)*log(c) + x*log(c)**2
```

3.233 $\int \frac{\log^2\left(c(bx^n)^p\right)}{x} dx$

Optimal. Leaf size=22

$$\frac{\log^3\left(c\left(bx^n\right)^p\right)}{3np}$$

[Out] 1/3*ln(c*(b*x^n)^p)^3/n/p

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2302, 30, 2445}

$$\frac{\log^3\left(c\left(bx^n\right)^p\right)}{3np}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(b*x^n)^p]^2/x,x]

[Out] Log[c*(b*x^n)^p]^3/(3*n*p)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2\left(c\left(bx^n\right)^p\right)}{x} dx &= \text{Subst}\left(\int \frac{\log^2\left(b^pcx^{np}\right)}{x} dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int x^2 dx, x, \log\left(b^pcx^{np}\right)\right)}{np}, b^pcx^{np}, c\left(bx^n\right)^p\right)\\
&= \frac{\log^3\left(c\left(bx^n\right)^p\right)}{3np}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log^3\left(c\left(bx^n\right)^p\right)}{3np}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]^2/x,x]
```

```
[Out] Log[c*(b*x^n)^p]^3/(3*n*p)
```

fricas [B] time = 0.68, size = 54, normalized size = 2.45

$$\frac{1}{3}\,n^2p^2\log(x)^3+\left(np^2\log(b)+np\log(c)\right)\log(x)^2+\left(p^2\log(b)^2+2\,p\log(b)\log(c)+\log(c)^2\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x,x, algorithm="fricas")
```

```
[Out] 1/3*n^2*p^2*log(x)^3 + (n*p^2*log(b) + n*p*log(c))*log(x)^2 + (p^2*log(b)^2
 + 2*p*log(b)*log(c) + log(c)^2)*log(x)
```

giac [B] time = 0.32, size = 59, normalized size = 2.68

$$\frac{1}{3}\,n^2p^2\log(x)^3+np^2\log(b)\log(x)^2+p^2\log(b)^2\log(x)+np\log(c)\log(x)^2+2\,p\log(b)\log(c)\log(x)+\log(c)^2\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x,x, algorithm="giac")
```

```
[Out] 1/3*n^2*p^2*log(x)^3 + n*p^2*log(b)*log(x)^2 + p^2*log(b)^2*log(x) + n*p*lo
g(c)*log(x)^2 + 2*p*log(b)*log(c)*log(x) + log(c)^2*log(x)
```

maple [A] time = 0.04, size = 21, normalized size = 0.95

$$\frac{\ln\left(c\,(b\,x^n)^p\right)^3}{3np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)^2/x,x)
```

```
[Out] 1/3*ln(c*(b*x^n)^p)^3/p/n
```

maxima [A] time = 1.14, size = 20, normalized size = 0.91

$$\frac{\log\left((bx^n)^p\,c\right)^3}{3\,np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*log((b*x^n)^p*c)^3/(n*p)
```

mupad [B] time = 3.75, size = 20, normalized size = 0.91

$$\frac{\ln\left(c\,(b\,x^n)^p\right)^3}{3\,n\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)^2/x,x)
```

```
[Out] log(c*(b*x^n)^p)^3/(3*n*p)
```

sympy [A] time = 1.57, size = 41, normalized size = 1.86

$$-\begin{cases} -\log(x)\log(b^p c)^2 & \text{for } n = 0 \\ -\log(c)^2\log(x) & \text{for } p = 0 \\ -\frac{\log\left(c(bx^n)^p\right)^3}{3np} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)**2/x,x)
```

```
[Out] -Piecewise((-log(x)*log(b**p*c)**2, Eq(n, 0)), (-log(c)**2*log(x), Eq(p, 0)
), (-log(c*(b*x**n)**p)**3/(3*n*p), True))
```

3.234 $\int \frac{\log^2\left(c(bx^n)^p\right)}{x^2} dx$

Optimal. Leaf size=46

$$-\frac{\log^2\left(c\left(bx^n\right)^p\right)}{x}-\frac{2np\log\left(c\left(bx^n\right)^p\right)}{x}-\frac{2n^2p^2}{x}$$

```
[Out] -2*n^2*p^2/x-2*n*p*ln(c*(b*x^n)^p)/x-ln(c*(b*x^n)^p)^2/x
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2305, 2304, 2445}

$$-\frac{\log^2\left(c\left(bx^n\right)^p\right)}{x}-\frac{2np\log\left(c\left(bx^n\right)^p\right)}{x}-\frac{2n^2p^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]^2/x^2,x]
```

```
[Out] (-2*n^2*p^2)/x - (2*n*p*Log[c*(b*x^n)^p])/x - Log[c*(b*x^n)^p]^2/x
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2\left(c\left(bx^n\right)^p\right)}{x^2} dx &= \text{Subst}\left(\int \frac{\log^2\left(b^pcx^{np}\right)}{x^2} dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\
&= -\frac{\log^2\left(c\left(bx^n\right)^p\right)}{x}+\text{Subst}\left((2np)\int \frac{\log\left(b^pcx^{np}\right)}{x^2} dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\
&= -\frac{2n^2p^2}{x}-\frac{2np\log\left(c\left(bx^n\right)^p\right)}{x}-\frac{\log^2\left(c\left(bx^n\right)^p\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.87

$$-\frac{\log^2\left(c\left(bx^n\right)^p\right)+2np\log\left(c\left(bx^n\right)^p\right)+2n^2p^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]^2/x^2,x]
```

```
[Out] -((2*n^2*p^2 + 2*n*p*Log[c*(b*x^n)^p] + Log[c*(b*x^n)^p]^2)/x)
```

fricas [A] time = 0.64, size = 81, normalized size = 1.76

$$-\frac{n^2p^2\log(x)^2 + 2\,n^2p^2 + 2\,np^2\log(b) + p^2\log(b)^2 + 2\left(np + p\log(b)\right)\log(c) + \log(c)^2 + 2\left(n^2p^2 + np^2\log(b) + \right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(n^2*p^2*log(x)^2 + 2*n^2*p^2 + 2*n*p^2*log(b) + p^2*log(b)^2 + 2*(n*p + p
*log(b))*log(c) + log(c)^2 + 2*(n^2*p^2 + n*p^2*log(b) + n*p*log(c))*log(x)
)/x
```

giac [A] time = 0.29, size = 90, normalized size = 1.96

$$-\frac{n^2p^2\log(x)^2}{x} - \frac{2\left(n^2p^2 + np^2\log(b) + np\log(c)\right)\log(x)}{x} - \frac{2\,n^2p^2 + 2\,np^2\log(b) + p^2\log(b)^2 + 2\,np\log(c) + 2\,p\,\mathrm{l}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^2,x, algorithm="giac")
```

```
[Out] -n^2*p^2*log(x)^2/x - 2*(n^2*p^2 + n*p^2*log(b) + n*p*log(c))*log(x)/x - (2
*n^2*p^2 + 2*n*p^2*log(b) + p^2*log(b)^2 + 2*n*p*log(c) + 2*p*log(b)*log(c)
 + log(c)^2)/x
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,(b\,x^n)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)^2/x^2,x)
```

```
[Out] int(ln(c*(b*x^n)^p)^2/x^2,x)
```

maxima [A] time = 1.14, size = 46, normalized size = 1.00

$$-\frac{2\,n^2p^2}{x} - \frac{2\,np\log\left((bx^n)^p\,c\right)}{x} - \frac{\log\left((bx^n)^p\,c\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*n^2*p^2/x - 2*n*p*log((b*x^n)^p*c)/x - log((b*x^n)^p*c)^2/x
```

mupad [B] time = 3.85, size = 40, normalized size = 0.87

$$-\frac{2\,n^2\,p^2 + 2\,n\,p\,\ln\left(c\,(b\,x^n)^p\right) + \ln\left(c\,(b\,x^n)^p\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)^2/x^2,x)
```

```
[Out] -(log(c*(b*x^n)^p)^2 + 2*n^2*p^2 + 2*n*p*log(c*(b*x^n)^p))/x
```

sympy [B] time = 1.04, size = 117, normalized size = 2.54

$$-\frac{n^2p^2\log(x)^2}{x}-\frac{2n^2p^2\log(x)}{x}-\frac{2n^2p^2}{x}-\frac{2np^2\log(b)\log(x)}{x}-\frac{2np^2\log(b)}{x}-\frac{2np\log(c)\log(x)}{x}-\frac{2np\log(c)}{x}-\frac{p^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)**2/x**2,x)
```

```
[Out] -n**2*p**2*log(x)**2/x - 2*n**2*p**2*log(x)/x - 2*n**2*p**2/x - 2*n*p**2*lo
g(b)*log(x)/x - 2*n*p**2*log(b)/x - 2*n*p*log(c)*log(x)/x - 2*n*p*log(c)/x
- p**2*log(b)**2/x - 2*p*log(b)*log(c)/x - log(c)**2/x
```

3.235 $\int \frac{\log^2(c(bx^n)^p)}{x^3} dx$

Optimal. Leaf size=52

$$-\frac{\log^2\left(c\left(bx^n\right)^p\right)}{2x^2}-\frac{np\log\left(c\left(bx^n\right)^p\right)}{2x^2}-\frac{n^2p^2}{4x^2}$$

```
[Out] -1/4*n^2*p^2/x^2-1/2*n*p*ln(c*(b*x^n)^p)/x^2-1/2*ln(c*(b*x^n)^p)^2/x^2
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2305, 2304, 2445}

$$-\frac{\log^2\left(c\left(bx^n\right)^p\right)}{2x^2}-\frac{np\log\left(c\left(bx^n\right)^p\right)}{2x^2}-\frac{n^2p^2}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]^2/x^3,x]
```

```
[Out] -(n^2*p^2)/(4*x^2) - (n*p*Log[c*(b*x^n)^p])/(2*x^2) - Log[c*(b*x^n)^p]^2/(2
*x^2)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2\left(c\left(bx^n\right)^p\right)}{x^3}\,dx &= \text{Subst}\left(\int \frac{\log^2\left(b^pcx^{np}\right)}{x^3}\,dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\
&= -\frac{\log^2\left(c\left(bx^n\right)^p\right)}{2x^2}+\text{Subst}\left((np)\int \frac{\log\left(b^pcx^{np}\right)}{x^3}\,dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\
&= -\frac{n^2p^2}{4x^2}-\frac{np\log\left(c\left(bx^n\right)^p\right)}{2x^2}-\frac{\log^2\left(c\left(bx^n\right)^p\right)}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.83

$$-\frac{2\log^2\left(c\left(bx^n\right)^p\right)+2np\log\left(c\left(bx^n\right)^p\right)+n^2p^2}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]^2/x^3,x]
```

```
[Out] -1/4*(n^2*p^2 + 2*n*p*Log[c*(b*x^n)^p] + 2*Log[c*(b*x^n)^p]^2)/x^2
```

fricas [A] time = 0.62, size = 87, normalized size = 1.67

$$-\frac{2\,n^2p^2\log(x)^2+n^2p^2+2\,np^2\log(b)+2\,p^2\log(b)^2+2\left(np+2\,p\log(b)\right)\log(c)+2\,\log(c)^2+2\left(n^2p^2+2\,np\right.}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*n^2*p^2*log(x)^2 + n^2*p^2 + 2*n*p^2*log(b) + 2*p^2*log(b)^2 + 2*(n
*p + 2*p*log(b))*log(c) + 2*log(c)^2 + 2*(n^2*p^2 + 2*n*p^2*log(b) + 2*n*p*
log(c))*log(x))/x^2
```

giac [B] time = 0.24, size = 94, normalized size = 1.81

$$-\frac{n^2p^2\log(x)^2}{2\,x^2}-\frac{\left(n^2p^2+2\,np^2\log(b)+2\,np\log(c)\right)\log(x)}{2\,x^2}-\frac{n^2p^2+2\,np^2\log(b)+2\,p^2\log(b)^2+2\,np\log(c)+}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*n^2*p^2*log(x)^2/x^2 - 1/2*(n^2*p^2 + 2*n*p^2*log(b) + 2*n*p*log(c))*l
og(x)/x^2 - 1/4*(n^2*p^2 + 2*n*p^2*log(b) + 2*p^2*log(b)^2 + 2*n*p*log(c) +
 4*p*log(b)*log(c) + 2*log(c)^2)/x^2
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\ln\left(c\,(b\,x^n)^p\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)^2/x^3,x)
```

```
[Out] int(ln(c*(b*x^n)^p)^2/x^3,x)
```

maxima [A] time = 1.14, size = 46, normalized size = 0.88

$$-\frac{n^2p^2}{4\,x^2}-\frac{np\log\left((bx^n)^p\,c\right)}{2\,x^2}-\frac{\log\left((bx^n)^p\,c\right)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*n^2*p^2/x^2 - 1/2*n*p*log((b*x^n)^p*c)/x^2 - 1/2*log((b*x^n)^p*c)^2/x^
2
```

mupad [B] time = 3.79, size = 46, normalized size = 0.88

$$-\frac{\ln\left(c\,(b\,x^n)^p\right)^2}{2\,x^2}-\frac{n^2\,p^2}{4\,x^2}-\frac{n\,p\,\ln\left(c\,(b\,x^n)^p\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)^2/x^3,x)
```

```
[Out] - log(c*(b*x^n)^p)^2/(2*x^2) - (n^2*p^2)/(4*x^2) - (n*p*log(c*(b*x^n)^p))/(
2*x^2)
```

sympy [B] time = 2.44, size = 134, normalized size = 2.58

$$-\frac{n^2p^2\log(x)^2}{2x^2}-\frac{n^2p^2\log(x)}{2x^2}-\frac{n^2p^2}{4x^2}-\frac{np^2\log(b)\log(x)}{x^2}-\frac{np^2\log(b)}{2x^2}-\frac{np\log(c)\log(x)}{x^2}-\frac{np\log(c)}{2x^2}-\frac{p^2\log(b)^2}{2x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)**2/x**3,x)
```

```
[Out] -n**2*p**2*log(x)**2/(2*x**2) - n**2*p**2*log(x)/(2*x**2) - n**2*p**2/(4*x*
*2) - n*p**2*log(b)*log(x)/x**2 - n*p**2*log(b)/(2*x**2) - n*p*log(c)*log(x
)/x**2 - n*p*log(c)/(2*x**2) - p**2*log(b)**2/(2*x**2) - p*log(b)*log(c)/x*
*2 - log(c)**2/(2*x**2)
```

3.236 $\int \frac{\log^2\left(c(bx^n)^p\right)}{x^4}\,dx$

Optimal. Leaf size=52

$$-\frac{\log^2\left(c\left(bx^n\right)^p\right)}{3x^3}-\frac{2np\log\left(c\left(bx^n\right)^p\right)}{9x^3}-\frac{2n^2p^2}{27x^3}$$

```
[Out] -2/27*n^2*p^2/x^3-2/9*n*p*ln(c*(b*x^n)^p)/x^3-1/3*ln(c*(b*x^n)^p)^2/x^3
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2305, 2304, 2445}

$$-\frac{\log^2\left(c\left(bx^n\right)^p\right)}{3x^3}-\frac{2np\log\left(c\left(bx^n\right)^p\right)}{9x^3}-\frac{2n^2p^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]^2/x^4,x]
```

```
[Out] (-2*n^2*p^2)/(27*x^3) - (2*n*p*Log[c*(b*x^n)^p])/(9*x^3) - Log[c*(b*x^n)^p]
^2/(3*x^3)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2\left(c\left(bx^n\right)^p\right)}{x^4}\,dx &= \text{Subst}\left(\int \frac{\log^2\left(b^pcx^{np}\right)}{x^4}\,dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\ &= -\frac{\log^2\left(c\left(bx^n\right)^p\right)}{3x^3}+\text{Subst}\left(\frac{1}{3}(2np)\int \frac{\log\left(b^pcx^{np}\right)}{x^4}\,dx, b^pcx^{np}, c\left(bx^n\right)^p\right)\\ &= -\frac{2n^2p^2}{27x^3}-\frac{2np\log\left(c\left(bx^n\right)^p\right)}{9x^3}-\frac{\log^2\left(c\left(bx^n\right)^p\right)}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 52, normalized size = 1.00

$$-\frac{\log^2\left(c\left(bx^n\right)^p\right)}{3x^3}-\frac{2np\log\left(c\left(bx^n\right)^p\right)}{9x^3}-\frac{2n^2p^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]^2/x^4,x]
```

```
[Out] (-2*n^2*p^2)/(27*x^3) - (2*n*p*Log[c*(b*x^n)^p])/(9*x^3) - Log[c*(b*x^n)^p]
^2/(3*x^3)
```

fricas [A] time = 0.51, size = 88, normalized size = 1.69

$$-\frac{9\,n^2p^2\log(x)^2+2\,n^2p^2+6\,np^2\log(b)+9\,p^2\log(b)^2+6\left(np+3\,p\log(b)\right)\log(c)+9\,\log(c)^2+6\left(n^2p^2+3\,np^2\right.}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(9*n^2*p^2*log(x)^2 + 2*n^2*p^2 + 6*n*p^2*log(b) + 9*p^2*log(b)^2 + 6
*(n*p + 3*p*log(b))*log(c) + 9*log(c)^2 + 6*(n^2*p^2 + 3*n*p^2*log(b) + 3*n
*p*log(c))*log(x))/x^3
```

giac [B] time = 0.24, size = 95, normalized size = 1.83

$$-\frac{n^2p^2\log(x)^2}{3\,x^3}-\frac{2\left(n^2p^2+3\,np^2\log(b)+3\,np\log(c)\right)\log(x)}{9\,x^3}-\frac{2\,n^2p^2+6\,np^2\log(b)+9\,p^2\log(b)^2+6\,np\log(c)+}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^4,x, algorithm="giac")
```

```
[Out] -1/3*n^2*p^2*log(x)^2/x^3 - 2/9*(n^2*p^2 + 3*n*p^2*log(b) + 3*n*p*log(c))*l
og(x)/x^3 - 1/27*(2*n^2*p^2 + 6*n*p^2*log(b) + 9*p^2*log(b)^2 + 6*n*p*log(c
) + 18*p*log(b)*log(c) + 9*log(c)^2)/x^3
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,(b\,x^n)^p\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)^2/x^4,x)
```

```
[Out] int(ln(c*(b*x^n)^p)^2/x^4,x)
```

maxima [A] time = 1.17, size = 46, normalized size = 0.88

$$-\frac{2\,n^2p^2}{27\,x^3}-\frac{2\,np\log\left((bx^n)^p\,c\right)}{9\,x^3}-\frac{\log\left((bx^n)^p\,c\right)^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^4,x, algorithm="maxima")
```

```
[Out] -2/27*n^2*p^2/x^3 - 2/9*n*p*log((b*x^n)^p*c)/x^3 - 1/3*log((b*x^n)^p*c)^2/x
^3
```

mupad [B] time = 3.88, size = 46, normalized size = 0.88

$$-\frac{\ln\left(c\,(b\,x^n)^p\right)^2}{3\,x^3}-\frac{2\,n^2\,p^2}{27\,x^3}-\frac{2\,n\,p\,\ln\left(c\,(b\,x^n)^p\right)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)^2/x^4,x)
```

```
[Out] - log(c*(b*x^n)^p)^2/(3*x^3) - (2*n^2*p^2)/(27*x^3) - (2*n*p*log(c*(b*x^n)^
p))/(9*x^3)
```

sympy [B] time = 5.23, size = 151, normalized size = 2.90

$$-\frac{n^2p^2\log{(x)}^2}{3x^3}-\frac{2n^2p^2\log{(x)}}{9x^3}-\frac{2n^2p^2}{27x^3}-\frac{2np^2\log{(b)}\log{(x)}}{3x^3}-\frac{2np^2\log{(b)}}{9x^3}-\frac{2np\log{(c)}\log{(x)}}{3x^3}-\frac{2np\log{(c)}}{9x^3}-p^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)**2/x**4,x)
```

```
[Out] -n**2*p**2*log(x)**2/(3*x**3) - 2*n**2*p**2*log(x)/(9*x**3) - 2*n**2*p**2/(
27*x**3) - 2*n*p**2*log(b)*log(x)/(3*x**3) - 2*n*p**2*log(b)/(9*x**3) - 2*n
*p*log(c)*log(x)/(3*x**3) - 2*n*p*log(c)/(9*x**3) - p**2*log(b)**2/(3*x**3)
 - 2*p*log(b)*log(c)/(3*x**3) - log(c)**2/(3*x**3)
```

3.237 $\int (ex)^q \left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=135

$$\frac{6b^2m^2n^2(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{e(q+1)^3}+\frac{(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^3}{e(q+1)}-\frac{3bmn(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}{e(q+1)^2}-\frac{6b^3}{}$$

```
[Out] -6*b^3*m^3*n^3*(e*x)^(1+q)/e/(1+q)^4+6*b^2*m^2*n^2*(e*x)^(1+q)*(a+b*ln(c*(d
*x^m)^n))/e/(1+q)^3-3*b*m*n*(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))^2/e/(1+q)^2+(
e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))^3/e/(1+q)
```

Rubi [A] time = 0.22, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2305, 2304, 2445}

$$\frac{6b^2m^2n^2(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{e(q+1)^3}+\frac{(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^3}{e(q+1)}-\frac{3bmn(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}{e(q+1)^2}-\frac{6b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^3,x]
```

```
[Out] (-6*b^3*m^3*n^3*(e*x)^(1 + q))/(e*(1 + q)^4) + (6*b^2*m^2*n^2*(e*x)^(1 + q)
*(a + b*Log[c*(d*x^m)^n]))/(e*(1 + q)^3) - (3*b*m*n*(e*x)^(1 + q)*(a + b*Lo
g[c*(d*x^m)^n])^2)/(e*(1 + q)^2) + ((e*x)^(1 + q)*(a + b*Log[c*(d*x^m)^n])^
3)/(e*(1 + q))
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int (ex)^q \left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^3 dx = \text{Subst}\left(\int (ex)^q \left(a+b\log\left(cd^nx^{mn}\right)\right)^3 dx, cd^nx^{mn}, c\left(dx^m\right)^n\right)$$
$$= \frac{(ex)^{1+q}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^3}{e(1+q)} - \text{Subst}\left(\frac{(3bmn)\int (ex)^q \left(a+b\log\left(cd^nx^m\right.\right.}{1+q}\right.$$
$$= -\frac{3bmn(ex)^{1+q}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}{e(1+q)^2} + \frac{(ex)^{1+q}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^3}{e(1+q)} +$$
$$= -\frac{6b^3m^3n^3(ex)^{1+q}}{e(1+q)^4} + \frac{6b^2m^2n^2(ex)^{1+q}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{e(1+q)^3} - \frac{3bmn(ex)^{1+}}{}$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.67

$$\frac{x(ex)^q\left(\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^3 - \frac{3bmn\left((q+1)^2\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2+2bmn\left(bmn-(q+1)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)\right)\right)}{(q+1)^3}\right)}{q+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^3,x]
```

```
[Out] (x*(e*x)^q*((a + b*Log[c*(d*x^m)^n])^3 - (3*b*m*n*((1 + q)^2*(a + b*Log[c*(
d*x^m)^n])^2 + 2*b*m*n*(b*m*n - (1 + q)*(a + b*Log[c*(d*x^m)^n]))))/(1 + q)
^3))/(1 + q)
```

fricas [B] time = 0.79, size = 1358, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*q^3 + 3*b^3*q^2 + 3*b^3*q + b^3)*x*log(c)^3 + (b^3*n^3*q^3 + 3*b^3*n^
3*q^2 + 3*b^3*n^3*q + b^3*n^3)*x*log(d)^3 + (b^3*m^3*n^3*q^3 + 3*b^3*m^3*n^
3*q^2 + 3*b^3*m^3*n^3*q + b^3*m^3*n^3)*x*log(x)^3 + 3*(a*b^2*q^3 - b^3*m*n
+ a*b^2 - (b^3*m*n - 3*a*b^2)*q^2 - (2*b^3*m*n - 3*a*b^2)*q)*x*log(c)^2 + 3
*(2*b^3*m^2*n^2 + a^2*b*q^3 - 2*a*b^2*m*n + a^2*b - (2*a*b^2*m*n - 3*a^2*b)
*q^2 + (2*b^3*m^2*n^2 - 4*a*b^2*m*n + 3*a^2*b)*q)*x*log(c) + 3*((b^3*n^2*q^
3 + 3*b^3*n^2*q^2 + 3*b^3*n^2*q + b^3*n^2)*x*log(c) + (a*b^2*n^2*q^3 - b^3*
m*n^3 + a*b^2*n^2 - (b^3*m*n^3 - 3*a*b^2*n^2)*q^2 - (2*b^3*m*n^3 - 3*a*b^2*
n^2)*q)*x)*log(d)^2 + 3*((b^3*m^2*n^2*q^3 + 3*b^3*m^2*n^2*q^2 + 3*b^3*m^2*n
^2*q + b^3*m^2*n^2)*x*log(c) + (b^3*m^2*n^3*q^3 + 3*b^3*m^2*n^3*q^2 + 3*b^3
*m^2*n^3*q + b^3*m^2*n^3)*x*log(d) + (a*b^2*m^2*n^2*q^3 - b^3*m^3*n^3 + a*b
^2*m^2*n^2 - (b^3*m^3*n^3 - 3*a*b^2*m^2*n^2)*q^2 - (2*b^3*m^3*n^3 - 3*a*b^2
*m^2*n^2)*q)*x)*log(x)^2 - (6*b^3*m^3*n^3 - 6*a*b^2*m^2*n^2 - a^3*q^3 + 3*a
^2*b*m*n - a^3 + 3*(a^2*b*m*n - a^3)*q^2 - 3*(2*a*b^2*m^2*n^2 - 2*a^2*b*m*n
 + a^3)*q)*x + 3*((b^3*n*q^3 + 3*b^3*n*q^2 + 3*b^3*n*q + b^3*n)*x*log(c)^2
+ 2*(a*b^2*n*q^3 - b^3*m*n^2 + a*b^2*n - (b^3*m*n^2 - 3*a*b^2*n)*q^2 - (2*b
^3*m*n^2 - 3*a*b^2*n)*q)*x*log(c) + (2*b^3*m^2*n^3 + a^2*b*n*q^3 - 2*a*b^2*
m*n^2 + a^2*b*n - (2*a*b^2*m*n^2 - 3*a^2*b*n)*q^2 + (2*b^3*m^2*n^3 - 4*a*b^
2*m*n^2 + 3*a^2*b*n)*q)*x)*log(d) + 3*((b^3*m*n*q^3 + 3*b^3*m*n*q^2 + 3*b^3
*m*n*q + b^3*m*n)*x*log(c)^2 + (b^3*m*n^3*q^3 + 3*b^3*m*n^3*q^2 + 3*b^3*m*n
^3*q + b^3*m*n^3)*x*log(d)^2 + 2*(a*b^2*m*n*q^3 - b^3*m^2*n^2 + a*b^2*m*n -
 (b^3*m^2*n^2 - 3*a*b^2*m*n)*q^2 - (2*b^3*m^2*n^2 - 3*a*b^2*m*n)*q)*x*log(c
) + (2*b^3*m^3*n^3 + a^2*b*m*n*q^3 - 2*a*b^2*m^2*n^2 + a^2*b*m*n - (2*a*b^2
```

```
*m^2*n^2 - 3*a^2*b*m*n)*q^2 + (2*b^3*m^3*n^3 - 4*a*b^2*m^2*n^2 + 3*a^2*b*m*
n)*q)*x + 2*((b^3*m*n^2*q^3 + 3*b^3*m*n^2*q^2 + 3*b^3*m*n^2*q + b^3*m*n^2)*
x*log(c) + (a*b^2*m*n^2*q^3 - b^3*m^2*n^3 + a*b^2*m*n^2 - (b^3*m^2*n^3 - 3*
a*b^2*m*n^2)*q^2 - (2*b^3*m^2*n^3 - 3*a*b^2*m*n^2)*q)*x)*log(d))*log(x))*e^
(q*log(e) + q*log(x))/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1)
```

giac [B] time = 0.49, size = 1811, normalized size = 13.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^3,x, algorithm="giac")
```

```
[Out] b^3*m^3*n^3*q^3*x*x^q*e^q*log(x)^3/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*b^3*
m^3*n^3*q^2*x*x^q*e^q*log(x)^3/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) - 3*b^3*m^3*
n^3*q^2*x*x^q*e^q*log(x)^2/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*b^3*m^2*n^3*
q^2*x*x^q*e^q*log(d)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*m^3*n^3*q*x*x
^q*e^q*log(x)^3/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) - 6*b^3*m^3*n^3*q*x*x^q*e^q
*log(x)^2/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*b^3*m^2*n^2*q^2*x*x^q*e^q*log
(c)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*m^2*n^3*q*x*x^q*e^q*log(d)*log
(x)^2/(q^3 + 3*q^2 + 3*q + 1) + b^3*m^3*n^3*x*x^q*e^q*log(x)^3/(q^4 + 4*q^3
 + 6*q^2 + 4*q + 1) + 6*b^3*m^3*n^3*q*x*x^q*e^q*log(x)/(q^4 + 4*q^3 + 6*q^2
 + 4*q + 1) - 6*b^3*m^2*n^3*q*x*x^q*e^q*log(d)*log(x)/(q^3 + 3*q^2 + 3*q +
1) + 3*b^3*m*n^3*q*x*x^q*e^q*log(d)^2*log(x)/(q^2 + 2*q + 1) - 3*b^3*m^3*n^
3*x*x^q*e^q*log(x)^2/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*a*b^2*m^2*n^2*q^2*
x*x^q*e^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*m^2*n^2*q*x*x^q*e^q*log(
c)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*m^2*n^3*x*x^q*e^q*log(d)*log(x)
^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*m^3*n^3*x*x^q*e^q*log(x)/(q^4 + 4*q^3 +
6*q^2 + 4*q + 1) - 6*b^3*m^2*n^2*q*x*x^q*e^q*log(c)*log(x)/(q^3 + 3*q^2 + 3
*q + 1) - 6*b^3*m^2*n^3*x*x^q*e^q*log(d)*log(x)/(q^3 + 3*q^2 + 3*q + 1) + 6
*b^3*m*n^2*q*x*x^q*e^q*log(c)*log(d)*log(x)/(q^2 + 2*q + 1) + 3*b^3*m*n^3*x
*x^q*e^q*log(d)^2*log(x)/(q^2 + 2*q + 1) + 6*a*b^2*m^2*n^2*q*x*x^q*e^q*log(
x)^2/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*m^2*n^2*x*x^q*e^q*log(c)*log(x)^2/(q^3
 + 3*q^2 + 3*q + 1) - 6*b^3*m^3*n^3*x*x^q*e^q/(q^4 + 4*q^3 + 6*q^2 + 4*q +
1) + 6*b^3*m^2*n^3*x*x^q*e^q*log(d)/(q^3 + 3*q^2 + 3*q + 1) - 3*b^3*m*n^3*x
*x^q*e^q*log(d)^2/(q^2 + 2*q + 1) + b^3*n^3*x*x^q*e^q*log(d)^3/(q + 1) - 6*
a*b^2*m^2*n^2*q*x*x^q*e^q*log(x)/(q^3 + 3*q^2 + 3*q + 1) - 6*b^3*m^2*n^2*x*
x^q*e^q*log(c)*log(x)/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*m*n*q*x*x^q*e^q*log(c
)^2*log(x)/(q^2 + 2*q + 1) + 6*a*b^2*m*n^2*q*x*x^q*e^q*log(d)*log(x)/(q^2 +
 2*q + 1) + 6*b^3*m*n^2*x*x^q*e^q*log(c)*log(d)*log(x)/(q^2 + 2*q + 1) + 3*
a*b^2*m^2*n^2*x*x^q*e^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*m^2*n^2*x*
x^q*e^q*log(c)/(q^3 + 3*q^2 + 3*q + 1) - 6*b^3*m*n^2*x*x^q*e^q*log(c)*log(d
)/(q^2 + 2*q + 1) + 3*b^3*n^2*x*x^q*e^q*log(c)*log(d)^2/(q + 1) - 6*a*b^2*m
^2*n^2*x*x^q*e^q*log(x)/(q^3 + 3*q^2 + 3*q + 1) + 6*a*b^2*m*n*q*x*x^q*e^q*l
og(c)*log(x)/(q^2 + 2*q + 1) + 3*b^3*m*n*x*x^q*e^q*log(c)^2*log(x)/(q^2 + 2
*q + 1) + 6*a*b^2*m*n^2*x*x^q*e^q*log(d)*log(x)/(q^2 + 2*q + 1) + 6*a*b^2*m
^2*n^2*x*x^q*e^q/(q^3 + 3*q^2 + 3*q + 1) - 3*b^3*m*n*x*x^q*e^q*log(c)^2/(q^
2 + 2*q + 1) - 6*a*b^2*m*n^2*x*x^q*e^q*log(d)/(q^2 + 2*q + 1) + 3*b^3*n*x*x
^q*e^q*log(c)^2*log(d)/(q + 1) + 3*a*b^2*n^2*x*x^q*e^q*log(d)^2/(q + 1) + 3
*a^2*b*m*n*q*x*x^q*e^q*log(x)/(q^2 + 2*q + 1) + 6*a*b^2*m*n*x*x^q*e^q*log(c
)*log(x)/(q^2 + 2*q + 1) - 6*a*b^2*m*n*x*x^q*e^q*log(c)/(q^2 + 2*q + 1) + b
^3*x*x^q*e^q*log(c)^3/(q + 1) + 6*a*b^2*n*x*x^q*e^q*log(c)*log(d)/(q + 1) +
 3*a^2*b*m*n*x*x^q*e^q*log(x)/(q^2 + 2*q + 1) - 3*a^2*b*m*n*x*x^q*e^q/(q^2
+ 2*q + 1) + 3*a*b^2*x*x^q*e^q*log(c)^2/(q + 1) + 3*a^2*b*n*x*x^q*e^q*log(d
)/(q + 1) + 3*a^2*b*x*x^q*e^q*log(c)/(q + 1) + a^3*x*x^q*e^q/(q + 1)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(d\, x^{m}\right)^{n}\right)+a \right)^{3} \left(e x \right)^{q}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^3,x)
```

```
[Out] int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^3,x)
```

maxima [B] time = 1.13, size = 282, normalized size = 2.09

$$-\frac{3\,a^2be^q mnxx^q}{\left(q+1\right)^2}+\frac{(ex)^{q+1}\,b^3\log\left(\left(dx^m\right)^n c\right)^3}{e\left(q+1\right)}+6\left(\frac{e^q m^2n^2xx^q}{\left(q+1\right)^3}-\frac{e^q mnxx^q\log\left(\left(dx^m\right)^n c\right)}{\left(q+1\right)^2}\right)ab^2-3\left(\frac{e^q mnxx^q\log\left((d\right.}{\left(q+1\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^3,x, algorithm="maxima")
```

```
[Out] -3*a^2*b*e^q*m*n*x*x^q/(q + 1)^2 + (e*x)^(q + 1)*b^3*log((d*x^m)^n*c)^3/(e*
(q + 1)) + 6*(e^q*m^2*n^2*x*x^q/(q + 1)^3 - e^q*m*n*x*x^q*log((d*x^m)^n*c)/
(q + 1)^2)*a*b^2 - 3*(e^q*m*n*x*x^q*log((d*x^m)^n*c)^2/(q + 1)^2 + 2*(e^(q
+ 1)*m^2*n^2*x*x^q/(q + 1)^3 - e^(q + 1)*m*n*x*x^q*log((d*x^m)^n*c)/(q + 1)
^2)*m*n/(e*(q + 1)))*b^3 + 3*(e*x)^(q + 1)*a*b^2*log((d*x^m)^n*c)^2/(e*(q +
 1)) + 3*(e*x)^(q + 1)*a^2*b*log((d*x^m)^n*c)/(e*(q + 1)) + (e*x)^(q + 1)*a
^3/(e*(q + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^q\,\left(a+b\,\ln\left(c\,(d\,x^m)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^3,x)
```

```
[Out] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^q\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n))**3,x)
```

```
[Out] Integral((e*x)**q*(a + b*log(c*(d*x**m)**n))**3, x)
```

3.238 $\int (ex)^q \left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2 dx$

Optimal. Leaf size=93

$$\frac{(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}{e(q+1)}-\frac{2bmn(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{e(q+1)^2}+\frac{2b^2m^2n^2(ex)^{q+1}}{e(q+1)^3}$$

```
[Out] 2*b^2*m^2*n^2*(e*x)^(1+q)/e/(1+q)^3-2*b*m*n*(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n
))/e/(1+q)^2+(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))^2/e/(1+q)
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2305, 2304, 2445}

$$\frac{(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}{e(q+1)}-\frac{2bmn(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{e(q+1)^2}+\frac{2b^2m^2n^2(ex)^{q+1}}{e(q+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^2,x]
```

```
[Out] (2*b^2*m^2*n^2*(e*x)^(1 + q))/(e*(1 + q)^3) - (2*b*m*n*(e*x)^(1 + q)*(a + b
*Log[c*(d*x^m)^n]))/(e*(1 + q)^2) + ((e*x)^(1 + q)*(a + b*Log[c*(d*x^m)^n])
^2)/(e*(1 + q))
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int (ex)^q\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2 dx=\text{Subst}\left(\int (ex)^q\left(a+b\log\left(cd^nx^{mn}\right)\right)^2 dx, cd^nx^{mn}, c\left(dx^m\right)^n\right)$$

$$=\frac{(ex)^{1+q}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}{e(1+q)}-\text{Subst}\left(\frac{(2bmn)\int (ex)^q\left(a+b\log\left(cd^nx^{mn}\right)\right)}{1+q}\right.$$

$$=\frac{2b^2m^2n^2(ex)^{1+q}}{e(1+q)^3}-\frac{2bmn(ex)^{1+q}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{e(1+q)^2}+\frac{(ex)^{1+q}\left(a+b\log\left(\right.\right.}{e(1+q)}$$

Mathematica [A] time = 0.04, size = 90, normalized size = 0.97

$$\frac{x(ex)^q\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}{q+1}-\frac{2bmnx^{-q}(ex)^q\left(\frac{x^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{q+1}-\frac{bmnx^{q+1}}{(q+1)^2}\right)}{q+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^2,x]
```

```
[Out] (x*(e*x)^q*(a + b*Log[c*(d*x^m)^n])^2)/(1 + q) - (2*b*m*n*(e*x)^q*(-((b*m*n
*x^(1 + q))/(1 + q)^2) + (x^(1 + q)*(a + b*Log[c*(d*x^m)^n]))/(1 + q)))/((1
 + q)*x^q)
```

fricas [B] time = 0.59, size = 391, normalized size = 4.20

$$\left(\left(b^2q^2+2\,b^2q+b^2\right)x\log(c)^2+\left(b^2n^2q^2+2\,b^2n^2q+b^2n^2\right)x\log(d)^2+\left(b^2m^2n^2q^2+2\,b^2m^2n^2q+b^2m^2n^2\right)x\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^2,x, algorithm="fricas")
```

```
[Out] ((b^2*q^2 + 2*b^2*q + b^2)*x*log(c)^2 + (b^2*n^2*q^2 + 2*b^2*n^2*q + b^2*n^
2)*x*log(d)^2 + (b^2*m^2*n^2*q^2 + 2*b^2*m^2*n^2*q + b^2*m^2*n^2)*x*log(x)^
2 - 2*(b^2*m*n - a*b*q^2 - a*b + (b^2*m*n - 2*a*b)*q)*x*log(c) + (2*b^2*m^2
*n^2 - 2*a*b*m*n + a^2*q^2 + a^2 - 2*(a*b*m*n - a^2)*q)*x + 2*((b^2*n*q^2 +
 2*b^2*n*q + b^2*n)*x*log(c) - (b^2*m*n^2 - a*b*n*q^2 - a*b*n + (b^2*m*n^2
- 2*a*b*n)*q)*x)*log(d) + 2*((b^2*m*n*q^2 + 2*b^2*m*n*q + b^2*m*n)*x*log(c)
 + (b^2*m*n^2*q^2 + 2*b^2*m*n^2*q + b^2*m*n^2)*x*log(d) - (b^2*m^2*n^2 - a*
b*m*n*q^2 - a*b*m*n + (b^2*m^2*n^2 - 2*a*b*m*n)*q)*x)*log(x))*e^(q*log(e) +
 q*log(x))/(q^3 + 3*q^2 + 3*q + 1)
```

giac [B] time = 0.35, size = 561, normalized size = 6.03

$$\frac{b^2m^2n^2q^2xx^qe^q\log(x)^2}{q^3+3\,q^2+3\,q+1}+\frac{2\,b^2m^2n^2qxx^qe^q\log(x)^2}{q^3+3\,q^2+3\,q+1}-\frac{2\,b^2m^2n^2qxx^qe^q\log(x)}{q^3+3\,q^2+3\,q+1}+\frac{2\,b^2mn^2qxx^qe^q\log(d)\log(x)}{q^2+2\,q+1}+\frac{b^2m}{q^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^2,x, algorithm="giac")
```

```
[Out] b^2*m^2*n^2*q^2*x*x^q*e^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 2*b^2*m^2*n^2*
q*x*x^q*e^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) - 2*b^2*m^2*n^2*q*x*x^q*e^q*lo
g(x)/(q^3 + 3*q^2 + 3*q + 1) + 2*b^2*m*n^2*q*x*x^q*e^q*log(d)*log(x)/(q^2 +
 2*q + 1) + b^2*m^2*n^2*x*x^q*e^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) - 2*b^2*
m^2*n^2*x*x^q*e^q*log(x)/(q^3 + 3*q^2 + 3*q + 1) + 2*b^2*m*n*q*x*x^q*e^q*lo
g(c)*log(x)/(q^2 + 2*q + 1) + 2*b^2*m*n^2*x*x^q*e^q*log(d)*log(x)/(q^2 + 2*
q + 1) + 2*b^2*m^2*n^2*x*x^q*e^q/(q^3 + 3*q^2 + 3*q + 1) - 2*b^2*m*n^2*x*x^
q*e^q*log(d)/(q^2 + 2*q + 1) + b^2*n^2*x*x^q*e^q*log(d)^2/(q + 1) + 2*a*b*m
*n*q*x*x^q*e^q*log(x)/(q^2 + 2*q + 1) + 2*b^2*m*n*x*x^q*e^q*log(c)*log(x)/(
q^2 + 2*q + 1) - 2*b^2*m*n*x*x^q*e^q*log(c)/(q^2 + 2*q + 1) + 2*b^2*n*x*x^q
*e^q*log(c)*log(d)/(q + 1) + 2*a*b*m*n*x*x^q*e^q*log(x)/(q^2 + 2*q + 1) - 2
*a*b*m*n*x*x^q*e^q/(q^2 + 2*q + 1) + b^2*x*x^q*e^q*log(c)^2/(q + 1) + 2*a*b
*n*x*x^q*e^q*log(d)/(q + 1) + 2*a*b*x*x^q*e^q*log(c)/(q + 1) + a^2*x*x^q*e^
q/(q + 1)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d\,x^m\right)^n\right)+a\right)^2(ex)^q\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(b*ln(c*(d*x^m)^n)+a)^2,x)
```

```
[Out] int((e*x)^q*(b*ln(c*(d*x^m)^n)+a)^2,x)
```

maxima [A] time = 1.05, size = 149, normalized size = 1.60

$$-\frac{2\,abe^{q}mnxx^{q}}{\left(q+1\right)^{2}}+2\left(\frac{e^{q}m^{2}n^{2}xx^{q}}{\left(q+1\right)^{3}}-\frac{e^{q}mnxx^{q}\log\left(\left(dx^{m}\right)^{n}c\right)}{\left(q+1\right)^{2}}\right)b^{2}+\frac{\left(ex\right)^{q+1}b^{2}\log\left(\left(dx^{m}\right)^{n}c\right)^{2}}{e\left(q+1\right)}+\frac{2\,\left(ex\right)^{q+1}ab\log\left(\left(dx^{m}\right)^{n}\right.}{e\left(q+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*e^q*m*n*x*x^q/(q + 1)^2 + 2*(e^q*m^2*n^2*x*x^q/(q + 1)^3 - e^q*m*n*x
*x^q*log((d*x^m)^n*c)/(q + 1)^2)*b^2 + (e*x)^(q + 1)*b^2*log((d*x^m)^n*c)^2
/(e*(q + 1)) + 2*(e*x)^(q + 1)*a*b*log((d*x^m)^n*c)/(e*(q + 1)) + (e*x)^(q
+ 1)*a^2/(e*(q + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^q\,\left(a+b\,\ln\left(c\,(d\,x^m)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^2,x)
```

```
[Out] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex\right)^{q}\left(a+b\log\left(c\left(dx^{m}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n))**2,x)
```

```
[Out] Integral((e*x)**q*(a + b*log(c*(d*x**m)**n))**2, x)
```

3.239 $\int (ex)^q \left(a+b\log\left(c\left(dx^m\right)^n\right)\right)\,dx$

Optimal. Leaf size=51

$$\frac{(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{e(q+1)}-\frac{bmn(ex)^{q+1}}{e(q+1)^2}$$

```
[Out] -b*m*n*(e*x)^(1+q)/e/(1+q)^2+(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))/e/(1+q)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2304, 2445}

$$\frac{(ex)^{q+1}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{e(q+1)}-\frac{bmn(ex)^{q+1}}{e(q+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n]),x]
```

```
[Out] -((b*m*n*(e*x)^(1 + q))/(e*(1 + q)^2)) + ((e*x)^(1 + q)*(a + b*Log[c*(d*x^m
)^n]))/(e*(1 + q))
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int (ex)^q\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)\,dx &= \text{Subst}\left(\int (ex)^q\left(a+b\log\left(cd^nx^{mn}\right)\right)\,dx,cd^nx^{mn},c\left(dx^m\right)^n\right)\\ &= -\frac{bmn(ex)^{1+q}}{e(1+q)^2}+\frac{(ex)^{1+q}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{e(1+q)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.73

$$\frac{x(ex)^q\left(aq+a+b(q+1)\log\left(c\left(dx^m\right)^n\right)-bmn\right)}{(q+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n]),x]
```

```
[Out] (x*(e*x)^q*(a - b*m*n + a*q + b*(1 + q)*Log[c*(d*x^m)^n]))/(1 + q)^2
```

fricas [A] time = 0.61, size = 72, normalized size = 1.41

$$\frac{\left(\left(bq+b\right)x\log(c)+\left(bnq+bn\right)x\log(d)+\left(bmnq+bmn\right)x\log(x)-\left(bmn-aq-a\right)x\right)e^{\left(q\log(e)+q\log(x)\right)}}{q^2+2\,q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n)),x, algorithm="fricas")
```

```
[Out] ((b*q + b)*x*log(c) + (b*n*q + b*n)*x*log(d) + (b*m*n*q + b*m*n)*x*log(x) -
 (b*m*n - a*q - a)*x)*e^(q*log(e) + q*log(x))/(q^2 + 2*q + 1)
```

giac [B] time = 0.34, size = 111, normalized size = 2.18

$$\frac{bmnqxx^qe^q\log(x)}{q^2+2q+1}+\frac{bmnxx^qe^q\log(x)}{q^2+2q+1}-\frac{bmnxx^qe^q}{q^2+2q+1}+\frac{bnxx^qe^q\log(d)}{q+1}+\frac{bxx^qe^q\log(c)}{q+1}+\frac{axx^qe^q}{q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n)),x, algorithm="giac")
```

```
[Out] b*m*n*q*x*x^q*e^q*log(x)/(q^2 + 2*q + 1) + b*m*n*x*x^q*e^q*log(x)/(q^2 + 2*
q + 1) - b*m*n*x*x^q*e^q/(q^2 + 2*q + 1) + b*n*x*x^q*e^q*log(d)/(q + 1) + b
*x*x^q*e^q*log(c)/(q + 1) + a*x*x^q*e^q/(q + 1)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\,(d\,x^m)^n\right)+a\right)(ex)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(b*ln(c*(d*x^m)^n)+a),x)
```

```
[Out] int((e*x)^q*(b*ln(c*(d*x^m)^n)+a),x)
```

maxima [A] time = 0.99, size = 62, normalized size = 1.22

$$-\frac{be^qmnxx^q}{\left(q+1\right)^2}+\frac{(ex)^{q+1}\,b\log\left((dx^m)^n\,c\right)}{e\left(q+1\right)}+\frac{(ex)^{q+1}\,a}{e\left(q+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n)),x, algorithm="maxima")
```

```
[Out] -b*e^q*m*n*x*x^q/(q + 1)^2 + (e*x)^(q + 1)*b*log((d*x^m)^n*c)/(e*(q + 1)) +
 (e*x)^(q + 1)*a/(e*(q + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (e\,x)^q\,\left(a+b\,\ln\left(c\,(d\,x^m)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a + b*log(c*(d*x^m)^n)),x)
```

```
[Out] int((e*x)^q*(a + b*log(c*(d*x^m)^n)), x)
```

sympy [A] time = 10.31, size = 112, normalized size = 2.20

$$a\left(\begin{cases}0^qx & \text{for}\ e=0\\ \frac{\begin{cases}\frac{(ex)^{q+1}}{q+1} & \text{for}\ q\neq -1\\ \log(ex) & \text{otherwise}\end{cases}}{e} & \text{otherwise}\end{cases}\right)-bmn\left(\begin{cases}0^qx & \text{for}\ \left(e=0\wedge q\neq -1\right)\vee e=0\\ \frac{\begin{cases}\frac{ee^qxx^q}{q+1} & \text{for}\ q\neq -1\\ \log(x) & \text{otherwise}\end{cases}}{eq+e} & \text{for}\ q>-\infty\wedge q<\infty\wedge q\neq -1\\ \frac{\log(ex)^2}{2e} & \text{otherwise}\end{cases}\right)+b\Bigg(\Bigg(\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n)),x)
```

```
[Out] a*Piecewise((0**q*x, Eq(e, 0)), (Piecewise(((e*x)**(q + 1)/(q + 1), Ne(q, -
1)), (log(e*x), True))/e, True)) - b*m*n*Piecewise((0**q*x, Eq(e, 0) | (Eq(
e, 0) & Ne(q, -1))), (Piecewise((e*e**q*x*x**q/(q + 1), Ne(q, -1)), (log(x)
, True))/(e*q + e), (q > -oo) & (q < oo) & Ne(q, -1)), (log(e*x)**2/(2*e),
True)) + b*Piecewise((0**q*x, Eq(e, 0)), (Piecewise(((e*x)**(q + 1)/(q + 1)
, Ne(q, -1)), (log(e*x), True))/e, True))*log(c*(d*x**m)**n)
```

3.240 $\int \frac{(ex)^q}{a+b\log\left(c(dx^m)^n\right)}\,dx$

Optimal. Leaf size=86

$$\frac{(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{q+1}{mn}}\text{Ei}\left(\frac{(q+1)\left(a+b\log\left(c(dx^m)^n\right)\right)}{bmn}\right)}{bemn}$$

```
[Out] (e*x)^(1+q)*Ei((1+q)*(a+b*ln(c*(d*x^m)^n))/b/m/n)/b/e/exp(a*(1+q)/b/m/n)/m/
n/((c*(d*x^m)^n)^((1+q)/m/n))
```

Rubi [A] time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2310, 2178, 2445}

$$\frac{(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{q+1}{mn}}\text{Ei}\left(\frac{(q+1)\left(a+b\log\left(c(dx^m)^n\right)\right)}{bmn}\right)}{bemn}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q/(a + b*Log[c*(d*x^m)^n]),x]
```

```
[Out] ((e*x)^(1 + q)*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)])/(
b*e*E^((a*(1 + q))/(b*m*n))*m*n*(c*(d*x^m)^n)^((1 + q)/(m*n)))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{(ex)^q}{a+b\log\left(c\,(dx^m)^n\right)}\,dx = \text{Subst}\left(\int \frac{(ex)^q}{a+b\log\left(cd^n x^{mn}\right)}\,dx, cd^n x^{mn}, c\,(dx^m)^n\right)$$
$$= \text{Subst}\left(\frac{\left((ex)^{1+q}\left(cd^n x^{mn}\right)^{-\frac{1+q}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{(1+q)x}{mn}}}{a+bx}\,dx, x, \log\left(cd^n x^{mn}\right)\right)}{emn}, cd^n x^{mn}, c\,(dx^m)^n\right)$$
$$= \frac{e^{-\frac{a(1+q)}{bmn}}(ex)^{1+q}\left(c\,(dx^m)^n\right)^{-\frac{1+q}{mn}}\text{Ei}\left(\frac{(1+q)\left(a+b\log\left(c(dx^m)^n\right)\right)}{bmn}\right)}{bemn}$$

Mathematica [A] time = 0.21, size = 85, normalized size = 0.99

$$\frac{x^{-q}(ex)^q\exp\left(-\frac{(q+1)\left(a+b\log\left(c(dx^m)^n\right)-bmn\log(x)\right)}{bmn}\right)\text{Ei}\left(\frac{(q+1)\left(a+b\log\left(c(dx^m)^n\right)\right)}{bmn}\right)}{bmn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q/(a + b*Log[c*(d*x^m)^n]),x]
```

```
[Out] ((e*x)^q*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)])/(b*E^((
(1 + q)*(a - b*m*n*Log[x] + b*Log[c*(d*x^m)^n]))/(b*m*n))*m*n*x^q)
```

fricas [A] time = 0.65, size = 105, normalized size = 1.22

$$\frac{\text{Ei}\left(\frac{aq+\left(bq+b\right)\log(c)+\left(bnq+bn\right)\log(d)+\left(bmnq+bmn\right)\log(x)+a}{bmn}\right)e^{\left(\frac{bmnq\log(e)-aq-\left(bq+b\right)\log(c)-\left(bnq+bn\right)\log(d)-a}{bmn}\right)}}{bmn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n)),x, algorithm="fricas")
```

```
[Out] Ei((a*q + (b*q + b)*log(c) + (b*n*q + b*n)*log(d) + (b*m*n*q + b*m*n)*log(x
) + a)/(b*m*n))*e^((b*m*n*q*log(e) - a*q - (b*q + b)*log(c) - (b*n*q + b*n)
*log(d) - a)/(b*m*n))/(b*m*n)
```

giac [A] time = 0.43, size = 140, normalized size = 1.63

$$\frac{\text{Ei}\left(q\log(x)+\frac{q\log(d)}{m}+\frac{q\log(c)}{mn}+\frac{\log(d)}{m}+\frac{aq}{bmn}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log(x)\right)e^{\left(q-\frac{aq}{bmn}-\frac{a}{bmn}\right)}}{bc^{\frac{q}{mn}}c^{\frac{1}{mn}}d^{\frac{q}{m}}d^{\left(\frac{1}{m}\right)}mn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n)),x, algorithm="giac")
```

```
[Out] Ei(q*log(x) + q*log(d)/m + q*log(c)/(m*n) + log(d)/m + a*q/(b*m*n) + log(c)
/(m*n) + a/(b*m*n) + log(x))*e^(q - a*q/(b*m*n) - a/(b*m*n))/(b*c^(q/(m*n))
*c^(1/(m*n))*d^(q/m)*d^(1/m)*m*n)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(ex)^q}{b\ln\left(c\,(d\,x^m)^n\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q/(b*ln(c*(d*x^m)^n)+a),x)
[Out] int((e*x)^q/(b*ln(c*(d*x^m)^n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^q}{b \log\left((dx^m)^n c\right) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n)),x, algorithm="maxima")
[Out] integrate((e*x)^q/(b*log((d*x^m)^n*c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^q}{a + b\,\ln\left(c\,(d\,x^m)^n\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q/(a + b*log(c*(d*x^m)^n)),x)
[Out] int((e*x)^q/(a + b*log(c*(d*x^m)^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^q}{a + b \log\left(c\,(dx^m)^n\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q/(a+b*ln(c*(d*x**m)**n)),x)
[Out] Integral((e*x)**q/(a + b*log(c*(d*x**m)**n)), x)
```

3.241 $\int \frac{(ex)^q}{\left(a+b\log\left(c(dx^m)^n\right)\right)^2} dx$

Optimal. Leaf size=127

$$\frac{(q+1)(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{q+1}{mn}}\text{Ei}\left(\frac{(q+1)\left(a+b\log\left(c(dx^m)^n\right)\right)}{bmn}\right)}{b^2em^2n^2}-\frac{(ex)^{q+1}}{bemn\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}$$

```
[Out] (1+q)*(e*x)^(1+q)*Ei((1+q)*(a+b*ln(c*(d*x^m)^n))/b/m/n)/b^2/e/exp(a*(1+q)/b
/m/n)/m^2/n^2/((c*(d*x^m)^n)^((1+q)/m/n))-(e*x)^(1+q)/b/e/m/n/(a+b*ln(c*(d*
x^m)^n))
```

Rubi [A] time = 0.24, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2306, 2310, 2178, 2445}

$$\frac{(q+1)(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{q+1}{mn}}\text{Ei}\left(\frac{(q+1)\left(a+b\log\left(c(dx^m)^n\right)\right)}{bmn}\right)}{b^2em^2n^2}-\frac{(ex)^{q+1}}{bemn\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q/(a + b*Log[c*(d*x^m)^n])^2,x]
```

```
[Out] ((1 + q)*(e*x)^(1 + q)*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*
m*n)])/(b^2*e*E^((a*(1 + q))/(b*m*n))*m^2*n^2*(c*(d*x^m)^n)^((1 + q)/(m*n))
) - (e*x)^(1 + q)/(b*e*m*n*(a + b*Log[c*(d*x^m)^n]))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{(ex)^q}{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^2}\,dx = \text{Subst}\left(\int \frac{(ex)^q}{\left(a+b\log\left(cd^n x^{mn}\right)\right)^2}\,dx, cd^n x^{mn}, c\left(dx^m\right)^n\right)$$

$$= -\frac{(ex)^{1+q}}{bemn\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)} + \text{Subst}\left(\frac{(1+q)\int \frac{(ex)^q}{a+b\log(cd^n x^{mn})}\,dx}{bmn}, cd^n x^{mn}, c\left(dx^m\right.\right.$$

$$= -\frac{(ex)^{1+q}}{bemn\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)} + \text{Subst}\left(\frac{\left((1+q)(ex)^{1+q}\left(cd^n x^{mn}\right)^{-\frac{1+q}{mn}}\right)\text{Subst}\left(\int \frac{e}{}\right.}{bem^2n^2}\right.$$

$$= \frac{e^{-\frac{a(1+q)}{bmn}}(1+q)(ex)^{1+q}\left(c\left(dx^m\right)^n\right)^{-\frac{1+q}{mn}}\text{Ei}\left(\frac{(1+q)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)}{b^2em^2n^2} - \frac{(ex)^{1+q}}{bemn\left(a+b\log\left(\right.\right.}$$

Mathematica [A] time = 0.31, size = 112, normalized size = 0.88

$$\frac{(ex)^q\left((q+1)x^{-q}\exp\left(-\frac{(q+1)\left(a+b\log\left(c\left(dx^m\right)^n\right)-bmn\log(x)\right)}{bmn}\right)\text{Ei}\left(\frac{(q+1)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)-\frac{bmnx}{a+b\log\left(c\left(dx^m\right)^n\right)}\right)}{b^2m^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q/(a + b*Log[c*(d*x^m)^n])^2,x]
```

```
[Out] ((e*x)^q*(((1 + q)*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)
])/(E^(((1 + q)*(a - b*m*n*Log[x] + b*Log[c*(d*x^m)^n]))/(b*m*n))*x^q) - (b
*m*n*x)/(a + b*Log[c*(d*x^m)^n])))/(b^2*m^2*n^2)
```

fricas [A] time = 0.66, size = 202, normalized size = 1.59

$$-\frac{bmnxe^{(q\log(e)+q\log(x))} - \left(aq+\left(bq+b\right)\log(c)+\left(bnq+bn\right)\log(d)+\left(bmnq+bmn\right)\log(x)+a\right)\text{Ei}\left(\frac{aq+\left(bq+b\right)\log(c}{}\right.}{b^3m^3n^3\log(x)+b^3m^2n^3\log(d)+b^3m^2n^2\log(c)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n))^2,x, algorithm="fricas")
```

```
[Out] -(b*m*n*x*e^(q*log(e) + q*log(x)) - (a*q + (b*q + b)*log(c) + (b*n*q + b*n)
*log(d) + (b*m*n*q + b*m*n)*log(x) + a)*Ei((a*q + (b*q + b)*log(c) + (b*n*q
 + b*n)*log(d) + (b*m*n*q + b*m*n)*log(x) + a)/(b*m*n))*e^((b*m*n*q*log(e)
- a*q - (b*q + b)*log(c) - (b*n*q + b*n)*log(d) - a)/(b*m*n)))/(b^3*m^3*n^3
*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)
```

giac [B] time = 0.84, size = 1540, normalized size = 12.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n))^2,x, algorithm="giac")
```

```
[Out] -b*m*n*x*x^q*e^q/(b^3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log
(c) + a*b^2*m^2*n^2) + b*m*n*q*Ei(q*log(x) + q*log(d)/m + q*log(c)/(m*n) +
log(d)/m + a*q/(b*m*n) + log(c)/(m*n) + a/(b*m*n) + log(x))*e^(q - a*q/(b*m
```

```
*n) - a/(b*m*n))*log(x)/((b^3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2
*n^2*log(c) + a*b^2*m^2*n^2)*c^(q/(m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m)) + b*n
*q*Ei(q*log(x) + q*log(d)/m + q*log(c)/(m*n) + log(d)/m + a*q/(b*m*n) + log
(c)/(m*n) + a/(b*m*n) + log(x))*e^(q - a*q/(b*m*n) - a/(b*m*n))*log(d)/((b^
3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)
*c^(q/(m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m)) + b*m*n*Ei(q*log(x) + q*log(d)/m
+ q*log(c)/(m*n) + log(d)/m + a*q/(b*m*n) + log(c)/(m*n) + a/(b*m*n) + log(
x))*e^(q - a*q/(b*m*n) - a/(b*m*n))*log(x)/((b^3*m^3*n^3*log(x) + b^3*m^2*n
^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)*c^(q/(m*n))*c^(1/(m*n))*d^(
q/m)*d^(1/m)) + b*q*Ei(q*log(x) + q*log(d)/m + q*log(c)/(m*n) + log(d)/m +
a*q/(b*m*n) + log(c)/(m*n) + a/(b*m*n) + log(x))*e^(q - a*q/(b*m*n) - a/(b*
m*n))*log(c)/((b^3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log(c)
 + a*b^2*m^2*n^2)*c^(q/(m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m)) + b*n*Ei(q*log(x
) + q*log(d)/m + q*log(c)/(m*n) + log(d)/m + a*q/(b*m*n) + log(c)/(m*n) + a
/(b*m*n) + log(x))*e^(q - a*q/(b*m*n) - a/(b*m*n))*log(d)/((b^3*m^3*n^3*log
(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)*c^(q/(m*n))*
c^(1/(m*n))*d^(q/m)*d^(1/m)) + a*q*Ei(q*log(x) + q*log(d)/m + q*log(c)/(m*n
) + log(d)/m + a*q/(b*m*n) + log(c)/(m*n) + a/(b*m*n) + log(x))*e^(q - a*q/
(b*m*n) - a/(b*m*n))/((b^3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^
2*log(c) + a*b^2*m^2*n^2)*c^(q/(m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m)) + b*Ei(q
*log(x) + q*log(d)/m + q*log(c)/(m*n) + log(d)/m + a*q/(b*m*n) + log(c)/(m*
n) + a/(b*m*n) + log(x))*e^(q - a*q/(b*m*n) - a/(b*m*n))*log(c)/((b^3*m^3*n
^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)*c^(q/(
m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m)) + a*Ei(q*log(x) + q*log(d)/m + q*log(c)/
(m*n) + log(d)/m + a*q/(b*m*n) + log(c)/(m*n) + a/(b*m*n) + log(x))*e^(q -
a*q/(b*m*n) - a/(b*m*n))/((b^3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^
2*n^2*log(c) + a*b^2*m^2*n^2)*c^(q/(m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m))
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(ex)^q}{\left(b\ln\left(c\left(d\,x^m\right)^n\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q/(b*ln(c*(d*x^m)^n)+a)^2,x)
```

```
[Out] int((e*x)^q/(b*ln(c*(d*x^m)^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$e^q(q+1)\int \frac{x^q}{b^2mn\log\left(\left(x^m\right)^n\right)+abmn+\left(mn^2\log(d)+mn\log(c)\right)b^2}\,dx-\frac{e^q x x^q}{b^2mn\log\left(\left(x^m\right)^n\right)+abmn+\left(mn^2\,\mathrm{lo}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n))^2,x, algorithm="maxima")
```

```
[Out] e^q*(q + 1)*integrate(x^q/(b^2*m*n*log((x^m)^n) + a*b*m*n + (m*n^2*log(d) +
 m*n*log(c))*b^2), x) - e^q*x*x^q/(b^2*m*n*log((x^m)^n) + a*b*m*n + (m*n^2*
log(d) + m*n*log(c))*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^q}{\left(a+b\,\ln\left(c\,\left(d\,x^m\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q/(a + b*log(c*(d*x^m)^n))^2,x)
```

```
[Out] int((e*x)^q/(a + b*log(c*(d*x^m)^n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^q}{\left(a + b\log\left(c\left(dx^m\right)^n\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q/(a+b*ln(c*(d*x**m)**n))**2,x)
```

```
[Out] Integral((e*x)**q/(a + b*log(c*(d*x**m)**n))**2, x)
```

3.242 $\int (ex)^q \left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\,dx$

Optimal. Leaf size=134

$$\frac{(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{q+1}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{(q+1)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{(q+1)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)}{e(q+1)}$$

```
[Out] (e*x)^(1+q)*GAMMA(1+p,-(1+q)*(a+b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)
^n))^p/e/exp(a*(1+q)/b/m/n)/(1+q)/((c*(d*x^m)^n)^((1+q)/m/n))/((-(1+q)*(a+b
*ln(c*(d*x^m)^n))/b/m/n)^p)
```

Rubi [A] time = 0.19, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2310, 2181, 2445}

$$\frac{(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{q+1}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{(q+1)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{(q+1)\left(a+b\log\left(c\left(d\right.\right.\right.}{bmn}\right.}{e(q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] ((e*x)^(1 + q)*Gamma[1 + p, -(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n))]*
(a + b*Log[c*(d*x^m)^n])^p)/(e*E^((a*(1 + q))/(b*m*n))*(1 + q)*(c*(d*x^m)^n
)^((1 + q)/(m*n))*(-(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int (ex)^q \left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\,dx = \text{Subst}\left(\int (ex)^q \left(a+b\log\left(cd^nx^{mn}\right)\right)^p\,dx, cd^nx^{mn}, c\left(dx^m\right)^n\right)$$

$$= \text{Subst}\left(\frac{\left((ex)^{1+q}\left(cd^nx^{mn}\right)^{-\frac{1+q}{mn}}\right)\text{Subst}\left(\int e^{\frac{(1+q)x}{mn}}(a+bx)^p\,dx, x, \log\left(cd^nx^{mn}\right)\right)}{emn},\right.$$

$$= \frac{e^{-\frac{a(1+q)}{bmn}}(ex)^{1+q}\left(c\left(dx^m\right)^n\right)^{-\frac{1+q}{mn}}\Gamma\left(1+p,-\frac{(1+q)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{e(1+q)}$$

Mathematica [A] time = 0.20, size = 133, normalized size = 0.99

$$\frac{x^{-q}(ex)^q\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\exp\left(-\frac{(q+1)\left(a+b\log\left(c\left(dx^m\right)^n\right)-bmn\log(x)\right)}{bmn}\right)\left(-\frac{(q+1)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{(q+1)\left(a+b\right.}{}\right.}{q+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] ((e*x)^q*Gamma[1 + p, -(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n))]*(a + b
*Log[c*(d*x^m)^n])^p)/(E^(((1 + q)*(a - b*m*n*Log[x] + b*Log[c*(d*x^m)^n]))
/(b*m*n))*(1 + q)*x^q*(-(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)))^p)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^q\left(b\log\left(\left(dx^m\right)^nc\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^q*(b*log((d*x^m)^n*c) + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^q\left(b\log\left(\left(dx^m\right)^nc\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^q*(b*log((d*x^m)^n*c) + a)^p, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (ex)^q\left(b\ln\left(c\left(d\,x^m\right)^n\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(b*ln(c*(d*x^m)^n)+a)^p,x)
```

```
[Out] int((e*x)^q*(b*ln(c*(d*x^m)^n)+a)^p,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^q\,{\left(a+b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^p,x)
```

```
[Out] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{q} \left(a + b \log{\left(c \left(dx^{m}\right)^{n} \right)}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n))**p,x)
```

```
[Out] Integral((e*x)**q*(a + b*log(c*(d*x**m)**n))**p, x)
```

3.243 $\int x^2 \left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\,dx$

Optimal. Leaf size=117

$$3^{-p-1}x^3e^{-\frac{3a}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{3}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)$$

```
[Out] 3^(-1-p)*x^3*GAMMA(1+p,-3*(a+b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n)
)^p/exp(3*a/b/m/n)/((c*(d*x^m)^n)^(3/m/n))/(((-a-b*ln(c*(d*x^m)^n))/b/m/n)^
p)
```

Rubi [A] time = 0.17, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2310, 2181, 2445}

$$3^{-p-1}x^3e^{-\frac{3a}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{3}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(dx\right.\right.\right.}{bmn}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*
Log[c*(d*x^m)^n])^p)/(E^((3*a)/(b*m*n))*(c*(d*x^m)^n)^(3/(m*n))*(-((a + b*L
og[c*(d*x^m)^n])/(b*m*n)))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\,dx &= \text{Subst}\left(\int x^2\left(a+b\log\left(cd^nx^{mn}\right)\right)^p\,dx, cd^nx^{mn}, c\left(dx^m\right)^n\right)\\
&= \text{Subst}\left(\frac{\left(x^3\left(cd^nx^{mn}\right)^{-\frac{3}{mn}}\right)\text{Subst}\left(\int e^{\frac{3x}{mn}}(a+bx)^p\,dx, x, \log\left(cd^nx^{mn}\right)\right)}{mn}, cd^nx^{mn}, c\right.\\
&= 3^{-1-p}e^{-\frac{3a}{bmn}}x^3\left(c\left(dx^m\right)^n\right)^{-\frac{3}{mn}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)\left(a+b\log\left(c\left(d\right.\right.\right.
\end{aligned}$$

Mathematica [A] time = 0.16, size = 117, normalized size = 1.00

$$3^{-p-1}x^3e^{-\frac{3a}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{3}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*
Log[c*(d*x^m)^n])^p)/(E^((3*a)/(b*m*n))*(c*(d*x^m)^n)^(3/(m*n))*(-((a + b*L
og[c*(d*x^m)^n])/(b*m*n)))^p)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\left(dx^m\right)^nc\right)+a\right)^px^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((d*x^m)^n*c) + a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(dx^m\right)^nc\right)+a\right)^px^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)^p*x^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^2\left(b\ln\left(c\left(d\,x^m\right)^n\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(d*x^m)^n)+a)^p,x)
```

```
[Out] int(x^2*(b*ln(c*(d*x^m)^n)+a)^p,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\,\ln\left(c\left(d\,x^m\right)^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d*x^m)^n))^p,x)
```

```
[Out] int(x^2*(a + b*log(c*(d*x^m)^n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \log \left(c \left(dx^m\right)^n\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d*x**m)**n))**p,x)
[Out] Integral(x**2*(a + b*log(c*(d*x**m)**n))**p, x)
```

3.244 $\int x\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\,dx$

Optimal. Leaf size=117

$$2^{-p-1}x^2e^{-\frac{2a}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{2}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)$$

```
[Out] 2^(-1-p)*x^2*GAMMA(1+p,-2*(a+b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n)
)^p/exp(2*a/b/m/n)/((c*(d*x^m)^n)^(2/m/n))/(((-a-b*ln(c*(d*x^m)^n))/b/m/n)^
p)
```

Rubi [A] time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2310, 2181, 2445}

$$2^{-p-1}x^2e^{-\frac{2a}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{2}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*
Log[c*(d*x^m)^n])^p)/(E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*(-((a + b*L
og[c*(d*x^m)^n])/(b*m*n)))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\,dx &= \text{Subst}\left(\int x\left(a+b\log\left(cd^nx^{mn}\right)\right)^p\,dx, cd^nx^{mn}, c\left(dx^m\right)^n\right)\\ &= \text{Subst}\left(\frac{\left(x^2\left(cd^nx^{mn}\right)^{-\frac{2}{mn}}\right)\text{Subst}\left(\int e^{\frac{2x}{mn}}(a+bx)^p\,dx, x, \log\left(cd^nx^{mn}\right)\right)}{mn}, cd^nx^{mn}\right.\\ &= 2^{-1-p}e^{-\frac{2a}{bmn}}x^2\left(c\left(dx^m\right)^n\right)^{-\frac{2}{mn}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)\left(a+b\log\left(c\right.\right.\end{aligned}$$

Mathematica [A] time = 0.14, size = 117, normalized size = 1.00

$$2^{-p-1}x^2e^{-\frac{2a}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{2}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*
Log[c*(d*x^m)^n])^p)/(E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*(-((a + b*L
og[c*(d*x^m)^n])/(b*m*n)))^p)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\log\left(\left(dx^m\right)^nc\right)+a\right)^px,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((d*x^m)^n*c) + a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(dx^m\right)^nc\right)+a\right)^px\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)^p*x, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x\left(b\ln\left(c\left(d\,x^m\right)^n\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(d*x^m)^n)+a)^p,x)
```

```
[Out] int(x*(b*ln(c*(d*x^m)^n)+a)^p,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,\ln\left(c\left(d\,x^m\right)^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d*x^m)^n))^p,x)
```

```
[Out] int(x*(a + b*log(c*(d*x^m)^n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(dx^{m}\right)^{n}\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d*x**m)**n))**p,x)
```

```
[Out] Integral(x*(a + b*log(c*(d*x**m)**n))**p, x)
```

3.245 $\int \left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\,dx$

Optimal. Leaf size=108

$$xe^{-\frac{a}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{1}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)$$

```
[Out] x*GAMMA(1+p,(-a-b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n))^p/exp(a/b/m
/n)/((c*(d*x^m)^n)^(1/m/n))/(((-a-b*ln(c*(d*x^m)^n))/b/m/n)^p)
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2300, 2181, 2445}

$$xe^{-\frac{a}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{1}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (x*Gamma[1 + p, -((a + b*Log[c*(d*x^m)^n])/(b*m*n))]*(a + b*Log[c*(d*x^m)^n
])^p)/(E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*(-((a + b*Log[c*(d*x^m)^n])/(b
*m*n)))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\,dx=\text{Subst}\left(\int \left(a+b\log\left(cd^nx^{mn}\right)\right)^p\,dx,cd^nx^{mn},c\left(dx^m\right)^n\right)$$

$$=\text{Subst}\left(\frac{\left(x\left(cd^nx^{mn}\right)^{-\frac{1}{mn}}\right)\text{Subst}\left(\int e^{\frac{x}{mn}}(a+bx)^p\,dx,x,\log\left(cd^nx^{mn}\right)\right)}{mn},cd^nx^{mn},c\left(dx\right.\right.$$

$$=e^{-\frac{a}{bmn}}x\left(c\left(dx^m\right)^n\right)^{-\frac{1}{mn}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{a}{-}\right.$$

Mathematica [A] time = 0.13, size = 108, normalized size = 1.00

$$xe^{-\frac{a}{bmn}}\left(c\left(dx^m\right)^n\right)^{-\frac{1}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (x*Gamma[1 + p, -((a + b*Log[c*(d*x^m)^n])/(b*m*n))]*(a + b*Log[c*(d*x^m)^n
])^p)/(E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*(-((a + b*Log[c*(d*x^m)^n])/(b
*m*n)))^p)
```

fricas [A] time = 0.92, size = 73, normalized size = 0.68

$$e^{\left(-\frac{bmnp\log\left(-\frac{1}{bmn}\right)+bn\log(d)+b\log(c)+a}{bmn}\right)}\Gamma\left(p+1,-\frac{bmn\log(x)+bn\log(d)+b\log(c)+a}{bmn}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

```
[Out] e^(-(b*m*n*p*log(-1/(b*m*n)) + b*n*log(d) + b*log(c) + a)/(b*m*n))*gamma(p
+ 1, -(b*m*n*log(x) + b*n*log(d) + b*log(c) + a)/(b*m*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(dx^m\right)^n c\right)+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)^p, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d\,x^m\right)^n\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d*x^m)^n)+a)^p,x)
```

```
[Out] int((b*ln(c*(d*x^m)^n)+a)^p,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+b\,\ln\left(c\left(d\,x^m\right)^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d*x^m)^n))^p,x)
```

```
[Out] int((a + b*log(c*(d*x^m)^n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(c\left(dx^{m}\right)^{n}\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d*x**m)**n))**p,x)
```

```
[Out] Integral((a + b*log(c*(d*x**m)**n))**p, x)
```

3.246 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx$

Optimal. Leaf size=33

$$\frac{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^{p+1}}{bmn(p+1)}$$

```
[Out] (a+b*ln(c*(d*x^m)^n))^(1+p)/b/m/n/(1+p)
```

Rubi [A] time = 0.09, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2302, 30, 2445}

$$\frac{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^{p+1}}{bmn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d*x^m)^n])^p/x,x]
```

```
[Out] (a + b*Log[c*(d*x^m)^n])^(1 + p)/(b*m*n*(1 + p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p}{x}\,dx &= \text{Subst}\left(\int \frac{\left(a+b\log\left(cd^nx^{mn}\right)\right)^p}{x}\,dx, cd^nx^{mn}, c\left(dx^m\right)^n\right)\\ &= \text{Subst}\left(\frac{\text{Subst}\left(\int x^p\,dx, x, a+b\log\left(cd^nx^{mn}\right)\right)}{bmn}, cd^nx^{mn}, c\left(dx^m\right)^n\right)\\ &= \frac{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^{1+p}}{bmn(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^{p+1}}{bmn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d*x^m)^n])^p/x,x]
```

```
[Out] (a + b*Log[c*(d*x^m)^n])^(1 + p)/(b*m*n*(1 + p))
```

fricas [A] time = 0.54, size = 49, normalized size = 1.48

$$\frac{\left(bmn\log(x) + bn\log(d) + b\log(c) + a\right)\left(bmn\log(x) + bn\log(d) + b\log(c) + a\right)^{p}}{bmnp + bmn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x,x, algorithm="fricas")
```

```
[Out] (b*m*n*log(x) + b*n*log(d) + b*log(c) + a)*(b*m*n*log(x) + b*n*log(d) + b*l
og(c) + a)^p/(b*m*n*p + b*m*n)
```

giac [A] time = 0.25, size = 36, normalized size = 1.09

$$\frac{\left(bmn\log(x) + bn\log(d) + b\log(c) + a\right)^{p+1}}{bmn\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x,x, algorithm="giac")
```

```
[Out] (b*m*n*log(x) + b*n*log(d) + b*log(c) + a)^(p + 1)/(b*m*n*(p + 1))
```

maple [A] time = 0.04, size = 34, normalized size = 1.03

$$\frac{\left(b\ln\left(c\,(d\,x^{m})^{n}\right) + a\right)^{p+1}}{\left(p+1\right)bmn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d*x^m)^n)+a)^p/x,x)
```

```
[Out] (b*ln(c*(d*x^m)^n)+a)^(p+1)/b/m/n/(p+1)
```

maxima [A] time = 1.25, size = 33, normalized size = 1.00

$$\frac{\left(b\log\left((dx^{m})^{n}c\right) + a\right)^{p+1}}{bmn\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x,x, algorithm="maxima")
```

```
[Out] (b*log((d*x^m)^n*c) + a)^(p + 1)/(b*m*n*(p + 1))
```

mupad [B] time = 4.07, size = 33, normalized size = 1.00

$$\frac{\left(a + b\,\ln\left(c\,(d\,x^{m})^{n}\right)\right)^{p+1}}{b\,m\,n\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d*x^m)^n))^p/x,x)
```

```
[Out] (a + b*log(c*(d*x^m)^n))^(p + 1)/(b*m*n*(p + 1))
```

sympy [A] time = 2.85, size = 80, normalized size = 2.42

$$-\begin{cases} -a^p \log(x) & \text{for } b = 0 \\ -\left(a + b\log(cd^n)\right)^p \log(x) & \text{for } m = 0 \\ -\left(a + b\log(c)\right)^p \log(x) & \text{for } n = 0 \\ -\frac{\begin{cases} \frac{\left(a+b\log\left(c(dx^m)^n\right)\right)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log\left(a + b\log\left(c\left(dx^m\right)^n\right)\right) & \text{otherwise} \end{cases}}{bmn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d*x**m)**n))**p/x,x)
```

```
[Out] -Piecewise((-a**p*log(x), Eq(b, 0)), (-(a + b*log(c*d**n))**p*log(x), Eq(m,
 0)), (-(a + b*log(c))**p*log(x), Eq(n, 0)), (-Piecewise(((a + b*log(c*(d*x
**m)**n))**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*log(c*(d*x**m)**n)), Tru
e))/(b*m*n), True))
```

3.247 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2} dx$

Optimal. Leaf size=107

$$-\frac{e^{\frac{a}{bmn}}\left(c\left(dx^m\right)^n\right)^{\frac{1}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(\frac{a+b\log\left(c(dx^m)^n\right)}{bmn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log\left(c(dx^m)^n\right)}{bmn}\right)}{x}$$

```
[Out] -exp(a/b/m/n)*(c*(d*x^m)^n)^(1/m/n)*GAMMA(1+p,(a+b*ln(c*(d*x^m)^n))/b/m/n)*
(a+b*ln(c*(d*x^m)^n))^p/x/(((a+b*ln(c*(d*x^m)^n))/b/m/n)^p)
```

Rubi [A] time = 0.16, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2310, 2181, 2445}

$$-\frac{e^{\frac{a}{bmn}}\left(c\left(dx^m\right)^n\right)^{\frac{1}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(\frac{a+b\log\left(c(dx^m)^n\right)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,\frac{a+b\log\left(c(dx^m)^n\right)}{bmn}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d*x^m)^n])^p/x^2,x]
```

```
[Out] -((E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*Gamma[1 + p, (a + b*Log[c*(d*x^m)^
n])/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x*((a + b*Log[c*(d*x^m)^n])/(b*m*
n))^p))
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p}{x^2}\,dx = \text{Subst}\left(\int \frac{\left(a+b\log\left(cd^nx^{mn}\right)\right)^p}{x^2}\,dx, cd^nx^{mn}, c\left(dx^m\right)^n\right)$$

$$= \text{Subst}\left(\frac{\left(cd^nx^{mn}\right)^{\frac{1}{mn}}\text{Subst}\left(\int e^{-\frac{x}{mn}}(a+bx)^p\,dx, x, \log\left(cd^nx^{mn}\right)\right)}{mnx}, cd^nx^{mn}, c\left(dx^{\right.}\right.$$

$$= -\frac{e^{\frac{a}{bmn}}\left(c\left(dx^m\right)^n\right)^{\frac{1}{mn}}\Gamma\left(1+p,\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(\frac{a+b\log\left(c\left(dx\right.\right.}{bmn}\right.}{x}$$

Mathematica [A] time = 0.14, size = 107, normalized size = 1.00

$$-\frac{e^{\frac{a}{bmn}}\left(c\left(dx^m\right)^n\right)^{\frac{1}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d*x^m)^n])^p/x^2,x]
```

```
[Out] -((E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*Gamma[1 + p, (a + b*Log[c*(d*x^m)^
n])/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x*((a + b*Log[c*(d*x^m)^n])/(b*m*
n))^p))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\left(dx^m\right)^n c\right)+a\right)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((d*x^m)^n*c) + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(dx^m\right)^n c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)^p/x^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d\,x^m\right)^n\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d*x^m)^n)+a)^p/x^2,x)
```

```
[Out] int((b*ln(c*(d*x^m)^n)+a)^p/x^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d*x^m)^n))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d*x^m)^n))^p/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log{\left(c \left(dx^{m}\right)^{n} \right)}\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d*x**m)**n))**p/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d*x**m)**n))**p/x**2, x)
```

3.248 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx$

Optimal. Leaf size=117

$$-\frac{2^{-p-1}e^{\frac{2a}{bmn}}\left(c\left(dx^m\right)^n\right)^{\frac{2}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(\frac{a+b\log\left(c(dx^m)^n\right)}{bmn}\right)^{-p}\Gamma\left(p+1,\frac{2\left(a+b\log\left(c(dx^m)^n\right)\right)}{bmn}\right)}{x^2}$$

```
[Out] -2^(-1-p)*exp(2*a/b/m/n)*(c*(d*x^m)^n)^(2/m/n)*GAMMA(1+p,2*(a+b*ln(c*(d*x^m
)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n))^p/x^2/(((a+b*ln(c*(d*x^m)^n))/b/m/n)^p)
```

Rubi [A] time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2310, 2181, 2445}

$$-\frac{2^{-p-1}e^{\frac{2a}{bmn}}\left(c\left(dx^m\right)^n\right)^{\frac{2}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(\frac{a+b\log\left(c(dx^m)^n\right)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,\frac{2\left(a+b\log\left(c(dx^m)^n\right)\right)}{bmn}\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d*x^m)^n])^p/x^3,x]
```

```
[Out] -((2^(-1 - p)*E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*Gamma[1 + p, (2*(a
+ b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x^2*((a + b*Lo
g[c*(d*x^m)^n])/(b*m*n))^p))
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p}{x^3}\,dx = \text{Subst}\left(\int \frac{\left(a+b\log\left(cd^nx^{mn}\right)\right)^p}{x^3}\,dx, cd^nx^{mn}, c\left(dx^m\right)^n\right)$$

$$= \text{Subst}\left(\frac{\left(cd^nx^{mn}\right)^{\frac{2}{mn}}\text{Subst}\left(\int e^{-\frac{2x}{mn}}(a+bx)^p\,dx, x, \log\left(cd^nx^{mn}\right)\right)}{mnx^2}, cd^nx^{mn}, c\left(dx^m\right)^n\right.$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bmn}}\left(c\left(dx^m\right)^n\right)^{\frac{2}{mn}}\Gamma\left(1+p, \frac{2\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(\frac{a+b\log\ldots}{\ldots}\right.}{x^2}$$

Mathematica [A] time = 0.14, size = 117, normalized size = 1.00

$$-\frac{2^{-p-1}e^{\frac{2a}{bmn}}\left(c\left(dx^m\right)^n\right)^{\frac{2}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(\frac{a+b\log\left(c\left(dx^m\right)^n\right)}{bmn}\right)^{-p}\Gamma\left(p+1, \frac{2\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{bmn}\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d*x^m)^n])^p/x^3,x]
```

```
[Out] -((2^(-1 - p)*E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*Gamma[1 + p, (2*(a
+ b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x^2*((a + b*Lo
g[c*(d*x^m)^n])/(b*m*n))^p))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\left(dx^m\right)^n c\right)+a\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log((d*x^m)^n*c) + a)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(dx^m\right)^n c\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)^p/x^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d\,x^m\right)^n\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d*x^m)^n)+a)^p/x^3,x)
```

```
[Out] int((b*ln(c*(d*x^m)^n)+a)^p/x^3,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d*x^m)^n))^p/x^3,x)
```

```
[Out] int((a + b*log(c*(d*x^m)^n))^p/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(dx^{m}\right)^{n}\right)\right)^{p}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d*x**m)**n))**p/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d*x**m)**n))**p/x**3, x)
```

3.249 $\int \frac{a+b\log\left(c(dx^m)^n\right)}{e+fx^2}\,dx$

Optimal. Leaf size=111

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{\sqrt{e}\sqrt{f}}-\frac{ibmn\text{Li}_2\left(-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{2\sqrt{e}\sqrt{f}}+\frac{ibmn\text{Li}_2\left(\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{2\sqrt{e}\sqrt{f}}$$

```
[Out] arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*(d*x^m)^n))/e^(1/2)/f^(1/2)-1/2*I*b*m*n
*polylog(2,-I*x*f^(1/2)/e^(1/2))/e^(1/2)/f^(1/2)+1/2*I*b*m*n*polylog(2,I*x*
f^(1/2)/e^(1/2))/e^(1/2)/f^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {205, 2324, 12, 4848, 2391, 2445}

$$-\frac{ibmn\text{PolyLog}\left(2,-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{2\sqrt{e}\sqrt{f}}+\frac{ibmn\text{PolyLog}\left(2,\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{2\sqrt{e}\sqrt{f}}+\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{\sqrt{e}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d*x^m)^n])/(e + f*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*(d*x^m)^n]))/(Sqrt[e]*Sqrt[f]) -
((I/2)*b*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]) + ((I/
2)*b*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2324

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(dx^m\right)^n\right)}{e+fx^2}\,dx = \text{Subst}\left(\int \frac{a+b\log\left(cd^nx^{mn}\right)}{e+fx^2}\,dx, cd^nx^{mn}, c\left(dx^m\right)^n\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{\sqrt{e}\sqrt{f}} - \text{Subst}\left((bmn)\int \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}x}\,dx, cd^nx^{mn}, c\right.$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{\sqrt{e}\sqrt{f}} - \text{Subst}\left(\frac{(bmn)\int \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{x}\,dx}{\sqrt{e}\sqrt{f}}, cd^nx^{mn}, c\left(dx\right.\right.$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{\sqrt{e}\sqrt{f}} - \text{Subst}\left(\frac{(ibmn)\int \frac{\log\left(1-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{x}\,dx}{2\sqrt{e}\sqrt{f}}, cd^nx^{mn}, c\left(\right.\right.$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}{\sqrt{e}\sqrt{f}} - \frac{ibmn\text{Li}_2\left(-\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{2\sqrt{e}\sqrt{f}} + \frac{ibmn\text{Li}_2\left(\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{2\sqrt{e}\sqrt{f}}$$

Mathematica [A] time = 0.09, size = 113, normalized size = 1.02

$$\frac{-\left(\left(\log\left(\frac{\sqrt{f}x}{\sqrt{-e}}+1\right)-\log\left(\frac{e\sqrt{f}x}{(-e)^{3/2}}+1\right)\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)\right)+bmn\text{Li}_2\left(\frac{\sqrt{f}x}{\sqrt{-e}}\right)-bmn\text{Li}_2\left(\frac{e\sqrt{f}x}{(-e)^{3/2}}\right)}{2\sqrt{-e}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d*x^m)^n])/(e + f*x^2),x]
```

```
[Out] (-((a + b*Log[c*(d*x^m)^n])*(Log[1 + (Sqrt[f]*x)/Sqrt[-e]] - Log[1 + (e*Sqr
t[f]*x)/(-e)^(3/2)])) + b*m*n*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]] - b*m*n*Poly
Log[2, (e*Sqrt[f]*x)/(-e)^(3/2)])/(2*Sqrt[-e]*Sqrt[f])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(\left(dx^m\right)^n c\right)+a}{fx^2+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] integral((b*log((d*x^m)^n*c) + a)/(f*x^2 + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log\left(\left(dx^m\right)^n c\right)+a}{fx^2+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))/(f*x^2+e),x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)/(f*x^2 + e), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{b \ln \left(c \left(d\,x^{m}\right)^{n}\right)+a}{f\,x^{2}+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d*x^m)^n)+a)/(f*x^2+e),x)
```

```
[Out] int((b*ln(c*(d*x^m)^n)+a)/(f*x^2+e),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{n \log(d) + \log(c) + \log\left(\left(x^{m}\right)^{n}\right)}{fx^{2} + e}\,dx + \frac{a \arctan\left(\frac{fx}{\sqrt{ef}}\right)}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] b*integrate((n*log(d) + log(c) + log((x^m)^n))/(f*x^2 + e), x) + a*arctan(f
*x/sqrt(e*f))/sqrt(e*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)}{f\,x^2+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d*x^m)^n))/(e + f*x^2),x)
```

```
[Out] int((a + b*log(c*(d*x^m)^n))/(e + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \log{\left(c \left(dx^{m}\right)^{n} \right)}}{e + fx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d*x**m)**n))/(f*x**2+e),x)
```

```
[Out] Integral((a + b*log(c*(d*x**m)**n))/(e + f*x**2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```